Magnetic properties of frustrated antiferromagnets. Phase transition into spin glass

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(Submitted 21 January 1987)
Zh. Eksp. Teor. Fiz. **93**, 1060–1073 (September 1987)

The transition of a frustrated Heisenberg antiferromagnet into a nonergodic state is investigated in the infinite-radius model. A phase diagram in an external magnetic field is constructed. Its distinctive feature is that the temperature of the transition into the nonergodic state increases with the field both in the collinear phase and after the overturning of the sublattices. The spin-flop transition line has a section in which the sublattice overturning is accompanied by a first-order transition from a nonergodic to an ergodic state. It is shown that the frustration leads to a growth of the transverse susceptibility as the temperature is lowered in the interior of the antiferromagnetic phase, and to other anomalies of the behavior of frustrated antiferromagnets.

INTRODUCTION

Many experimental investigations of the properties of disordered (both Heisenberg and Ising) antiferromagnets and of the transition of these antiferromagnets into a nonergodic state have been recently reported.[1–11] A theory of Ising disordered antiferromagnets was developed in Ref. 13 in the framework of the Sherrington-Kirkpatrick infinite-range model.[12] In the nonergodic state, both the antiferromagnetic and the spin-glass order parameters differ from zero. This state was therefore named antiferromagnetic spin glass (AFSG). The distinction of AFSG is manifested primarily in its behavior in an external magnetic field, which suppresses both the long-range ferromagnetic order and the spin-glass state. Therefore, depending on the relation between the parameters, a magnetic field can not only decrease but also increase the temperature interval in which spin glass exists.

The experimental data[3,10] show that the temperature $T_g(H)$ of the transition into AFSG does indeed increase strongly with the field in both Ising and Heisenberg antiferromagnets.

The distinctive features of the transition into AFSG in Heisenberg systems are due first to feasibility, in general, of transverse freezing and second to the fact that the transition can take place from an spin-flopped state, in which the sublatice moments are not collinear. This raises two questions: how does the function $T_g(H)$ behave in the collinear and spin-flopped phases, and what singularities appear on the phase diagram when the line of the continuous phase transition into the nonergodic state $T_g(H)$ intersects the line of the spin flop, which is a first-order phase transition? We shall show that in both the collinear and the spin-flopped phases the region in which AFSG can exist can increase with increase of the magnetic field. According to experiment,[10] the relative growth of the temperature $T_g(H)$ with increase of the field is larger the smaller the disorder. On the line separating these phases there exists a region of fields and temperatures where a first-order transition into a nonergodic state takes place simultaneously with the jump of the moments.

In an ergodic antiferromagnetic state, regardless of the relations between the parameters, experiments[5–7,15] have shown that disorder leads to an increase of the transverse susceptibility with decrease of temperature. In some cases the longitudinal susceptibility can also increase near the Néel temperature $T_N (T \lesssim T_N)$. Increase of the disorder leads to a decrease of the sublattice flopping field (at the same Néel temperature) and narrows down the magnetic-field region in which long-range antiferromagnetic order exists.

1. EQUATIONS OF STATE

Consider a Heisenberg antiferromagnet in which the spins are distributed among two subsystems. We assume that the exchange interaction energies V_{ij} and J_{ij} inside and between the subsystems do not depend on the distances between the spins and have a normal distribution with mean values V_0/N and $-J_0/N$, and respective variances $V/N^{1/2}$ and $J/N^{1/2}$, where N is the total number of spins in each subsystem. The spin Hamiltonian of the system is

$$\mathscr{H} = -\sum_{i,j} J_{ij}(\mathbf{S}_{1i}\mathbf{S}_{2j}) - \frac{1}{2}\sum_{p}\sum_{i,j} V_{ij}(\mathbf{S}_{pi}\mathbf{S}_{pj}) - D\sum_{i}\sum_{p=1,2}(S_{pi}^{z})^2 - H\sum_{i,p} S_{pi}, \quad (1)$$

where $D > 0$ is the magnetic-anisotropy constant, H is the external field, and the subscripts $p = 1$ and 2 number the subsystems.

In Appendix 1 we obtain by the replica method a system of equations of state for the "sublattice" magnetizations $\mathbf{m}_p$, the Edwards-Anderson parameters $q_p^{\mu\nu}$, and the quadrupolarity parameters $Q_p^{\mu\nu}$ (μ and ν are the spin vector indices):

$$\mathbf{m}_p = \langle\langle \mathbf{S}_p \rangle_T \rangle_c, \quad q_p^{\mu\nu} = \langle\langle S_p^{\mu}\rangle_T \langle S_p^{\nu}\rangle_T\rangle_c, \quad Q_p^{\mu\nu} = \langle\langle S_p^{\mu} S_p^{\nu}\rangle_T\rangle_c, \quad (2)$$

where the thermodynamic averaging $\langle \dots \rangle_T$ is carried out with the effective Hamiltonian.

$$\mathscr{H}_{eff}=-\sum_{p\neq p'}(V_0\mathbf{m}_p-J_0\mathbf{m}_{p'}+\mathbf{H}+J\mathbf{t}_{p'}+V\mathbf{t}_p)\mathbf{S}_p$$
$$-\sum_{\substack{p\neq p'\\ \mu,\nu}}\Big[\frac{J^2}{2T}(Q_{p'}{}^{\mu\nu}-q_{p'}{}^{\mu\nu})+\frac{V^2}{2T}(Q_p{}^{\mu\nu}-q_p{}^{\mu\nu})\Big]S_p{}^\mu S_p{}^\nu-D\sum_p(S_p{}^z)^2,\qquad H^\mu=H\delta_{\mu z}\tag{3}$$

and the configuration averaging $\langle\ldots\rangle$ is over the Gaussian distribution of the random fields:

$$P(\mathbf{t}_1,\mathbf{t}_2)=P(\mathbf{t}_1)P(\mathbf{t}_2),$$
$$P(\mathbf{t}_p)=\frac{(\det\hat{q}_p{}^{-1})^{1/2}}{(2\pi)^{3/2}}\exp\left(-\frac{1}{2}\mathbf{t}_p\hat{q}_p{}^{-1}\mathbf{t}_p\right).\tag{4}$$

These are the equations of state of a frustrated antiferromagnet and are valid above the temperature $T_g(H)$ of the transition into the nonergodic phase and on the transition line itself.

If the field is weaker than the flopping field H_c, the sublattice magnetizations are parallel to the anisotropy axis z, while the tensors $\hat{q}_p$ and $\hat{Q}_p$ are diagonal and isotropic in the transverse plane. In this case Eqs. (2)–(4) are simplified. The expression for the effective Hamiltonian takes, apart from a constant, the form

$$\mathscr{H}_{eff}=-\sum_p[\mathbf{a}_p\mathbf{S}_p+b_p(S_p{}^z)^2],\tag{5}$$

where the components of the vectors $\mathbf{a}_p$ are

$$a_p{}^\mu=H^\mu-J_0m_{p'}{}^\mu+V_0m_p{}^\mu+J(q_{p'}{}^{\mu\mu})^{1/2}t_{p'}{}^\mu+V(q_p{}^{\mu\mu})^{1/2}t_p{}^\mu,\qquad p\neq p',\tag{6}$$

and the effective anisotropy parameters are

$$b_p=D+\frac{J^2}{2T}[(Q_{p'}{}^{\parallel}-q_{p'}{}^{\parallel})-(Q_{p'}{}^{\perp}-q_{p'}{}^{\perp})]+\frac{V^2}{2T}[(Q_p{}^{\parallel}-q_p{}^{\parallel})-(Q_p{}^{\perp}-q_p{}^{\perp})],\qquad p'\neq p.\tag{7}$$

Here $q^\parallel=q^{zz}$, $q^\perp=q^{xx}=q^{yy}$, and similarly for Q. In the change from (3) to (5)–(7) we have used the substitution

$$t_p{}^\mu\to t_p{}^\mu(q_p{}^{\mu\mu})^{1/2},$$

so that the distribution functions are

$$P(\mathbf{t}_p)=(2\pi)^{-3/2}\exp(-t_p{}^2/2).$$

Rotating the coordinate frame in the $(t_p^\mu,t_{p'}^\mu)$ plane through an angle φ such that

$$\operatorname{tg}\varphi=(V/J)^{1/2}(q_p{}^\mu/q_{p'}{}^\mu)^{1/2},$$

we rewrite a_p^μ in the form

$$a_p{}^\mu=H^\mu-J_0m_{p'}{}^\mu+V_0m_p{}^\mu+(J^2q_{p'}{}^{\mu\mu}+V^2q_p{}^{\mu\mu})^{1/2}t_p{}^\mu.\tag{8}$$

All the configuration averagings are then reduced to integration with respect to one vector $\mathbf{t}$. For example,

$$\langle\langle S_p{}^\mu\rangle_T\rangle_c=\frac{1}{(2\pi)^{3/2}}\int dt\,e^{-t^2/2}\left\{\int d\mathbf{S}_pS_p{}^\mu\exp(\mathbf{a}_p\mathbf{S}_p+b_p(S_p{}^z)^2)\right\}\times\left[\int d\mathbf{S}_p\exp(\mathbf{a}_p\mathbf{S}_p+b_p(S_p{}^z)^2)\right]^{-1}.\tag{9}$$

The averaging for $\hat{q}$ and $\hat{Q}$ in Eqs. (2) should be taken to have the same meaning.

2. TRANSITION TEMPERATURE

The temperature of the transition to the nonergodic state is determined by the vanishing of one of the eigenvalues of the stability matrix[16–18]

$$\lim_{n\to0}\hat{M}=-\lim_{n\to0}\frac{\partial^2\Phi}{\partial y^{\mu\nu}_{\alpha\beta,p}\,\partial y^{\mu'\nu'}_{\gamma\delta,p'}}.\tag{10}$$

As shown in Appendix 2, the eigenvalues λ of the matrix $\hat{M}$ can be obtained from the condition that the following system of equations be solvable:

$$\frac{V^2}{T^2}\sum_{\mu'\nu'}\langle\chi_p^{\mu\mu'}\chi_p^{\nu\nu'}\rangle_cx_p^{\mu'\nu'}+\frac{J^2}{T^2}\sum_{\mu'\nu'}\langle\chi_p^{\mu\mu'}\chi_p^{\nu\nu'}\rangle_cx_{p'}^{\mu'\nu'}=(1-\lambda)x_p{}^{\mu\nu},\quad p'\neq p,\tag{11}$$

with the susceptibilities given by

$$\chi_p{}^{\mu\mu'}=\langle S_p{}^\mu S_p{}^{\mu'}\rangle_T-\langle S_p{}^\mu\rangle_T\langle S_p{}^{\mu'}\rangle_T.\tag{12}$$

Let the external magnetic field be weaker than the sublattice flopping field. Then, depending on the value of D, there are two possibilities: a) A phase transition to a nonergodic state in which only the longitudinal components of the tensor $q^{\mu\nu}$ differ from zero (if D exceeds a certain value determined by the relation between the mean values of the exchange intergrals and their variances). b) Nonergodicity and freezing of the transverse spin components take place simultaneously at the transition point, i.e., $q_p^{xx}=q_p^{yy}\neq0$ below T_g (Refs. 17 and 18). In either case, however, if the field is weaker than the sublattice-flopping field, we have $q_p^{\mu\nu}=0$ $(\mu,\nu=x,y)$ at $T\geqslant T_g$, and therefore the system (11) breaks up into three systems for the longitudinal, transverse, and mixed components:

$$\frac{V^2}{T^2}\langle(\chi_p{}^{\mu\mu})^2\rangle_cx_p{}^{\mu\mu}+\frac{J^2}{T^2}\langle(\chi_p{}^{\mu\mu})^2\rangle_cx_{p'}{}^{\mu\mu}=(1-\lambda)x_p{}^{\mu\mu},\tag{13}$$

$$\frac{V^2}{T^2}\langle\chi_p{}^{xx}\chi_p{}^{zz}\rangle_cx_p{}^{xz}+\frac{J^2}{T^2}\langle\chi_p{}^{xx}\chi_p{}^{zz}\rangle_cx_{p'}{}^{xz}=(1-\lambda)x_p{}^{xz}.\tag{14}$$

Since

$$\langle\chi_p{}^{xx}\chi_p{}^{zz}\rangle_c<\max(\langle(\chi_p{}^\perp)^2\rangle_c,\langle(\chi_p{}^\parallel)^2\rangle_c),$$

it can be seen that the "danger" to the ergodic state lies in the eigenvalues obtained from the solution of one of the systems (13). Putting $\lambda=0$ in the systems (13), we obtain from their solvability condition equations for the temperature of the transition to the nonergodic state:

$$T_\mu{}^2={}^1/_2\{V^2[\langle(\chi_1{}^{\mu\mu})^2\rangle_c+\langle(\chi_2{}^{\mu\mu})^2\rangle_c]+[V^4(\langle(\chi_1{}^{\mu\mu})^2\rangle_c-\langle(\chi_2{}^{\mu\mu})^2\rangle_c)^2+4J^4\langle(\chi_1{}^{\mu\mu})^2\rangle_c\langle(\chi_2{}^{\mu\mu})^2\rangle_c]^{1/2}\}.\tag{15}$$

The transition to the nonergodic phase takes place at the higher of the two temperatures ($T_\parallel$ and $T_\perp$) defined by Eq. (15).

It is known that in pure and ferromagnetic spin glasses a magnetic field always suppresses the nonergodic phase. It was shown in Refs. 13 and 14 that in a frustrated Ising antiferromagnet the temperature T_g can increase with the magnetic field, i.e., an external field expands the region of existence of the nonergodic phase. The reason is that the magnetic field decreases the total field acting on one of the sublattices. It can be seen from (15) that this conclusion remains valid also for a Heisenberg antiferromagnet, regardless of which of the temperatures ($T_{\parallel}$ or $T_{\perp}$) coincides with T_g. Let, for example, the interaction between the sublattices be non-fluctuating ($J/J_0 \to 0$, $J_0 = \text{const}$). Then

$$T_\mu^2 = V^2 \max(\langle(\chi_1^{\mu\mu})^2\rangle_c, \quad \langle(\chi_2^{\mu\mu})^2\rangle_c).$$

The sublattic with the highest susceptibility will be the one acted upon by a molecular field directed counter to the external one. Since this susceptibility increases with increase of the external field, T_g likewise increases with H.

The character of the $T_g(H)$ dependence in various limiting cases, both before and after the sublattice flopping, will be considered in detail below. We shall see that T_g can increase with the field also in the case when the principal role is played by fluctuations of the interaction between the sublattices.

After the sublattice flopping, when their magnetizations are noncollinear, the analysis of the system (11) is extremely complicated. A complete analysis, capable of constructing a phase diagram in fields both higher and lower than the flopping field, will therefore be carried out for the $T_g(H)$ dependence in the simplest case of a weakly frustrated antiferromagnet.

3. PHASE DIAGRAM IN THE CASE OF WEAK DISORDER

We begin the study of the phase diagram with a calculation of the effective energies of the anisotropy b_p. It is easily seen that if J, $V \ll J_0 + V_0$, and the temperature $T \ll T_N = J_0 + V_0$, in the configuration averaging the main contribution to the integrals for $\hat{q}$ and $\hat{Q}$ are made by values $t \approx 1$, so that the terms containing J and V can be neglected in the calculation of $\hat{q}$ and $\hat{Q}$. It turns out then that

$$Q_p^{\parallel} - q_p^{\parallel} \approx T^2 (J_0 + V_0)^{-2},$$

and $Q_p^{\perp} \approx T(J_0 + V_0)^{-1}$. Recall that at $T \gg T_g$ the transverse component $q_p^{\perp} = 0$. As a result we have

$$b_p = D - U^2/U_0,$$

where

$$U^2 = J^2 + V^2, \quad U_0 = J_0 + V_0.$$

In the following subsections a) and b) of the present section we assume initially that $D > U^2/U_0$. This restriction on D is not stringent and can be met even if $D \ll J$, $V \ll J_0$, V_0; we have thus an effective anisotropy of the "easy axis" type: $b_p \approx D > 0$.

a) Let the magnetic field be weaker than the flopping field $H_c = 2S(DJ_0)^{1/2}$. It can be shown that since the effective anisotropy $b_p > 0$, the transverse correlator is $\langle \chi_\perp^2 \rangle_c \sim (T/D)^2$ and the eigenvalue λ_T does not vanish at any temperature. Thus, T_g coincides with $T_{\parallel}$ of (15). We shall show presently that T_g is exponentially smaller than J or V, meaning, also D. In the calculation of the thermodynamic mean values in expressions of type (9) for $\chi_{\parallel}$ it is therefore necessary to integrate with respect to S near the values $S_z = \pm S$. Carrying out next configuration averaging just as in the Ising model, we obtain an expression for T_g (Ref. 19):

$$T_g(H) = T_g(0) \exp\left(-\frac{H^2}{2U^2S^2}\right) \times \frac{1}{U^2}\left\{V^2 \operatorname{ch}\frac{HU_0}{U^2S} + \left(J^4 + V^4 \operatorname{sh}^2\frac{HU_0}{U^2S}\right)^{1/2}\right\},$$

$$T_g(0) = \frac{2}{3}\left(\frac{2}{\pi}\right)^{1/2} US^2 \exp\left(-\frac{U_0^2}{2U^2}\right). \qquad (16)$$

If

$$V \gg J \exp(-H_cU_0/2U^2S + H_c^2/4U^2S^2), \qquad (17)$$

then

$$T_g(H_c) = T_g(0)\, V^2U^{-2} \exp[H_cU_0/U^2S - H_c^2/2U^2S^2] \qquad (18)$$

and $T_g(H_c) \gg T_g(0)$, i.e., the temperature of the transition to the nonergodic state increases strongly with H. Since $D \ll J_0$, the criterion (17) is almost always satisfied. If, however, $V > J^2/U_0$, then T_g increases monotonically with the field all the way to the flopping.

b) The magnetic field exceeds the flopping field. Since the disorder is weak, the sublattice magnetization is the same as in a pure antiferromagnet:

$$m_1^z = m_2^z = SH/H_0, \quad m_1^x = -m_2^x = S(1 - H^2/H_0^2)^{1/2}, \quad m_p^y = 0,$$

and the sublattice collapse field is $H_0 = 2(J_0 - D)S$.

The tensor $\hat{q}$ is not diagonal in this case, and in view of the small disorder we have

$$q_p^{\mu\nu} = m_p^\mu m_p^\nu. \qquad (19)$$

Since the matrices $\hat{q}_p$ cannot be inverted, it is inconvenient to use Eq. (4). It is simplest to turn again to Eq. (A8). Expressing the second sum in (A9) in the form

$$\frac{1}{2}\sum_{p \neq p'} \frac{J^2}{2T}\left(\sum_\alpha \mathbf{S}_p^\alpha \mathbf{m}_{p'}^\alpha\right)^2 + \frac{V^2}{2T}\left(\sum_\alpha \mathbf{S}_p^\alpha \mathbf{m}_p^\alpha\right)^2, \qquad (20)$$

we obtain for the effective Hamiltonian, with the aid of the transformation (A9) for a scalar quantity, the expression (5), but now the fields $\mathbf{a}_p$ are equal to

$$\mathbf{a}_p = \mathbf{H} - J_0\mathbf{m}_{p'} + V_0\mathbf{m}_p + Jt_{p'}\mathbf{m}_{p'} + Vt_p\mathbf{m}_p, \quad p' \neq p. \qquad (21)$$

We assume as before that $D > U^2/U_0$ and therefore retain only D in the effective anisotropy. The same considerations as above lead to the conclusion that the phase transition is connected with the longitudinal spin correlator. Since $D \gg T$, we obtain by using the saddle-point method to calculate the thermodynamic mean values

$$\langle(\chi_1^{zz})^2\rangle_c = \langle(\chi_2^{zz})^2\rangle_c = S^4\left\langle \xi_0^4 \operatorname{ch}^{-4}\frac{a_z\xi_0S}{T}\right\rangle_c, \qquad (22)$$

where the saddle-point value ξ_0 of the cosine of the angle between the spin $\mathbf{S}$ and the z axis is given by

$$2DS\xi_0 + a_z - \xi_0 a_x(1 - \xi_0^2)^{-1/2} = 0. \qquad (23)$$

The transition temperature (15) is equal to

$$T_g^2 = U^2\langle(\chi_2^{zz})^2\rangle_c = U^4S^4\left\langle \xi_0^4 \operatorname{ch}^{-4}\frac{a_z\xi_0S}{T}\right\rangle_c. \qquad (24)$$

Before giving the calculated T_g, we advance qualitative arguments that explain the character of the $T_g(H)$ dependence after the flopping. The main contribution to (22) is made by configurations in which $a_z\xi_0 \approx T$. It can be seen from (24) that in all cases the dependence of the characteris-

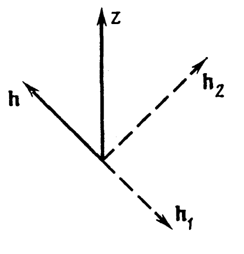

FIG. 1. Fields acting on the spins of the first sublattice. The thick arrow shows the average total field $\mathbf{h} = \mathbf{H} - J_0\mathbf{m}_2 + V_0\mathbf{m}_1$, the dashed arrows shows the molecular fields that fluctuate in space, from the directions of the spins of the first ($h_1 = Vq^{1/2}t_2$) and second ($h_2 = jq^{1/2}t_1$) sublattices.

tic values of ξ_0 on T is in all cases weaker that $T^{1/2}$ otherwise Eq. (24) has no solutions. It can be verified by direct calculations that ξ_0 does not depend on T at all, meaning that $a_z \approx T$. If we now neglect a_z in (23), we find that

$$\xi_0 = \begin{cases} 0, & a_x > 2DS \\ 1 - a_x^2/(2DS)^2, & a_x < 2DS \end{cases}. \tag{25}$$

Thus, if $J_0 m_x \gg 2DS$, the configurations of importance are those in which not only a_z but also a_x is much smaller than in a pure antiferromagnet. Random fields that can lead to such a weakening of the field **a** are shown in Fig. 1. It can be seen from the figure, just as from Eq. (21), that at $V = 0$ these conditions cannot be met simultaneously. This means that if there are no fluctuations of the exchange between the sublattices, the temperature T_g jumps down to zero at $H = H_c$ and is equal to zero all the way to fields close to H_0 when the condition $J_0 m_x \gg 2DS$ is not met. In such strong fields, a_x is small in all the configurations, so that Eq. (24) can be satisfied. The temperature T_g increases with H in such strong fields up to $H = H_0$, and then drops just as in a ferromagnet.

If $V \neq 0$, the conditions under which the components of the vector **a** are small can be met. The temperature T_g is therefore no longer zero in the flopped phase far from the collapse field, although the $T_g(H)$ plot does cross the spin-flop curve at lower temperatures than in the collinear phase (see Fig. 2). With increase of field, m_x decreases and the transition temperature increases monotonically in the entire field interval $H_c < H < H_0$.

The actual calculation of the configuration mean values leads to the following relations.

For fields far from H_0, when the following condition is met:

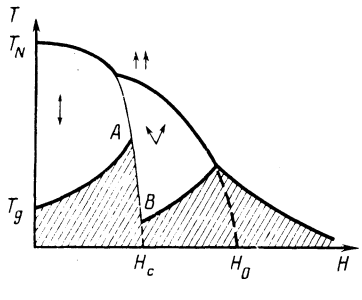

FIG. 2. Phase diagram on (T,H) plane. The shaded region of the diagram corresponds to the nonergodic phase under the $T_g(H)$ line. The thin line is a first-order phase-transition curve that coincides with $T_g(H)$ on the segment AB. The dashed lines are the suggested phase-transition lines in the nonergodic phase.

$$\eta_0 = m_x\left(\frac{J^2 U_0}{DU^2} + \frac{V^2 - J^2}{U^2}\right) > 1,$$

the transition temperature is

$$T_g = \left(\frac{2}{\pi}\right)^{1/2} \frac{U^2 S^{1/2}}{J m_z} \frac{V^4 m_x^{3/2}}{D^{1/2} U_0^{5/2}}$$
$$\times \exp\left[-\frac{1}{2V^2}\left(U_0 - D\left(1 + \frac{S}{m_x}\right)\right)^2 - \frac{D^2}{2J^2}\left(1 - \frac{S}{m_x}\right)^2\right], \tag{26}$$

and close to H_0, when $\eta_0 < 1$,

$$T_g = \frac{2}{3}\left(\frac{2}{\pi}\right)^{1/2} US^2(1-\eta_0)^{3/2} \exp\left[-\frac{(U_0 - 2D)^2}{2U^2}\right]. \tag{27}$$

The general form of the phase diagram is shown in Fig. 2. It can be seen that (26) that, accurate to the leading term in the argument of the exponential,

$$T_g^{>}(H_c) \equiv T_g(H \to H_c + 0) \approx \exp(-U_0^2/2V^2) \tag{28}$$

and is consequently exponentially smaller than $T_g(0)$. At the same time $T_g^{<} \equiv T_g(H \to H_c - 0)$ exceeds $T_g(0)$, so that at $H = H_c$, in accordance with the qualitative arguments advanced above, $T_g(H)$ decreases jumpwise. If $V = 0$, the temperature $T_g^{>}(H_c)$ goes to zero.

It follows from (26) that the transition temperature $T_g(H)$ increases with the field exponentially in almost the entire interval from H_c to H_0, except for a narrow interval near H_0 where m_x is small: $m_x < DU^2/U_0 J^2 \ll 1$. Near H_0 the temperature $T_g(H)$ increases according to (27) as a power of H. At the intersection point of the $T_g(H)$ and $T_N(H)$ curves Eq. (27) is continued by the known expression of de Almeida and Thouless[16] for the temperature of the transition to the paramagnetic state. In our case this expression is of the form

$$T_g(H > H_0) = \frac{2}{3}\left(\frac{2}{\pi}\right)^{1/2} US^2 \exp\left(-\frac{(H - J_0 S + V_0 S)^2}{2U^2 S^2}\right). \tag{29}$$

It is seen from (27) and (29) that the $T_g(H)$ curve has a kink at the point where it crosses the $T_N(H)$ curve. This fact was noted by us earlier[14] for the Ising model. The ratio

$$T_g(H_0)/T_g(0) \approx \exp[(2D(U_0 - D))U^{-2}] \gg 1$$

obtains independently of the relation between J and V.

It follows from our results that in systems where the long-range magnetic order changes jumpwise the transition to the spin-glass state can also be jumplike rather than of third order as usual. The jumplike onset of nonergodicity on the phase diagram of Fig. 2 should take place on going from flopped antiferromagnetic to collinear on section AB of the phase diagram.

c) Similar calculations show that the relation between D and $U^2 S/U_0$ is of no importance for the general character of the $T_g(H)$ dependence. Although the transition temperature $T_g(H)$ at $D \ll U^2 S/U_0$ is determined not by the longitudinal but by the transverse susceptibility, it increases with the field as before both prior to and after the sublattice flopping. In the field H_c, the temperature $T_g(H)$ drops jumpwise.

The behavior of $T_g(H)$ in the case of weak anisotropy will be considered in greater detail in the next section for the opposite limiting case of a strongly frustrated antiferromagnet.

4. FIELD DEPENDENCE OF T_g IN A STRONGLY FRUSTRATED ANTIFERROMAGNET

If the anisotropy is weak, the transition temperature is determined by the transverse susceptibility. In a field $H < H_c$ (the question of the value of H_c in a strongly frustrated antiferromagnet will be considered below) the transition temperature is determined by Eq. (15), in which $\langle (\chi_p^{\mu\mu})^2\rangle_c$ must be replaced by[1)]

$$\langle(\chi_p{}^\perp)^2\rangle_c = \frac{1}{2}\int_{-\infty}^{\infty}\frac{dt}{\sqrt{2\pi}}e^{-t^2/2}\left\{\int_{-3^{1/2}}^{3^{1/2}} dS^z(3-(S^z)^2)\times\exp\left(\frac{a_p}{T}S^z+\frac{b_p}{T}(S^z)^2\right)\times\left[\int_{-3^{1/2}}^{3^{1/2}} dS^z\exp\left(\frac{a_p}{T}S^z+\frac{b_p}{T}(S^z)^2\right]^{-1}\right\}^2, \quad (30)$$

where

$$a_p = H - J_0 m_{p'}{}^z + V_0 m_p{}^z + (J^2 q_{p'}{}^{\parallel} + V^2 q_p{}^{\parallel})^{1/2} t,$$
$$b_p = D + \frac{J^2}{2T}(X_{p'} - q_{p'}{}^{\parallel}) + \frac{V^2}{2T}(X_p - q_p{}^{\parallel}),$$
$$X_p = \frac{1}{2}(Q_p{}^{\parallel} - 1). \quad (31)$$

We use in (31) the fact that $\Sigma_\mu Q_p^{\mu\mu} = 3$.

It follows from (30) and (31) that in the case of strong disorder, when $\rho = (U_0 - U)/U_0 \ll 1$, we have in weak fields

$$m_{1,2}^z = \pm 2^{1/2}\rho + H\frac{U_0}{2J_0}\left(1-\frac{\rho}{2J_0}\right) \mp \frac{H^2}{8\cdot 2^{1/2}\rho J_0^2},$$
$$q_{1,2}^{\parallel} = \rho \pm H\frac{U^2 A(U_0^2\rho J^{-2})}{2^{1/2}J_0^2 J^2}\rho + 0.333\frac{H^2}{J_0^2},$$
$$q_{1,2}^{\perp} = 0,$$
$$X_{1,2} = D + \frac{\rho^2}{2} \pm \frac{H}{J_0}\frac{2^{1/2}U^2\rho}{5U_0^2+3(J^2-V^2)} + 0.333\rho\frac{H^2}{J_0^2}. \quad (32)$$

The function $A(U_0^2\rho/J^2)$, whose exact form will not be written out, takes on in limiting cases the values

$$A\left(\frac{U_0^2\rho}{J^2}\right) \approx \begin{cases} 1, & U_0^2\rho \ll J^2 \\ \rho^{-1}, & U_0^2\rho \gg J^2. \end{cases}$$

Substituting (32) in (31) and (30) and expanding the arguments of the exponentials in powers of a_p/T and b_p/T, we get

$$\langle(\chi_{1,2}^{\perp})^2\rangle_c = 1 - 2D - \frac{23}{25}\rho^2 - 2^{3/2}\frac{H\rho}{J_0}\left[\frac{1}{5U_0^2+3(J^2-V^2)} + \frac{6\rho(J^2-V^2)}{25J^2}A\left(\frac{\rho U_0^2}{J^2}\right)\right] - 0.614\rho\frac{H^2}{J_0^2}. \quad (33)$$

It follows from (33) and (15) that the temperature $T_g(H)$ increases with the field if $J \ll V\rho^{1/2}$ and decreases in the case of the opposite inequality. From this and from the results of the preceding section it follows that in real systems, when $U \approx U_0$ and $\rho \approx 1$, the temperature T_g in a collinear antiferromagent increases with the field if $J \lesssim V$, regardless of whether the replica symmetry is broken first for the longitudinal or transverse components of the tensor q. Note that in the considered case $\rho \ll 1$ the relative change of the temperature T_g in the field is small: with parametric accuracy, T_g is independent of the field all the way to $H = H_c$.

5. MAGNETIC PROPERTIES OF ERGODIC PHASE

a) Near the Néel temperature $T_N = U_0$, when the quantities $m_{1,2}$, $q_{1,2}$, and $X_{1,2}$ are small, we have for the longitudinal and transverse susceptibilities of a weakly ionized antiferromagnet

$$\chi_{\parallel,\perp} = \Pi_{\parallel,\perp}/[T + (J_0 - V_0)\Pi_{\parallel,\perp}]. \quad (34)$$

Here

$$\Pi_\parallel = 1 - \frac{1}{T^2}\left[U^2 q_0 - \frac{6}{5}U^2 X_0 + \frac{3}{5}U_0^2 m_0^2 - \frac{2U_0^2 m_0^2(J^2-V^2)}{U^2+J^2-V^2} + \frac{12}{5}\frac{U_0^2 m_0^2(J^2-V^2)}{5U^2+3(J^2-V^2)}\right], \quad (35)$$

$$\Pi_\perp = 1 - \frac{3}{5}\frac{U^2}{T^2}X_0 - \frac{U_0^2}{5T^2}m_0^2, \quad (36)$$

where m_0, q_0, and X_0 are the magnetization, the Edwards-Anderson parameter, and the sublattice-quadrupolarity parameter in a zero magnetic field.

Simple calculations show that

$$q_0 = \frac{(\tilde{U}_0 m_0)^2}{1-\tilde{U}^2}, \quad X_0 = \frac{(\tilde{U}_0 m_0)^2}{5-3\tilde{U}^2}, \quad \tau \equiv \frac{T_N - T}{T},$$
$$(\tilde{U}_0 m_0)^2 = 5(1-3\tilde{U}^2)(1-\tilde{U}^2)(5+11\tilde{U}^2-6\tilde{U}^4)^{-1}\tau. \quad (37)$$

The tilde marks quantities measured in units of T_N. We have hence for the susceptibilities

$$\chi_\parallel = \frac{1}{2J_0}\left\{1 + \tau\frac{U_0}{J_0}\frac{(1-\tilde{U}^2)(5-3\tilde{U}^2)}{5+11\tilde{U}^2-6\tilde{U}^4}\frac{6\tilde{W}^4+11\tilde{W}^2-5}{(1+\tilde{W}^2)(5+3\tilde{W}^2)}\right\},$$
$$\tilde{W}^2 \equiv \tilde{J}^2 - \tilde{V}^2, \quad (38)$$

$$\chi_\perp = \frac{1}{2J_0}\left\{1 + \frac{\tilde{U}^2(8-3\tilde{U}^2)}{5+11\tilde{U}^2-6\tilde{U}^4}\tau\right\}. \quad (39)$$

At $V = V_0 = 0$ Eqs. (38) and (39) coincide with those obtained by us earlier.[20]

The coeffficient of τ in (39) is positive at any value of $\tilde{U}$, i.e., at any degree of disorder. Consequently, the transverse susceptibility of a frustrated antiferromagnet in the ordered phase always increases with decrease of temperature, at any rate near T_N, and if $\tilde{U} \approx 1$ the coefficient of τ contains no small quantities whatever. The increase of the transverse susceptibility in disordered antiferromagnets were experimentally observed in Refs. 5–7 and 15 in the interval $T_N > T > T_g$.

The behavior of the longitudinal susceptibility is more complicated. The coefficient of τ in (38) is positive when $\tilde{W}^2 > 0.35$. Since the condition for the existence of an antiferromagnetic phase is $\tilde{U}^2 \lesssim 1$, the sign of the coefficient of τ can be positive only if $\tilde{V}^2 < 0.31$. If these two inequalities are satisfied, i.e., the intersublattice interaction is sufficiently strongly frustrated, and the intrasublattice interaction sufficiently weakly, the longitudinal susceptibility also increases with decrease of the temperature near T_N. These conditions are quite stringent, so that in most cases the transverse susceptibility is expected to increase with lowering of the temperature, and the longitudinal to decrease.

b) Sublattice flopping field. The flopping field is determined by the condition that the collinear phase be stable to transverse fluctuations of the sublattice moments, i.e., by the pole of the transverse susceptibility $\chi_\perp$. In the ergodic phase, $q_\perp$ is always zero. For the sublattice susceptibilities $\chi_p^\perp$, which add up to $\chi^\perp$, we obtain therefore the equations

$$\chi_p{}^\perp=\frac{1}{2}\left(1-\frac{J_0}{T}\chi_{p'}{}^\perp+\frac{V_0}{T}\chi_p{}^\perp\right)Q_p{}^\perp,\quad p'\neq p, \tag{40}$$

whose solution has a pole at

$$\left(1-\frac{1}{2}\frac{V_0}{T}Q_1{}^\perp\right)\left(1-\frac{1}{2}\frac{V_0}{T}Q_2{}^\perp\right)-\frac{1}{4}\frac{J_0^2}{T^2}Q_1{}^\perp Q_2{}^\perp=0. \tag{41}$$

Relation (41) determines the flopping field.

Near T_N we obtain

$$H_c{}^2=12\frac{DJ_0T_N{}^2}{(U_0+2V_0)^2}\left[\frac{1}{1-\tilde U^2}-\frac{\tilde W^2(19+9\tilde W^2)(5-3\tilde U^2)}{5(1+\tilde W^2)(5+3\tilde W^2)}\right]^{-1}. \tag{42}$$

With increase of the disorder, the field H_c decreases. In strongly frustrated antiferromagnets, when $1-\tilde U^2\ll 1$, the flopping field is parametrically small:

$$H_c=\frac{2T_N(3DJ_0)^{1/2}}{U_0+2V_0}(1-\tilde U^2)^{1/2}. \tag{43}$$

The magnetic field region in which long-range antiferomagnetic order exists decreases simultaneously with increase of U. In weak magnetic fields, the derivative is

$$\frac{dT_N}{dH^2}=-\frac{(U_0+2V_0)^2}{16J_0{}^2U_0}\frac{1}{1-\tilde U^2}.$$

APPENDIX I

Introducing replicas in the usual fashion, we rewrite the expression for the free energy in the form

$$f=-T\lim_{\substack{n\to0\\N\to\infty}}\frac{1}{n}\left\{\int \mathrm{Sp}\exp\left[\frac{1}{T}\sum_{(i,j)}\sum_\alpha J_{ij}\mathbf{S}_{1i}{}^\alpha\mathbf{S}_{2j}{}^\alpha+\frac{1}{T}\sum_{(i,j)}\sum_{\alpha,p}V_{ij}\mathbf{S}_{pi}{}^\alpha\mathbf{S}_{pj}{}^\alpha+\frac{D}{T}\sum_{i,p,\alpha}(S_{pi}{}^{z\alpha})^2+\frac{H}{T}\sum_{i,p,\alpha}S_{pi}{}^{z\alpha}\right]\prod_{(i,j)}P(J_{ij})dJ_{ij}\prod_{(i,j)}P(V_{ij})dV_{ij}-1\right\}. \tag{A1}$$

The parentheses (i,j) denote different spin pairs, α is the replica index, n is the number of replicas, and $P(J_{ij})$ and $P(V_{ij})$ are distribution functions:

$$P(J_{ij})=\left(\frac{N}{2\pi J^2}\right)^{1/2}\exp\left(-\frac{(J_{ij}+J_0/N)^2}{2J^2}N\right),$$
$$P(V_{ij})=\left(\frac{N}{2\pi V^2}\right)^{1/2}\exp\left(-\frac{(V_{ij}-V_0/N)^2}{2V^2}N\right). \tag{A2}$$

Averaging over the distribution of the exchange integrals, we have

$$f=-T\lim_{\substack{n\to0\\N\to\infty}}\frac{1}{Nn}\left\{\mathrm{Sp}\exp\left[\frac{J_0}{2T}\sum_\alpha\left(\sum_{i,p}\mathbf{S}_{p,i}^\alpha\right)^2-\frac{J_0+V_0}{2T}\sum_\alpha\sum_p\left(\sum_i\mathbf{S}_{pi}{}^\alpha\right)^2+\frac{J^2}{4T^2}\sum_{\alpha,\beta}\left(\sum_{i,p}S_{pi}{}^{\mu\alpha}S_{pi}{}^{\nu\beta}\right)^2+\frac{V^2-J^2}{4T^2}\sum_{\alpha\beta,p}\left(\sum_i\sum_{\mu\nu}S_{pi}{}^{\mu\alpha}S_{pi}{}^{\nu\beta}\right)^2+\frac{D}{T}\sum_{i,p,\alpha}(S_{pi}{}^{z\alpha})^2+\frac{H}{T}\sum_{i,p,\alpha}S_{pi}{}^{z\alpha}\right]-1\right\}. \tag{A3}$$

Here $\mu,\nu=x,y,z$ are vector indices.

Applying the Hubbard-Stratonovich transformation, we obtain

$$f=-T\lim_{\substack{n\to0\\N\to\infty}}\frac{1}{Nn}\left\{\int\mathrm{Sp}\exp\left[-\frac{N}{2}\sum_p(\mathbf{x}_p{}^2+\mathrm{Sp}\,\hat y_p{}^2)-\frac{N}{2}(\mathbf{x}_3{}^2+\mathrm{Sp}\,\hat y_3{}^2)+i\left(\frac{J_0}{T}\right)^{1/2}\sum_i\mathbf{s}_{pi}\mathbf{x}_3+\left(\frac{J_0+V_0}{T}\right)^{1/2}\sum_{p,i}\mathbf{s}_{pi}\mathbf{x}_p+\frac{J}{2^{1/2}T}\sum_{p,i}\mathbf{s}_{pi}\hat y_3\mathbf{s}_{pi}+i\left(\frac{J^2-V^2}{2T^2}\right)^{1/2}\sum_{pi}\mathbf{s}_{pi}\hat y_p\mathbf{s}_{pi}+\frac{D}{T}\sum_{p,i}(s_{pi}{}^z)^2+\frac{H}{T}\sum_{p,i}s_{pi}{}^z\right]-1\right\}\prod_p d\mathbf{x}_p\,d\mathbf{x}_3\prod_p d\hat y_p\,d\hat y_3. \tag{A4}$$

Here $p=1$ or 2, and $\mathbf{x}_p$, $\mathbf{x}_3$, $\hat y_p$, $\hat y_3$ are vectors and tensors in the direct product of the replica and spin space, and $\mathbf{s}_{pi}$ is the spin vector in this space and has components $S_{pi}^{\mu\alpha}$. The integration is over all the components of the vectors $\mathbf{x}$ and of the tensors $\hat y$.

Summing over i, we express f in the form

$$f=-T\lim_{\substack{N\to\infty\\n\to0}}\frac{1}{Nn}\left\{\int\prod_p d\mathbf{x}_p\,d\mathbf{x}_3\prod_p d\hat y_p\,d\hat y_3\times\left(\frac{N}{2\pi}\right)^{9n(3n+1)/2}\exp[N\Phi(\mathbf{x}_p,\mathbf{x}_3,\hat y_p,\hat y_3)]-1\right\}, \tag{A5}$$

where

$$\Phi(\mathbf{x}_p,\mathbf{x}_3,\hat y_p,\hat y_3)=-\frac{1}{2}\sum_p(\mathbf{x}_p{}^2+\mathrm{Sp}\,\hat y_p{}^2)-\frac{1}{2}(\mathbf{x}_3{}^2+\mathrm{Sp}\,\hat y_3{}^2)+\ln\mathrm{Sp}\exp\left[i\left(\frac{J_0}{T}\right)^{1/2}\sum_p\mathbf{s}_p\mathbf{x}_3+\left(\frac{J_0+V_0}{T}\right)^{1/2}\sum_p\mathbf{s}_p\mathbf{x}_p+\frac{J}{T}\sum\mathbf{s}_p\hat y_3\mathbf{s}_p+i\left(\frac{J^2-V^2}{T^2}\right)^{1/2}\sum_p\mathbf{s}_p\hat y_p\mathbf{s}_p+\frac{D}{T}\sum_p(s_p{}^z)^2+\frac{H}{T}\sum_p s_p{}^z\right]. \tag{A6}$$

We calculate the free energy by the saddle-point method in the replica-symmetric approximation, i.e., assuming that in the saddle point the components $y_{p,\alpha\alpha}^{\mu\nu}$ and $y_{3,\alpha\alpha}^{\mu\nu}$ which are diagonal in the replica are independent of α, while $y_{p,\alpha\beta}^{\mu\nu}$ and $y_{3,\alpha\beta}^{\mu\nu}$ are independent of the choice of the (α,β) pair. Introducing next the quantities

$$\mathbf{m}_p=\left(\frac{T}{J_0-V_0}\right)^{1/2}\mathbf{x}_p{}^\alpha,\qquad q_p{}^{\mu\nu}=-i\left(\frac{2T}{J^2-V^2}\right)^{1/2}y_{p,\alpha\beta}^{\mu\nu},$$
$$Q_p{}^{\mu\nu}=-i\left(\frac{2T}{J^2-V^2}\right)^{1/2}y_{p,\alpha\alpha}^{\mu\nu}$$

and similarly for x_3 and y_3, we obtain the following saddle-point equations:

$$\mathbf{m}_p = \lim_{n\to 0}\frac{1}{n}\sum_{\alpha}\langle \mathbf{S}_p{}^\alpha\rangle,$$

$$q_p{}^{\mu\nu} = \lim_{n\to 0}\sum_{(\alpha,\beta)}\frac{2}{n(n+1)}\langle S_{p,\alpha}S^{\nu}_{p,\beta}\rangle,$$

$$Q_p{}^{\mu\nu} = \lim_{n\to 0}\sum_{\alpha}\frac{1}{n}\langle S^{\mu}_{p,\alpha}S^{\nu}_{p,\alpha}\rangle. \qquad (A7)$$

The averaging in (A7) is carried out with the effective energy

$$\Psi=-\sum_{p\neq p'}\sum_{\alpha}\mathbf{S}_p{}^\alpha(\mathbf{H}+V_0\mathbf{m}_p-J_0\mathbf{m}_{p'})$$

$$-\sum \mathbf{S}_p{}^\alpha\left(\frac{J^2}{2T}\hat{q}_{p'}+\frac{V^2}{2T}\hat{q}_p\right)\mathbf{S}_p{}^\beta$$

$$-\sum_{p\neq p'}\mathbf{S}_p{}^\alpha\left(\frac{J^2}{2T}(\hat{Q}_{p'}-\hat{q}_{p'})+\frac{V^2}{2T}(\hat{Q}_p-\hat{q}_p)\right)\mathbf{S}_p{}^\alpha-D\sum_{\alpha,p}(S_p{}^{z\alpha})^2. \qquad (A8)$$

Expression (A8) can be so transformed that only the spin variables pertaining to one replica are left. We use for this purpose the identity

$$\exp\frac{1}{2}\mathbf{x}\hat{\Lambda}\mathbf{x}=\frac{(\det\hat{\lambda})^{1/2}}{(2\pi)^{3/2}}\int e^{\mathbf{t}\mathbf{x}-{}^1/{}_2\mathbf{t}\hat{\lambda}\mathbf{t}}\,dt, \qquad (A9)$$

where $\hat{\lambda}=\hat{\Lambda}^{-1}$. Taking after the transformation the limit as $n\to 0$, we obtain the system (2)–(4) given in the text for the equations of state of a frustrated antiferromagnet.

APPENDIX 2

According to (A6), the matrix $\hat{M}$ has the following elements:

$$M^{(\alpha'\beta'),\mu'\nu',3}_{(\alpha\beta),\mu\nu,3}=-\frac{\partial^2\Phi}{\partial y^{\mu\nu}_{3,\alpha\beta}\partial y^{\mu\nu}_{3,\alpha'\beta'}}=\delta_{(\alpha\beta),(\alpha'\beta')}\delta_{\mu\nu,\mu'\nu'}$$

$$-\frac{J^2}{2T^2}\sum_{pp'}(\langle S^{\alpha}_{p\mu}S^{\nu}_{p\beta}S^{\mu'}_{p'\alpha'}S^{\nu'}_{p'\beta'}\rangle_T-\langle S^{\mu}_{p\alpha}S^{\nu}_{p\beta}\rangle_T\langle S^{\mu'}_{p\alpha'}S^{\nu'}_{p\beta'}\rangle_T), \qquad (A10)$$

$$M^{\alpha'\beta',\mu'\nu',p'}_{\alpha\beta,\mu\nu,p}=\delta_{\alpha\beta,\alpha'\beta'}\delta_{\mu\nu,\mu'\nu'}\delta_{pp'}$$

$$+\frac{J^2-V^2}{2T^2}(\langle S^{\mu}_{p\alpha}S^{\nu}_{p\beta}S^{\mu'}_{p'\alpha'}S^{\nu'}_{p'\beta'}\rangle_T-\langle S^{\mu}_{p\alpha}S^{\nu}_{p\beta}\rangle_T\langle S^{\mu'}_{p'\alpha'}S^{\nu'}_{p'\beta'}\rangle_T), \qquad (A11)$$

$$M^{(\alpha'\beta'),\mu'\nu',p}_{(\alpha\beta),\mu\nu,3}=-i\frac{J(J^2-V^2)^{1/2}}{2T^2}\sum_{p'}(\langle S^{\alpha}_{p'\mu}S^{\beta}_{p'\nu}S^{\alpha'}_{p\mu'}S^{\beta'}_{p\nu'}\rangle_T$$

$$-\langle S^{\alpha}_{p'\mu}S^{\beta}_{p'\nu}\rangle_T\langle S^{\alpha'}_{p\mu'}S^{\beta'}_{p\nu'}\rangle_T). \qquad (A12)$$

The symmetry of these matrix elements in replica space leads, just as in Refs. 16 and 17, to the following structure of the eigenfunctions $\eta^{\mu\nu}_{(\alpha\beta),p}$, of the matrix $\hat{M}$:

if all the (α,β) pairs take on a fixed value (γ,ε)

Therefore, in the limit as $n\to 0$ the problem reduces to a determination of the eigenvalues of a matrix $\hat{C}$ with elements

$$C^{\mu'\nu',p'}_{\mu\nu,p}=\delta_{p,p'}\left\{\delta_{\mu\nu,\mu'\nu'}+\frac{J^2-V^2}{2T^2}\langle\chi_p^{\mu\mu'}\chi_p^{\nu\nu'}\rangle_c\right\},$$

$$C^{\mu'\nu',3}_{\mu\nu,3}=\delta_{\mu\nu,\mu'\nu'}-\frac{J^2}{2T^2}\sum_p\langle\chi_p^{\mu\mu'}\chi_p^{\nu\nu'}\rangle_c.$$

$$C^{\mu'\nu',3}_{\mu\nu,p}=-i\frac{J(J^2-V^2)^{1/2}}{2T^2}\langle\chi_p^{\mu\mu'}\chi_p^{\nu\nu'}\rangle_c.$$

It can be shown that one of the eigenvalues is always equal to unity, and the others are determined by the system of equations (11) of the main text.

CONCLUSION

We have obtained in this paper general equations that describe a frustrated antiferromagnet and the stability limit of the ergodic state. Analysis of the phase diagram in simple limiting cases of weak and strong disorder has revealed the qualitative distinctive features of the behavior of the considered systems, features which we assume to be preserved also in a real situation. We have in mind here the increase of T_g with increase of the magnetic field both before and after the sublattice flopping, the jumplike decrease of $T_g(H)$ on intersecting the line of the magnetic first-order phase transitions, and the jumplike onset (vanishing) of nonergodicity on some segment of this line. These phenomena are brought about not by specific relations between the parameters, but by qualitative factors, viz., by the decrease of the total field on one of the sublattices prior to the flopping, and by the appearance, after the flopping, of a molecular-field component perpendicular to the anisotropy axis and to the external field.

Naturally, to observe these phenomena it is necessary to carry out the measurements on single crystals. Unfortunately, polycrystals were investigated in Ref. 10, where an increase of T_g with H was observed.

We discuss, finally, the role of random-field effects. It is known that a magnetic field $H>H_c$ applied parallel to the anisotropy axis of a disordered antiferromagnet leads to the appearance of effective Ising random fields that act on the antiferromagnetic order parameter **l** (Ref. 21). If these field are "turned on" at $T>T_N$, lowering the temperature leads to freezing of a metastable state without a long-range magnetic order.[22] The magnetic field must therefore be turned on after the transition into the antiferromagnetic phase. On the other hand if $H>H_c$, the external field is perpendicular to the antiferromagnetic order parameter, so that no random fields are produced at all. An investigation of the phase diagram in the flopped phase is from this viewpoint most desirable.

We note finally that the conclusion that the transverse susceptibility increases with decrease of temperature at $T\leqslant T_N$ is of general character. It was obtained, on the one hand, for an antiferromagnet with arbitrary disorder, and on the other, for the case when, just as after the flopping, no random fields capable of altering the character of the dependence of $\chi_\perp$ on T are produced.

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Translated by J. G. Adashko