Autoresonance in electron cyclotron heating of a plasma

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It is shown that autoresonance acceleration can play an important role in electron cyclotron heating of a plasma. It is apparently responsible for the formation of a ring of high-energy electrons in electron-cyclotron-resonance heating of a plasma in open traps and in bump toruses. It is also found that autoresonance effects can weaken the heating of the bulk of the electrons confined in magnetic traps.

INTRODUCTION

The operation of a number of charged-particle accelerators is based on the autoresonance (autophasing) phenomenon, i.e., on automatic preservation of the condition of cyclotron resonance when the system parameters vary slowly. Thus, an increase of the magnetic field strength or a lowering of the frequency of the acclerating rf field leads to an increase of the particle energy in accordance with the resonance condition $\omega = \omega_e^0 mc^2/\varepsilon$. Here ε is the particle energy (to be specific, we consider electrons), ω the frequency of the accelerating rf field, $\omega_e^0 = eB_0/mc$ the cyclotron frequency calculated for the electron rest mass, and B_0 the magnetic field induction.

The magnetic field in most plasma traps is stationary,[1)] and fixed-frequency oscillations are used for rf heating of the plasma. Under these conditions, autoresonance acceleration becomes possible as a result of the inhomogeneity of the magnetic field of the trap, as the charged particle move towards the increasing magnetic field.

In our opinion, autoresonance acceleration is the cause of the appearance of a group of high-energy electrons ($\varepsilon - mc^2 \gtrsim 100$ keV) in electron cyclotron resonance (ECR) heating of a plasma in a magnetic trap. Investigations of open traps and bumpy toruses have revealed that these electrons form an annular layer located near the intersection of the resonance surface ($\omega = \omega_e(\mathbf{r})$) with the surface of the minima of the magnetic field on the force lines $[(\mathbf{B}_0\nabla)\mathbf{B}_0 = 0]$, see e.g., Refs. 2–4.

We show in the present paper that if the cyclotron resonance condition is met in at a point of minimum magnetic field on some force line, the electrons moving along this force lines with sufficiently low velocity are particularly easily "captured" into autoresonance. They are then localized in the vicinity of the magnetic-field-minimum point. The low-frequency oscillations, which are almost always spontaneously excited in magnetic traps, should cause the autoresonance electrons to drift across the magnetic field. Drift in the direction of the stronger field is accompanied by an increase of the electron energy. As a result, a layer of high-energy electrons is produced near the part of the $(\mathbf{B}_0\nabla)\mathbf{B}_0 = 0$ surface where $\omega_e^0 > \omega$. The magnetic-field geometry of open traps and bumpy toruses is such that this layer should be annular.

At a sufficiently high amplitude of the rf field, autoresonance influences the ECR heating of the bulk of the electrons that oscillate freely along the inhomogeneous magnetic field of the trap. If such an electron moves towards the stronger field, it can be "captured" into autoresonance on passing through the electron-cyclotron resonance zone. The autoresonance acceleration process, however, is reversible and the electron gives up energy to the rf field as it moves back towards the weaker magnetic field. The resultant energy changes is zero at the same accuracy with which is conserved the adiabatic invariant that characterizes the autoresonance state (see the main text below). It is important that the electron capture into autoresonance is accompanied on the phase plane by a passage of the trajectory through the separatrix of the finite and infinite trajectories. Such transitions can alter the adiabatic invariant (see Refs. 5–7). According to Refs. 6–8, after many passages through the cyclotron-resonance zone the changes of the adiabatic invariant, and with them the changes of the energy of the electron motion across the magnetic field, $\varepsilon_\perp \propto \mu$, turns out to be uncorrelated. (Here, $\mu = p_\perp^2/2m\omega_e^0$ is the value of the adiabatic invariant in the absence of an rf field.) Diffusion in μ should therefore set in. The diffusion coefficient decreases with increase of the rf field amplitude[2)]: $D_\mu \propto E^{-1}$.

In the case of a weak rf field, when no new adiabatic invariant is produced, we have $D_\mu \propto E^2$ (see, e.g., Ref. 11). There should therefore exist an optimal rf field amplitude such that the diffusion coefficient is a maximum and, consequently, ECR heating is most effective. We estimate in this paper the optimal value of E.

1. TRANSVERSE ADIABATIC INVARIANT IN THE ABSENCE OF AN RF FIELD

The electron motion in a stationary magnetic field in the presence of rf oscillations is described by the Hamiltonian (see, e.g., Ref. 12)

$$H = \varepsilon - \mu\omega + (2ecB_0\mu)^{1/2}(eA/\varepsilon)\cos\Phi. \tag{1}$$

We use the following notation:

$$\varepsilon = [(mc^2)^2 + 2ecB_0\mu + (p_\parallel c)^2]^{1/2}$$

is the electron energy, A is the rf-field vector-potential amplitude, $\Phi = \theta - \omega t$, and θ is the phase of the Larmor rotation of the electron. We consider the simplest case of oscillations with $\omega \approx \omega_e$, right-hand polarization of the electric field vector, and with a wavelength in the direction transverse to the magnetic field much longer than the Larmor radius of the electrons. The Doppler effect is disregarded. The canonically conjugate variables in (1) are μ, Φ and $p_\parallel$, z, where z is the coordinate along the stationary magnetic field. A canonical transformation $\theta \to \Phi = \theta - \omega t$ of the phase led to the appearance of the second term in (1).

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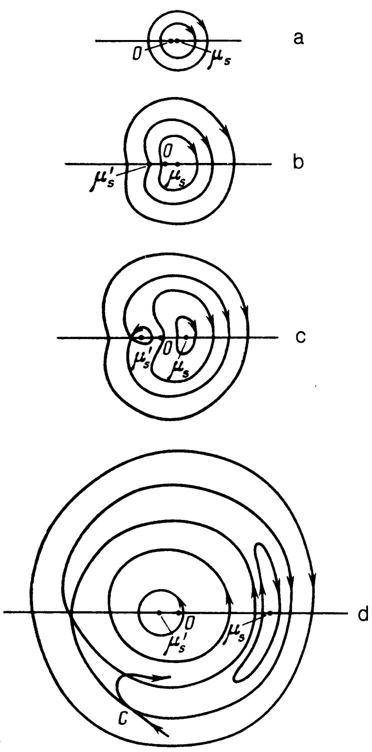

FIG. 1. Phase portrait of system described by the Hamiltonian (1): a—$\omega_e^* < \omega$; b—$\omega_e^* - \omega = \frac{3}{2}\omega(eA/\varepsilon^*)^{2/3}$; c—$\omega_e^* - \omega = \frac{3}{2}\omega(eA/\varepsilon^*)^{2/3} + \delta\omega$, $\delta\omega \ll \omega(eA/\varepsilon^*)^{2/3}$; d—$\omega_e^* - \omega \gg \omega(eA/\varepsilon^*)^{2/3}$.

than π, during the time of passage of the electron along the trap so that the variables μ and Φ can be regarded as fast, and $p_\parallel$ and z as slow.

The picture of the phase trajectories on the plane of the fast variables at given values of the slow ones is shown in Fig. 1. At $\omega_e^* < \omega$ the phase trajectories are close to circles with center at the point $(\mu_s \approx (eA)^2\omega/2\varepsilon^*(\omega_e^* - \omega)^2, 0)$, see Fig. 1a. Here

$$\omega_e^* = \omega_e(\varepsilon^*),\quad \varepsilon^* = [(mc^2)^2 + (p_\parallel c)^2]^{1/2}.$$

With increase of ω_e^* the phase trajectories are "compressed" from the left, and at $\omega_e^* = \omega[1 + 3(eA/\varepsilon^*)^{2/3}/2]$ a new stationary point is created $(\mu_s' \approx (\varepsilon^*/2\omega)(eA/\varepsilon^*)^{2/3}, \pi)$, see Fig. 1b. With further increase of ω_e^* the region occupied by the trajectories surrounding the point (μ_s', π) broaden, and the point itself approaches the origin. At the same time, the stationary point $(\mu_s, 0)$ shifts towards larger values of μ (see Figs. 1c, d). Thus, at $\omega_e^* - \omega \gg \omega(eA/\varepsilon^*)^{2/3}$ we have

$$\mu_s \approx \frac{\varepsilon^*}{2\omega_e^*}\left[\left(\frac{\omega_e^*}{\omega}\right)^2 - 1\right],$$
$$\mu_s' \approx \frac{(eA)^2}{2\varepsilon^*}\,\frac{\omega}{(\omega_e^* - \omega)^2}.$$

The autoresonantly accelerated electrons are those moving along the orbits surrounding the point $(\mu_s, 0)$.

If $\omega_e^* - \omega \gg \omega(eA/\varepsilon^*)^{2/3}$, the Hamiltonian (1) takes in the region $\mu \approx \mu_s$ the form

$$H \approx H_0(p_\parallel, z) - \alpha(\mu - \mu_s)^2 + \beta\cos\Phi, \tag{2}$$

where

$$H_0(p_\parallel, z) = \frac{mc^2}{2}\left(\frac{\omega_e^0}{\omega} + \frac{\omega}{\omega_e^0}\right) + \frac{p_\parallel^2}{2m}\,\frac{\omega}{\omega_e^0}, \tag{3}$$

$$\alpha = \omega^3/2\omega_e^0 mc^2,$$
$$\beta = (e\omega A/c)(2\mu_s/m\omega_e^0)^{1/2}.$$

Rapid motion over the (μ, Φ) phase plane is characterized by an adiabatic invariant $I = \oint \mu d\Phi$. In the region $\mu \approx \mu_s$, using the expressions for the adiabatic invariant of a nonlinear pendulum (see, e.g., Ref. 5), we get

$$I = 2\pi\mu_s \pm 8v^{1/2}\varkappa E(\varkappa^{-1}),$$
$$I = 16v^{1/2}[E(\varkappa) + (\varkappa^2 - 1)K(\varkappa)], \tag{4}$$

where K and E are coplete elliptic integrals, $\varkappa = (w/2v)^{1/2}$, $w = (H_0 - H + \beta)/2\alpha$, $v = \beta/2\alpha$. The upper expressions in (4) should be used for trajectories encircling the origin and on which the phase Φ changes without limit; the lower expression should be used for trajectories with bounded variation of Φ. By analogy with the nonlinear pendulum, we refer to the first regime as rotational and to the second as vibrational.

Relations (4) express I in the form of a function of $\omega_e^0(z)$, $p_\parallel$, and $\varkappa(\omega_e^0, p_\parallel, H)$. If (4) is used to find the inverse function $H[\omega_e^0(z), p_\parallel, \varkappa(\omega_e^0(z), p_\parallel, I)]$, we obtain a Hamiltonian that describes slow motion along the magnetic field, averaged over fast (transverse) oscillations,

$$\bar{H} = H_0(z, p_\parallel) + \beta[1 - 2\varkappa^2(z, p_\parallel, I)]. \tag{5}$$

We determine now the conditions under which an adiabatic invariant I exists, i.e., the longitudinal motion is slow compared with the transverse. The electron oscillates along the magnetic trap at the so-called bounce frequency $\omega_b \sim p_\parallel/mL_0$, where L_0 is the characteristic scale of variation of the magnetic field. When the electron is far from the cyclotron resonance point z_s $(\omega_e(z_s) = \omega)$, the adiabaticity condition is of the form $|\omega_e - \omega| \gg \max(|\dot{\omega}_e|^{1/2}, |\ddot{\omega}_e|^{1/3})$. (The quantity $\dot{\omega}_e$ vanishes if the electron is stopped at the cyclotron-resonance point or if this point coincides with the point where the function $\omega_e(z)$ is a minimum). The foregoing condition is obviously violated when the electron lands in the electron band, where $\omega_e \approx \omega$. The motion, however, remains adiabatic also in this case if the frequency Ω of the transverse oscillations under the action of the rf field exceeds $\max(|\dot{\omega}_e|^{1/2}, |\ddot{\omega}_e|^{1/3})$, considerably. Using (1) and (2), we obtain $\Omega \sim \omega(Ep_\perp/B_0 mc)^{1/2}$. If the point z is displaced far enough from the magnetic-field minimum $(\omega_e(z_s - \omega_{e,\min} \gtrsim \Omega)$, the adiabaticity condition becomes

$$\frac{E}{B_0} \gg \max\left\{\frac{c}{L_s\omega}\frac{p_\parallel}{p_\perp},\ \left(\frac{c}{L_s\omega}\right)^{4/3}\left(\frac{p_\perp}{mc}\right)^{1/3}\right\}. \tag{6}$$

The values of $\dot{\omega}_e$ and $\ddot{\omega}_e$ were estimated here with the aid of the Hamiltonian $\bar{H} \approx H_0$ [see Eqs. (5) and (3)]:

$$|\dot{\omega}_e/\omega| \sim p_\parallel/mL_s\omega,\quad |\ddot{\omega}_e/\omega| \sim (p_\perp/mL_s\omega)^2,$$

where

$$L_s = |(d\omega/dz)/\omega|^{-1}_{z=z_s}.$$

At $\omega_e(z_s) - \omega_{e,\min} \lesssim \Omega$ the motion is adiabatic if

$$\frac{E}{B_0} \gg \left(\frac{c}{L_0\omega}\right)^{4/3}\frac{p_\parallel^{4/3}}{p_\perp(mc)^{1/3}}. \tag{7}$$

We have used here the estimate $|\ddot{\omega}_e/\omega| \sim (p_\parallel/mL_0\omega)^2$, where

$$L_0 = |(d^2\omega/dz^2)/2\omega|^{-1/2}_{z=z_s}.$$

2. HEATING UPON MOTION ALONG THE MAGNETIC FIELD

a) Consider the motion of an electron whose initial-momentum transverse component is small enough: $p_{\perp 0}/mc \ll (E/B_0)^{1/3}$. We assume that at the initial instant the electron was in a region where $\omega > \omega_e$. By virtue of the conservation of the adiabatic invariant, the electrons considered always move in a small vicinity of a stationary point $(\mu_s, 0)$, see Fig. 1. Their motion can therefore be described by the Hamiltonian

$$H_s(p_\parallel, z) \approx \varepsilon_s - \mu_s\omega + (2ecB_0\mu_s)eA/\varepsilon_s, \tag{8}$$

where

$$\varepsilon_s = [(mc^2)^2 + 2ecB_0\mu_s + (p_\parallel c)^2]^{1/2},$$

and μ_s is defined by the equation $(\partial H/\partial\mu)_{\Phi=0} = 0$.

The advance of the electron into the strong-field region is accompanied by a growth of $\mu \approx \mu_s$, and consequently also by a diamagnetic force $F = -\mu ecB_0'/\varepsilon$, that pushes the electron into the region of the weak magnetic magnetic field. The maximum value of μ depends on the longitudinal component of the initial momentum.

In the weakly relativistic case Eq. (8) leads to simple expressions for $\mu_{\max}$. Thus, if $p_{0\parallel}/mc \ll (E/B_0)^{2/3}$, the electron is reflected from the region of the strong magnetic field (the magnetic mirror) without reaching the cyclotron resonance point, in which case ($\mu_{\max} \approx (p_{\parallel 0}^4/2m^3c^2\omega)(B_0/E)^2(p_{\perp\max}/mc \approx (p_{\parallel 0}/mc)^2B_0/E)$. If $p_{0\parallel}/mc \gg (E/B_0)^{2/3}$, we have $\mu_{\max} \approx p_{\parallel 0}c/\omega(p_{\perp\max}/mc \approx (2p_{\parallel 0}/mc)^{1/2}, \varepsilon_{\perp\max} \approx (2\varepsilon_{\parallel 0}mc^2)^{1/2})$. The processes considered, however, are reversible—their sequence is reversed after reflection from the magnetic mirror. As a result, μ returns to its initial value μ_0.

Thus, whereas in the case of a weak rf field an electron passing through the cyclotron resonance region acquires a finite energy increment $\Delta\varepsilon \propto E$ (see, e.g., Ref. 11), under conditions (7), which ensure the existence of a transverse adiabatic invariance, the resultant energy increment turns out to be zero.

Assume now that an electron with $p_\perp/mc \ll (E/B_0)^{1/3}$ is located at the initial region in the region of the strong magnetic field $(\omega_e^* - \omega \gg \omega(eA/\omega_e^*)^{2/3})$. Such electrons are located on the phase plane in the vicinity of the stationary point (μ_s', π). When the electron is displaced into the weak-field region the point (μ_s', π) comes close to the hyperbolic (see Fig. 1). After the coalescence of the points, the electrons considered turn out to be on the phase plane on a certain orbit that encircles the point μ_s, and in this case $p_\perp \sim mc(E/B_0)^{1/3}$. When the electron returns to the strong-field region, the probability of landing in the place where a new stationary point is produced, i.e., of having $\Phi = \pi$, is negligibly small. The electron has therefore a probability $P \approx 1$ of remaining on the orbit encircling the point $(\mu_s, 0)$ and will accordingly have $p_\perp \sim mc(E/B_0)^{1/3}$.

b) Consider now electrons with $p_\perp/mc \gg (E/B_0)^{1/3}$ and assume that as they move along an inhomogeneous magnetic field they enter a region in which the condition $(\omega_e^* - \omega) > \frac{3}{2}\omega(eA/\varepsilon^*)^{2/3}$ is met. The phase trajectories of such electrons can cross on the (μ, Φ) plane the separatrices of the regions of rotational and vibrational motion. In accordance with Refs. 5–7, the transitions of a phase trajectory through a separatrix should change the adiabatic invariant by $\Delta I \ll I$. Methods for calculating ΔI if the parameters of the system are linear in time have by now been developed. The decisive factor in our problem is the temporal dependence of the quantity $\omega_e[z(t)] - \omega$. It can be regarded as linear if the cyclotron-resonance point is far both from the minimum point of $\omega_e(z)$ and from the electron stopping point $(p_\parallel = 0)$.

The value of ΔI depends substantially on the phase Φ at which the passage through the separatrix takes place. In the case of multiple transitions, the variation of I becomes random.[6–8]

A simple expression for ΔI can be obtained in the region $\mu \gg (mc^2/\omega)(E/B_0)^{2/3}$ $((p_\perp/mc) \gg (E/B_0)^{1/3})$, where the picture of the phase trajectories on the (μ, Φ) plane is close to the phase portrait of a nonlinear pendulum (see Fig. 1). In this region, not all the electrons are captured by oscillations on passage through the cyclotron-resonance point. Assuming a uniform distribution over the phases, the capture probability is

$$P = S_1/(S_0 + {}^1/_2 S_1),$$

where

$$S_0 = 2\pi\mu_s, \quad S_1 = 16(\beta/2\alpha)^{1/2} = 16(ecA)^{1/2}(2\mu_s m\omega_e^0)^{1/4}/\omega$$

is the area of the vibrational region. Calculating S_0 and S_1 from the Hamiltonian $H \approx H_0$ [see (3) and (5)], we get

$$P \approx \frac{1}{4}\frac{S_1}{S_0}\left\{1 + \frac{1}{2}\left[1 - \left(\frac{\omega}{\omega_e^*}\right)^2\right]\right\} \sim \left(\frac{E}{B_0}\right)^{1/2}\left(\frac{mc}{p_\perp}\right)^{1/2} \ll 1.$$

Thus, on passage of the high-energy electrons through resonance the capture probability is low and most electrons go over from the region of the backward rotation $(\dot\Phi = \omega_e - \omega < 0)$ into the forward region $(\dot\Phi = \omega_e - \omega > 0)$ along trajectories of type C in Fig. 1.

We calculate ΔI using the results of Ref. 6. For an electron that passes twice through resonance (on moving towards the stronger field and back) we obtain in the first-order approximation

$$\Delta I = \frac{\mu_s}{(2\alpha\beta)^{1/2}}\left\{2(\xi' - \xi)\ln\left(\frac{\pi|\dot\mu_s|}{16\beta}\right) + \ln\left(\frac{1-\xi}{1-\xi'}\right) + 2\ln\left[\frac{\Gamma(\xi')}{\Gamma(\xi)}\right]\right\},$$

where ξ and ξ' depend on the values of the phase Φ at the instant of the transition through the separatrix of the finite and infinite motions in Fig. 1. In accordance with Refs. 6–8, the quantities ξ and ξ' can be regarded as random, independent, and uniformly distributed on the segment (0,1). The quantity ΔI can then also be regarded as random with zero mean value and with a variance

$$\sigma_\mu^2 \sim \left(\frac{p_\parallel c^2}{L_s\omega^2}\right)^2 \frac{mc}{p_\perp}\frac{B_0}{E}\ln^2\left(\frac{p_\perp}{p_\parallel}\frac{E}{B_0}\frac{L_s\omega}{c}\right).$$

The diffusion coefficient is then equal to $D_\mu = \omega_b\sigma_\mu^2$.

It is known (see, e.g., Ref. 11) that at sufficiently low rf field amplitudes, when the adiabaticity condition is not met, we have

$$D_\mu \approx (cp_\perp E/B_0)^2(\pi L_s m/p_\parallel\omega)\omega_b.$$

Thus, the diffusion coefficient is a maximum on the boundary of the adiabaticity region defined by conditions (6) and

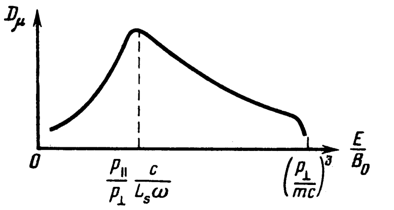

FIG. 2.

(7) (see Fig. 2). Note that at $(p_\parallel/p_\perp)(c/L_s\omega) > (p_\perp/mc)^3$ the diffusion coefficient vanishes if $E/B_0 > (p_\parallel/p_\perp)(c/L_s\omega)$.

The diffusion can be substantially enhanced by heating with oscillations having a more complicated spectrum, say a combination of two monochromatic oscillations with frequencies that differ by $\Delta\omega \ll \omega$. The amplitude of the resultant rf field will vary on account of the beats. If an electron moving towards the stronger magnetic field passes through the resonance region during the period with the rf field amplitude increases, the capture probability increases. The captured electron acquires more energy in accordance with the autoresonance condition $\varepsilon = mc^2\omega_e^0/\omega$. The decrease of the amplitude of the RF field stops the autoresonant acceleration. While the electron is in the autoresonance state, its energy increases by $\Delta\varepsilon = mc^2(\omega_{e,2}^0 - \omega_{e,1}^0)/\omega$, where $\omega_{e,1}^0$ ($\omega_{e,2}^0$) is the cyclotron frequency at the instant of going "into" ("out of") autoresonance. It is assumed that $\omega_{e,2}^0 > \omega_{e,1}^0$. Since transitions with $\omega_{e,2}^0 < \omega_{e,1}^0$, are also possible, the electron heating has the character of diffusion in ε, the diffusion rate is maximal[3)] at $\Delta\omega \sim \omega_b$.

It is possible that the described heating mechanism was observed in experiments[4] in which the heating became more effective when the rf oscillation spectrum was more complicated.

In conclusion, we find the maximum energy that an electron can acquire by ECR heating. The heating is accounted for in the Hamiltonian (1) by the last (small) term $\sim cp_\perp eA/\varepsilon$. Since it is resonant—secular, it entails energy changes $\delta\varepsilon \gg cp_\perp eA/\varepsilon$. To determine them we can assume approximately that the motion is over the surface $H^{(0)} = \varepsilon - \mu\omega = \text{const}$. Combining this relation with the cyclotron-resonance condition $\omega = ecB_0/\varepsilon$, we get

$$\mu = [(H^{(0)})^2 - (mc^2)^2 - (p_\parallel c)^2]^{1/2}/\omega.$$

The quantity μ, and with it the electron energy $\varepsilon = H^{(0)} + \mu\omega$ is a maximum if the resonance condition is met at the instant when the electron motion along the magnetic field is halted ($p_\parallel = 0$):

$$\mu_{max} = \left\{2\mu_0\frac{mc^2}{\omega}\left[\frac{\overset{0}{\omega}_{e,max}}{\omega} - \left(1 + \frac{\mu_0\overset{0}{\omega}_{e,max}}{mc^2}\right)^{1/2}\right] + \mu_0^2\right\}^{1/2}, \quad (9)$$

where μ_0 is the initial value of the magnetic moment and $\omega_{e,\max}^0$ is the maximum cyclotron frequency reached in the motion along the trap prior to turning-on the rf field.

In the weakly relativistic case it is convenient to represent (9) in the form $\mu_{\max} \approx (2\varepsilon_{\parallel s} mc^2)^{1/2}/\omega$, and correspondingly $\varepsilon_{\max} - mc^2 \approx (2\varepsilon_{\parallel s} mc^2)^{1/2}$. Note that the same expression for $\varepsilon_{\max}$ was obtained in Ref. 12 for the motion of an electron in a weak rf field. $\varepsilon_{\parallel s}$ in the expressions for $\mu_{\max}$ and $\varepsilon_{\max}$, is the initial energy of the longitudinal motion of the electrons at the instant of passage through the resonance point. If $\varepsilon_{\parallel s}$ is assumed equal to the energy of electrons produced by ionization ($\varepsilon_{\parallel s} \sim 10$ eV), which corresponds to the conditions of Refs. 2–4, we obtain $\varepsilon_{\max} - mc^2 \sim 1$ keV.

3. HEATING IN MOTION ACROSS THE MAGNETIC FIELD

In motion described by the Hamiltonian (1), the electron cannot reach the energies ~ 100 keV observed in the experiments.[2–4] We therefore widen the scope of the problem by invoking a new factor, viz., motion across the magnetic field.

Assume that the resonance condition $\omega = \omega_e(z)$ is met on some force line of the magnetic field at the point where $\omega_e(z)$ is a minimum. An electron with a sufficiently small longitudinal momentum component $p_\parallel \ll p_\perp (E/B_0)^{1/4}(p_\perp/mc)^{1/4}$, moving near the minimum of the magnetic field, is autoresonant. Low-frequency oscillations that cause drift across the magnetic field are spontaneously excited in practically all magnetic fields. If the drift velocity is directed towards increasing B_0 and the motion is slow enough $(\max(|\dot\omega_e|^{1/2}, |\ddot\omega_e|^{1/3}) \ll \Omega)$, the energy of the autoresonant electrons will increase in accordance with the condition $\varepsilon = mc^2\omega_e^0/\omega$.

Drift in low-frequency oscillations is as a rule random. By virtue of the autoresonance condition, the spatial diffusion should be accompanied by a diffusion with respect to μ:

$$D_\mu \approx D(\varepsilon/\omega L_\perp)^2,$$

where D is the coefficient of spatial diffusion and $L_\perp$ is the characteristic scale of the transverse inhomogeneity of the magnetic field.

We consider now longitudinal motion of autoresonance electrons. They are localized in the vicinity of the minimum of the magnetic field, where $\omega_e^0(z) \approx \omega_{e,\min}^0[1 + (z/L_0)^2]$. Assuming that $\omega_{e,\min}^0 = \omega\varepsilon/mc^2$, we get from (3)

$$\bar H \approx H_0 \approx \frac{c}{2\varepsilon}\left[p_\perp^2\left(\frac{z}{L_0}\right)^2 + p_\parallel^2\right]. \quad (10)$$

Assuming that in the case of drift across the magnetic field the longitudinal adiabatic invariant $I_\parallel = \oint p_\parallel\, dz$ is covered along with the transverse one, we obtain from (10) $z_{\max} = (I_\parallel L_0/\pi p_\perp)^{1/2}$. This expression shows that with increase of $p_\perp$ the swing of the electron oscillations along the magnetic field decreases. Thus, autoresonance heating should lead to formation of a thin layer of high-energy electrons localized near the surface $(\mathbf{B}_0\nabla)\mathbf{B}_0 = 0$.

In experiments[2–4] on the spatial distribution of high-energy electrons, the intersection of the cyclotron-resonance surface with the surface $(\mathbf{B}_0\nabla)\mathbf{B}_0 = 0$ is a circle. In this case, the high-energy electrons should form an annular layer, as was in fact observed in Refs. 2–4. In view of the substantial transverse inhomogeneity of the magnetic field in Refs. 2–4, the drift of the autoresonance electrons was perfectly capable of imparting to them an energy ~ 100 keV.

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[1)] A plasma trap with a magnetic field that increases with time, placed in an rf electromagnetic field (plasma synchrotron), was investigated in Ref. 1.

[2)] The decrease of the effectiveness of the resonant cyclotron interaction with increase of the rf-field amplitude, due to the onset of a new adiabatic invariant, was noted in Refs. 9 and 10. These references dealt with ion-cyclotron oscillations propagating across a magnetic field, and relativistic effects were disregarded.

[3)] A similar diffusion-enhancement mechanism was considered in Ref. 13, where heating of a hydrogen plasma in oscillations with $k_{\parallel} = 0$ was considered.

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