Density of states of two-dimensional electrons in a transverse magnetic field and screening of a random potential

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The density of states of two-dimensional ($2D$) electrons in a transverse magnetic field is investigated by measuring the spectra of the radiative recombination of the $2D$ electrons with nonequilibrium photoexcited holes in silicon metal-insulator-semiconductor (MIS) structures. The structure of the Landau levels is investigated by determining the intervalley and spin splittings, and it is shown that interelectron interactions enhance these splittings by almost an order of magnitude. Oscillations of the peaks of the density of states (DS) on the Landau levels are observed as functions of the filling of the levels by electrons (of the filling factor). For half-integer filling, the width of the DS peak is a minimum and depends on the magnetic field and on the electron mobility in accordance with the theory for short-range random scatterers. For completely filled Landau levels, the DS peak width has a maximum determined by the amplitude of the large-scale fluctuations of the random potential. The amplitude and the linear scale of the fluctuations are determined from the dependence of the widths of the completely filled Landau levels on the magnetic field and on the quantum number. It is shown that in the case of complete level filling the density of states in the gaps of the energy spectrum is not exponentially small, owing to the long-period fluctuations of the potential of the charged centers. The oscillatory behavior of the SD on the Landau levels as a function of the filling factor, and the behavior of the line widths as functions of the temperature, are attributed to the screening of these fluctuations by the electrons. The $2D$-electron DS measured by independent methods (using magnetooptics and thermally activated conductivity) are quantitatively compared. It is shown that in the gaps between the Landau levels these quantities and their dependences on the magnetic field are in good agreement.

§1. INTRODUCTION

The distribution of the density of electron states in the presence of a random potential due to defects is at present a central point in the discussion of the energy spectra of a two-dimensional ($2D$) system in a transverse magnetic field. It is customarily assumed that the density of states of an ideal $2D$ electron layer placed in a transverse quantizing magnetic field constitutes a set of equidistant δ functions with spacing equal to the cyclotron energy $\hbar\omega$. Each Landau level is strongly degenerate, and the multiplicity of this degeneracy (disregarding spin and valley-orbit degeneracy) is equal to $n_0 = 1/2\pi l_H^2$, where $l_H = (\hbar/eH)^{1/2}$ is the magnetic length. A random potential (of an inhomogeneity, of an impurity, etc.) lifts the Landau-level degeneracy; this is the cause of their finite width Γ. If this width is related to the short-range potential of the scatterers, the density of states on the Landau-level wings decreases like

$$D(E) \sim \exp[-(E/\Gamma)^2]$$

and if

$$\hbar\omega_c \gg \Gamma = \hbar\omega_c (\pi\mu H/2)^{-1/2}$$

it should be vanishingly small in the gaps of the energy spectrum.[1] On the basis of recent determinations of the electron states on the Fermi level by measuring the oscillatory dependences of the magnetization,[2] of the electronic specific heat,[3] and also the thermoactivation conductivity,[4,5] it was concluded that the density of states between the Landau levels is exponentially small and is an appreciable fraction of D at $H = 0$. This conclusion contradicts directly the results of Ref. 1.

It was recently proposed to determine the density of $2D$ electron states in a transverse magnetic field by a method based on measurements of the luminescence spectra connected with radiative recombination of $2D$ electrons with nonequilibrium photoexcited holes in Si(100) MIS structures.[7,8] In this method one measures the energy distribution of the density of states $D(E)$ at a fixed filling of the Landau level by electrons, Refs. 3 and 4, as well as 8 and 9, deal with the density of states dn/dE_F on the Fermi level, called also the thermodynamic density of states (TDS). When account is taken of the electron-electron interaction and of the ensuing screening of the random potential, these state densities differ.[10] It is possible to track, by the spectroscopic method, the variation of the energy distribution of the density of states as a function of the filling factor $\nu = n_S h/eH$ (n_S is the electron density). We have reported in a short letter[7] that the width of the DS on a Landau level oscillates when the filling factor is varied, and it was suggested that these oscillations are due to screening of the random potential. We show in the present paper that the oscillations of the energy with change of ν are due to the screening of the large-scale fluctuations of the potential of the charged centers in the insulator SiO_2 (§5). From the dependences of the width s of the completely filled Landau levels on the magnetic field and on the quantum number (N), we determine the amplitude and the linear scale of the long-period fluctuations of the random potential (§7) and show that the presence of just these fluctuations is the cause of the nonexponential smallness of the

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DS in the gaps between the Landau levels when the latter are completely filled (§8). We investigate also the structure of the Landau levels, viz., we determine the spin and intervalley splittings and find that these splittings oscillate strongly as functions of the filling factor; these oscillations are explained in terms of the electron-electron interaction. Finally, we study the influence of a short-range potential on the widths of the half-filled Landau levels, when the large-scale fluctuations are screened (§6). The results of our experiments are compared with the theory, recently developed in Ref. 11, of nonlinear screening of a random potential of charged impurities.[11]

§2. EXPERIMENTAL TECHNIQUE AND MIS STRUCTURES

We used in the investigations several MIS transistors produced on the (001) surface of boron-doped silicon. The structures contained a semitransparent gate of 0.3 to 4 mm^2 area for the passage of an LG-106 argon laser beam that generated nonequilibrium electron-hole (e-h) pairs near the Si–SiO_2 interface. The characteristic power density used in our experiment was always about 10^{-3} W/cm^2, since the dependence of the ratio of the integrated radiation intensity of the 2D electrons to the bulk recombination has a sharp maximum at this power.

All the experiments were performed with the structures placed in an optical cryostat equipped with a solenoid that permitted operation in magnetic fields up to 8.5 T at $T = 1.5$ K. The radiation spectra were measured in a direction perpendicular to H (Voigt geometry). The sample mountings could be rotated to vary and control the angle between the normal to the interface and the magnetic field. The spectral instrument used was a DFS double monochromator providing a sufficiently high spectral purity and free range (~ 10 Å/mm) in the operating region. In view of its extremely low intensity, the investigated recombination radiation was recorded with a photon-counting system and a photomultiplier (S-1 cathode cooled to 77 K) followed by signal storage with a multichannel analyzer, thereby improving substantially the signal/noise ratio. The spectral sensitivity of the photomultiplier in the measured radiative-recombination band varied monotonically and decreased on the long-wave edges of the spectra (by about 40% at an approximate bandwidth 10 meV). A computer was used for the reduction of all the spectra as well as to account for the photomultiplier sensitivity.

It is important to emphasize that the spectroscopic and magneto-transport measurements of the MIS transistors were performed simultaneously. This made it possible to determine and compare important structure parameters (n_S, μ, N_A) and physical quantities obtained by independent methods. Thus, for example, we compare in §8 the absolute values of the density of states in the gaps btween the Landau levels, obtained by optical spectroscopy and from an analysis of thermally activated magnetoconductivity.

We investigated five MIS transistors with semitransparent gates, produced on the (001) surface of boron-doped silicon. Table I lists the structure parameters: the maximum 2D-electron mobility and $T = 1.5$ K; the density N_S^* at which the maximum mobility is realized; the oxide-layer thickness d_{SiO_2}; the density n_+ of the positive charge in the oxide, obtained from the dependence of the threshold voltage on the substrate bias voltage[12]; and the structure geometry—rectangular (R) with five potential contacts or annular (C, Corbino disks). The table lists also the parameters that describe the amplitude (ΔE) and the linear scale (d) of the long-period fluctuations of the random defect potential, as found by optical spectroscopy for each of the structures. The method of determining ΔE, Q, and d and a discussion of the obtained values are given in §7. The magneto-transport measurements were made both with direct and with 19-Hz alternating current. The measurement current and voltage, 10^{-7} A/cm and 10^{-3} V/cm were chosen to prevent overheating of the 2D-electron system. The remaining details of the experiment can be found in Ref. 12.

§3. SPECTRA OF RADIATIVE RECOMBINATION OF 2D ELECTRONS AND NONEQUILIBRIUM HOLES

It is known that optical-spectroscopy method determines most directly the energy distribution of the density of single-particle states, including those under the Fermi surface. This is the primary difference from the other experimental procedures used to determine the DS of 2D electrons. Indeed, methods based on the study of the magnetization,[2] of the electronic specific heat,[3] of the magnetocapacitance,[8] of thermally activated conductivity,[4] and others claim to yield a value of dn_S/dE_F, measured on the Fermi surface, different from $D(E_F)$. All this makes it essential to obtain information from optical measurements.

As reported in Ref. 12, there is practically no depletion layer under photoexcitation conditions, and the 2D-electron system turns out to be in the immediate vicinity of photoexcited carriers, this being the cause of radiation due to recombination of 2D electrons with nonequilibrium holes. In the case considered, the recombination (line $2D_e$ in Fig. 1) is not direct in either momentum or coordinate space, and its intensity is weaker by 2–3 orders than the corresponding volume recombination lines (line BE of Fig. 1 corresponds to recombination of excitons bound on boron atoms). With-

TABLE I.

№	μ^* $m^2/V\cdot s$	n^*, 10^{11} cm^{-2}	ΔE, meV	Q, meV	d^*, Å	d^{**}, Å	N_A, 10^{14} cm^{-3}	n_+, 10^{12}	d_{SiO_2}, Å	Geometry
1	3.2	3.0	3.0	3.5	350	400	7.0	2.0	1300	C
2	2.6	4.0	3.2	3.5	320	350	9.2	2.5	1350	C
3	1.8	7.2	3.2	3.7	270	290	7.0	3.1	1300	C
4	1.3	9.0	3.6	4.0	230	250	8.3	4.1	1900	R
5	0.6	9.2	5.3	–	–	–	7.3	4.3	1950	R

Note. The value of d^* was obtained from the $\Gamma(H, N)$ dependence using (5). The value of d^{**} was obtained from the intersection of the $\Gamma_{min}(N)$ and $\Gamma_{max}(N)$ plots. No well resolved Landau-level emission spectrum pattern was obtained for sample No. 3 in a magnetic field $H \leqslant 8$ T, so that the parameters Q and d could not be determined.

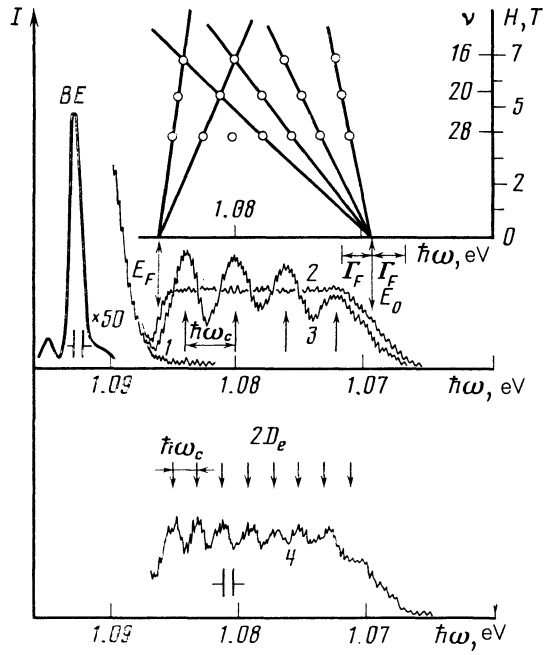

FIG. 1. Spectra of radiative recombination of $2D$ electrons with photoexcited holes ($2D_e$ lines), measured (for sample No. 1) at $W = 10^{-3}$ W/cm, $T = 1.6$ K, $n_S = 2.7\cdot 10^{12}$ cm^{-2} in magnetic fields $H = 0$ (spectrum 2), $H = 7$ and perpendicular to the $2D$ layer (spectrum 3, $\nu = 16$, $N = 4$) and $H = 7$ T and the field making an angle 60° with the normal (spectrum 4, $\nu = 32$, $N = 8$). Spectrum 1 was obtained under conditions when $n_S = 0$ ($V_g < V_T$). The line BE corresponds to emission of excited bounds on boron atoms; $\hbar\omega_c$ is the cyclotron splitting. In the upper part of the figure is shown the fan of Landau levels (spectral position of the lines) constructed at $n_S = 2.7\cdot 10^{12}$ cm^{-3} under condition of complete filling of four, five, and seven levels. Extrapolations to $H \to 0$ determine the position of the bottom of the size-quantized band E_0 and of the Fermi energy E_F.

out repeating the details of the recombination mechanism, which is described in detail in Ref. 12, we list the main properties of the line of radiative recombination of $2D$ electrons with photoexcited holes in the absence of a magnetic field.

1. Under laser illumination and at gate voltages $V_g > V_T$ (V_T is the threshold voltage), a new line, $2D_e$, appears in the radiation spectrum (Fig. 1., spectrum 2).

2. The integrated intensity of the $2D_e$ line is weaker by 2–3 orders than that for bulk recombination (see Fig. 1).

3. The $2D_e$ line shape is a step function of the energy (spectrum 2 of Fig. 1), in accord with the fact that at $H = 0$ the density of states of the $2D$ electrons is independent of E.

4. The width of the $2D_e$ line increases linearly with increase of the electron density n_S; this yields a mass $m_d \approx 0.2m_0$ for the density of states.

5. The violet boundary of the $2D_e$ spectrum is practically independent of V_g and is located near 1.0865 eV at an excitation power $W \approx 10^{-3}$ W/cm^2.

6. The recombination of the $2D$ elecrons with the photoexcited holes proceeds with participation of *TO* and *TA* phonons, and is observed in the corresponding spectrum components. The recombination probability is therefore independent of the energies of the recombining particles (the matrix elements M_{TO} and M_{TA} of the transitions are constant).

7. Extremely weak radiation of the $2D_e$ line is observed also in the zero-phonon region of the spectrum and is due to transfer of a near-Brillouin momentum to the impurity center or to the surface.

8. The emission spectrum is a convolution of the distribution functions of the $2D$ electrons and of nonequilibrium holes, and the width of the hole distribution does not exceed 8 K.

9. The TO and TA phonon components of the $2D_e$ line are polarized in the $2D$ plane, because only two out of the six electron valleys on the [001] axes are populated by $2D$ electrons.

10. The characteristic recombination time of the $2D$ electrons is 10^{-3} s.

11. If the $2D$ electrons are strongly localized on fluctuations of the random potential (region of very low densities n_S) the $2D_e$ line is strongly broadened and its width is a measure of the amplitude of the random-potential fluctuations.

12. When n_S is increased and the $2D_e$-electron system becomes conducting and the $2D_e$ line becomes very narrow and polarized, in view of the screening of the random-potential fluctuations.

We have previously reported[7] an investigation of the properties of the $2D_e$ line and established the following facts:

1. Application of a magnetic field strictly perpendicular to the $2D$ layer causes the $2D_e$ line to split in accordance with the number of filled Landau levels. Thus, at $n_S = 2.7\cdot 10^{12}$ cm^{-2} and $H = 7$ T, when four Landau levels are completely filled, the spectrum reveals four equidistant lines separated by the cyclotron energy $\hbar\omega_c = 4$ meV (see spectrum 3 of Fig. 1).

2. To demonstrate the two-dimensional character of the electron system, we used the method of deflecting the magnetic field from the normal towards the $2D$ layer. Thus, at an angle $\theta = 60°$, $n_S = 2.7\cdot 10^{12}$ cm^{-2}, and $H = 7$ T the filling factor was exactly doubled and eight Landau levels appeared below the Fermi surface. The spectrum revealed in this case eight lines at half the spacing. At $H = 7$ T but with the $2D$ layer parallel ($H \| [010]$), no Landau-level picture is observed.

3. The Landau-level "fan" corresponding to the positions of the radiation lines measured at $n_S = \text{const}$ in different magnetic fields and under conditions of integer filling, makes it possible to determine the position of the size-quantization level E_0 and the Fermi energy (see the upper part of Fig. 1.

4. At $T = 1.5$ K, $2D$-electron radiation in a strong transverse magnetic is observed only for electrons with spin $S_z = +\frac{1}{2}$ (i.e., at $2 + 4M < \nu < 4 + 4M$, where M is an integer).

5. The *TO*-phonon components of a $2D_e$ line in a transverse magnetic field is practically completely polarized in the H direction.

6. According to the selection rules of Table II and also the spectrum properties noted in items 4 and 5, it follows that at $T = 1.6$ K the $2D$ electrons recombine with nonequilibrium holes having a projection $J_z = -\frac{3}{2}$. Optical transitions for electrons with $S_z = -\frac{1}{2}$ are forbidden.

7. The Landau-level width oscillates strongly when ν is varied.

Figure 2 shows the variation of the recombination spectral line positions with change of the density n_S or of the filling factor ν. It shows also the measured dependences (dark squares) of the position of the bottom of the band E_0 and of the Fermi energy E_F with change of n_S. The figure

TABLE II. Selection rules for indirect transitions in Si($\Gamma_8^+ - \Delta_6$).

Phonons	J_z, S_z						
	$^3/_2$; $-^1/_2$	$-^3/_2$; $^1/_2$	$^1/_2$; $^1/_2$	$-^1/_2$; $-^1/_2$	$^1/_2$; $-^1/_2$	$-^1/_2$; $^1/_2$	$-^3/_2$; $-^1/_2$ $+^3/_2$; $+^1/_2$
LA	$-\gamma e_+ u_{LA}$	$-\gamma e_- u_{LA}$	$\frac{\gamma}{\sqrt{3}} e_+ u_{LA}$	$\frac{\gamma}{\sqrt{3}} e_- u_{LA}$	0	0	0
LO	$-\eta e_- u_{LO}$	$\eta e_+ u_{LO}$	$\frac{1}{\sqrt{3}} \eta e_- u_{LO}$	$-\frac{\eta}{\sqrt{3}} e_+ u_{LO}$	$\sqrt{2/3}\lambda e_z u_{LO}$	$-\sqrt{2/3}\lambda e_z u_{LO}$	0
TO, TA	$-\alpha e_z u_-$	$\alpha e_z u_+$	$\frac{\alpha}{\sqrt{3}} e_z u_-$	$-\frac{\alpha}{\sqrt{3}} e_z u_+$	$-\sqrt{2/3}\beta \cdot (e_+ u_- + e_- u_+)$	$\sqrt{2/3}\beta \cdot (e_+ u_- + e_- u_+)$	0

Note. The z axis is parallel to H. J_z and S_z are the projections of the angular momenta of the holes ($J_z = \pm 3/2, \pm 1/2$) and electrons ($s_z = \pm 1/2$), u_i are the displacement amplitudes; $\alpha, \beta, \eta, \lambda, \gamma$—constants, e—polarization vector, $e_\pm = e_x + ie_y$ (Ref. 30).

shows, first of all, that a series of Landau levels spaced $\hbar\omega_c$ apart is observed in a transverse field $H = 7$ T. The spectral positions of the lines corresponding to the Landau levels in the interior below the Fermi surface are linear in n_S and correspond to the change in the position $E_0(n_S)$ of the bottom of the band. Two radiation lines corresponding to recombination of $2D$ electrons from different valleys are observed near the Fermi surface in the region of odd integer ν (they are shown in detail in the inset of Fig. 2; see also §4). It is of interest to investigate how the behavior of the spectral positions of the recombination lines are affected by the Hall-resistance quantization[13] due to the fact that at integer values of ν the Fermi level turns out to be in the region of localized states in a gap between Landau levels. It was noted in Ref. 14 that, at integer ν, pinning of the spectral position of the recombination lines is observed in quantum wells of GaAs–AlGaAs heterostructures. As seen from Fig. 2, in our experiment the pinning of the spectral position of the $2D_e$ line is observed for half-integer ν, and not at all for integer

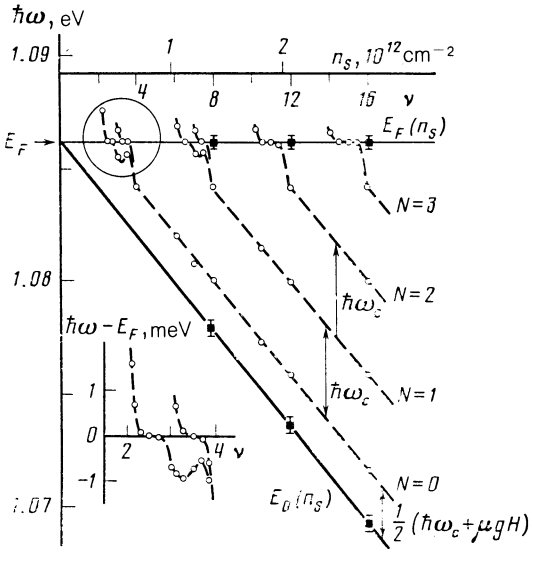

FIG. 2. Spectral locations of the $2D$-electron radiation lines, measured for sample No. 3 at $W = 10^{-3}$ W/cm^2. $T = 1.6$ K and $H = 7$ T, as functions of the density n_S. The dark squares correspond to the energy position of the Fermi energy E_F and of the bottom of the band E_0, both measured for different n_S according to the Landau-level fan, under conditions of complete filling. The region $2 < \nu < 4$ is shown in a larger scale in the inset. The observed line splitting is due to the non-equivalence of the electron valleys.

filling. These facts can be readily attributed to the low DS on the Fermi level at integer ν, and to the fact that E_F varies then strongly with n_S, whereas at half-integer ν the value of $D(E_F)$ is large and the changes of E_F are small.

It is seen even from the indicated properties of the radiative recombination of $2D$ electrons with photoexcited holes that the optical-spectroscopy method uncovers new possibilities of studying the energy spectrum of a system of $2D$ electrons and of its variation with the magnetic field, with the Landau-level filling factor, with the temperature, and with the quality of the structure. In the next section we demonstrate the use of the method to determine the spin and intervalley splittings.

§4. STRUCTURE OF LANDAU LEVELS

Spin and intervalley splittings

We know that the electron energy spectrum is fourfold degenerate in a system of $2D$ electrons on the (001) surface of silicon, owing to the presence of two equivalent electron valleys and of spin. It was observed in the very earliest experiments,[15] however, that this degeneracy is lifted in a perpendicular magnetic field and that the energy spectrum of the $2D$ electrons becomes fully discrete. This is manifested, for example, in Shubnikov–de Haas oscillations where, as each of the four Landau sublevels is filled (i.e., at $\nu = 1, 2, 3, 4$), magnetoresistance minima are observed, due to the fact that the Fermi level lands in the region of localized states in gaps between quantum sublevels. Various methods were used to determine the spin and intervalley gaps, based on the study of the singularities of the magnetoconductivity in oblique field H (Refs. 16–19), of the thermally activated magnetoconductivity,[20] and of the contact potential difference.[21] In all the indicated studies, however, the splitting was measured indirectly and fitting parameters and assumptions requiring special corroboration were used. The use of optical-spectroscopy method permits direct measurements of the splittings regarded as the energy intervals between the corresponding lines in the radiative-recombination spectra (in analogy with the measurement of the cyclotron splitting, see Fig. 1).

a) Intervalley splitting ΔE_V. Intervalley splitting was calculated in Refs. 22–26 without resorting to interelectron (e-e) interaction effects, and the obtained value of ΔE_V^0 was determined by the gate field:

$$\Delta E_V^0 = \alpha(\partial V/\partial z) \sim n_S. \qquad (1)$$

where $\alpha = \text{const}$ and $(\partial V/\partial z)$ is the gradient of the potential. Splitting in these theories is of electrostatic origin and occurs because two electron valleys are not equivalent with respect to the electric-field direction. In Refs. 27 and 28, on the other hand, it was noted that intervalley splitting in a perpendicular magnetic field should increase considerably on account of the contribution ΔE_V^* from the interaction of the 2*D* electrons. We have thus for the total intervalley splitting (ΔE_V)

$$\Delta E_V = \Delta E_V^0 + \Delta E_V^*. \qquad (2)$$

It is important that the contributions ΔE_V^0 and ΔE_V^* have different dependences on H, n_S and ν. The change of the radiative-recombination spectra as the electron valleys become filled was reported in Ref. 29. At $2 < \nu < 3$ the radiation spectrum shows one line due to recombination of 2*D* electrons from a lower electron valley, having a spin $+\frac{1}{2}$ and belonging to the ground Landau level $N = 0$. At $\nu > 3$, filling of a valley with $S_z = +\frac{1}{2}$ begins and one more line appears in the radiation spectrum, with a relative integrated intensity corresponding to the deviation of ν from 3. The energy interval between the lines corresponds to the intervalley splitting ΔE_V. The value of ΔE_V depends quite strongly on ν, decreasing with filling of the succeeding Landau level.[29] The maximum of ΔE_V is reached as $\nu \to 3$ and amounts to ≈ 1.5 meV, substantially higher than the values determined in other experiments[19,21] and calculated within the framework of theories[22–26] in which no account is taken of *e-e* interaction. At the same time, the $\Delta E_V(\nu)$ dependence observed in our experiment, as well as the absolute value of ΔE_V as $\nu \to 3$, agree splendidly with the prediction of the theories of Refs. 27 and 28, according to which the enhancement of ΔE_V^* is due to the fact that different-valley electrons having different quantum numbers are not prevented by the Pauli principle from approaching one another. A detailed investigation of the dependence of the intervalley splitting on the parameters ν, H, N, and n_S was published in Ref. 29.

b) Spin splitting ΔE_S. As noted above, at low temperatures and in strong magnetic fields, the recombination spectrum has only one 2*D*-electron spin component with projection $S_z = +\frac{1}{2}$ and there is no radiation from electrons with $S_z = -\frac{1}{2}$. The reason is that the Zeeman effect splits the hole level into a quartet, and at low temperatures all the nonequilibrium holes are in a ground state wth $J_z = -\frac{3}{2}$ during their lifetime. Optical transitions with participation of electrons having $S_z = -\frac{1}{2}$ and of holes with $J_z = -\frac{3}{2}$ are forbidden by the selection rules (see Table II), and no corresponding radiation is observed. Observation of recombination of 2*D* electrons with $S_z = -\frac{1}{2}$ (and hence determination of the spin splitting $\Delta E_S \equiv \mu_B g_e H$) requires the appearance of a noticeable fraction of holes with $J_z = -\frac{1}{2}$, and then, according to the selection rules, additional radiation should appear in the same π polarization. Since the density ratio of the holes with $J_z = -\frac{3}{2}(n_{-3/2})$ and $J_z = -\frac{1}{2}(n_{-1/2})$ under equilibrium condition is equal to

$$(n_{-3/2})/(n_{-1/2}) \sim \exp(-\mu_B g_h H/kT), \qquad (3)$$

where μ_B is the Bohr magneton and g_h is the *g* factor of the nonequilibrium holes, it follows that to observe recombinations of electrons with $S_z = -\frac{1}{2}$ it is necessary to decrease the ratio H/T.

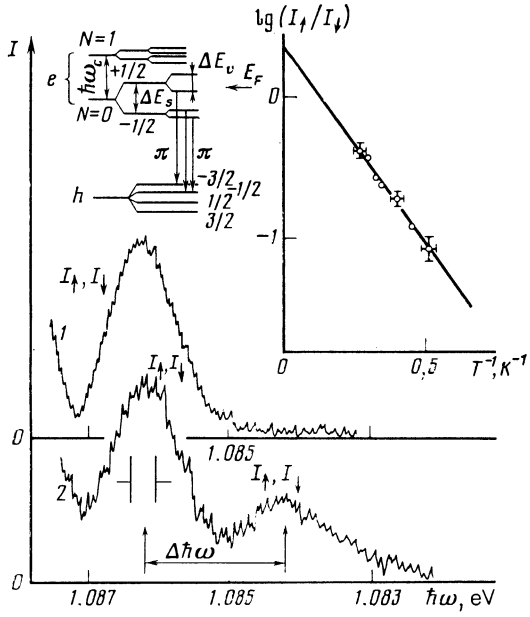

FIG. 3. Spectra of radiative recombination of *D* electrons, measured for sample No. 2 at $H = 6$ T, $\nu = 2.5$, $W = 10^{-3}$ W/cm^2 and at temperatures 1.6 K (spectrum 1) and 3.3. K (spectrum 2). The value of $\Delta\hbar\omega$ is determined by the spin splitting ΔE_S ($g_e \approx 8.6$). The diagram shows the level splittings of the electrons and holes in a magnetic field and the possible optical transition. The inset shows the temperature dependence of the intensity ratio of lines $I_\uparrow$ and $I_\downarrow$.

As seen from Fig. 3, at $H = 6$ T, $\nu = 2.5$ and $T = 1.6$ K one observes in the radiation spectrum only one line corresponding to recombination of 2*D* electrons from the lower valley and having $S_z = +\frac{1}{2}$ with hole having $J_z = -3/2$. When the temperature is raised to 3.3 K, an additional radiation line appears in the low-energy region of the spectrum and is due to interaction between electrons having $S_z = -\frac{1}{2}$ and holes having $J_z = -\frac{1}{2}$ (spectrum 2 of Fig. 3). It is easily seen from the scheme shown in Fig. 3 that the energy interval between the lines ($\Delta\hbar\omega$) is determined by the spin splitting:

$$\Delta\hbar\omega = \mu_B g_e H - {}^1/_2 \Delta E_V - g_h \mu_B H. \qquad (4)$$

since ΔE_V can be determined independently at $\nu = 3.5$, when both electron valleys appear, and it can be assumed from symmetry considerations that $\Delta E_V(\nu = 2.5) = \Delta E_V(\nu = 3.5) = 0.8$ meV, to determine ΔE_S it remains to find g_h. As seen from (3), the parameter g_h can be found from the temperature dependence of the ratio of the integrated intensities of the lines due to recombination of 2*D* electrons with spins $S_z = +\frac{1}{2}(I_\uparrow)$ and $S_z = -\frac{1}{2}(I_\downarrow)$. This dependence is shown with the appropriate coordinates in the inset of Fig. 3, and yields the value $g_h = 1.6$, which agrees well with the *g* factor given in Refs. 30–32 for the *g* factor of the holes in silicon. Since all the quantities in (4) are known, the *g* factors of the 2*D* electrons can be determined at $H = 6$ T, $\nu = 2.5$ and $T = 3.3$ K. It was found to be $g_e = 8.6$, substantially higher than the electron bulk *g* factor $g_e^0 = 2$. This enhancement of g_e for 2*D* electrons, just as in the case of intervalley splitting, is undoubtedly due to *e-e* interaction effects.[33] Preceding experiments[17,34] also yielded data attesting to a noticeable increase of the 2*D*-electron *g* factor compared with $g_e^0 = 2$, but this is the first time that a rise to as much as $g_e = 8.6$ has been observed.

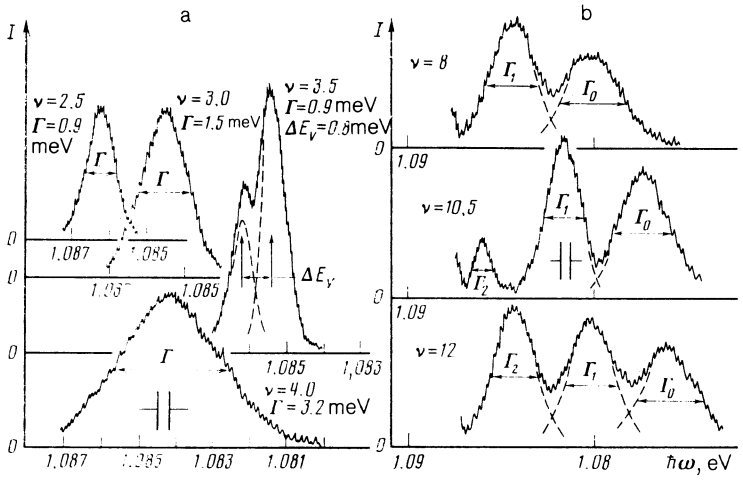

FIG. 4. 2*D*-electron radiative recombination spectra obtained for sample No. 1 at $H = 7$ T, $T = 1.6$ K, $W = 10^{-3}$ W/cm^2 at different fillings of the Landau levels: a—spectra of one ($N = 0$) Landau level; b—spectra of several (up to $N = 2$) Landau levels.

§5. SCREENING OF RANDOM-POTENTIAL FLUCTUATIONS

The screening of the random potential due to defects manifests itself most pronouncedly in the oscillatory behavior of the recombination-radiation line width on Landau levels when their filling is changed, and also in singularities of the temperature dependence of the recombination spectra. It is expedient to consider these effects separately.

a. Oscillations of Landau-level widths with change of filling factor. The recombination-radiation spectra were investigated with one ($N = 0$; $2 < \nu < 4$) as well as several ($N = 3$; $8 < \nu < 12$) Landau levels filled. Recall that only one 2*D*-electron spin projection $S_z = +\frac{1}{2}$ appears in the radiation spectra in a strong magnetic field at $T = 1.5$ K. The radiation is therefore recorded under these conditions for fillings satisfying the condition $2 + 4M < \nu < 4(M + 1)$, where $M = 0, 1, 2, \ldots$. Figure 4a shows plots of the spectra corresponding to fillings of the lowest Landau level: $\nu = 2.5$, 3, 3.5, and 4.5. The doublet character of the spectrum at $\nu = 3.5$ is connected with the valley-orbit splitting ΔE_ν. At $\nu = 3.5$ the lower-energy state corresponding to the projection $S_z = +\frac{1}{2}$ is completely filled, and the upper, separated by an distance ΔE_ν and having the same spin projection is half-populated by the electrons. Therefore the intensity of the short-wave doublet component corresponding to the upper split-off state is one-half as weak as the long-wave component (the contours of the components of this doublet are separated in Fig. 4a by dashed lines).

Comparing the recombination spectra in Fig. 4a, it is easily seen that the linewidths have a nonmonotonic dependence on the filling of the electronic states: in the case of complete filling (integer ν) the lines are much wider than under conditions of half-integer filling. A qualitatively similar behavior when ν is varied is exhibited by the recombination spectra when several Landau levels are completely filled and the population of the level highest in energy is varied. Figure 4b shows plots of the spectra measured with complete filling of the two lowest Landau levels ($N = 0$, 1) and with different fillings of the upper level ($N = 3$). In this case, too, the widths of all the spectral lines have a nonmonotonic dependence on ν: the lines are much broader for integer ν than for half-integer filling. It is seen in addition that when several Landau levels are filled the linewidths increase monotonically with decrease of the quantum number N.

The oscillatory behavior of the recombination linewidths as functions of ν when dealing with one and with several Landau levels is illustrated in Figs. 5a and 5b. The oscillations have the largest swing when working with one (lowest) Landau level. When several Landau levels are filled, the oscillations are similar, but their depth is smaller the deeper the quantum state below the Fermi level (the smaller N). The observed oscillations of the linewidths in the recombination spectra are a direct consequence of the oscillatory behavior of widths of the state-density peaks on the Landau levels as functions of the electron density. This phenomenon stems undoubtedly from the screening of the random potential.

Let us examine this question in greater detail. For integer ν, the Fermi level is exactly in the middle of the energy-spectrum gap. The electrons fill completely the Landau levels and take therefore no part in the screening. More accurately speaking, the screening has in this case a weakened, dielectric character. In the absence of screening, the Landau levels "follow-up" the potential relief due to the long-period fluctuations (see Fig. 6a). If it is assumed that these fluctuations are due with charged defects in the SiO_2 insulator layer, the nonequilibrium holes with which the 2*D* electrons recombine are less subject to the action of these fluctuations, since they are significantly farther from the Si–SiO_2 interface ($z \approx 10^2$ Å). It is natural then to conlcude that the radiation linewidth for integer filling is determined by the amplitude of these charge fluctuations in the insulator,

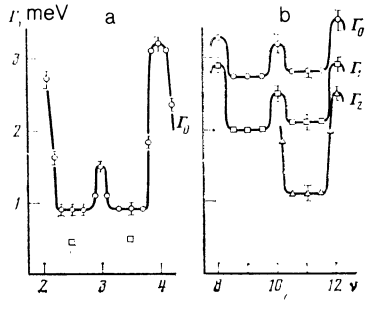

FIG. 5. Dependence of Landau-level width Γ on the filling factor ν, measured for sample No. 1 at $H = 7$ T and $T = 1.6$ K for Landau levels numbered $N = 0$ (a) and $N = 2$ (b). The squares in Fig. a correspond to Γ values obtained by approximating the $\Gamma_{min}(H/\mu)$ dependence for given H and μ.

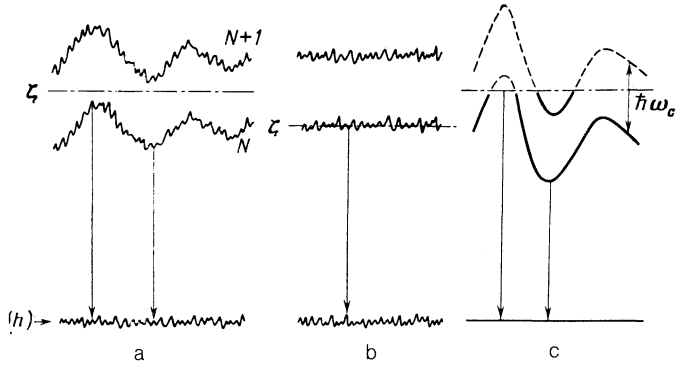

FIG. 6. Change of energy of 2*D*-electrons of the Landau levels N and $(N+1)$ and of the nonequilibrium holes in the volume (h) vs the coordinate along the 2*D* plane under conditions of integer filling at $Q<\hbar\omega_c$ (a) and $Q>\hbar\omega_c$ (c), and also of half-integer filling (b), when the large-scale fluctuations of the potential are screened.

since the recombination is with all the electrons whose energy distribution within the limits of a given level accords with the potential relief (see Fig. 6a). The linewidths obtained by us for several MIS structures with a completely filled lowest Landau level are practically equal to the linewidths ΔE measured under conditions of strong localization at $H=0$, widths that likewise accord with the amplitude of the large-scale fluctuations.[12] These results are summarized in Table I.

It is useful to point out that in the case of cyclotron resonance (CR), in contrast to the luminescence method considered here, large-scale fluctuations of a random potential do not contribute at d/l_H to the CR linewidth.

For half-integer filling, the Fermi level coincides with the peak of the density of states (with the maximum of the line in the recombination spectrum). The electronic states in this region of the energy spectrum are extended. The electrons now screen most effectively the large-scale fluctuations of the random potential, and as a consequence the associated potential relief turns out to be smoothed out (see Fig. 6b). The width of the luminescence peak should therefore become narrower, in full agreement with experiment. The finite width of the SD peak should be determined under these conditions by small random scatterers that remain unscreened.[35,36] In our experiment, the value of Γ for half-integer ν and large μ can be determined by the energy distribution of the nonequilibrium holes with which the electrons recombine (this will be treated in greater detail in §6). If account is taken of the broadening connected with the holes, it turns out that the true width of the luminescence peaks at half-integer ν that reflect the widths of the DS peaks on the Landau levels should be much narrower (the corresponding values of obtained by approximation are designated by squares in Fig. 5). This means that the true swing of the DS peak-width oscillations as ν is varied is approximately three times larger than that observed in experiment (Fig. 5).

Oscillations of Γ with change of the filling were predicted in Refs. 37–40. The nature of oscillations was attributed to periodic variation of the screening radius and its gist was that the screening effects and, for example, the Thomas-Fermi constant, are determined by the density of states on the Fermi surface,[1] which oscillates with change of the filling of the Landau levels by the electrons. No analytic expressions, however, are given for the $\Gamma(\nu)$ dependence in these papers.

A theory was recently developed for nonlinear screening, by 2*D* electrons, of the potential of charged centers that are randomly distributed in a volume and have a density N_q (Ref. 11). A Coulomb potential is assumed for the centers, and the screening is described by considering the fluctuations of the charge in a volume with a characteristic linear dimension L. For integer filling of the Landau levels ($n_S = n_S^0 = veH/h$ with ν an integer) there is no screening. Screening appears when at a deviation δn_S of the 2*D*-electron density from n_S^0 ($\delta n_S \equiv n_S - n_S^0$). If, in the scale of L_c, the number of additional electrons in the $\delta n_S L_c^2$ plane is equal to the fluctuation of the number of charged centers in the volume $(N_q L_c^3)^{1/2}$, the random potential is completely screened by the electrons at all lengths $L > L_c = N_q/\delta n_S^2$. A change δn_S in the number of electrons leads to a change of the screening length, and in this sense the scale L_c is the nonlinear-screening radius. Shklovskiĭ and Éfros[11] obtained the Fermi-level position and of the state-density peak width on the Landau level as functions of the electron density in a magnetic field. An important consequence of the theory of nonlinear screening is that the density of states in the gaps between the Landau levels is not exponentially small, as might be expected for a short-range potential[1,35] or if the charged centers lie in a 2*D* plane.[41] The reason is that the bulk charged centers play a more important role than the surface ones, for when δn_S is decreased the production of the random potential brings into play new centers in a layer having a rapidly increasing thickness L_c.

The theory of Ref. 11 explains qualitatively the principal experimental results connected with oscillations of the density of states when the electron density is increased; these effects are revealed by the luminescence spectra. From Ref. 11, in particular, it follows that when the amplitude of the large-scale fluctuations is of the order of the corresponding gap in the energy spectrum ($\hbar\omega_c$, ΔE_S or ΔE_V), two neighboring Landau sublevels begin to participate in the screening (see Fig. 6c). As a result, the state-density peak width ceases to increase and is bounded from above by the size of the gap. It follows from experiment (see Fig. 5a) that when a Landau level is completely filled ($\nu=4$) the luminescence linewidth is substantially larger than at $\nu=3$, in accordance with the fact that the energy gap at $\nu=4$($\hbar\omega_c = 4$ meV) is substantially larger than $\nu=3$($\Delta E_V = 1.5$ meV, Ref. 29). Using the equations of the theory of Ref. 11 and the parameters of the investigated MIS structures, one can estimate the amplitude and the linear scale of the random-potential fluctuations. The results of these estimates wil be given in §7.

b. Temperature dependence of the Landau-level widths. As already noted, if N Landau levels are completely filled in a system of 2*D* electrons in a perpendicular magnetic field, the electrons are uniformly distributed over the system area and there is no screening. Screening sets in if some number δn_S of additional electrons (or holes) is added to the system. Production of electrons and holes can be ensured not only by changing n_S or H, but also by raising the temperature at constant values of n_S and H. One should therefore expect in experiment a narrowing of the radiation lines corresponding to integer filling of the Landau levels (i.e., at $n_S = veH/h$ with ν an integer) as the temperature is raised. Since the densities of the thermally activated electrons and holes are

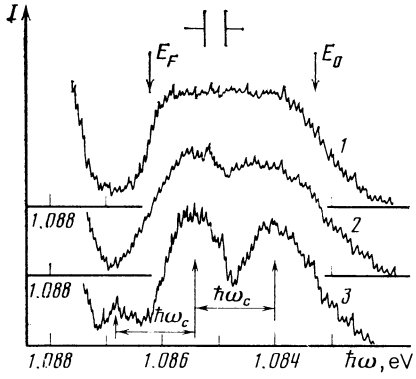

FIG. 7. Radiative recombination spectra of $2D$ electrons, measured on structure No. 1 at $n_S = 4.9\cdot10^{11}$ cm^{-2}, $\mu = 2.6\cdot10^4$ cm^2/V·s, W = 10^{-3} W/cm^2 and various values of H and T: $H = 0$, $T = 1.5$ K (spectrum 1); $H = 2.5$ T ($\nu = 8.0$), $T = 1.5$ K (spectrum 2); $H = 2.5$ T, $T = 3.5$ K (spectrum 3).

$$n_e = n_h \sim \exp(-\hbar\omega_c/2kT),$$

it follows that to observe this effect it is necessary that the experimental values of $\hbar\omega_c$ and kT be comparable.

Figure 7 shows the radiation spectra obtained at $n_S = 4.9\cdot10^{11}$ cm^{-2} and $\mu = 2.6\cdot10^4$ cm^2/V·s in magnetic fields $H = 0$ (spectrum 1) and $H = 2{,}5$ T (spectrum 2) at a temperature $T = 1.5$ K. We see first of all that even though the parameter μH exceeds unity substantially ($\mu H = 6.5$ in the experiment), no pattern of Landau levels is manifested in the radiation spectrum. The reason is that the level width (which can be determined from the high-energy edge of the line $2D_e$) is practically equal to the cyclotron splitting. This property is common to all investigated structures: in weak magnetic fields the luminescence-peak width turned out to be close to $\hbar\omega_c$, and with increase of H, when $\hbar\omega_c$ comes closer to the amplitude of the fluctuations of the random potential Q, the value of Γ tends to saturate and the Landau levels are well resolved. This behavior agrees with the theory of nonlinear screening of large-scale fluctuations of a random potential.[11]

Spectrum 3 of Fig. 7 corresponds to the case of high temperatures ($T = 3.5$ K). It shows clearly how the Landau levels become narrower with rise of temperature, when electrons and holes appear on the sublevels. The temperature-induced filling of the higher-lying level is accompanied by the appearance of an additional radiation line shifted by an amount $\hbar\omega_c$ towards higher energies (see Fig. 7, spectrum 3). The narrowing of the level with increase of T means a decrease of the density of states in the gaps. Such a behavior of the DS with increase of T was observed in Ref. 42.

Thus, changes of the Landau-level width as functions of ν and T are attributed to screening of large-scale fluctuation of the random potential of the defects, and agree qualitatively with the deductions of the theory.[11,37–40]

§6. LUMINESCENCE-PEAK WIDTH FOR HALF-INTEGER FILLING OF LANDAU LEVEL

In the case of half-integer filling of the quantum states, the large-scale fluctuations of the potential are screened and the width of the state-density peak narrows down to a minimum value Γ_{min}. If interaction with the short-range scatterers becomes decisive, the value of Γ_{min} should depend,

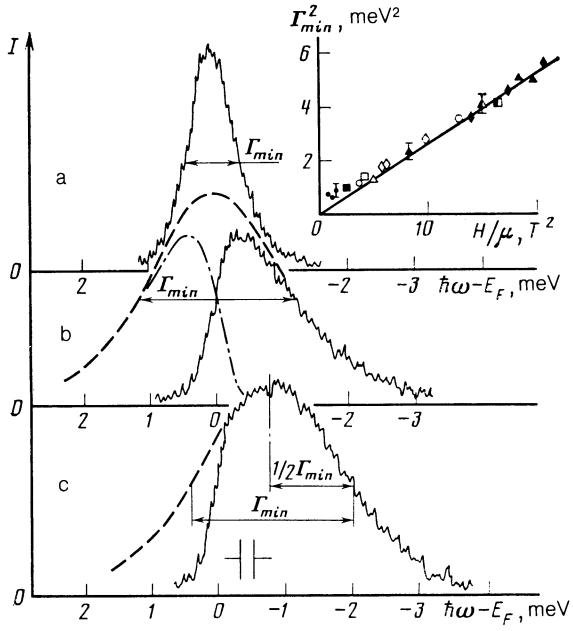

FIG. 8. Radiative recombination spectra of $2D$ electrons, measured at $H = 7$ T and $T = 1.6$ K for structures No. 1 and No. 4 under conditions of incomplete filling of the Landau levels: a) structure No 1; $\nu = 2.5$; $\mu H = 22.0$; b) structure No. 4; $\nu = 2.5$; $\mu H = 2.3$; c) structure No. 4; $\nu = 2.75$; $\mu H = 2.4$. The dashed line shows the energy distribution of the density of states on the Landau level, obtained from the radiation line contour. The inset shows the dependence of Γ_{min} on the parameter H/μ, measured for four different structures (different-shape symbols) and different numbers N (light and dark symbols).

according to the theory,[35,36] on a single parameter H/μ. It is therefore of interest to track the variation of the state-density peak width, which can be found by analyzing the luminescence line, when the parameter H/μ is varied.

We discuss first the luminescence line shape. For half-integer filling, the Fermi level coincides with the peak of the density of states and, in contrast to integer filling of a quantum level, the luminescence line should become in this case strongly asymmetric. The reason is that at sufficiently low temperatures ($kT \ll \Gamma_{min}$) the electron states are populated only below the Fermi levels, and consequently the emission line should fall off steeply towards higher energies. The state-density peak should remain symmetric in this case, and its width should be approximately double the width of the asymmetric luminescence line. In experiment, however, an asymmetric emission line at half-integer ν is observed only when the dimensionless parameter μH does not exceed unity greatly (see Fig. 8). This figure shows also how to reconstruct from the asymmetric contour of the luminescence line from the symmetric contour of the state-density peak (dashed line) and estimate its width. It can be seen that the shape of the state-density peak at $\nu = 2.5$ is obtained by adding two contours, the experimental luminescence-line contour and its mirror image, whereas at $\nu = 2.75$ it is obtained from the low-energy part of the spectrum. We follow hereafter mainly this procedure of determining the state-density peaks from the radiation-line contour (the Landau-level widths at $\nu = 2.5$ and at $\nu = 2.75$ were always practically equal).

The asymmetry is lost with increase of the parameter μH, and the luminescence line becomes practically symmet-

ric already at $\mu H > 15$ (Fig. 8a). This is readily understood by recognizing that the observed radiation spectrum is a convolution of the distributions of electrons and nonequilibrium holes. A symmetric contour is produced under conditions when the hole-distribution width exceeds Γ_{min}.

The dependence of the state-density peak width for half-integer filling on the parameter H/μ is plotted in the inset of Fig. 8, with Γ_{min} and H/μ as the coordinates. The experimental points of this plot were obtained for different structures, and also for different quantum numbers N (represented by different symbols). Nevertheless, all are described by a universal dependence on the parameter H/μ, well approximated by a straight line. This behavior indicates that for half-integer ν the state-density peak width is governed by scattering from small-scale fluctuations. In addition, it follows from the observation results that the peak widths are independent of the number of the quantum state. The absolute values of Γ_{min}, obtained in experiment for specified H and μ, are close to the values calculated in Refs. 34 and 35.

At low values of the parameter H/μ (see Fig. 8), the experimental $\Gamma^2_{min}(H/\mu)$ dependence deviates noticeably from linearity and as $H/\mu \to 0$ the symmetric luminescence line has a finite width. This quantity, as noted above, is directly connected with the width of the energy distribution of the nonequilibrium holes, found to equal 0.8 meV.

§7. DENSITY OF STATES FOR INTEGER FILLING

Determination of the amplitude and scale of long-period fluctuations

It was already noted in §5 that for integer filling of the Landau level and if the level width is much less than the gap energy ($\hbar\omega_c \gg \Gamma$) the density of states within a single level reflects an energy distribution of large-scale fluctuations of a random potential, so that the fluctuation amplitude Q can be determined from widths of the recombination lines in the luminescence spectrum. It must be borne in mind, however, that the electrons are localized on the random-potential distributions if the fluctuation scale exceeds the electron cyclotron radius $l_H(2N+1)^{1/2}$ (Ref. 1), where l_H is the magnetic length. Therefore only if the conditions $d \gg l_H(2N+1)^{1/2}$ and $\hbar\omega_c \gg Q$ are met will the Landau-level width equal the fluctuation amplitude. In the more general case $d \gtrsim l_H(2N+1)^{1/2}$ the width Γ is less than Q and depends on the ratio $l_H(2N+1)^{1/2}/d$ like[43,44]

$$\Gamma = Q[1+(4N+2)l_H^2/d^2]^{-1/2}. \quad (5)$$

Expression (5) was obtained for the Landau ground level in Ref. 43 and for higher N in Ref. 44. Using Eq. (5) we can determine, from an analysis of the luminescence spectra, the parameters Q and d for different structures. It must be borne in mind here that Eq. (5) was obtained without allowance for screening effects and is therefore valid, first, for integer filling and, second, if $\Gamma < \hbar\omega_c$, i.e., when the structure of the Landau levels is well resolved in the spectrum. Figure 9a shows a plot of $\Gamma(n, H)$, with Γ^{-2} and $(2N+1)H^{-1}$ as the coordinates, for two structures. The points on these plots were obtained for different values of N, and are all well described by a single straight line. The slope of this line, in the coordinates indicated, yields the value of d, and the intercept of the line with the ordinate axis yields Q. It is interesting that the parameters d and Q obtained for two structures of different quality were different, whereas for a structure of worse quality (judging from the parameters μ^*, n_S^*, n_+) the amplitude Q of the fluctuations turned out to be larger and the scale d smaller.

The scale of the long-period fluctuations of the random potential of the defects can be estimated independently from other measurements. Comparing at constant H the ratios of the Landau-level widths at integer (Γ_{max}) and half-integer (Γ_{min}) filling with increase of the quantum number N, one can hope that the observed Γ_{min} and Γ_{max} will come closer together. Indeed, the cyclotron radius increases with increase of N. Therefore, for cyclotron orbits $l_H(2N+1)^{1/2} > d$, the fluctuations of a potential with linear dimension d become small-scale. The difference between Γ_{min} and Γ_{max} should vanish as a result.

Figure 9b shows, for the same two structures as in Fig. 9a, the changes of Γ_{min} and Γ_{max} in a field 7 T with increase of N. The growth of Γ_{min} is attributed to a decrease of the $2D$-electron mobility with increase of n_S. It is seen that as N increases the values of Γ_{min} and Γ_{max} come closer and become equal at $N = N_{cr}$. It can be assumed that at this value of the quantum number the fluctuation scale d and the electron cyclotron radius $l_H(2N+1)^{1/2}$ become equal, and d can be estimated as a result. The values of d obtained by this

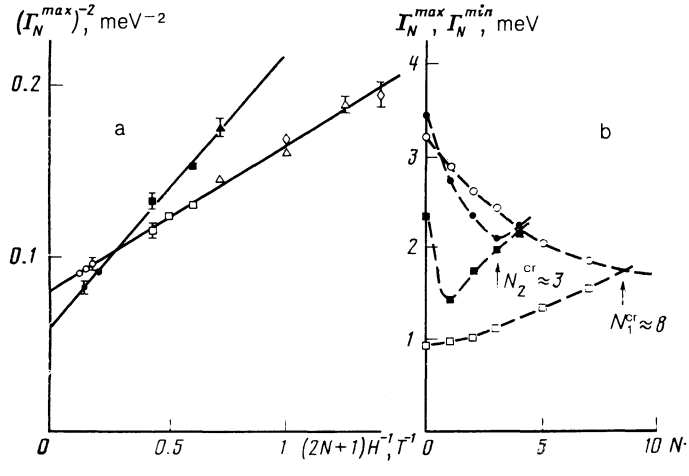

FIG. 9. Width Γ_N^{max} of the Nth Landau level, measured for integer filling of N levels in different magnetic fields for structures No. 1 (light symbols) and No. 4 (dark), vs the parameter $(2N+1)H^{-1}$. Different numbers N are marked by different symbols: $N=0$ (circles), $N=1$ (squares), $N=2$ (triangles), $N=3$ (diamonds), $Q_1 = 3.5$ meV, $d_1 = 350$ Å, $Q_2 = 4.0$ meV, $d_2 = 230$ Å; b) Dependences of Γ_N^{max} (circles) and Γ_N^{min} (squares) on the Landau level number N, measured at $H = 7$ T and $T = 1.6$ K for structures No. 1 (light symbols) and No. 4 (dark), $d_1 \approx 400$ Å, $d_2 \approx 250$ Å.

procedure for four MIS structures are listed in Table I together with the values of d determined with the aid of Eq. (3) from the $\Gamma(H, N)$ dependence. Comparison of these values of d shows that different methods give close values for the scale of the large-period fluctuations of the potential in all the investigated structures. As for the amplitudes of the fluctuations, we note that the values of ΔE obtained at $H=0$ and the values of Q measured using the dependence of H are in good agreement. It follows also from the table that the larger the charge in the oxide the smaller the scale d of the fluctuations and the larger their amplitude Q. This indicates most likely that the large-scale fluctuations of the potential stem from the space charge concentrated in the oxide in a layer of the order of d from the Si–SiO_2 boundary. In this case it follows from theory[11] that the scales and amplitudes of the potential fluctuations are bounded from above by the condition $L_c = N_q/\delta n_S^2 = d$ so that as $\delta n_S \to 0$ neither L_c nor $Q = e^2(\varkappa L_e)^{-1}(N_q L_c^3)^{1/2}$ (Ref. 11) increases without limit, but tends to a finite value. Estimates based on theoretical equations[11] are in fair agreement with our present experimental results ($Q_{\text{theor}} \approx 2$ meV).

One of the most substantial questions that can be raised concerning the method of determining the parameters Q and d is that the long-period part of the relief of the conduction band near the interface can be partially repeated in the interior of the semiconductor, i.e., in that region where nonequilibrium holes are located. Indeed, the system of $2D$ electrons screens well the external electric field due to the total charge of the system, but the alternating-sign part of the long-period charge fluctuations is poorly screened. To determine the extent to which the long-period part of the fluctuation potential due to the charged center in the insulator SiO_2 extends from the interface to the interior of the semiconductor, where the nonequilibrium holes are located, we solved the following problem.[1)] The large-scale fluctuations of the potential were simulated by a system of alternating unlike unit charges located at the centers of cubes with bases on the interface. The distance between like charges (d) determines in such a model the scale of the fluctuations. We were interested in the change of the ratio of the amplitude Q_1 of the electrostatic-potential on the interface to the amplitude Q_2 at a distance $z = 100$ Å from the interface, where the nonequilibrium holes are located. The values of Q_2/Q_1 for the scales $d = 200$, 400, 800, and 1600 Å were respectively 0.01, 0.1, 0.3, and 0.58. It follows hence that the broadening of the hole levels on account of potential fluctuations with linear scales $d < 500$ Å is negligible and can neither influence the interpretation or change strongly the values estimated by the spectroscopic method. The same conclusion can be drawn also from other considerations. Assume that the fluctuations of the potential in the valence band, where the nonequilibrium holes are located, could exert an influence. This means that the optically measured luminescence linewidth would be substantially less than the actual Landau-level width Γ. Yet it follows from the nonlinear-screening premises that Γ cannot exceed $\hbar\omega$, and hence all the linewidths observed optically for integer filling would always be much smaller than $\hbar\omega_c$, at variance from experiment (see Fig. 7). It must therefore be concluded that the real values of d in the investigated structures are not too large, so that such fluctuations of the random potential do not act strongly on the valence band in which the holes are located. On the other hand, it is also clear that potential fluctuations with scales $\gtrsim 10^3$ Å are almost fully duplicated in the valence-band relief at distances $z = 100$ Å from the interface. Such fluctuation scales should therefore hardly be manifested in the considered luminescence method.

The use of the optical-spectroscopy method has thus not only enabled us to assess the influence of long-period fluctuations of the random potential of the defects on the Landau-level fluctuations, but also made it possible to determine the amplitude and scale of these fluctuations from the dependences of the widths of the Landau peaks on the magnetic field and on the quantum number.

§8. ENERGY DISTRIBUTION OF THE DENSITY OF STATES FOR INTEGER FILLING

It was already noted that various methods are used to determine the density of states of $2D$ electrons under condition of complete filling of the Landau levels (when the quantum Hall effect is observed[13]). These methods are based on magnetization,[2] electronic specific heat,[3] magnetocapacitance,[8,9] thermally activated conductivity,[4,5] and the gate current.[9] These diverse experimental methods led to one and the same conclusion: the density of states in any gap of the energy spectrum is not small and constitutes an appreciable fraction of the density of states at $H=0$. The authors of Refs. 2–4 arrived at the conclusion that the DS of $2D$ electrons are superpositions of a system of narrow Gaussian peaks on the Landau levels and on a background DS that is independent of the energy. We have proposed a qualitative explanation[5,7,45] based on oscillations, as functions of ν, of the widths of the Landau levels. According to this explanation, the Landau levels with integer filling are greatly broadened by the large-scale fluctuations of the random potential, and the density of states between the levels is therefore high. For half-integer filling, when the Fermi level is in a region of extended states and the large-scale fluctuations are screened, the widths of the Landau levels are greatly decreased and the DS turns out to be very large on the Fermi level and exponentially small in the gaps between the Landau levels.

It is of interest to compare quantitatively the $2D$-electron DS measured by entirely different experiments—by magnetooptics and thermally activated magnetoconductance. It must first of all be borne in mind that different quantities are studied by these two methods. Whereas optical spectroscopy of $2D$ electrons makes it possible to study the entire $D(E)$ dependence and yields the quantity $D(E_F)$, in all other experiments one determines only the thermodynamic DS, i.e., the quantity dn_S/dE_F, which is in general not equal to $D(E_F)$. At the same time it can be expected that in the case when E_F lands at the minimum of $D(E)$, the values of $D(E_F)$ and dn_S/dE_F should be equal,[10] so that they should be compared just for integer ν.

We turn now to the quantitative determination of $D(E)$ from the radiative recombination spectra.[12] This method is based on a comparison of the radiation spectrum measured in a magnetic field for complete filling of the N Landau level, on the one hand, and the emission spectrum measured at the same density n_S and at $H=0$. The energy distribution of the radiation intensity $I(E)$ is determined by the convolution of the electron distribution function

$$F_e(E) = D(E) f(E), \quad (f(E) = \theta(E - E_F))$$

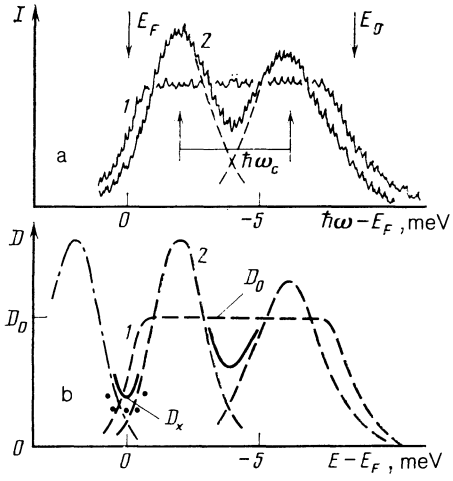

FIG. 10. 2*D*-electron recombination spectra measured for structure No. 2 at $H = 0$ (spectrum 1) and $H = 7$ T, $\nu = 8$ (spectrum 2) and at $W = 10^{-3}$ W/cm^2, $T = 1.6$ K, $n_S = 1.35 \cdot 10^{12}$ cm^{-2}, $\mu = 1.6 \cdot 10^4$ cm^2/V·s (a) and energy distribution of the density of states on the Landau levels (dashed line) and overall $D(E)$ dependence (solid), obtained from the contours of the spectra (b) for $H = 7$ T, $\nu = 8$, $\mu H \approx 11$. The points show the dependence of the quantity dn_S/dE_F, measured simultaneously, at the same parameters, by the method of thermally activated magnetoconductance.

with that of the holes $F_h(E)$, and the width is $F_h(E) \approx 10$ K, which is much less than the characteristic electron energies E_F. We have therefore

$$I(E) = AD(E).$$

To determine the constant that relates the intensity with the DS, we recall that at $H = 0$ the shape of the $2D_d$ line is that of a step function of the energy, a reflection of the constancy of the DS of 2*D* electrons at $D(E) = 4m/2\pi\hbar^2 = D_0$. This circumstance enables us to establish on the intensity scale an absolute value of the DS, equal to D_0, and then determine quantitatively the functions $D(E)$ and the quantities $D(E_F)$, by comparing the radiation spectrum obtained in a magnetic field with the spectrum measured at the same 2*D*-electron density but at $H = 0$. The spectra compared must have equal integrated intensities, since the value of n_S is the same in both cases.

Figure 10 shows how the density of states and its energy dependence, and also the value of $D(E_F)$ (see Fig. 10b) can be calculated from the radiation spectra obtained for $n_S = 1.36 \cdot 10^{12}$ cm^2 and $T = 1.6$ K at $H = 0$ (spectrum 1 of Fig. 10a) and at $H = 7$ T ($\nu = 8$, spectrum 2). Note that under these conditions no additional complications are introduced by the intervalley and spin splittings, since radiation from only one spin component of the electron is observed at low temperatures, and the intervalley splitting is practically zero, $\Delta E_V = 0.3$ meV (Ref. 29). The dashed line in Fig. 10b shows the contours of the Landau levels which, on convolution with the function $f_h(E)$ (measured separately at $\mu H = 50$, when $T_{min} \to 0$) describe exactly the radiation spectrum 2 of Fig. 10a. The dash-dot curve in Fig. 10b shows the next unfilled Landau level, and the solid curve shows the total density of states in the gaps between the levels. It can be seen that under the indicated conditions ($H = 7$ T, $\nu = 8$, $\mu H = 11$) we have $D(E_F) = 0.32$.

We have measured at the same time, for the same structures, the thermodynamic density of state by the method of activated magnetoconductance.[4,5] Since we were interested in the value of the thermodynamic density of states strictly halfway between the Landau levels (inasmuch as only in the region of the minimum of $D(E)$ should the values of $D(E_F)$ and dn_S/dE be close), the determination of this quantity, as shown in Ref. 5, calls for allowance for both the electron and the hole contributions to the magnetoconductance. The temperature dependence of the magnetoconductance, measured in the region of the minimum of σ_{xx}, is described by the expression[5]

$$\sigma_{xx}(\delta n_S) \sim \exp(-W/kT)\,\mathrm{ch}(\delta E_F/kT),$$

where δE is the shift of the Fermi level from the position corresponding to the middle of the gap following a density change δn_S compared with the density n_S^0 corresponding to complete filling of the Landau levels, and W is the activation energy and is equal to half the energy gap. The thermodynamic density of states is obtained from such measurements in the form of the ratio $\delta n_S/\delta E_F$. Under the conditions of Fig. 11b, when the magnetooptic measurements yield the value $D(E_F) = 0.32 \cdot D_0$, the thermodynamic density of states determined from an analysis of the thermally activated magnetoconductance turned out to be somewhat lower, $dn_S/dE_F = 0.25 \cdot D_0$ (the variation of dn_S/dE_F with E_F is also shown in Fig. 10b).

The quantities $D(E_F)$ and dn_S/dE_F determined under conditions of complete filling of the Landau levels by entirely different experimental methods are thus in good agreement. The small discrepancy between them can be attributed to several factors, but it is more important that when the parameters μ and H or the structure are varied the values of $D(E_F)$ and dn_S/dE_F (for integer ν) vary in like manner.

The dependence of the density of states between Landau levels on the magnetic field should be interpreted as follows: In the magnetic-field region in which $\mu H \gg 1$ but the cyclotron splitting is less than the amplitude of the large-scale fluctuations Q of the random potential, the level width is found to be of the order $\hbar\omega_c$, owing to the nonlinear screening, and the DS in the gap constitutes an appreciable fraction of D_0. So long as the condition $\hbar\omega_c \leqslant Q$ is met, the density of the state in the gap decreases with increase of the magnetic field not exponentially but less strongly, for example as a power law.[5,11] On the other hand, when the magnetic field reaches a value such that $\hbar\omega_c$ becomes larger than Q, the width Γ saturates and tends to Q, while the density of states in the gaps decreases exponentially with increase of the magnetic field. In this manner it is possible to estimate the amplitude of the fluctuations of the random potential not only by optical methods but also, for example, with the aid of thermally activated magnetoconductance.

§9. CONCLUSION

The spectroscopic method, based on the study of radiative recombination of two-dimensional electrons with nonequilibrium holes, was thus found to be productive in the determination of the density of states of 2*D* electrons in a transverse quantizing magnetic field, with silicon MIS structures as the example. The principal usefulness of this method, which distinguishes it from those heretofore employed, is that it makes possible a direct experimental determination of the energy distribution of the density of states of 2*D* elec-

trons as the quantum levels become filled with electrons. The use of these capabilities has made it possible to establish that the width of the Landau levels oscillates as a function of the filling factor, and to explaim why the density of states in the gaps of the energy spectrum, when the Landau levels are completely filled, is not exponentially small, but constitutes a significant fraction of the SD at $H = 0$. It was possible to demonstrate with the aid of this method that the oscillations of the density of states are the result of screening of the large-scale fluctuations of the random potential, and to determine the magnitude and linear scale of these fluctuations.

The spectroscopic method may also find use in the solution of other problems connected with the energy spectra of two-dimensional electron systems. The most timely among them is that of the spectra of incompressible Fermi liquids that appear under condition of the fractional quantum Hall effect.[46] We hope to use this method not only to determine the magnitudes of the Coulomb gaps in the spectrum of the excitations (quasielectrons and quasiholes) of incompressible liquids, but also to determine the influence exerted on these gaps by disorder, temperature and hierarchy of the states.

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