Generation of radiation in quantum transitions of electrons in a strong magnetic field

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Quantum-mechanical formulas are used to investigate the photon emission probabilities in spontaneous transitions of electrons between Landau levels in a strong magnetic field $B \sim B_c$. Estimates are obtained for the characteristic times that describe the change in the state of electrons in these transitions. The spectra and polarizations of quantum synchrotron radiation and of stationary synchrotron cooling radiation are calculated

1. INTRODUCTION

It has now been established that the magnetic field on the surface of a neutron star is close to the critical value $B_c = m^2c^3/e\hbar = 4.4 \times 10^{13}$G. Spontaneous transitions of electrons between Landau levels in such a strong field are significantly different from the case where $B \ll B_c$. Studies of such transition $B \lesssim B_c$ are essential for elucidating physical mechanisms responsible for the generation of x-ray and radiation in neutron stars, and for comparing observed properties of this radiation with physical conditions prevailing in the generation region.

Single-photon transitions of electrons in a strong magnetic field were examined in Refs. 1–4. The analysis given in Ref. 2 is subject to the error noted in Ref. 4, which means that all the results reported in the extensive series of papers subsequent to Ref. 2 are incorrect. Some authors (see, for example, Ref. 3) have used relativistically incorrect wave functions (this is discussed in Ref. 4), which in turn has meant that spin effects that are important in the quantum region could not be investigated. A completely relativistic quantum-mechanical analysis of single-photon transitions was carried out in Refs. 1 and 4, but the properties of these transitions in strong magnetic fields were not investigated numerically. Analysis of two-photon synchrotron radiation[5] has shown that single-photon transitions are the main process responsible for the generation of radiation in a magnetic field. The special case of quantum synchrotron cooling of electrons was considered in Ref. 6 on the basis of numerical Monte Carlo calculations for a large number of single-photon transitions.

In this paper, we investigate single-photon quantum transitions of electrons in a magnetic field, and the basic properties of the generated radiation. By incorporating these results in the transfer problem that takes into account not only generation but also absorption, splitting, and scattering of photons, we can investigate, within the framework of a particular model, the emission of radiation by plasmas in ultrastrong magnetic fields. In Sec. 2, we consider the dependence of the transition probabilities on the magnetic field, the quantum numbers in initial and final states, and the direction of propagation and polarization of photons. In Sec. 3, we investigate the decay of excited states, and estimate the basic characteristic times governing their evolution. In Sec. 4, the calculated spontaneous transition probabilities are used to investigate the spectra and the polarization of stationary quantum synchrotron radiation (SSR). The spectra deduced from classical formulas (see, for example, Ref. 7) differ appreciably from these results because of the presence of quantum recoil and spin-reversal effects.

2. PROBABILITY OF QUANTUM TRANSITIONS OF ELECTRONS IN A MAGNETIC FIELD

The energy of an electron in a magnetic field

$$E_n = (1+p^2+2nB)^{1/2} \tag{1}$$

is determined by its longitudinal momentum p and principal quantum number $n = 0, 1, 2, \ldots$ (here and henceforth we use the system of units in which $\hbar = 1$, $mc^2 = 1$, and $B_c = 1$). Apart from the quantum numbers p and n, a "pure" state of an electron is characterized by the spin component s along the magnetic field ($s = \pm 1$ means that the spin is aligned parallel or antiparalled to the magnetic field, respectively). We know that $n = 0$ and $s = -1$ in the ground state. The amplitude for a spontaneous transition from the state $\zeta_n\,(n,s)$ to the state $\zeta_m = (m,s')$ with the emission of one photon is

$$S(\zeta_n \to \zeta_m) = ie \int d^4x\, \bar{\psi}_m^{(s')}(x) A^\mu(x) \gamma_\mu \psi_n^{(s)}(x), \tag{2}$$

where the photon wave function is given by

$$A^\mu(x) = e_\mu \frac{e^{i\mathbf{kx}}}{(2\omega L^3)^{1/2}} \tag{3}$$

and corresponds to a plane wave with a wave 4-vector $(\omega, \mathbf{k})$ and the polarization 4-vector $e_\mu = (0, \mathbf{e})$.

It is common to distinguish between two photon polarization states: the state with the vector $\mathbf{e}^{(1)}$ lying in the $(\mathbf{k}, \mathbf{B})$ plane and that with $\mathbf{e}^{(2)}$ perpendicular to this plane. In a rectangular coordinate frame, with the z axis lying along the magnetic field $\mathbf{B}$, these vectors can be assumed without loss of generality to be $\mathbf{k} = (0, \omega\sin\theta, \omega\cos\theta)$, $\mathbf{e}^{(1)} = (0, -\cos\theta, \sin\theta)$, $\mathbf{e}^{(2)} = (1, 0, 0)$. The vectors $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ correspond to the polarization of two normal waves in polarized vacuum.[8]

The wave function $\psi_n{}^{(s)}(x)$ is obtained by applying the Lorentz transformation to the solution of the Dirac equation in a magnetic field in the case of an electron with $p = 0$ (Ref. 4):

$$\psi_n^{(s)}(x) = \left(\frac{E_{n0}+1}{2E_{n0}}\right)^{1/2} \frac{u_n^{(s)}(x) \exp(-iE_n t)}{[2E_n(E_n+E_{n0})]^{1/2}}, \tag{4}$$

where $E_{n0} = (1+2nB)^{1/2}$ and the spinors

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$$u_n^{(+1)}(x)=\begin{vmatrix} (E_n+E_{n0})\,\chi_{n-1} \\ -\dfrac{p\,(2nB)^{1/2}}{E_{n0}+1}\chi_n \\ p\chi_{n-1} \\ \dfrac{(E_n+E_{n0})(2nB)^{1/2}}{E_{n0}+1}\chi_n \end{vmatrix},$$

$$u_n^{(-1)}(x)=\begin{vmatrix} \dfrac{p\,(2nB)^{1/2}}{E_{n0}+1}\chi_{n-1} \\ (E_n+E_{n0})\,\chi_n \\ \dfrac{(E_n+E_{n0})(2nB)^{1/2}}{E_{n0}+1}\chi_{n-1} \\ -p\chi_n \end{vmatrix} \qquad (5)$$

can be expressed in terms of the scalar function

$$\chi_n(x)=\frac{i^n}{(\lambda^2\pi)^{1/4}(2^n n!)^{1/2}L} \exp\left[-\frac{(x-a)^2}{2\lambda^2}-i\frac{ay}{\lambda^2}+ipz\right]H_n\left(\frac{x-a}{\lambda}\right). \qquad (6)$$

In these expressions, a is the x coordinate of the center of rotation of the electron, $\lambda = \lambda_c/B^{1/2}$, λ_c is the Compton wavelength, and H_n is the Hermite polynominal. The normalization of the wave function (4) corresponds to one elementary charge in L^3.

The transition rate between the state $\zeta_n = (n, p, s)$ and the state $\zeta_n = (m, q, s')$ with the emission of the photon $(\omega, \mathbf{k})$ with the polarization $\mathbf{e}^{(j)}$ into the solid angle $d\Omega$, subject to the conservation laws $E_n = \omega + E_m$ and $p = q + k_z$ is

$$dR(\zeta_n\to\zeta_m;\mathbf{k},\mathbf{e}^{(j)})=\frac{\alpha}{16\pi}\frac{(E_n-E_m)}{E_mE_{m0}E_nE_{n0}} \cdot\frac{(E_{n0}+1)\,(E_{m0}+1)}{(E_n+E_{n0})\,(E_m+E_{m0})\,(1-q\cos\theta/E_m)}\,|I_j(n,s\to m,s')|^2\,d\Omega, \qquad (7)$$

where $\alpha = 1/137$ and

$$|I_1(n,s=\pm1\to m,s'=s)|^2$$
$$=\Big[D_1\cos\theta\Big(B_nM\Big(m-\frac{1}{2}-\frac{s}{2},\ n-\frac{1}{2}+\frac{s}{2}\Big) + B_mM\Big(m-\frac{1}{2}+\frac{s}{2},\ n-\frac{1}{2}-\frac{s}{2}\Big)\Big)$$
$$-D_2\sin\theta\Big(B_nB_mM\Big(m-\frac{1}{2}+\frac{s}{2},\ n-\frac{1}{2}+\frac{s}{2}\Big) + M\Big(m-\frac{1}{2}-\frac{s}{2},\ n-\frac{1}{2}-\frac{s}{2}\Big)\Big)\Big]^2, \qquad (8)$$

$$|I_2(n,s=\pm1\to m,s'=s)|^2=D_1^2\Big[B_mM\Big(m-\frac{1}{2}+\frac{s}{2},\ n-\frac{1}{2}-\frac{s}{2}\Big)-B_nM\Big(m-\frac{1}{2}-\frac{s}{2},\ n-\frac{1}{2}+\frac{s}{2}\Big)\Big]^2, \qquad (9)$$

$$|I_1(n,s=\pm1\to m,s'=-s)|^2$$
$$=\Big[D_3\cos\theta\Big(M\Big(m-\frac{1}{2}+\frac{s}{2},\ n-\frac{1}{2}-\frac{s}{2}\Big) - M\Big(m-\frac{1}{2}-\frac{s}{2},\ n-\frac{1}{2}+\frac{s}{2}\Big)B_nB_m\Big)$$
$$-D_4\sin\theta\Big(B_nM\Big(m-\frac{1}{2}+\frac{s}{2},\ n-\frac{1}{2}+\frac{s}{2}\Big) - B_mM\Big(m-\frac{1}{2}-\frac{s}{2},\ n-\frac{1}{2}-\frac{s}{2}\Big)\Big)\Big]^2. \qquad (10)$$

$$|I_2(n,s=\pm1\to m,s'=-s)|^2$$
$$=D_3^2\Big[M\Big(m-\frac{1}{2}+\frac{s}{2},\ n-\frac{1}{2}-\frac{s}{2}\Big) + B_nB_mM\Big(m-\frac{1}{2}-\frac{s}{2},\ n-\frac{1}{2}+\frac{s}{2}\Big)\Big]^2. \qquad (11)$$

In these expressions,

$$D_{1,4}=A_mA_n\mp pq,\quad D_{2,3}=qA_n\pm pA_m,\quad A_n=E_{n0}+E_n,$$
$$B_n=(2nB)^{1/2}/(E_{n0}+1),$$
$$M(m,n)=\exp(-\xi/2)\,\xi^{(n-m)/2}(m!/n!)^{1/2}L_m^{n-m}(\xi),$$

where L is the generalized Laguerre polynomial and $\xi = \omega^2\sin^2\theta/2B$.

The total rate of the $\zeta_n\to\zeta_m$ quantum transition is obtained (7) by integrating over the directions of emission of the photon and summing over the polarizations:

$$R(\zeta_n\to\zeta_m)=\sum_{j=1,2}\int_{4\pi}dR(\zeta_n\to\zeta_m;\mathbf{k},\mathbf{e}^{(j)}). \qquad (12)$$

Two types of transition, corresponding to different initial and final spin states, can occur between the states ζ_n and ζ_m: direct transitions ($\pm1\to\pm1$) and transitions with spin flip ($\pm1\to\mp1$). The rates of all these transitions are comparable for $B\sim1$ (see below). Since $m = 0$ in the ground state, the only possible spin component is (-1), and the only possible $\zeta_n\to\zeta_0$ transitions are $(\pm1\to-1)$.

For $nB\ll1$, approximate expressions for the transition rates follow from (12):

$$R(n,-1\to m,-1)=\alpha\frac{(2B)^{n-m+1}(n-m)^{2(n-m)}(n-m+1)\,n!}{(2n-2m+1)!\,m!}, \qquad (13)$$

$$R(n,+1\to m,+1)=\frac{m}{n}R(n,-1\to m,-1), \qquad (14)$$

$$R(n,+1\to m,-1) =\alpha\frac{(2B)^{n-m+2}(n-m)^{2(n-m)+2}(n-m+1)\,(n-1)!}{4(2n-2m+1)!\,m!} \qquad (15)$$

$$R(n,-1\to m,+1)\approx\alpha\frac{(2B)^{n-m+4}(n-m)^{2(n-m+1)}n!}{32(2n-2m+1)!\,(m-1)!}. \qquad (16)$$

It follows from (13)–(16) that, for $nB\ll1$,

$$R(n,-1\to m,-1)\sim R(n,+1\to m,+1) \gg R(n,+1\to m,-1)\gg R(n,-1\to m,+1).$$

This special case corresponds to the nonrelativistic classical limit in which transitions with spin reversal are suppressed.

The probability of emission of a photon in a quantum transition depends on the direction of its wave vector and polarization. For a given initial quantum number n, the probability of emission of a photon energy ω_{nm} depends on the "harmonic number" $\nu = n - m$ (Fig. 1). For $nB\ll1$, the angular distribution of radiation with $\nu = 1$ is described by the well-known $1+\cos^2\theta$ law; when $\nu>1$, the angular dis-

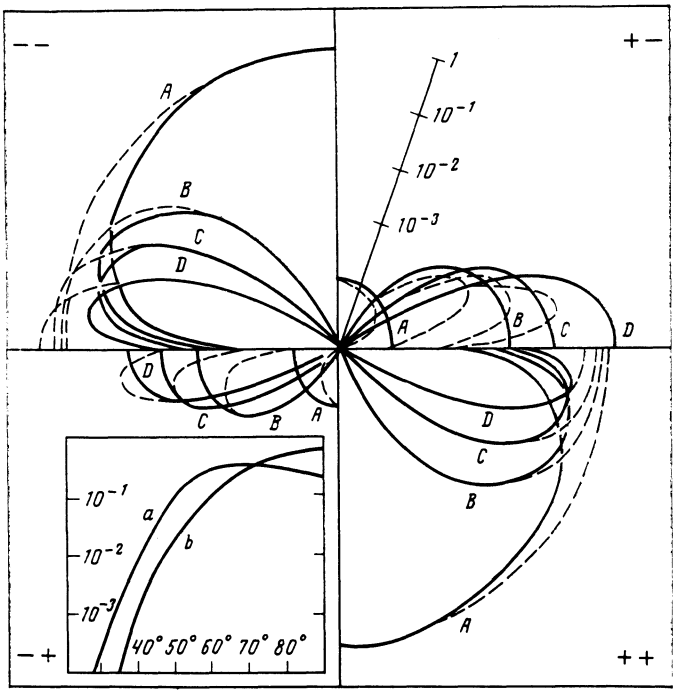

FIG. 1. Angular distribution of radiation emitted in different types of transition for $B = 1$, $n = 30$, and $p = 0$ (relative units). Curves A, B, C, D correspond to the $\nu = 1, 10, 20, 28$ harmonics, respectively; the solid curve corresponds to polarization in the $(\mathbf{k}, \mathbf{B})$ plane, and the broken curve to polarization at right angle to this plane. The broadening of the distribution in the $(30, \pm 1) \to (20, \pm 1)$ transition in magnetic fields $B = 1$ (a) and 10^{-2} (b) is illustrated in the insert.

tribution of the cyclotron harmonics is $\sim (\sin\theta)^{2(\nu - 1)}$. According to (13)–(16), the intensity of the $\nu > 1$ harmonics decreases in this approximation in proportion to $(Bn)^{\nu}\nu^{2\nu}$ for transitions without spin reversal (when $n \gg \nu \gg 1$). The contribution of transitions with spin reversal to the emission of radiation into a given harmonic can be neglected.

For $B \sim 1$, the relative contribution of higher-order harmonics to the spontaneous transition probability is found to be higher (for $\theta \lesssim 90°$ it may even predominate), and transitions with and without spin reversal may become comparable in intensity. An important feature of transitions in a strong field is the "broadening" of the angular distributions of the higher harmonics as compared with the weak-field case (Fig. 1). This is due to the significant influence of recoil effects.

At angles close to the direction of the magnetic field, the two linear polarizations of all harmonics with $\nu \gg 1$ occur with practically equal probability, and the radiation as a whole is unpolarized. As $\theta \to 90°$, the polarization of radiation due to transitions without spin reversal increases and finally reached 100% at $\theta = 90°$ (Fig. 1). The polarization vector is then perpendicular to the $(\mathbf{k}, \mathbf{B})$ plane. Conversely, for transition with spin reversal $(+1 \to -1)$, the linear polarization with vector $\mathbf{e}$ in the $(\mathbf{k}, \mathbf{B})$ plane rises to 100% as $\theta \to 90°$. For transitions with spin reversal $(-1 \to +1)$, the degree of polarization may vary nonmonotonically as θ increases from 0 to 90°: at first, the polarization is perpendicular to the $(\mathbf{k}, \mathbf{B})$ plane, but is then replaced with polarization parallel to this plane; it reaches 100% when $\theta = 90°$.

The total rates of transitions (12) from the state ζ_n depend significantly on the magnetic field (Fig. 2a). In a weak magnetic field with $nB \ll 1$, direct transitions $(\pm 1 \to \pm 1)$ with the emission of low harmonics $(1 \leqslant \nu \leqslant n)$ have the maximum rate, whereas transitions with spin reversal are strongly suppressed [see (13)–(16)]. As ν increases, the rate of the $(+1 \to +1)$ transitions decreases as compared with the rate of $(-1 \to -1)$ transitions: for $\nu \lesssim n$, the first of them turns out to be even lower than the rate of transitions with spin reversal $(+1 \to -1)$. This is so because the direct transition from the $(n, +1)$ state to the ground

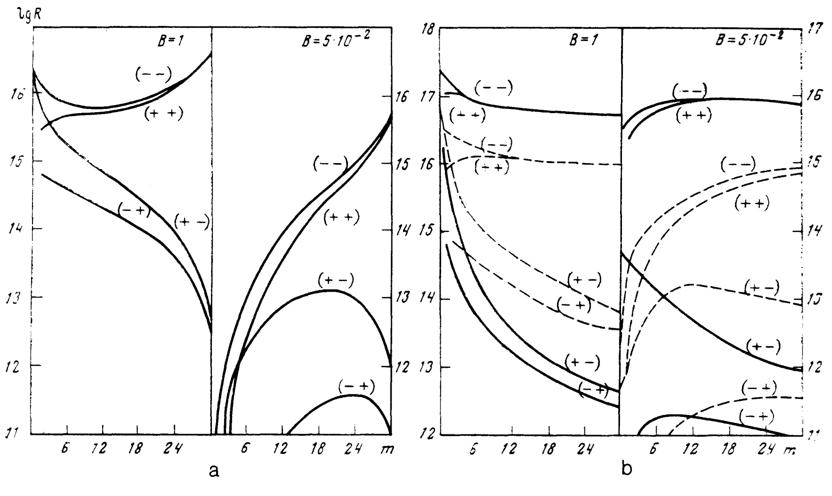

FIG. 2. Rates (in s^{-1}) of $(n, s \to m, s')$ transitions: (a) for $n = 30$ and different s and s', as shown against the figures; (b) for constant $\nu = 1$ (solid curve) and $\nu = 10$ (broken curve) and different final quantum number m.

state $(0, -1)$ is forbidden. We therefore conclude that, even in a weak field, quantum effects may have a significant influence on the emission of the higher harmonics.

In a strong magnetic field, the rates of transitions without spin reversal for given n and different ν are found to differ by less than an order of magnitude (Fig. 2a). For the lower harmonics ($1 \leqslant \nu \leqslant n$), the rates of transition with spin reversal are low (similarly to the weak-field case). They increase with increasing ν, and become comparable with the rates of direct transitions for the higher harmonics ($\nu \lesssim n$).

The rates of transitions corresponding to emission into a given harmonic depend on the final quantum number m (Fig. 2b). When $(\nu + m)B \ll 1$, the rates of direct transitions increase with increasing m, but the rates of transitions with spin reversal are found to decrease [see (13)–(16)]. For $(\nu + m)B \gtrsim 1$, the increase in these rates is replaced by a slow reduction. In the classical nonrelativistic limit $(\nu + m)B \ll 1$, and for constant m, the transition rates decrease with increasing ν. For $B \ll 1$, the emission of higher harmonics can be the dominant effect only for $(m + \nu)B \gtrsim 1$. In a strong magnetic field $B \sim 1$, the rates of direct transitions and transitions with spin reversal are found to decrease monotonically with increasing m, and this is much more rapid in the second case.

3. DECAY OF EXCITED ELECTRON STATES

An electron that is initially in an excited state $\zeta_n = (n,s)$ eventually reaches the ground state ζ_0 after successive spontaneous transitions. In general, the complete description of the transition involves a summation of the squares of the probability amplitudes for the $\zeta_n \to \zeta_0$ transitions with the emission of $J = 1$, 2, 3 or more photons.[9,10] The main contribution to the probability of a J-photon transition is provided by the resonant part for which the energies of all the emitted photons are equal to differences between quantum levels $E_n > \dots E_i > \dots > E_0$. The spin degeneracy of energy states means that the expression for the total probability of the $\zeta_n \to \zeta_0$ J-photon transition contains terms due to spin interference. However, it can be shown that these terms vanish after integration over the photon directions of propagation. This ensures that the probability of the $\zeta_n \to \zeta_0$ J-photon transition can be expressed in terms of the product of the probabilities of independent single-photon transitions between "pure" states.

It follows that the total rate of decay of an excited state $\zeta_n\,(n,s)$ is determined by the sum of all single-photon transitions to all states ζ_m with $m < n$:

$$R(\zeta_n) = \sum_{\substack{m<n \\ s'=\pm 1}} R(\zeta_n \to \zeta_m). \tag{17}$$

The lifetime of the state ζ_n is $\tau_1(\zeta_n) = R^{-1}(\zeta_n)$.

The relaxation time from the initial state ζ_n is also physically meaningful: it is the time τ_2 in which the probability of finding the excited state ζ_m with $0 < m \leqslant n$ falls by a factor of e. Obviously, $\tau_2(\zeta_n)$ is always greater than $\tau_1(\zeta_n)$. The time $\tau_2(\zeta_n)$ is determined from the condition

$$[1 - b_{\zeta_0}(t=\tau_2)]^{-1} = e, \tag{18}$$

where $b_{\zeta_0}(t)$ is the probability that the particle is in the state ζ_0 at time t. In general, the determination of $\tau_2(\zeta_n)$ requires the solution of the following set of transport equations:

$$\dot{b}_{\zeta_k}(t) = -R(\zeta_k)\, b_{\zeta_k}(t) + \sum_{\substack{n \geqslant l \geqslant k+1 \\ s'=\pm 1}} R(\zeta_l \to \zeta_k)\, b_{\zeta_l}(t) \tag{19}$$

for all the intermediate states $\zeta_k = (k,s), k = n, n-1, \dots, 0$.

In the nonrelativistic approximation $nB \ll 1$ [see (13)–(16)], we can put

$$R(l,-1 \to l-1,-1) \approx lR(1,-1 \to 0,-1) = lR_0,$$

for electric dipole transitions, and the rates of other transitions can be assumed to be zero. In the case of the initial state $\zeta_n^{(-)} = (n, -1)$, the set of equations given by (19) has the simple solution

$$b_{\zeta_k}(t) = C_n^k e^{-kR_0 t}(1 - e^{-R_0 t})^{n-k}, \tag{20}$$

where C_n^k is the number of combinations of k from n. It follows from (20) that

$$\tau_2(\zeta_n^{(-)}) \approx \frac{1}{R_0} \ln \frac{1}{1 - (1 - e^{-1})^{1/n}}. \tag{21}$$

With $nB \ll 1$, but $n \gg 1$, we have $\tau_2/\tau_1 \simeq n \ln n$.

The electron cooling time in a magnetic field, $\tau_3(\zeta_n)$, can be determined from the condition for the mean transverse energy to fall by a factor of e in the $\zeta_n \to \zeta_0$ transition:

$$E_n b_{\zeta_n}(t=0) \Big/ \Big(E_n b_{\zeta_n}(t=\tau_3) + \sum_{\substack{k<n \\ s'=\pm 1}} E_k b_{\zeta_k}(t=\tau_3) \Big) = e. \tag{22}$$

Like $\tau_2(\zeta_n)$, this time can be found by solving (19). In the nonrelativistic limit, $nB \ll 1$, the cooling time from the initial state $\zeta_n^{(-)}$ is independent of the initial quantum number and $\tau_3(\zeta_n^{(-)}) \simeq R_0^{-1}$.

The times τ_2 and τ_3 for the states $\zeta_n^{(+)}$ can also be estimated in the nonrelativistic limit $nB \ll 1$, but we must then take into account transitions with spin reversal $(1, +1 \to 0, -1)$ whose rate can be estimated as $\sim BR_0$. For $n \gg 1$, the time $\tau_2(\zeta_n^{(+)})$ is greater than $\tau_2(\zeta_n^{(-)})$ by a factor of about B^{-1} (since a transition with spin reversal should take place), and the estimated $\tau_3(\zeta_n^{(+)})$ does not differ from $\tau_3(\zeta_n^{(-)})$.

Figure 3 shows the τ_1, τ_2, and τ_3 contours for different values of B and E_n (the longitudinal momentum in the initial state ζ_n is assumed to be zero). It is clear that, when $n = 1$, these times are equal for each of the two spin states. For constant B, the increase in E_n leads to a reduction in τ_1: the corresponding contours deflect towards lower fields. The bending of these curves for $nB \gtrsim 1$ is due to relativistic effects. The maximum difference between the values of τ_1 for $\zeta_n^{(-)}$ and $\zeta_n^{(+)}$ is observed for $n = 1$, and the corresponding contours approach one another as n increases.

In contrast to τ_1, the relaxation time τ_2 increases with increasing E_n: the τ_2 contours bend toward stronger fields. This can be readily calculated, but only in the nonrelativistic limit [see (21)]: the corresponding curve segments are shown in Fig. 3. In the same limit, the τ_3 contours are satisfactorily described by the curves obtained as a result of classical estimates.[11]

In the general case, $nB \gtrsim 1$, the relaxation time τ_2 and the cooling time τ_3 must be estimated with allowance for the contributions of all the intermediate spontaneous transitions. For given initial state ζ_n, there are 3^{n-1} possible transitions to the ground state. It is convenient to introduce the so-called "transition channel" K corresponding to a particu-

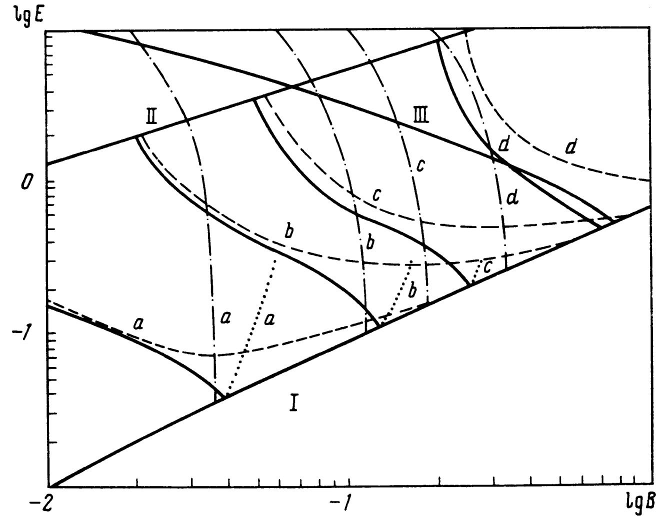

FIG. 3. Lines of equal τ_1, τ_2, and τ_3 for different values of B and $E=(1+2nB)^{1/2}-1$ (a—10^{-16}, b—10^{-17}, c—3.7×10^{-18}, d—10^{-18} s: the solid and broken curves correspond, respectively, to $\tau_1(n,-1)$ and $\tau_1(n,+1)$; the dot-dash curve represents τ_3 estimated from the classical formula, dotted curve—τ_2 estimated for $nB\ll 1$. The condition $n=1$ corresponds to the boundary I whereas the possibilities of the calculation ($n=200$) are indicated by boundary II. Curve III shows the region in which the approximation involving partial allowance for recoil $\langle q_{n0}^2\rangle \leqslant 0.1$ is valid (see text).

lar sequence of single-photon transitions between "pure" intermediate states $\{\zeta_i^{(K)}\}$. The probability that a given channel will be present can be represented by

$$W_K(\zeta_n)=\prod_{\zeta_i\in K} R(\zeta_n\to\zeta_i)/R(\zeta_n). \tag{23}$$

The concept of the transition channel helps in obtaining relatively simple formulas for calculating emission spectra due to the spontaneous decay of excited states, in which the contribution of a large number of possible single-photon transitions is taken into account.

4. PROPERTIES OF PHOTONS EMITTED BY ELECTRONS IN A MAGNETIC FIELD

It is usual to consider two types of emission by electrons in a magnetic field: SSR and synchrotron cooling radiation (SCR). In the case of SSR, it is assumed that some physical process (e.g., collisions) maintains the stationary particle energy distribution $F(n,p)$. In the thermal equilibrium, this distribution is a function of temperature: $F=F_T(n,p)$. The spectral density of the SSR photon number emitted by $\mathcal{N}_e$ electrons per unit time can be written in the form

$$\frac{dN}{d\omega\, d\Omega\, dt} =\mathcal{N}_e \sum_{\substack{n>0\\ s=\pm1}} \int dpF(n,p) \sum_{\substack{m<n\\ s'=\pm1}} R(\zeta_n\to\zeta_m)\,\delta(\omega-\omega_{nm}(p)). \tag{24}$$

The distribution function for the radiating electrons can be regarded as stationary if the thermalization time τ_0 is much less than the lifetimes of all the excited states.

In the case of SCR, it is assumed that, initially ($t=0$), the ensemble is characterized by a distribution function $F_0(n,p)$. For $t>0$, the excited states decay spontaneously and, after a sufficiently long interval of time, $t\gg\tau_2$, all the electrons are in the ground state The spectral density of the SCR photon number emitted by the $\mathcal{N}_e$ excited electrons in a time $t\gg\tau_2$ is

$$\frac{dN}{d\omega\, d\Omega}=\mathcal{N}_e \sum_{\substack{n>0\\ s=\pm1}} \int dp\,F_0(n,p) \sum_{\{K\}} W_K(\zeta_n) \sum_{\{\zeta_i\in K\}} \frac{dR(\zeta_i\to\zeta_{i-1})}{R(\zeta_i\to\zeta_{i-1})\,d\Omega}\,\delta(\omega-\omega_{i,i-1}(p)). \tag{25}$$

If the quantum levels of an ensemble of electrons are excited with characteristic time $\tau_0\gg\tau_2$, then an individual electron will occupy the ground state practically at all times, and the fraction of excited particles in the ensemble is $\sim\tau_2/\tau_0\ll 1$. SCR generation is then a stationary process: the spectral density $dN/d\omega d\Omega dt$ is then obtained from (25) by replacing the distribution over the initial states, $F_0(n,p)$, with the probability $\dot F(n,p)$ of excitation to the state ζ_n per particle per unit time.

For thermal synchrotron radiation (TSR), the total power radiated by the ensemble of electrons at a temperature T is $\dot E_{\rm TSR}\sim\mathcal{N}_e T\tau_3^{-1}$. The power radiated in the form of stationary SCR is determined by the charateristic level excitation time: $\dot E_{\rm SCR}\sim\mathcal{N}_e T\tau_0^{-1}\sim \dot E_{\rm SCR}\tau_3/\tau_0$. Since $\tau_2\gtrsim\tau_3$ holds, the condition for the generation of stationary SCR, $\tau_2\ll\tau_0$, leads to $\dot E_{\rm SCR}\ll\dot E_{\rm TSR}$.

Known probabilities $R(\zeta_n\to\zeta_m)$ of quantum transitions can be used to calculate the intensity, spectrum, and polarization of TSR for any T and B. Figure 4 shows the TSR spectrum for $B=0.03$ and 0.1 at the temperature $T=0.2$ for $\theta=90°$. As the field decreases, the shape of the spectrum approaches the curve deduced in Ref. 7 by classical calculation. For "quantum values" of the parameters B and T, three factors have an important influence on the spectrum: (1) the dependence of the excited-state population on n (when $E_n\gtrsim T$, the population decreases exponentially), (2) transitions with spin reversal, and (3) quantum recoil effects (see also Ref. 12).

Direct quantum mechanical calculations on SCR are difficult: the distribution function for the intermediate states over the longitudinal momenta is determined by the totality of recoils due to all photons emitted in all previous transitions. Allowance for these recoils becomes significant when the photon energy is comparable with the initial electron energy. A transition from a state ζ_n with $p=0$ to a state with $m<n$ can occur as a result of direct $\zeta_n\to\zeta_m$ transition, or as a result of a cascade $\zeta_n\to\zeta_l\to\ldots\zeta_m$. The total probability of a cascade transition can be greater than the direct transition probability. The spread of longitudinal momenta in a level m is determined by the total mean square recoil momentum:

$$\langle q_{nm}^2\rangle=\sum_{\{K\}} W_K(\zeta_n\to\zeta_m)\langle q_{nm}^2\rangle_K, \tag{26}$$

where the contributions $\langle q_{nm}{}^2\rangle_K$ are due to all the $\zeta_n\to\zeta_m$ transition channels. Suppose that, for some channels, $\langle q_{nm}{}^2\rangle_K\gtrsim 1$. When the probability of such channels is low, the quantity given by (26) can be much greater than unity. This means that, in the calculation of the decay of the intermediate state ζ_n, the corresponding longitudinal momentum can be set equal to zero.

For a fixed initial state ζ_n, the momentum $\langle q_{nm}{}^2\rangle$ that

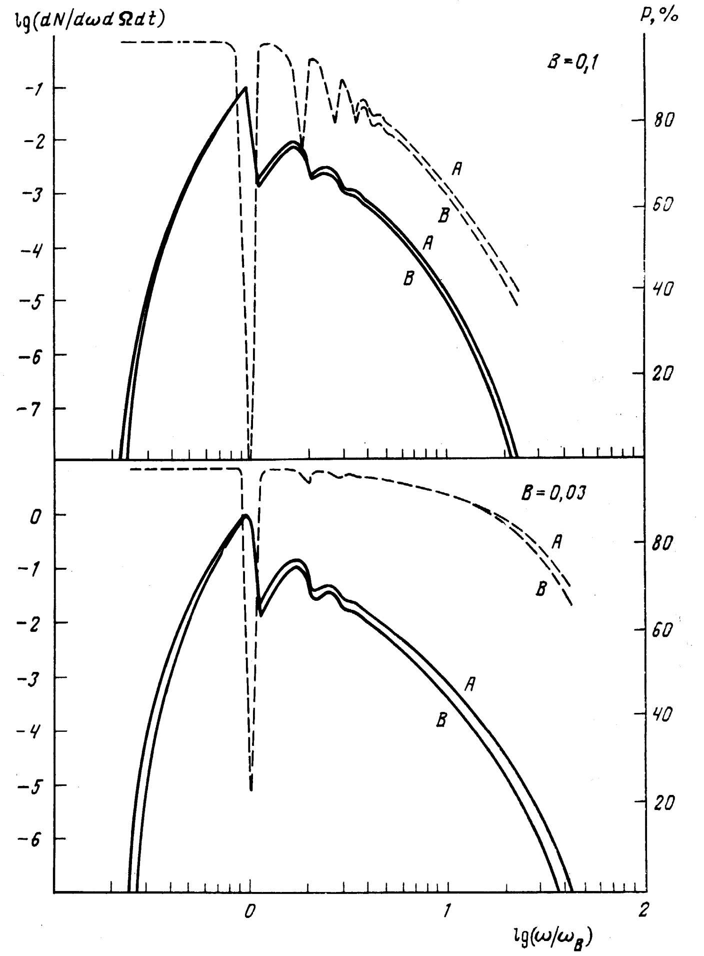

FIG. 4. The spectrum of thermal synchrotron radiation (A) and thermal synchrotron cooling radiation (B) for $T = 0.2$ and $\theta = 90°$, calculated for a step of $\Delta\omega/\omega = 0.01$. The dashed line represents the degree of linear polarization P; ω_B is the cyclotron frequency (in the chosen units, $\omega_B = B$).

"accumulates" in the intermediate state ζ_m increases with decreasing m, reaching a maximum for $m = 0$. Consequently, the condition $\langle q_{n\,0}{}^2\rangle \ll 1$ determines the attainable limit ($n = n_{\max}$) of the approximation relying on the partial inclusion of recoil, in which the recoil momentum is taken into account when the characteristics of the radiated photons are calculated, but is ignored in the description of the intermediate electron states. When the initial state ζ_n is characterized by mean momentum $\langle p_n \rangle = 0$ and mean square momentum $\langle p_n{}^2\rangle$, partial inclusion of the recoil effect is possible when the more general condition $\langle q_{n\,0}{}^2\rangle \ll \max(1, \langle p_n{}^2\rangle)$ is satisfied, where $\langle q_{n\,0}{}^2\rangle$ corresponds to the decay of the state ζ_n with $p = 0$ [see (26)]. In the approximation in which recoil is partially taken into account, expressions (25) and (26) can be transformed to a recursive form.

Figure 4 shows the SCR spectra calculated in the above approximation for $B = 0.03$ and 0.1 with $T = 0.2$. Comparison of the SCR and TSR spectra for given values of B and T shows that the latter spectrum is softer. This is not unexpected, since for each photon with energy corresponding to a higher harmonic there is in the second case a larger number of photons corresponding to lower harmonics. In the classical region $B \ll 1$, the SCR and the TSR spectra differ to a much greater extent than in the quantum region (see Ref. 7). For given B, the difference between the quantum SCR and TSR spectra increases with increasing temperature.

An important feature of synchrotron radiation is its polarization. In the nonrelativistic limit, $nB \ll 1$ holds, and the emission of photons is largely due to electric transitions ($n, \pm 1 \to m, \pm 1$) (see Figs. 1 and 2a). At $\theta = 90°$, this radiation is polarized at right angles to the magnetic field. For $nB \gtrsim 1$, the radiation includes an appreciable contribution due to magnetic transitions with spin reversal. At $\theta = 90°$, this radiation is polarized along the direction of the magnetic field (Fig. 1). The quantum effect due to spin reversal is thus seen to lead to the depolariztion of SCR and TSR.

In the case of the lower harmonics, depolarization appears in the form of narrow features on the spectrum (see Fig. 4).

Depolarization is a maximum in the right-hand wing of the profile of the first harmonic, which corresponds to the $\zeta_n \to \zeta_{n-1}$ transitions between the lowest-lying levels with $n \gtrsim 1$. As n increases, the energy of photons radiated as a result of $n \to n - 1$ transitions shifts toward the left-hand low-frequency wing of the profile. The probability of transitions with spin reversal is then found to fall rapidly (Fig. 2b), so that depolarization is reduced. In the case of the higher harmonics, which merge into the continuous spectrum, spin reversal leads to overall depolarization (Fig. 4), and the degree of linear polarization decreases with increasing radiation frequency.

5. DISCUSSION AND CONCLUSIONS

The quantum approach is essential when it is necessary to take into account the quantized motion of electrons, the quantum recoil, and the spin reversal accompanying the emission of a photon. Whether or not the first condition is satisfied depends on the average quantum number $\bar{n}$ evaluated over the electron ensemble. When $\bar{n} \gg 1$, quantization of the motion of electrons can be ignored. This inequality can also be written in the form $B \ll \bar{E}_n (\bar{E}_n - 1)$ or $B \ll T(1+T)$ where $\bar{E}_n$ is the average energy and $T = \bar{E}_n - 1$ is the average temperature of the particles.

Recoil and spin reversal play an important role when the final state is close to the ground state. When such transitions play are important in decays of excited states with $n \gg 1$, quantum-mechanical calculations are essential for determining all the basic characteristics of synchrotron radiation. When low harmonics with $\nu \ll n$ predominate in decays for levels with $n \gg 1$, estimates of integrated quantities can be deduced from classical formulas. However, the quantum-mechanical calculation is always essential for the description of the properties of the higher harmonics with $\nu \lesssim n$.

The synchrotron radiation flux density has a maximum at the energy $\sim B\bar{E}^2{}_n$ (Ref. 13) in both the classical and the quasi-quantum approximations. When this energy corresponds to harmonics with $\nu \ll \bar{n}$, for which the energy of the emitted photons is $\ll E_n - 1$, the classical formulas are valid. Hence the second condition for the validity of a classical approximation can be written in the form $B\bar{E}^2{}_n \ll \bar{E}_n - 1$ [or, in the case of a thermal distribution, $B(1+T)^2 \ll T$]. It is clear that the second of the above two conditions is the more stringent, and it is indeed this condition that determines the limit for the validity of the classical approximation (see Ref. 7). When $B \gtrsim T(1+T)^{-2}$, quantum transitions must be examined when synchrotron radiation due to an ensemble of electrons is investigated.

The evolution of the excited states of electrons is conveniently described in terms of the level lifetime, the excited-state relaxation time, and the energy loss time. The conditions for the generation of synchrotron radiation can be found by comparing these quantities with the characteristic time for the excitation of transverse degrees of freedom in the ensemble. The difference between the SCR and TSR spectra in the quantum region of parameter value is smaller than in the classical region. The principal difference is due to the radiated power: when the characteristic time for the excitation of transverse degrees of freedom is $\tau_0 \gg \tau_2$, the power carried by stationary SCR is lower by the factor τ_3/τ_0 than the power carried by TSR.

Transitions with spin reversal lead to strong depolarization of the radiation. The main difference between classical and quantum synchrotron radiation from electrons in a strong magnetic field is due to polarization.

Calculations of quantum transitions of electrons (and positrons) in a strong magnetic field can be used to determine the characteristic lifetime and the evolution of excited states, as well as the intensity, spectrum, angular distribution, and polarization of the generated radiation. These results are important for constructing physical models of many astrophysical objects, including radio pulsars, gamma-pulsars, and sources of gamma-ray bursts, thought to be from neutron stars with surface magnetic fields $\gtrsim 10^{12}$ G.

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