Parallel pumping of spin waves with a magnetic-field drift

V. V. Zautkin, V. S. L'vov, B. I. Orel, E. V. Podivilov, and V. B. Cherepanov

Institute of Automation and Electrometry, Siberian Branch, Academy of Sciences of the USSR

(Submitted 25 July 1986)
Zh. Eksp. Teor. Fiz. **93**, 1269–1280 (October 1987)

The parametric excitation of spin waves and their above-threshold behavior have been studied experimentally and theoretically in a situation in which the resonant frequency is caused to drift by an oscillating external magnetic field. It is shown in particular that the nature of the excitation depends strongly on the sign of the drift velocity: For $\dot{H} > 0$ the excitation is soft, while for $\dot{H} < 0$ it is hard. The susceptibility of parametric spin waves has been measured at various values of the drift rate and the pump amplitude. In the case of a positive drift ($\dot{H} > 0$) the results agree well, qualitatively and quantitatively, with the theory proposed by Podivilov and Cherepanov {Zh. Eksp. Teor. Fiz. **90**, 767 (1986) [Sov. Phys. JETP **63**, 447 (1986)]}. In the case of a negative drift there are significant discrepancies between experiment and the theory based on the approximation of a self-consistent field. It is shown that these discrepancies can be ascribed to collisions of parametric spin waves, whose role increases for $\dot{H} < 0$. The behavior of self-oscillations as a function of the sign and magnitude of the drift rate has also been studied.

INTRODUCTION

Research on large-amplitude parametric spin waves (reviewed in Ref. 1) has shown that their interaction plays a decisive role, giving rise to many nonlinear effects such as multistep excitation, amplitude limiting, self-oscillations of the magnetization, and changes in the spectrum of spin waves. A change in the spectrum of spin waves, $\Delta\omega_{\mathbf{k}}$, accompanying their interaction was first observed by Prozorova and Smirnov,[2] who carried out quantitative measurements of $\Delta\omega_{\mathbf{k}}$ and used the results to experimentally determine the coefficients in the interaction Hamiltonian of the spin waves. Zhirnyuk *et al.*[3] subsequently observed an anomalous broadening, a distortion of the shape of a packet of parametric spin waves, and a shift of the center of the packet with respect to half the pump frequency. According to the present understanding,[4] these unusual effects result from a drift of the natural frequencies of the parametric spin waves, which causes them to deviate from resonance with the pump as the amplitude of the spin waves increases. The reason for the drift is the multistep nature of the relaxation of the spin waves, which has the result that nonequilibrium spin waves accumulate at one stage of the relaxation and influence the spectrum of parametric spin waves. It is found that the natural frequencies of the spin waves drift for a comparatively long time, since the time scale of the buildup of nonequilibrium spin waves may be more than an order of magnitude longer than the relaxation time of the parametric spin waves, according to Ref. 5. As a result, as the natural frequency $\omega_{\mathbf{k}}$ of the spin waves drifts in k space, a drifting packet of parametric spin waves forms, localized near the surface $\omega_{\mathbf{k}} = \omega_p/2$. Podivilov and Cherepanov[4] showed that the behavior of the parametric spin waves depends strongly on the sign of the rate of the frequency drift and the amplitude of the four-magnon interaction, $S_{\mathbf{k},\mathbf{k}'}$. Under the condition $S(\partial\omega_{\mathbf{k}}/\partial t) > 0$ the number of parametric spin waves falls to the thermal level as the drift rate $|\partial\omega_{\mathbf{k}}/\partial t|$ increases. For $S(\partial\omega_{\mathbf{k}}/\partial t) < 0$ the number of parametric spin waves increases; in addition, some new stationary states arise; these new states are qualitatively different from the ground state of parametric spin waves in an S-theory without drift.

In this paper we report a study of the parametric excitation of spin waves in a situation in which their natural frequencies are deliberately caused to drift, while the magnitude and time dependence of the drift rate and also other parameters can be varied over broad ranges. In Sec. 1 we derive a theoretical expression for the number of parametric spin waves as a function of the pump power and the drift rate in the steady state, assuming parameter values close to the experimental values. We find the frequencies of collective oscillations and the conditions for stability of a steady-state packet of parametric spin waves. In Sec. 2 we describe our experimental apparatus and measurement procedure. In Sec. 3 we examine the nature of the parametric excitation of spin waves in a drifting magnetic field, $H = H_0 + \delta H(t)$ in a ferrite (yttrium iron garnet) with $S > 0$. The results show that for $\dot{H} < 0$ the excitation is hard, while for $\dot{H} > 0$ it is soft. The threshold pump power has been measured at various drift rates. In Sec. 4 we study the above-threshold state of a system of parametric spin waves in the cases of a sawtooth drift ($\dot{H} = \text{const}$) and a sinusoidal drift of the magnetic field. We find the behavior of the amplitude of the parametric spin waves and the susceptibility as functions of the drift rate and the pump power. We study the behavior of self-oscillations of the parametric spin waves as a function of the drift rate for various orientations of the magnetization with respect to the crystallographic axes. Finally, in Sec. 5, we compare the experimental results with the theoretical predictions, and we show that in the case of a positive drift ($S\dot{H} > 0$) the behavior of the parametric spin waves agrees well with the results of Ref. 4, while in the case of a negative drift the scattering by one another of the parametric spin waves plays an important role along with the self-consistent renormalization of the pump (the S-theory).

1. QUALITATIVE ANALYSIS OF THE PARAMETRIC EXCITATION OF WAVES WITH A FREQUENCY DRIFT

The k-space distribution of parametric spin waves is described by the equations of the S-theory.[1] Introducing the variables $\varepsilon_{\mathbf{k}} = \omega_{\mathbf{k}} - \omega_p/2$ (the deviation of the natural frequency from resonance) and $\Omega = (\theta_{\mathbf{k}}, \varphi_{\mathbf{k}})$ (the angular co-

 0038-5646/87/100717-08$04.00

ordinates of the wave vector), we write these equations in the form

$$\partial n_{\varepsilon,\Omega}/\partial t+2\gamma_\Omega n_{\varepsilon,\Omega}+2\,\mathrm{Im}(P_\Omega^*\sigma_{\varepsilon,\Omega})=2\gamma_\Omega n_\Omega^0, \tag{1.1}$$

$$\partial\sigma_{\varepsilon,\Omega}/\partial t+2\{\gamma_\Omega+i[\tilde{\varepsilon}_\Omega+g_\Omega(t)]\}\sigma_{\varepsilon,\Omega}+2iP_\Omega n_{\varepsilon,\Omega}=0,$$

$$P_\Omega=hV_\Omega+\int S_{\Omega,\Omega'}\sigma_{\varepsilon',\Omega'}\,d\varepsilon'\,d\Omega'. \tag{1.2}$$

Here P_Ω is the renormalized pump; h is the amplitude of the pump field; v_Ω is the coefficient of the coupling of the parametric spin waves with the pump; $S_{\Omega,\Omega'}$ is the amplitude of the four-wave mixing of pairs of parametric spin waves;

$$g_\Omega(t)=u_\Omega\delta H(t),\quad u_\Omega=\partial\omega_{\mathbf{k}}/\partial H \tag{1.3}$$

is the frequency drift due to the change in the magnetic field, $\delta H(t)$;

$$\tilde{\varepsilon}_\Omega=\varepsilon+2\int T_{\Omega,\Omega'}n_{\varepsilon',\Omega'}\,d\varepsilon'\,d\Omega' \tag{1.4}$$

is the deviation of the natural frequency from resonance when the nonlinear interaction is taken into account; and n_Ω^0 is the intensity of thermal noise.

In the steady state we have $g(t)=0$, so for $|P_\Omega|>\gamma_\Omega$ the solution of system of (1.1)–(1.4) increases exponentially with a growth rate

$$\nu_{\Omega,\varepsilon}=-\gamma_\Omega+(|P_\Omega|^2-\tilde{\varepsilon}_\Omega^2)^{1/2}. \tag{1.5}$$

The range of instability as a function of the frequencies ω_k is determined by the inequality $\nu_\Omega>0$, so we find

$$|\tilde{\varepsilon}|<(|P_\Omega|^2-\gamma_\Omega^2)^{1/2}. \tag{1.6}$$

In the case of a linear frequency drift $[g(t)=\alpha t$ or $\delta\dot{H}=\dot{H}=\alpha/u_\Omega=\text{const}]$ packets of parametric spin waves may exist that maintain their shape while their frequency $\omega_{\mathbf{k}}$ changes at a constant rate. In other words, Eqs. (1.1)–(1.4) have a steady-state solution of the form $n(\Omega,\varepsilon,t)=n_\Omega(\varepsilon+\alpha t)$. For a qualitative analysis of the shape of this packet we can assume that the drift is slow, and we can ignore the retardation of the phase of the pairs. The evolution equations (1.1) then simplify to

$$\frac{1}{2}\frac{\partial n_\varepsilon}{\partial t}=\nu_\varepsilon(t)n_\varepsilon+\gamma n^0, \tag{1.7}$$

where $\nu_\varepsilon(t)$ is the instantaneous value of the instability growth rate for a wave with a fixed $|k|$, given by

$$\nu_\varepsilon(t)=-\gamma+[|P|^2-(\varepsilon+\alpha t)^2]^{1/2}. \tag{1.8}$$

In drifting, some waves enter the instability region near the resonant surface at each instant, while others are leaving it. The incoming waves grow exponentially with the growth rate $\nu_\varepsilon(t)$ given by (1.8), starting from the level of thermal fluctuations. They traverse the instability region (1.6) in a time

$$\tau_0=2(|P_\Omega|^2-\gamma_\Omega^2)^{1/2}/|\alpha|, \tag{1.9}$$

and their level increases to

$$n_{max}\approx n^0\exp\left(\int_{-\varepsilon/|\alpha|-\tau_0}^{-\varepsilon/|\alpha|+\tau_0}2\nu(t)\,dt\right)\approx n^0\exp[{}^4/_3(|P|^2-\gamma^2)^{3/2}|\alpha|\gamma], \tag{1.10}$$

according to (1.7) and (1.8). These waves then emerge from the instability region and are damped by a factor of e over a time τ_1, with an average damping rate $\bar{\nu}\approx\nu(t_0+\tau_1)$, where $\nu(t)$ is given by (1.8), and τ_0 is the time corresponding to the crossing of the rear boundary of the instability region $[\nu(t_0)=0]$. We then find the following estimates for $\bar{\nu}$ and τ_1:

$$\bar{\nu}\approx|\alpha|\tau_1(|P|^2-\gamma^2)^{1/2}/\gamma,\quad \tau_1\approx[\gamma/|\alpha|(|P|^2-\gamma^2)^{1/2}]^{1/2}. \tag{1.11}$$

The maximum of the wave packet is determined by expression (1.10) and lies at the rear boundary of the instability region, (1.6):

$$\Delta\omega_{\mathbf{k}}=\frac{\alpha}{|\alpha|}(|P_\Omega|^2-\gamma_\Omega^2)^{1/2}. \tag{1.12}$$

From (1.9) and (1.11) we see that under the condition

$$|\alpha|\gamma\ll(|P|^2-\gamma^2)^{3/2} \tag{1.13}$$

the width of the packet, $\delta=|\alpha|\tau_1$, is much smaller than its shift $|\Delta\omega_{\mathbf{k}}|$, given by (1.12). The integral number of parametric spin waves, N, can be estimated quite easily:

$$N\sim n_{max}\delta\sim n^0\exp[{}^4/_3(|P|^2-\gamma^2)^{3/2}/|\alpha|\gamma]. \tag{1.14}$$

At our level of rigor, the coefficient of the exponential function in (1.14) is not determined; the only point of importance for our purposes is that the number of parametric spin waves is proportional to the thermal noise level n^0. From Eqs. (1.1) in the case of a slow drift we can easily derive an expression for the phase difference between the self-consistent pump $P_{\mathbf{k}}$ and the anomalous correlation functions $\sigma_{\mathbf{k}}$:

$$\mathrm{Re}(P_{\mathbf{k}}^*\sigma_{\mathbf{k}})\approx-\Delta\omega_{\mathbf{k}}n_{\mathbf{k}}. \tag{1.15}$$

We can now determine the number of parametric spin waves as a function of the pump power, h^2. Specifically, we can put expression (1.2) in the form

$$|hV_{\mathbf{k}}|^2=|P_{\mathbf{k}}|^2+\left|\sum_{\mathbf{k}'}S_{\mathbf{k}\mathbf{k}'}\sigma_{\mathbf{k}'}\right|^2-2\,\mathrm{Re}\left(P_{\mathbf{k}}^*\sum_{\mathbf{k}'}S_{\mathbf{k}\mathbf{k}'}\sigma_{\mathbf{k}'}\right), \tag{1.16}$$

from which we find, using (1.15),

$$SN=SN_S-\Delta\omega_{\mathbf{k}},\quad |SN_S|=(|hV|^2-\gamma^2)^{1/2}. \tag{1.17}$$

We see that in the case of a positive drift ($S\Delta\omega_{\mathbf{k}}>0$) the number of waves drops to a level lower than that in the steady state, $N=N_S$, while in the case of a negative drift ($S\Delta\omega_{\mathbf{k}}<0$) the number of waves instead increases. To determine the explicit dependence of the number of waves on the drift rate and the pump power we need to express $\Delta\omega_{\mathbf{k}}$ in terms of these parameters by means of (1.14) and (1.12). We find

$$SN\approx SN_S-\left\{\frac{3}{4}\gamma u|\dot{H}|\ln\left[\frac{(|hV|^2-\gamma^2)^{1/2}}{\gamma\xi}\right]\right\}^{1/3}\mathrm{sign}(\dot{H}), \tag{1.18}$$

where the small parameter ξ characterizes the intensity of the thermal noise and is given by[1]

$$\xi=8\pi^2\frac{k^2}{v}|S|\gamma\frac{T}{\omega_p}, \tag{1.19}$$

where k and v are the wave vector and group velocity of the parametric spin waves, and T is the temperature. Under the

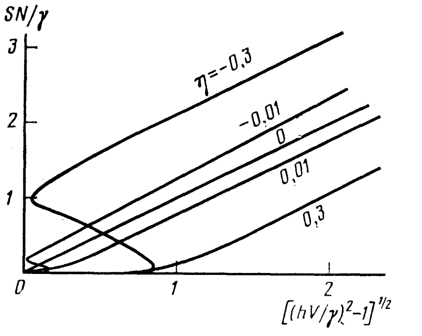

FIG. 1. Theoretical curves of the amplitude of the parametric spin waves as a function of the pump amplitude.

experimental conditions the value of ξ was 10^{-2}–10^{-3}. Expression (1.18), derived through simple estimates, breaks down at a high drift velocity and when the pump is just slightly above the threshold. Podivilov and Cherepanov[4] have derived a complete solution of the problem for an arbitrary drift velocity. In the present paper we show plots of the number of parametric spin waves, N, versus the pump power h^2 for several values of the drift rate (Fig. 1). We see that in the case of a positive drift the number of these waves decreases with increasing drift rate, and the excitation becomes softer. In the case of a negative drift, the dependence of N on h^2 becomes multivalued; hysteresis arises. As was shown in Ref. 4, the field h_1 corresponding to the direct jump in the amplitude is

$$\frac{|h_1V|^2}{\gamma^2}=1+\left[\frac{3}{4}\frac{u|\dot{H}|}{\gamma^2}\ln(\xi^{-1})\right]^{2/3}. \tag{1.20}$$

Podivilov and Cherepanov[4] also studied the stability of the steady-state solutions which they found. They derived the frequencies of uniform collective oscillations of the parametric spin waves:

$$\Omega_0^{\pm}\approx-i\gamma\pm[4(SN+\Delta\omega_k)(2T+S)N-\gamma^2]^{1/2}, \tag{1.21}$$

where $\Delta\omega_k$ is the deviation of the center of the packet from the resonant frequency, given by (1.12). These expressions can also be derived from the S-theory equations without drift, for a narrow packet of parametric spin waves localized on the surface $\omega_k=\omega_p/2+\Delta\omega_k$. The same can be said of relation (1.15), which relates the normal and anomalous correlation functions of parametric spin waves. In those cases in which the number of parametric spin waves is large in comparison with the thermal noise, the role played by the drift is considerably reduced to one of modifying the conditions for external stability: Instead of $\omega_k=\omega_p/2$ we find $\omega_k=\omega_p/2+\Delta\omega_k$, where the position of the center of the packet, $\Delta\omega_k$, is given by (1.12).

In the absence of a drift ($\Delta\omega_k=0$) a steady state is stable under the condition[1]

$$S(2T+S)>0. \tag{1.22}$$

We see that from (1.21) that the stability condition (Im $\Omega_0^{\pm}<0$) in this case holds even if the drift is positive. If the drift is negative, the stability is disrupted when $SN+\Delta\omega_k$ vanishes. In the opposite limit from (1.22), a steady state without a drift is unstable.[1] It can be seen from (1.21) that a negative drift of sufficient magnitude can stablize the steady state.[4]

2. EXPERIMENTAL PROCEDURE

The parametric excitation of spin waves is carried out by a method of parallel pumping in samples of yttrium iron garnet single crystals. The sample is placed in a rectangular cavity which is critically matched with a waveguide system, along which microwave pulses at a frequency of 9.4 GHz, with a repetition period of 20 ms, propagate from a magnetron. The threshold, the excitation level, and the time-varying characteristics of the system of parametric spin waves are found from the changes in the pulsed signal reflected from the cavity, which is displayed on an oscilloscope after detection.

A constant drift of the natural frequencies of the spin waves during the pump pulse is introduced by introducing a deliberate drift of the magnetizing field, $H(t)=H_0+\delta H(t)$. For this purpose a coil is inserted into the cavity; the axis of the coil is oriented parallel to the static magnetic field H_0. The current from a sawtooth voltage generator, operating in a regime of single pulses synchronized with the microwave pump pulse, is passed through the coil. The length of the current pulses is chosen so that the total change in the field at the given drift rate $\dot{H}$ is less than a few oersteds and has little effect on the threshold, the steady-state susceptibility, or other parameters of the spin waves. The shape of the current pulse in the coil is monitored on the oscilloscope screen while the signal reflected from the resonator is being observed. Where necessary, the time at which the field drift begins can be shifted from the time at which the pump is turned on, through the use of a delay line.

A constant drift $H(t)$ can also be imposed by sending 50 Hz line current through the modulating coils of an electromagnet. By turning on the pump for a brief time interval whenever the current crosses zero, we obtain nearly linear modulation of H during the microwave pulse. The change in the drift rate $\dot{H}$ in this case is regulated by simply varying the modulation amplitude. In the present study we used both methods to create a drift of the magnetizing field, with the goal of eliminating as far as possible the error associated with the calibration of the rate of change of the magnetic field. The block diagram of the experimental apparatus is similar to that shown in Ref. 6.

3. NATURE OF THE PARAMETRIC EXCITATION

As the pump field amplitude h is increased gradually, a point is ultimately reached at which a "spall" appears in the pulse reflected from the resonator: The pulse acquires a characteristic step-shaped distortion. Figure 2 shows an experimental curve of the threshold found from the distortion of the shape of the reflected signal as a function of the drift rate. The asymmetry of the threshold curve with respect to the sign of the drift reflects a sharp difference in the nature of the excitation of the parametric spin waves. In the case of a negative drift ($\dot{H}<0$) the excitation becomes harder (in the case of a positive drift, softer) than in the absence of a drift. These results imply that in the former case the step in the reflected pulse becomes steeper and higher and appears abruptly, while in the former case the step decreases gradually with increasing $\dot{H}$, to the point that it ultimately disappears entirely.

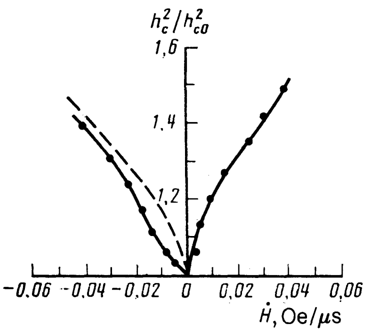

FIG. 2. Solid line—threshold power versus the magnetic-field drift rate, $H = H_c - 100$ Oe; dashed line—theoretical behavior of the cutoff field h_1 (H_c is the threshold pump field).

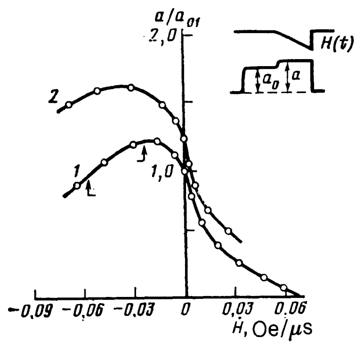

FIG. 4. Relative change in the amplitude of the reflected signal versus the magnetic-field drift rate at $H = H_c - 100$ Oe. 1— $(h/h_{c0})^2 = 3$ dB; 2— $(h/h_{c0})^2 = 4$ dB.

The hardness of the excitation of the parametric spin waves due to the particular form of the $N(h)$ dependence at $\dot H < 0$ should be accompanied by hysteresis of the threshold. It can be seen from Fig. 1 that as the pump is reduced the excitation of parametric spin waves should be disrupted for $h_2 < h_1$; i.e., the threshold for the disappearance of the parametric spin waves is lower than the threshold for their appearance. The ratio $(h_1 - h_2)/h_1$ can be used to characterize the magnitude of the hardness of the excitation. To measure this quantity, we can abruptly reduce the power of the microwave signal after the step, i.e., after the establishment of some above-threshold state of the parametric spin waves during the same pump pulse. By adjusting the magnitude of this jump, we can reach the point h_2 and detect the disappearance of the parametric spin waves at subsequent times. To control the power we placed an absorbing cell (consisting of a waveguide section with a microwave diode) in the microwave line. In the "on" state, the diode limits the power transmitted through itself, and the degree of this limitation is determined by the voltage applied to it. In our case, the diode was controlled by pulses synchronized with the pump, with an adjustable time delay. The results of the measurements of the hardness are shown in Fig. 3. In accordance with the theory (Sec. 1, Ref. 4), the stiffness increases as the field drift rate becomes more negative.

In the case of positive drift ($\dot H > 0$) the experimental results on the threshold reported above are in complete qualitative agreement with the theoretical predictions, but it is difficult to make a quantitative comparison in this case since the sensitivity of the apparatus is not known sufficiently accurately. In the case of a negative drift ($\dot H < 0$) it is possible to compare the measured threshold with the field predicted by the theory, h_1 (see (1.20)]. Figure 2 shows, along with the experimental dependence of the threshold on the magnitude of the drift, a plot of the cutoff field h_1 calculated from (1.20) with $\dot H < 0$ (dashed line). We see from this figure that the theory agrees well with experiment at $|\dot H| \gtrsim 5 \cdot 10^3$ Oe/s. The theoretical results deviate somewhat from the experimental results at a low drift; the reason for this discrepancy will be discussed in Sec. 5.

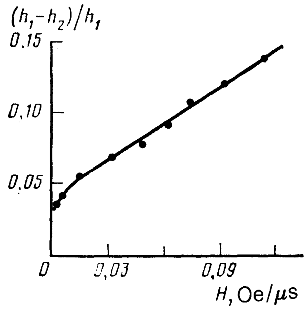

FIG. 3. Hardness of the excitation of the parametric spin waves as a function of the rate of the negative field drift ($H = H_c - 100$ Oe).

4. ABOVE-THRESHOLD STATE AND SELF-OSCILLATIONS: EXPERIMENTAL RESULTS

1. Steady state. According to the theory (Fig. 1), the number of parametric spin waves is a multivalued function of the drift rate and the pump amplitude for $\dot H < 0$, and the steady state may depend on the initial conditions in the case of a field drift at a constant rate. In different experiments in which the frequency drift was begun before and after the beginning of the pump pulse, however, the measured steady-state amplitude of the parametric spin waves was found to be identical. Figure 4 shows some typical results on the amplitude of the steady-state signal as a function of the field drift rate $\dot H$ at a fixed pump power and with $\mathbf{M} \| \langle 100 \rangle$, in which case there are no self-oscillations at $\dot H = 0$. Curves 1 and 2 in Fig. 5 show $\ln|1 - a/a_0|$ versus $\ln|\dot H|$ for the cases of posi-

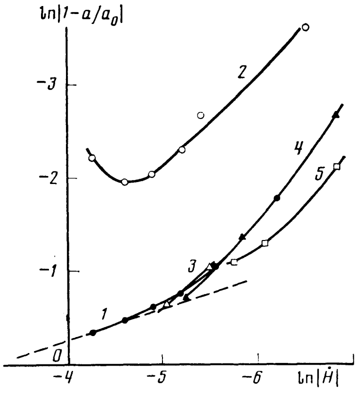

FIG. 5. Amplitude of the parametric spin waves versus the drift rate, in logarithmic coordinates. 1 (●)—positive steady-state drift (sawtooth modulation); 2 (○)—negative steady-state drift; 3 (△)—negative time-varying drift (sinusoidal modulation; see also Fig. 7) ($H_m = 2.7$ Oe); 4 (▲)—the same, for $H_m = 0.8$ Oe; 5 (×)—the same, for $H_m = 0.35$ Oe ($\dot H$ is expressed in units of oersteds per microsecond).

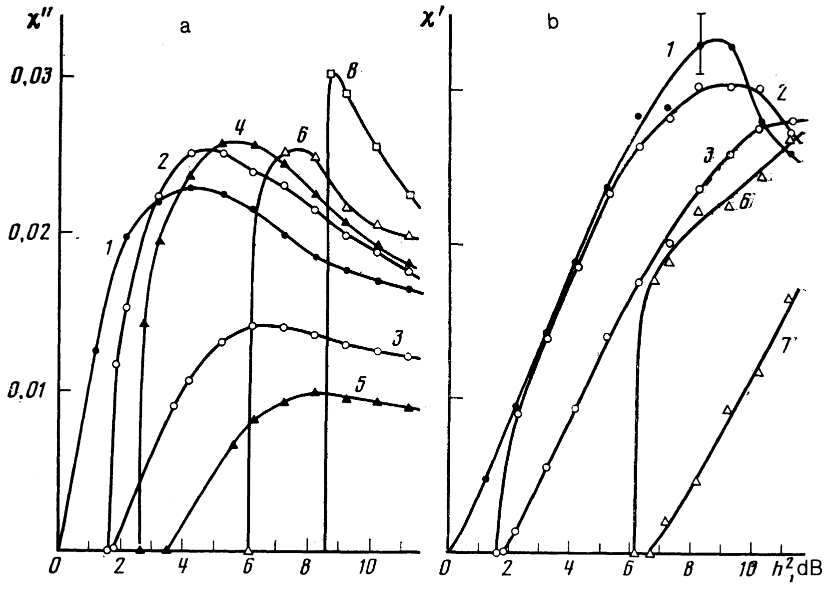

FIG. 6. The complex microwave permittivity χ versus the pump power. a: Imaginary part, χ''. b: Real part, χ'. 1—$\dot{H}=0$; 2— $-$ 0.045 Oe/μs; 3—0.045; 4— $-$ 0.09; 5—0.09; 6— $-$ 0.45; 7—0.45; 8— $-$ 0.9 ($H = H_c - 100$ Oe).

tive and negative steady-state drifts. We see that for $\dot{H} \gtrsim 5 \cdot 10^3$ Oe/s the reflected signal and thus the number of parametric spin waves have the behavior $1 - a/a_0 \approx A\,|\dot{H}\,|^{1/3}$. Numerical estimates of the coefficient A from (1.18) show that its experimental value differs from the theoretical value by no more than a factor of two.

2. Nonlinear susceptibility. By measuring the magnitude of the reflected signal at a given incident power we can determine the reflection coefficient at the cavity. Knowing the quality factor of the cavity, we can then calculate the susceptibility of the sample with respect to the microwave signal.[7] Figure 6a shows experimental results of χ'' versus the pump power h^2 for various values of the drift rate $\dot{H}$. These curves show that the maximum value of the susceptibility, $\chi''_{\max}$, increases as the drift rate becomes more negative, reaching $\sim 1.3\chi''^{0}_{\max}$ at $\dot{H} \approx 1$ Oe/μs. Here we can also clearly see the effect of a change in the hardness of the excitation of the system of parametric spin waves; the effect is manifested as a change in the steepness of the initial part of the $\chi''(h^2)$ curve. Figure 6b shows measurements of the real part of the steady-state microwave susceptibility, χ', versus the same experimental parameters.

3. Transients. A study of the transients under drift conditions yields further information on the nature of the above-threshold state. When the drift is turned on rapidly, the derivative $\ddot{H}$ is discontinuous; this discontinuity may complicate the transients. We accordingly studied the dependence $N(\dot{H})$ as the drift rate was varied continuously from zero to some given value. It is sufficient here, for example, to impose a drift which has the harmonic behavior $\delta H(t) = H_m \cos(\Omega_m t)$ with a modulation frequency $\Omega_m \ll \gamma$, where γ is the spin-wave damping. The initial region will correspond to the imposed drift. The experimental procedure remains the same, except that the current from a standard signal generator is passed through the modulation coil inside the resonator.

The inset in Fig. 7 shows an oscillogram on which one trace shows the signal proportional to the sinusoidal modulation field, while the second trace, synchronized with the first, shows the signal reflected from the pump resonator, i.e., a signal proportional to the amplitude of the parametric spin waves. The pronounced asymmetry of the reflected signal is evidence of a minimum on the $N(\dot{H})$ curve at $\dot{H} < 0$, while the asymmetry with respect to the points $\dot{H}_{\max}$ is evidence of hysteresis in this dependence. If the modulation frequency or amplitude is increased, the drift rate attained increases proportionately, and this asymmetry is enhanced. The behavior of the amplitude over one modulation period is not reproduced exactly. Figure 7 shows the reflected signal versus the instantaneous drift rate $\dot{H}$ for $\Omega_m = 2\pi \cdot 2 \cdot 10^3\,\mathrm{s}^{-1}$ and for various values of H_m. As the rate of a negative drift is increased, the amplitude of the parametric spin waves increases rapidly (the peaks) and then drops off to a level close

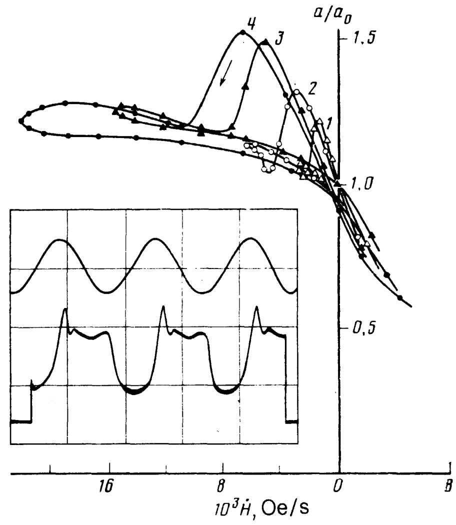

FIG. 7. Relative change in the amplitude of the reflected signal as a function of the instantaneous drift rate in the case of a sinusoidal modulation [$H = H_c - 100$ Oe, $\Omega_m/2\pi = 2$ kHz $(h/h_{c0})^2 = 2.5$ dB]. 1—$H_m = 0.35$ Oe; 2—0.8; 3—1.9; 4—2.7. The inset is an oscillogram of the modulation of the magnetic field and of the signal reflected from the resonator for $H = H_c - 100$ and $\Omega_m/2\pi = 2$ kHz.

to the steady-state level in the case of a sawtooth drift. Figure 5 shows parts of the same curves, whose right-hand slopes are shown in logarithmic coordinates (curves 3–5). We see that curves 3–5, obtained during a time-varying negative drift, lie far from curve 2, which corresponds to a steady-state negative drift and close to curve 1, obtained for a steady-state positive drift. We recall that according to the simple theory [Eq. (1.18)] we would have

$$1-a/a_0 \approx 1-(N/N_0)^2 \sim |\dot{H}|^{1/3}\,\mathrm{sign}(\dot{H})$$

so the curves of the positive and negative drift should coincide in the coordinates of Fig. 5. Again in the case of a sawtooth modulation we observe a peak on the curve of $\chi''(t)$ in the transients just after the pump is turned on. The amplitude of this peak is substantially greater than the steady-state amplitude after the steady state, χ''. These features will be discussed in Sec. 5.

As the static magnetizing field and the initial extent to which the critical value is exceeded for the system of parametric spin waves change, the oscilloscope traces and the corresponding curves in the figures change slightly in shape, but the basic features—the abrupt change in the amplitude and the hysteresis—persist down to fields $H = H_c - 260$ Oe. In weaker fields, these processes are not manifested. The effect of sinusoidal modulation on the parametric excitation of spin waves has also been studied in detail previously,[8–10] and this effect is presently used quite frequently as a sensitive and precise method for determining the threshold for a parametric instability, the relaxation parameters, the parametric spin wave interaction constants, and so forth.[9,10] The effects described in this section of the paper were not examined in those previous studies since the modulation frequencies in those experiments were at least two orders of magnitude higher than the modulation frequency in our experiment. At such high frequencies, the time variation of the excitation suppresses all other factors, and the distinctive shape of the response of the system of parametric spin waves to modulation, expressed as a distortion of its shape, disappears.

4. Self-oscillations of the magnetization. The effect of a drift of the magnetic field on self-oscillations in a system of parametric spin waves was studied by magnetizing an yttrium iron garnet crystal along the ⟨111⟩ axis. The self-oscillations are known to be particularly intense in this case, because the system is unstable with respect to the excitation of a spatially uniform zeroth axial mode of collective oscillations of the parametric spin wave system.[11] This series of experiments revealed that inducing a positive drift of the field does not alter the nature of self-oscillations, which initially arise—as in the absence of a drift, at a comparatively slight interval above the critical value, on the order of 0.5 dB—as oscillations with a regular harmonic shape. As the pump power is increased further, this regularity is quickly disrupted, and the oscillations become stochastic. The only effect of a positive drift in this case is that these characteristic changes occur at slightly higher pump levels. This effect can be explained in a natural way in terms of a change in the functional dependence $N(h)$.

A negative drift affects the stability in a different way. Figure 8 shows a series of oscilloscope traces of the reflected pulse which were observed in succession on the screen of an oscilloscope as the drift rate $\dot{H}$ was increased; the drift in this case was turned on for a certain time interval during the pump pulse. These traces show that the average level of excitation of the parametric spin waves depends on $\dot{H}$ in the same way as in the stable situation, shown in Fig. 4. We also see that in the present case the fact of the stability and the nature of the self-oscillations depend on $\dot{H}$. For example, on a trace with a drift rate $\dot{H}_3$, despite the high average excitation level, the random self-oscillations convert into regular oscillations. On the trace with $\dot{H}_4$, with the same average steady-state level of parametric spin waves, there are no self-oscillations for the duration of the negative drift. The oscillations are excited to a significant amplitude in all other time intervals with $\dot{H} = 0$. Accordingly, for the case $\mathbf{M}\|\langle 111\rangle$ the states which are stable with respect to self-oscillations are those which correspond in Fig. 4 to the left-hand slope of the plot of the signal amplitude versus the drift rate, while the unstable states are those with points on the right-hand slope of this dependence, including the region $\dot{H} > 0$.

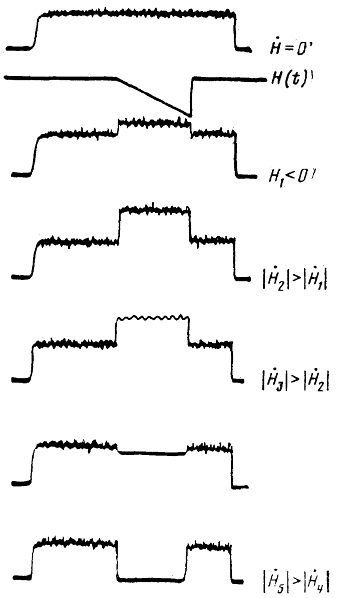

FIG. 8. Series of oscilloscope traces as the magnetic-field drift rate is successively increased with $\mathbf{M}\|\langle 111\rangle$. The second trace from the bottom corresponds to $|\dot{H}_4| > |\dot{H}_3|$.

A change in the stability of the system of parametric spin waves also occurs in the case $\mathbf{M}\|\langle 100\rangle$. Here we observe the opposite transition: As the drift rate becomes more negative, the stable state gives way to an unstable state, and self-oscillations appear. Although the amplitude of these oscillations is considerably lower than in the case $\mathbf{M}\|\langle 111\rangle$, they can be reliably detected directly on the oscilloscope screen. The change in stability occurs immediately after the $N(\dot{H})$ maximum is reached. As the drift rate becomes more negative, and the signal amplitude decreases to a value below $N(0)$, the self-oscillations again disappear. The region in which they exist is shown by the arrows in Fig. 4.

5. DISCUSSION OF RESULTS

Most of the experimental results described in the Secs. 3 and 4 agree qualitatively with the theoretical predictions of Ref. 4, and in the case of the positive drift there is also a good quantitative agreement. In the case of a negative drift, the situation is extremely complicated. On the one hand, we see the hardness of the excitation and the hysteresis which are predicted by the theory, and we also see the increase in the amplitude of the parametric spin waves with increasing $|\dot{H}|$ at low drift rates. On the other hand, as $|\dot{H}|$ is increased further the amplitude of the parametric spin waves decreases, in contradiction of the predictions.[4] We also see a peak whose duration is significantly longer (by a factor ~ 10) than the relaxation time of the parametric spin waves, and its amplitude increases monotonically with increasing $|\dot{H}|$. The qualitative behavior of the collective oscillations agrees entirely with the theoretical predictions, except in a narrow interval of negative drift rates, where small self-oscillations are observed (Sec. 4, Subsec. 4).

It is natural to suggest that in the case of a negative drift we are going beyond the range of applicability of the theory of Ref. 4. Specifically, the S-theory equations which were used in Ref. 4 were derived in assuming a self-consistent field, which ignores the scattering of parametric spin waves by one another. However, it has been established quite solidly[12,13] that in the absence of a drift such a scattering would not determine the shape of the packet, although it would have a slight effect on the total number of parametric spin waves, N. In particular, the distribution function of the parametric spin waves in k space would be the square of a Lorentzian function,[12]

$$n_k = \frac{32}{3\pi^2} \frac{\gamma^2 |\Delta S|^2 N^3}{k^3[(\omega_k - \omega_p/2)^2 + \nu_k^2]^2} \delta\left(\theta - \frac{\theta}{2}\right), \tag{5.1}$$

where $\Delta S = S_2 - S_{-2}$, and S_m is the axial Fourier harmonic of index m of matrix element $S_{\mathbf{k},\mathbf{k}'}$. The width of the packet in k space, Δk, is

$$\Delta k = \nu_k / v \tag{5.2}$$

(v is the group velocity of the parametric spin waves). The quantity $\nu_{\mathbf{k}}$—the width of the packet in eigenfrequencies $\omega_{\mathbf{k}}$, set by the scattering of parametric spin waves by each other—is[13]

$$\nu_{\mathbf{k}} \approx \left[\frac{16}{3\pi} \frac{\gamma^2 |\Delta S|^2 N^2}{kv}\right]^{1/3}. \tag{5.3}$$

At $H = H_c - 100$ Oe, $\mathbf{M} \| \langle 100 \rangle$, we thus find

$$\nu_k \approx {}^1/_4 \gamma (|hV|^2/\gamma^2 - 1)^{1/3}. \tag{5.4}$$

It is not difficult to see that the discrepancies with the theory at low drift rates which were pointed out in Secs. 3 and 4 occur in the case in which the deviation of a packet drifting in k space from the resonant frequency, $\Delta\omega_{\mathbf{k}}$, given by (1.12), turns out to be smaller than or on the order of the width of the packet of parametric spin waves, $\nu_{\mathbf{k}}$, given by (5.4). Under our experimental conditions, $\Delta\omega_{\mathbf{k}}$ in (1.2) is twice $\nu_{\mathbf{k}}$ in (5.4) at $\dot{H} \approx 5 \cdot 10^3$ Oe/s. As a result, in the case $\Delta\omega_{\mathbf{k}} \lesssim \nu_{\mathbf{k}}$ the dependence of the number of parametric spin waves, N, on drift rate (1.18) becomes smoothed over.

In addition to the change in the shape of the central peak of the packet, the scattering of parametric spin waves by one another leads to yet another, subtler, effect: Under drift conditions, the thermal noise grows exponentially, reaching the range of the parametric instability $|\omega_{\mathbf{k}} - \omega_p/2| < \Delta\omega_{\mathbf{k}}$, which largely determines the shape of the spin-wave packet. The scattering of the spin waves, however, may cause the tail of an intense packet of spin waves to exceed the thermal noise at the boundary of the instability region. We can estimate the ratio of the intrinsic noise to the thermal noise at this boundary:

$$\frac{n_k}{n_k^0} \approx \left(\frac{\gamma}{2\Delta\omega_{\mathbf{k}}}\right)^4 \frac{\gamma}{|S| n_{\mathbf{k}}^0 k^3} \left|\frac{\Delta S}{S}\right|^2 \left(\frac{|hV|^2}{\gamma^2} - 1\right)^{3/2}. \tag{5.5}$$

We can find the particular drift rate at which the intrinsic noise becomes comparable to the thermal noise. According to (5.5), (1.12), and (1.18), this event occurs at

$$\dot{H} = \dot{H}_* \approx \frac{\gamma^2}{6g} \left(\ln \left|\frac{\gamma}{SN_T}\right|\right)^{-1} \left|\frac{\Delta S}{S}\right|^{3/2} \left(\frac{\gamma}{|S| n_{\mathbf{k}}^0 k^3}\right)^{3/4} \left|\frac{SN}{\gamma}\right|^{9/4}. \tag{5.6}$$

At a point 2.5 dB above the threshold ($|hV|^2/\gamma^2 \approx 2$) we have $\dot{H}_* \approx 2.5 \cdot 10^4$ Oe/s. For this drift rate we find $\Delta\omega_{\mathbf{k}} \approx \gamma_{\mathbf{k}}$ from (5.5). At lower drift velocities, the intrinsic noise at the boundary of the instability region exceeds the thermal noise. In this case, in estimating the effect of the intrinsic noise we should replace the thermal noise in (1.18) by the sum of the thermal and intrinsic noises. As a result, the total number of parametric spin waves becomes smaller than those given in Ref. 4. At higher drift rates, the deviation of the center of the spin-wave packet from resonance exceeds the damping: $\Delta\omega_{\mathbf{k}} > \gamma_{\mathbf{k}}$. As was shown in Ref. 4, in this case the spin-wave width caused by the drift becomes comparable to the deviation from resonance, $\Delta\omega_{\mathbf{k}}$. The intrinsic noise is proportional to the convolution of three functions $n_{\mathbf{k}}$ (Ref. 12), so it has a characteristic width $3\Delta\omega_{\mathbf{k}}$. Consequently, for $\Delta\omega \gtrsim \gamma_{\mathbf{k}}$ it is large in comparison with the thermal noise in the instability region. The approximation of a self-consistent field thus gives a poor description of the steady state of parametric spin waves both in the case of a slow negative drift ($\Delta\omega_{\mathbf{k}} < \gamma_{\mathbf{k}}$) and in the case of a fast negative drift ($\Delta\omega_{\mathbf{k}} > \gamma_{\mathbf{k}}$). However, the initial stage of the parametric instability in the case of a negative drift (the peak) is described correctly in the self-consistent-field approximation[4] (Sec. 4). We believe that the peak appears because the self-consistent field approximation incorporates the strongest mechanisms for the interaction of parametric spin waves,[1,2] so in the first stage of the onset of the parametric instability one can ignore the mutual scattering of the spin waves. However, the state which arises is found to be unstable with respect to frequency broadening of the packet because of this scattering, and over a time on the order of $\tau \sim \gamma/\nu^2$ (Ref. 13) the steady state observed experimentally is established. In the case of a sinusoidal drift, the onset of this instability cannot compensate for the increase in the amplitude of the parametric spin waves due to the constantly increasing drift rate in the case of an increasing negative drift rate $|\dot{H}|$ ($\dot{H} < 0$). Accordingly, the peak height in the case of a sinusoidal drift is described considerably better by the theory of Ref. 4 than in the case of sawtooth drift.

In the case of a positive drift, at values of $\dot{H}$ satisfying $\Delta\omega_{\mathbf{k}} > \nu_{\mathbf{k}}$, the amplitude of the parametric spin waves falls off sharply according to (1.18). The intensity of the intrinsic noise varies as N^3, so the effects of the scattering of the spin waves by each other are inconsequential. Accordingly, the behavior of the parametric spin waves observed experimentally with a drifting magnetic field can be explained in a natural way on the basis of the existing theoretical understanding.[4,12]

It is a pleasure to express our gratitude to G. A. Melkov for useful discussions of this study.

[1]V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys. Usp. **17**, 896 (1974)].
[2]L. A. Prozorova and A. I. Smirnov, Zh. Eksp. Teor. Fiz. **74**, 1554 (1978) [Sov. Phys. JETP **47**, 812 (1978)].
[3]V. S. Zhirnyuk, G. A. Melkov, and V. B. Cherepanov, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 149 (1980) [JETP Lett. **32**, 137 (1980)].
[4]E. V. Podivilov and V. B. Cherepanov, Zh. Eksp. Teor. Fiz. **90**, 767 (1986) [Sov. Phys. JETP **63**, 447 (1986)].
[5]V. B. Cherepanov, Zh. Eksp. Teor. Fiz. **91**, 235 (1986) [Sov. Phys. JETP **64**, 136 (1986)].
[6]V. V. Zautkin, B. I. Orel, and V. B. Cherepanov, Zh. Eksp. Teor. Fiz. **85**, 708 (1983) [Sov. Phys. JETP **58**, 414 (1983)].
[7]A. G. Gurevich, *Magnitnyĭ rezonans v ferritakh i antiferromagnetikakh* (*Magnetic Resonances in Ferrites and Antiferromagnets*), Nauka, Moscow, 1973.
[8]V. V. Zautkin, V. S. L'vov, B. I. Orel, and S. S. Starobinets, Zh. Eksp. Teor. Fiz. **72**, 272 (1977) [Sov. Phys. JETP **45**, 143 (1977)].
[9]A. V. Andrienko, V. I. Ozhogin, V. L. Safonov, and A. Yu. Yakubovskiĭ, Zh. Eksp. Teor. Fiz. **84**, 1158 (1983) [Sov. Phys. JETP **57**, 673 (1983)].
[10]A. V. Andrienko, V. I. Ozhogin, V. L. Safonov, and A. Yu. Yakubovskiĭ, Zh. Eksp. Teor. Fiz. **84**, 1474 (1983) [Sov. Phys. JETP **57**, 858 (1983)].
[11]V. S. L'vov, S. L. Musher, and S. S. Starobinets, Zh. Eksp. Teor. Fiz. **64**, 1074 (1973) [Sov. Phys. JETP **37**, 546 (1973)]; V. V. Zautkin and S. S. Starobinets, Fiz. Tverd. Tela (Leningrad) **16**, 678 (1974) [Sov. Phys. Solid State **16**, 446 (1974)].
[12]V. S. L'vov, Zh. Eksp. Teor. Fiz. **69**, 2079 (1975) [Sov. Phys. JETP **42**, 1057 (1975)].
[13]V. S. L'vov and V. B. Cherepanov, Zh. Eksp. Teor. Fiz. **75**, 1114 (1978) [Sov. Phys. JETP **48**, 561 (1978)].

Translated by Dave Parsons