Multivalley two-dimensional electron systems in a strong magnetic field

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The properties of a two-dimensional system in a strong magnetic field are studied with allowance for interaction between the particles. In the presence of two equivalent valleys the symmetry of the Hamiltonian is shown to determine in an essential way the properties of magnetic excitons. The character of symmetry depends on intervalley scattering of the particles and on the symmetry of the dispersion law $\varepsilon(\mathbf{k})$ near the minimum. In the case when intervalley scattering is predominant ($E_{ex}(0) < 0$), gapless collective excitations with a linear dispersion law occur. A similar situation may take place for electrons in a GaAs heterojunction.

INTRODUCTION

Two-dimensional electron system in a strong magnetic field H are the subject of intensive theoretical and experimental investigations (see, e.g., reviews, Ref. 1 and Ref. 2). Although the important role of the electron interaction is widely known (especially for the fractional filling of Landau levels), its rigorous account remains an unsolved problem.

In a recent paper[3] are discussed, properties of silicon-based MIS structures with surface orientation (110) such that when partial lifting of degeneracy is taken into account two-dimensional electrons have, owing to time reversal, two degenerate valleys[1] with energy minima at points $\pm\mathbf{Q}/2$, which we shall consider located on the y axis in the plane of the boundary (the z axis is the normal to the boundary). Electron-electron interaction plays a crucial role in forming the ground state of such a system, and two possibilities shall be considered: electrons fill only one valley or are evenly distributed between both valleys (in Ref. 3 this is a result of interaction between electrons and impurities). The aim of the present paper is, first, to point out a connection between Ref. 3 and earlier work,[4–9] in which magnetic excitons were studied in two-dimensional systems in a strong magnetic field, so that some striking properties of these excitons find a natural explanation in the symmetry analysis suggested in Ref. 3. Second, we investigate various physical effects not connected with the presence of impurities, but which determine the character of the ground state and the collective excitations in such a system. We obtain also concrete expressions for various physical quantities near $\nu = 1$ (ν is the filling factor of a Landau level). In addition, we show that a similar physical picture may be realized also in a heterojunction.

1. BASIC PHYSICAL MODEL

Our initial premise is that in the two-dimensional system there is a crystalline order leading to a dispersion law $\varepsilon(\mathbf{k})$ of two-dimensional electrons as a function of their quasimomentum $\mathbf{k}$, which assumes values in a two-dimensional Brillouin zone. The function $\varepsilon(\mathbf{k})$ is even ($\varepsilon(\mathbf{k}) = \varepsilon(-\mathbf{k})$) and has equal minima at $\mathbf{k} = (0, \pm\mathbf{Q}/2)$.

Assuming that the magnetic length $l_H = (\hbar c/eH)^{1/2}$ is much larger than the atomic distances of the order of Q^{-1}, we obtain the Hamiltonian of interacting two-dimensional electrons, neglecting exponentially small terms of order $\exp(-l_H^2 Q^2)$.

As is known (see, e.g., Ref. 10), in the presence of a vector potential $\mathbf{A}$ the appropriate basis consists of functions $\exp(i\mathbf{k}\cdot\boldsymbol{\rho})u_{\mathbf{k}-e\mathbf{A}/c}(\boldsymbol{\rho})$, where $u_{\mathbf{k}}(\boldsymbol{\rho})$ are Bloch functions of the two-dimensional crystal (everywhere below we are not interested in the dependence of the wave functions on the transverse coordinate assuming this dependence to be known). We shall try to find the wave functions in the magnetic field in the form $\psi(\boldsymbol{\rho}) = f(\boldsymbol{\rho})u_{\mathbf{k}-e\mathbf{A}/c}(\boldsymbol{\rho})$, where the function $f(\boldsymbol{\rho})$ satisfies the effective Schrödinger equation in the crystal (the electron charge is equal to $-e$):

$$\hat{\varepsilon}(-i\hbar\nabla + e\mathbf{A}/c)f(\boldsymbol{\rho}) = Ef(\boldsymbol{\rho}).$$

Here $\varepsilon(\mathbf{k})$ is a periodic function of its argument with the periods of the reciprocal lattice. For low-energy states only the minima of $\varepsilon(\mathbf{k})$ are relevant. These minima are located far away from each other, and, neglecting the overlap of the wave functions (of the order $\exp(-l_H^2 Q^2)$), we obtain the wave functions of the lowest levels in a magnetic field with a vector potential $\mathbf{A} = (0, Hx, 0)$ (in dimensionless units, i.e., $x \to l_H x$, $y \to l_H y$, $k_y \to k/l_H$):

$$\psi_{n,k,\tau}(\boldsymbol{\rho}) = L_y^{-1/2}\exp[iy(k+\tau Q/2)]\Phi_{n,\tau}(x+k)u_{0,\tau Q/2}(\boldsymbol{\rho}), \quad (1.1)$$

where the index n numbers Landau levels, k is the quasimomentum in the y-direction, the index $\tau = \pm 1$ numbers valleys, and L_y is the length of the sample in the y direction. The normalized functions $\Phi_{n,\tau}(x)$ are the oscillator functions, and $\Phi_{n,\tau}(x)$ and $\Phi_{n,-\tau}(x)$ coincide only in the parabolic approximation in $\varepsilon(\mathbf{k})$ near a minimum, while in the general case they satisfy the relation

$$\Phi_{n,\tau}(x) = \Phi_{n,-\tau}(-x). \quad (1.2)$$

since the function $\varepsilon(\mathbf{k})$ is even.

When the interaction is neglected, the energy of an electron depends only on the number of the Landau level, and we shall regard the quantity $\Delta E_n = E_{n+1} - E_n = \hbar\omega_c$ (ω_c is the cyclotron frequency defined by $\varepsilon(\mathbf{k})$) as large compared to the characteristic interaction energy ($e^2/l_H \ll \hbar\omega_c$), and restrict ourselves to electron states within the nth Landau level. In this case states with $n' < n$ are completely occupied, while states with $n' > n$ are empty.

Introducing Fermi creation and annihilation operators $\hat{a}_{k,\tau}^{+}$ and $\hat{a}_{k,\tau}$ (we omit the index n), we obtain for a pair interaction with a potential $V(r)$, after integrating over the variable y, the following expression for the interaction Hamiltonian ($L_y = 1$):

$$\hat{H}_{int}=\frac{(2\pi)^2}{2}\sum_{\substack{\{k,\tau\}\\ b_y,b_y',q_y}}\int V(x-x',y)$$

$$\times e^{-iq_y y}\,dy\,dx\,dx'\,\delta(k_1-k_4+(\tau_1-\tau_4)Q/2$$

$$-q_y-b_y)\,\delta(k_2-k_3+(\tau_2-\tau_3)Q/2+q_y-b_y')$$

$$\times\Phi_{n,\tau_1}(x+k_1)\,\Phi_{n,\tau_4}(x+k_4)$$

$$\times\Phi_{n,\tau_2}(x'+k_2)\,\Phi_{n,\tau_3}(x'+k_3)$$

$$\times N_{\tau_1\tau_4}(x,b_y)\,N_{\tau_2\tau_3}(x',b_y')\,\hat{a}^{+}_{k_1,\tau_1}\hat{a}^{+}_{k_2,\tau_2}\hat{a}_{k_3,\tau_3}\hat{a}_{k_4,\tau_4}, \qquad (1.3)$$

where b_y are reciprocal-lattice vectors, $\{k,\tau\}$ stands for $k_1\tau_1$, $k_2\tau_2$, $k_3\tau_3$, and $k_4\tau_4$, and the quantities $N_{\tau_i\tau_j}(x,b_y)$ are defined through the relation

$$u^{*}_{0,\tau_i Q/2}(\rho)\,u_{0,\tau_j Q/2}(\rho)=\sum_{b_y} e^{iyb_y}N_{\tau_i\tau_j}(x,b_y).$$

It is essential that when exponentially small terms are neglected, the arguments of the functions $\Phi_{n,\tau}(x+k)$ and $\Phi_{n,\tau'}(x+k')$ must be equal to within l_H. Therefore only terms with

$$\tau_1=\tau_4,\quad \tau_2=\tau_3;\quad \tau_1=\tau_3,\quad \tau_2=\tau_4, \qquad (1.4)$$

are relevant in (1.3), and $b_y=-b'_y$ (the quantity Q is incommensurate with b_y). Thus, with exponential accuracy, the interaction Hamiltonian is

$$\hat{H}_{int}=\frac{(2\pi)^2}{2}\sum_{\{k,\tau\},b_y,q_y}\int V(x-x',y)$$

$$\times e^{-iyq_y}\,dy\,dx\,dx'\,\delta(k_1+k_2-k_3-k_4)$$

$$\times\delta(k_1-k_4+(\tau_1-\tau_4)Q/2-q_y-b_y)\,\Phi_{n,\tau_1}(x+k_1)$$

$$\times\Phi_{n,\tau_4}(x+k_4)\,N_{\tau_1\tau_4}(x,b_y)$$

$$\times\Phi_{n,\tau_2}(x'+k_2)\,\Phi_{n,\tau_3}(x'+k_3)\,N_{\tau_2\tau_3}(x',-b_y)\,\hat{a}^{+}_{k_1,\tau_1}\,\hat{a}^{+}_{k_2,\tau_2}\,\hat{a}_{k_3,\tau_3}\,\hat{a}_{k_4,\tau_4} \qquad (1.5)$$

(when the relations (1.4) are taken into account).

Thus, it is seen that allowance for umklapp processes with exponential accuracy modifies only slightly the form of the interaction, without causing any qualitative changes (since the quasimomentum conservation law has the usual form). We therefore omit the umklapp processes and set $b_y=0$. The interaction may be expressed in the form

$$\hat{H}_{int}=\frac{1}{2}\sum_{k_1,k_2}\int\frac{d\mathbf{q}}{(2\pi)^2}[V(q)\,A_{11}\hat{a}_{k_1}{}^{+}\hat{a}_{k_2}{}^{+}\hat{a}_{k_2+q_y}\hat{a}_{k_1-q_y}$$

$$+V(q)\,A_{22}\hat{b}_{k_1}{}^{+}\hat{b}_{k_1}^{+}\hat{b}_{k_2+q_y}\hat{b}_{k_1-q_y}$$

$$+2V(q)\,A_{12}\hat{a}_{k_1}{}^{+}\hat{b}_{k_2}{}^{+}\hat{b}_{k_2+q_y}\hat{a}_{k_1-q_y}+2B_{12}\hat{a}_{k_1}{}^{+}\hat{b}_{k_2}{}^{+}\hat{a}_{k_2+q_y}\hat{b}_{k_1-q_y}], \qquad (1.6)$$

where the operators $\hat{a}_k=\hat{a}_{k,+1}$ and $\hat{b}_k=\hat{a}_{k,-1}$, $V(q)$ is the Fourier component of the interaction potential, and

$$A_{ij}=\int dx\,dx'\,e^{iq_x(x-x')}\Phi_{n,\tau_i}(x+k_1)$$

$$\times\Phi_{n,\tau_i}(x+k_1-q_y)\,\Phi_{n,\tau_j}(x'+k_2)$$

$$\times\Phi_{n,\tau_j}(x'+k_2+q_y)\,N_{\tau_i\tau_i}(x,0)\,N_{\tau_j\tau_j}(x',0). \qquad (1.7)$$

Let us show that $A_{11}=A_{22}$. To this end, we note that in the expansion

$$N_{\tau_i\tau_i}(x,0)\,N_{\tau_j\tau_j}(x',0)=\sum_{b_x,b_x'}e^{ib_x x+ib_x'x'}\tilde{N}_{\tau_i\tau_i}(b_x)\,\tilde{N}_{\tau_j\tau_j}(b_x'),$$

where b_x are reciprocal-lattice vectors, the contribution of the terms with $b_x\neq-b'_x$ to the integral (1.7) may be neglected with exponential accuracy. Further, performing the transformation

$$x=\xi-k_1+q_y/2,\qquad x'=\xi'-k_2-q_y/2,$$

we obtain

$$A_{\tau\tau}=\sum_{b_x}\exp[i(q_x+b_x)(k_2-k_1+q_y)]$$

$$\times\int\exp[i(q_x+b_x)(\xi-\xi')]\,d\xi\,d\xi'$$

$$\times\tilde{N}_{\tau\tau}(b_x)\,\tilde{N}_{\tau\tau}(-b_x)\,\Phi_{n,\tau}(\xi+q_y/2)\,\Phi_{n,\tau}(\xi-q_y/2)$$

$$\times\Phi_{n,\tau}(\xi'+q_y/2)\,\Phi_{n,\tau}(\xi'-q_y/2).$$

Interchanging variables $\xi\leftrightarrow-\xi'$ and taking into account that the quantities $\tilde{N}_{\tau\tau}$ do not depend on τ and the condition (1.2) is satisfied, we obtain $A_{11}=A_{22}$. Below we shall omit the factors $\tilde{N}_{\tau\tau}(b_x)$ with $b_x\neq0$, and regard the constants $\tilde{N}_{\tau\tau}(0)$ as included in the form of the interaction. In this case

$$A_{11}=A_{22}=\exp[iq_x(k_2-k_1+q_y)]\,|w_n(\mathbf{q})|^2, \qquad (1.8)$$

where

$$w_n(\mathbf{q})=\int e^{iq_x\xi}\,\Phi_{n,+1}(\xi+q_y/2)\,\Phi_{n,+1}(\xi-q_y/2)\,d\xi. \qquad (1.9)$$

In similar fashion it can be shown that the quantities A_{12} and B_{12} are given by

$$A_{12}=\exp[iq_x(k_2-k_1+q_y)]\,w_n^2(\mathbf{q}), \qquad (1.10)$$

$$B_{12}=\exp[iq_x(k_2-k_1+q_y)]\,V(\mathbf{q}+\mathbf{Q})\,|w_n(\mathbf{q})|^2. \qquad (1.11)$$

The last term in the Hamiltonian (1.6) may be transformed by using the intermediate integration

$$\int d\mathbf{q}B_{12}(\mathbf{q})\exp[iq_x(k_2-k_1+q_y)]\,\hat{a}_{k_1}{}^{+}\hat{b}_{k_2}{}^{+}\hat{a}_{k_2+q_y}\hat{b}_{k_1-q_y}$$

$$=\int B_{12}(\mathbf{q}_1)\exp[i\boldsymbol{\alpha}(\mathbf{q}-\mathbf{q}_1)]\frac{d\boldsymbol{\alpha}}{(2\pi)^2}\,d\mathbf{q}\,d\mathbf{q}_1$$

$$\exp[iq_x(k_2-k_1+q_y)]\,\hat{a}_{k_1}{}^{+}\hat{b}_{k_2}{}^{+}\hat{a}_{k_2+q_y}\hat{b}_{k_1-q_y}.$$

Carrying out integration over the variable q_x, we obtain the condition $k_2+q_y=k_1-\alpha_x$, and, relabeling variables, we combine the last two terms in the expression (1.6). Thus, the interaction Hamiltonian finally takes the form

$$\hat{H}_{int}=\sum_{k_1,k_2}\frac{1}{2}\int\frac{d\mathbf{q}}{(2\pi)^2}$$

$$\times e^{iq_x(k_2-k_1+q_y)}\{V_1(\mathbf{q})[\hat{a}_{k_1}{}^{+}\hat{a}_{k_2}{}^{+}\hat{a}_{k_3}\hat{a}_{k_4}+(\hat{a}\to\hat{b})]$$

$$+V_2(\mathbf{q})\,\hat{a}_{k_1}{}^{+}\hat{b}_{k_2}{}^{+}\hat{b}_{k_3}\hat{a}_{k_4}\},$$

$$k_3=k_2+q_2,\quad k_4=k_1-q_y, \qquad (1.12)$$

where the effective interaction potentials are

$$V_1(\mathbf{q})=V(q)\,|w_n(\mathbf{q})|^2,$$

$$V_2(\mathbf{q})=V(q)\,w_n^2(\mathbf{q})-\int\frac{d\mathbf{q}_1}{2\pi}V(\mathbf{q}_1+\mathbf{Q})\,|w_n(\mathbf{q}_1)|^2\,e^{i[\mathbf{q}\mathbf{q}_1]\mathbf{n}}. \qquad (1.13)$$

Here $\mathbf{n} = \mathbf{H}/H$.

The Hamiltonian (1.12) is valid with exponential accuracy in the variable $\exp(-l^2_H/a^2)$, where a is the atomic length. The concrete form of the quantities $V_{1,2}(\mathbf{q})$ depends on the form of the Bloch functions and an allowance for the umklapp processes. However, the symmetry properties of the Hamiltonian, which we shall discuss in the next section, do not depend on these functions and processes.

2. THERMODYNAMIC PROPERTIES OF THE SYSTEM

The difference between the effective interaction potentials V_1 and V_2 [Eq. (1.13)] is due to the transfer of a momentum of order Q and to the asymmetry of $\varepsilon(\mathbf{k})$ near an energy minimum. Both factors are powerlike small in the parameter a/l_H. The model with $V_1 = V_2$ is the most symmetric one, and admits an exact solution for the ground state of the system.[3,8] In this case, the Hamiltonian may be written in the form

$$\hat{H}^{(0)}_{int} = \frac{1}{2}\sum_{k_1,\,k_2}\int V_1(\mathbf{q})\frac{d\mathbf{q}}{(2\pi)^2} \times e^{iq_x(k_2-k_1+q_y)}(\hat{A}_{k_1}{}^{+}\hat{1}\,(\hat{A}_{k_2}{}^{+}\hat{1}\,\hat{A}_{k_2+q_y})\,\hat{A}_{k_1-q_y}), \tag{2.1}$$

where $\hat{A}_k = (\hat{a}_k, \hat{b}_k)$, and $\hat{1}$ is the unit matrix in the index τ. If one introduces, following Ref. 3, the "spin" operators

$$\hat{S}_i = \frac{1}{2}\sum_k(\hat{A}_k{}^{+}\hat{\sigma}_i\hat{A}_k), \quad i = x, y, z, \tag{2.2}$$

where $\hat{\sigma}_i$ are the Pauli matrices, then the Hamiltonian (2.1) commutes with all the operators $\hat{S}_i$.

One may choose as the ground state, e.g., the state with the maximum value of the operator

$$\hat{S}_z = \frac{1}{2}\sum_k(\hat{a}_k{}^{+}\hat{a}_k - \hat{b}_k{}^{+}\hat{b}_k), \tag{2.3}$$

when all electrons are in one valley and fill it completely. Owing to the isotropy (2.1) in the "isospin" space of the two valleys, any state obtainable as a result of a rotation in the isospin space has the same energy. The corresponding general expression for the wave function depends on two "angles" φ and θ and has the form[3]

$$\Psi_{gr} = \prod_k[\cos(\theta/2)\hat{a}_k{}^{+} + e^{i\varphi}\sin(\theta/2)\hat{b}_k{}^{+}]\,|0\rangle, \tag{2.4}$$

where $|0\rangle$ is the vacuum with respect to electrons.

If one proceeds from a state with a completely filled Landau level in one valley, these results may be interpreted in terms of "magnetic" excitons. Acting on the wave function by the operator

$$\hat{S}_- = \hat{S}_x - i\hat{S}_y = \sum_k \hat{b}_k{}^{+}\hat{a}_k, \tag{2.5}$$

we arrive at a state that contains one magnetic exciton with zero momentum, and for the isotropic Hamiltonian (2.1) the energy of formation of such an exciton is zero.

The Hamiltonians considered in this paper, are totally symmetric with respect to electrons in both valleys. In such systems a Bose condensate of excitons with zero momentum[4,8] corresponds to one of the degenerate states (2.4), and these excitons form an ideal noninteracting gas. This striking situation, for the first time observed in Ref. 4, is a consequence of symmetry—isotropy in the isospin space of a Hamiltonian of the form (2.1), which describes an electroneutral system, if one assumes $S_z = 0$ and an electron density equal to 1/2.

Our aim is the investigation of the thermodynamics and the collective excitations for the Hamiltonian (1.12) with $V_1 \neq V_2$. We note that such a Hamiltonian commutes with the operator $\hat{S}_z$, since it conserves the number of electrons in any valley. Depending on the sign of the energy of the exciton, there will be realized for a completely filled valley a ground state with either the minimum or the maximum value of S_z, and in view of the smallness of the terms that violate the complete isotropy, the account of the electron-electron interaction will be exact in the leading order in the small anisotropy.

The energy of formation of one exciton can be found by examining the commutator $[\hat{H}_{int}, \hat{S}_-]$. Omitting simple calculations (see also Ref. 7–9), we obtain the expression for the energy of an exciton with zero momentum:

$$E_{ex}(0) = \left\{\frac{1}{2\pi}V(Q) - \frac{1}{(2\pi)^2}\int V(\mathbf{q}+\mathbf{Q})\,|w_n(\mathbf{q})|^2\,d\mathbf{q}\right\} + 2\int\frac{d\mathbf{q}}{(2\pi)^2}V(q)\,[\operatorname{Im} w_n(\mathbf{q})]^2. \tag{2.6}$$

The term in braces may be readily computed in the parabolic approximation to the electron energy near a minimum:

$$\varepsilon(\mathbf{k}) = k_x^2/2m_x + k_y^2/2m_y.$$

In the case of silicon, for the plane (110), the masses are[1] $m_x = m_t$ and $m_y = (m_t + m_l)/2$. The oscillator function is

$$\Phi_n(x) = (\pi^{1/2}\cdot 2^n n!)^{-1/2}e^{-\beta^2x^2/2}H_n(\beta x), \tag{2.7}$$

where $\beta = (m_x/m_y)^{1/4}$, and $H_n(z)$ is a Hermite polynomial. Substituting this function in (1.9) and performing necessary integrations, we find that the first term of the expansion in the small parameter $(Ql_H)^{-1}$ of the expression for $E_{ex}(0)$ in the braces (2.6) takes for the potential $V(q) = 2\pi e^2/\varepsilon q$ the form

$$\frac{-e^2(2n+1)m_l\beta^2}{2\varepsilon m_t l_H}\frac{1}{(Ql_H)^3}. \tag{2.8}$$

Thus, intervalley interaction favors formation of excitons, while the nonparabolicity of the energy near the bottom of the valley (the last term in (2.6)) acts in the opposite direction. Although the first term is of the order $(a/l_H)^3$, and the second of the order $(a/l_H)^2$, numerical coefficients may change their ratio. It is very difficult to make a more exact statement, since the calculation in reality must include the wave functions in the direction of the normal to the surface, and also the true spectrum $\varepsilon(\mathbf{k})$ of two-dimensional electrons, which is not known accurately enough.

A heterojunction without inversion may be another example of energywise favorable formation of excitons. In this case the spin-orbit interaction lifts the two-fold degeneracy.[11,12] In the presence of a magnetic field the spectrum consists of two branches:

$$\varepsilon_s{}^{\pm} = \hbar\omega^*[s \pm (\delta^2 + \gamma^2 s)^{1/2}].$$

Here s are integers,

$$\omega^* = \frac{eH}{m^* c}, \quad \delta = \frac{1}{2}\left(1 - \frac{gm^*}{2m_0}\right), \quad \gamma^2 = \frac{4\Delta}{\hbar\omega^*},$$

where m^* is the effective mass, m_0 is the mass of a free electron, g is the g factor, and Δ is the parameter of spin-orbit coupling. For such a system, the part of the exciton energy due to the interaction of particles is equal to[13]

$$E_{ex}(0) \approx -\alpha e^2 \gamma^2 / \varepsilon l_H,$$

where α is a numerical parameter. This energy must be compared with the energy loss due the transition of an electron to an other energy level. In this case transitions within one energy branch (e.g., the transition $(s, +) \to (s+1, +)$) correspond to cyclotron resonance whose energy is much larger than the particle-interaction energy. Transitions between levels $(s, +) \to (s+1, -)$ correspond to the spin resonance, whose energy is

$$\hbar\omega_{SR} \approx \hbar\omega^* \frac{gm^*}{2m_0}.$$

For electrons in GaAs, $\hbar\omega_{SR} \ll \hbar\omega^*$, and presumably a situation with $E_{ex}(0) + \hbar\omega_{SR} < 0$ may occur, i.e., formation of excitons is energywise favored.

In the calculation of thermodynamic properties we shall consider hereafter the case when $E_{ex}(0) < 0$ and Bose condensation of electrons sets in. Corresponding calculations may be easily carried out in the generalized Hartree-Fock approximation. We note that when one Landau level is completely filled in the isotropic case (when $E_{ex}(0) = 0$) we know the exact wave function of the ground state (2.4), and calculations by the Hartree-Fock method will correspond to the first order of the perturbation theory in the small anistropy, which yields $E_{ex}(0) \neq 0$.

We introduce the temperature Green functions:

$$\begin{aligned}
G_{11}(k, \tau; k', \tau') &= -\langle T\hat{a}_k(\tau)\hat{a}_{k'}^{+}(\tau')\rangle = -\delta_{k,k'} g_{11}(\tau-\tau'),\\
G_{22}(k, \tau; k', \tau') &= -\langle T\hat{b}_k(\tau)\hat{b}_{k'}^{+}(\tau')\rangle = -\delta_{k,k'} g_{22}(\tau-\tau'),\\
G_{12}(k, \tau; k', \tau') &= -\langle T\hat{a}_k(\tau)\hat{b}_{k'}^{+}(\tau')\rangle = -\delta_{k,k'} g_{12}(\tau-\tau'),\\
G_{21}(k, \tau; k', \tau') &= -\langle T\hat{b}_k(\tau)\hat{a}_{k'}^{+}(\tau')\rangle = -\delta_{k,k'} g_{21}(\tau-\tau').
\end{aligned} \tag{2.9}$$

($\hat{a}_k(\tau) = e^{\tau\hat{H}}\hat{a}_k e^{-\tau\hat{H}}$, the averaging is over the Gibbs distribution).

Restricting oneself to the lowest orders of perturbation theory (or splitting the highest Green functions, as is done in the theory of superconductivity), it is not difficult to obtain the self-consistent-field equations:

$$\begin{aligned}
-\frac{\partial}{\partial\tau} g_{11}(\tau-\tau') &= \delta(\tau-\tau') + \xi_1 g_{11}(\tau-\tau') + \Delta g_{21}(\tau-\tau'),\\
-\frac{\partial}{\partial\tau} g_{22}(\tau-\tau') &= \delta(\tau-\tau') + \xi_2 g_{22}(\tau-\tau') + \Delta^* g_{12}(\tau-\tau'),\\
-\frac{\partial}{\partial\tau} g_{12}(\tau-\tau') &= \xi_1 g_{12}(\tau-\tau') + \Delta g_{22}(\tau-\tau'),\\
-\frac{\partial}{\partial\tau} g_{21}(\tau-\tau') &= \xi_2 g_{21}(\tau-\tau') + \Delta^* g_{11}(\tau-\tau').
\end{aligned} \tag{2.10}$$

Here, the following notation has been introduced:

$$\xi_{1,2} = -E\psi_{1,2} - E_Q\psi_{2,1} - \mu, \quad \Delta = -E_0\chi, \tag{2.11}$$

where μ is the chemical potential, and

$$\psi_1 = \langle \hat{a}_k^{+}\hat{a}_k\rangle, \quad \psi_2 = \langle \hat{b}_k^{+}\hat{b}_k\rangle, \quad \chi = \langle \hat{b}_k^{+}\hat{a}_k\rangle. \tag{2.12}$$

The energy parameters in (2.10) are

$$\begin{aligned}
E &= \int \frac{d\mathbf{q}}{(2\pi)^2} V(q)\,|w_n(\mathbf{q})|^2,\\
E_Q &= \int \frac{d\mathbf{q}}{(2\pi)^2} V(\mathbf{q}+\mathbf{Q})\,|w_n(\mathbf{q})|^2,\\
E_0 &= \int \frac{d\mathbf{q}}{(2\pi)^2} V(q)\,\{[\mathrm{Re}\, w_n(\mathbf{q})]^2 - [\mathrm{Im}\, w_n(\mathbf{q})]^2\} - \frac{V(Q)}{2\pi}.
\end{aligned} \tag{2.13}$$

We note that in the derivation of Eqs. (2.10) we subtracted, as usual, the energy of interaction with the background positive-charge and put $V(q=0) = 0$.

In the formulae presented above we regarded all the averages as independent of k, which corresponds to a spatially uniform state. However, it should be remembered that the momenta k are measured from the bottom of the corresponding valleys, which is equivalent to density modulation with the wave vector Q:

$$2\pi\langle\hat{\psi}^{+}(\rho)\hat{\psi}(\rho)\rangle = \psi_1 + \psi_2 + 2\,\mathrm{Re}(\chi e^{iQy}),$$

i.e., we have a charge density wave, and the condition $\psi_1 + \psi_2 = \nu$ yields only a spatially averaged density ν.

Equations (2.10) may readily be solved by a Fourier transform with respect to τ. As a result, we obtain ($\omega = \pi T(2n+1)$)

$$\begin{aligned}
g_{11(22)}(\omega) &= \frac{i\omega - \xi_{2(1)}}{(i\omega-\xi_1)(i\omega-\xi_2) - |\Delta|^2},\\
g_{12(21)}(\omega) &= \frac{\Delta(\Delta^*)}{(i\omega-\xi_1)(i\omega-\xi_2) - |\Delta|^2}.
\end{aligned} \tag{2.14}$$

On calculating the quantities (2.12) with the help of (2.14), and substituting them in (2.11), we arrive at the following relations:

$$\begin{aligned}
&\xi_1 = \xi_2 = \xi, \quad \psi_1 = \psi_2 = \nu/2,\\
&\nu = f\left(\frac{\xi - |\Delta|}{T}\right) + f\left(\frac{\xi + |\Delta|}{T}\right),\\
&1 = \frac{E_0}{2|\Delta|}\left[f\left(\frac{\xi - |\Delta|}{T}\right) - f\left(\frac{\xi + |\Delta|}{T}\right)\right],
\end{aligned} \tag{2.15}$$

where $f(x) = [e^x + 1]^{-1}$ is the Fermi distribution function.

Equations (2.15) implicitly define the quantities $\mu(\nu, T)$, and $\Delta(\nu, T)$ and thus all the thermodynamic quantities. For the phase-transition temperature we obtain from (2.15), letting $|\Delta|$ tend to zero:

$$T_c = E_0\nu(2-\nu)/4. \tag{2.16}$$

The analysis carried out so far pertained to the case, $E_{ex}(0) < 0$. In the opposite case $E_{ex}(0) > 0$, no charge density wave is formed and at low temperatures predominant filling of one valley is favorable, so that one has to look for a solution of equations (2.10) with $\psi_1 \neq \psi_2$ and $\chi = 0$. Now the consistency conditions take the form

$$\psi_1 + \psi_2 = \nu, \quad f(\xi_1/T) = \psi_1, \quad f(\xi_2/T) = \psi_2 \tag{2.17}$$

and they define the chemical potential $\mu(\nu, T)$. In this case the critical temperature T_c', at which the intervalley symmetry is spontaneously broken, is given by the expression

$$T_c' = (E - E_Q)\nu(2-\nu)/4. \tag{2.18}$$

It is seen from (2.16) and (2.18) that in all cases T_c (T_c')

reaches a maximum at $\nu = 1$. Of course, the expressions obtained here are only the first terms of an expansion in a/l_H and may be valid only in a small vicinity of $\nu = 1$.

3. COLLECTIVE EXCITATIONS

Since dissipation is usually determined by soft modes, it is of interest to find the energies of the collective excitations. If $E_{ex}(0) > 0$, the excitons themselves are these collective excitations, and we may use the results of Ref. 9 for the energy of an exciton as a function of its momentum:

$$E_{ex}(\mathbf{p}) \approx -\int \frac{d\mathbf{q}}{(2\pi)^2} [V(q) e^{i\mathbf{q}\mathbf{p}} w_n^2(\mathbf{q}) - V(q) | w_n(\mathbf{q}) |^2 + V(\mathbf{q}+\mathbf{Q}) | w_n(\mathbf{q}) |^2] + \frac{1}{2\pi} V(Q) | w_n(\mathbf{p}) |^2. \quad (3.1)$$

From this it is evident that the expansion of the excition energy in the momentum begins from the quadratic terms (in the vicinity of an energy minimum).

In the case $E_{ex}(0) < 0$ the collective excitations are Goldstone modes, which correspond to the independence of the ground-state energy of the phase of the charge-density wave or, in terms of excitons, of the phase of the Bose condensate of excitons. We note that this is due to the commutation of the interaction Hamiltonian with the operator $\hat{S}_z$.

To find the collective excitations we have to consider equations for two-particle Green functions obtained in the ladder approximation or, since we are interested only in the position of the poles, consider homogeneous equations for the variation $\delta G(k,\tau; k',\tau')$ of one-particle Green functions whose matrix form is[14]

$$\delta\hat{G} = \hat{G}\delta\Sigma\hat{G}. \quad (3.2)$$

Here $\hat{G}$ is the matrix Green function with components defined according to (2.14) and $\delta\hat{\Sigma}$ is the variation of the mass operator in the self-consistent-field equations.

Equation (3.2) corresponds to diagrams, which are depicted in Fig. 1, where the thick lines represent the quantities $\delta\hat{G}$, and the thin lines the Green functions $\hat{G}$ (2.14). The dash corresponds to the interaction matrix, which defines the Hamiltonian (1.12). Since Σ contains only equal-time Green functions, we can assume $\tau = \tau'$ in the left-hand side of (3.2).

We have quite a complicated system of four integral equations. It is convenient to represent the functions δG in the form

$$\delta G(k, \tau; k', \tau) = \int \delta\tilde{G}(k-k', \varkappa; \tau, \tau) e^{i\varkappa(k+k')/2} \frac{d\varkappa}{2\pi},$$

by introducing exciton variables $\varkappa$ and $k - k'$ corresponding to the exciton momentum $\mathbf{p}$. In the new variables the integral equations become algebraic for the matrix $\delta\hat{G}$. After taking the Fourier transform in the variable τ we obtain the following system of equations:

$$\delta\tilde{G}_{ij}(\omega, p) = T \sum_{\omega_1} g_{ik}(\omega+\omega_1) \delta\Sigma_{kl}(\omega, p) g_{lj}(\omega_1). \quad (3.3)$$

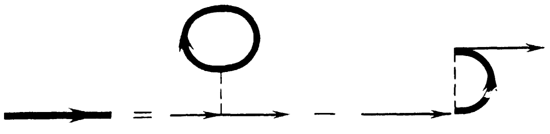

FIG. 1

The variations $\delta\Sigma$ themselves are readily expressed in terms of the variations $\delta\tilde{G}$:

$$\begin{gathered}
\delta\Sigma_{11} = \left[\frac{V_1(\mathbf{p})}{2\pi} - E_1(\mathbf{p})\right] \delta\tilde{G}_{11} + \frac{V_2(\mathbf{p})}{2\pi} \delta\tilde{G}_{22}, \\
\delta\Sigma_{22} = \left[\frac{V_1(\mathbf{p})}{2\pi} - E_1(\mathbf{p})\right] \delta\tilde{G}_{22} + \frac{V_2(\mathbf{p})}{2\pi} \delta\tilde{G}_{11}, \qquad (3.4) \\
\delta\Sigma_{12} = -E_2(\mathbf{p}) \delta\tilde{G}_{12}, \quad \delta\Sigma_{21} = -E_2(\mathbf{p}) \delta\tilde{G}_{21}.
\end{gathered}$$

Here we have used the notation:

$$E_{1,2} = \int \frac{d\mathbf{q}}{(2\pi)^2} V_{1,2}(\mathbf{q}) e^{i\mathbf{q}\mathbf{p}}. \quad (3.5)$$

Sums over frequency ω_1 are easily calculated if the expressions (2.14) for the functions $g_{ij}(\omega)$ are used, and it turns out that the right-hand sides of the equations for $\delta\tilde{G}$ contain only two combinations: $\alpha = \delta\tilde{G}_{11} - \delta\tilde{G}_{22}$ and $\beta = \Delta^* \delta\tilde{G}_{12} - \Delta\delta\tilde{G}_{21}$. The final equations for them are

$$\begin{aligned}
\alpha &= \frac{4|\Delta|^2}{E_0(\omega^2+4|\Delta|^2)} \left[E_1(\mathbf{p}) + \frac{V_2(\mathbf{p}) - V_1(\mathbf{p})}{2\pi}\right] \times\alpha - \frac{2i\omega E_2(\mathbf{p})}{E_0(\omega^2+4|\Delta|^2)} \beta, \\
\beta &= \frac{-2i\omega|\Delta|^2}{E_0(\omega^2+4|\Delta|^2)} \left[E_1(\mathbf{p}) + \frac{V_2(\mathbf{p}) - V_1(\mathbf{p})}{2\pi}\right] \times\alpha + \frac{4|\Delta|^2 E_2(\mathbf{p})}{E_0(\omega^2+4|\Delta|^2)} \beta. \qquad (3.6)
\end{aligned}$$

Setting the determinant of this system equal to zero, we obtain equations for the spectrum of the collective excitations $\omega(\mathbf{p})$ $(E_0 = E_2(0))$:

$$\begin{aligned}
\omega_1^2(\mathbf{p}) &= 4|\Delta|^2, \\
\omega_0^2 &= \frac{4|\Delta|^2}{E_2^2(0)} [E_2(0) - E_2(\mathbf{p})] \times\left[E_2(0) - E_1(\mathbf{p}) + \frac{V_1(\mathbf{p}) - V_2(\mathbf{p})}{2\pi}\right]. \qquad (3.7)
\end{aligned}$$

The frequencies of the collective excitations appearing in (3.7) were obtained by analytic continuation of the Matsubara frequencies contained in the definition of $\delta\tilde{G}_{ij}$.

It is necessary to make here the following remark. The equations for the functions $\delta\tilde{G}_{ij}$ have been obtained in the approximation of the generalized Hartree-Fock method. One may say immediately that, more than likely, this method is not applicable near the critical temperature T_c (2.16), about which one can presumably only assert that $T_c \sim E$, where E is the ionization energy of an exciton (with a possible denominator in the form of a logarithm of the ratio of the exciton ionization energy to the anisotropy energy (see, e.g., Ref. 15), and T_c has a maximum near $\nu = 1$. However, the circumstance that the Hartree-Fock approximation is asymptotically exact in the limit $T \to 0$ in the case[4] $V_1 = V_2$ allows to hope that in the presence of a small anisotropy in the Hamiltonian our treatment is also valid in the limit $T \to 0$.

Gapless collective excitations at small momenta p correspond to a weak modulation of the charge-density wave. With the aid of (3.7), neglecting the asymmetry of $\varepsilon(\mathbf{k})$, we obtain in the limit $p \to 0$

$$\begin{aligned}
\omega_0(\mathbf{p}) &= c(\beta^{-2} p_x^2 + \beta^2 p_y^2)^{1/2}, \\
c &\approx \frac{e^2}{\varepsilon\hbar} \frac{1}{(Ql_H)^{3/2}}. \qquad (3.8)
\end{aligned}$$

We emphasize that in the case when the effect of asymmetry of $\varepsilon(\mathbf{k})$ near the minimum is predominant, the spectrum of the collective excitations begins from the gap $E_{ex}(0)$ given by the formula (2.6).

It is clear that low-lying modes will be responsible for various dissipative processes. In the present paper we do not consider them.

CONCLUSION

The system considered in the present paper, of two-dimensional electrons with two valleys in a strong magnetic field with filling factors near to unity, and with the intervalley scattering and nonparabolicity of the spectrum neglected, is a rare example of a problem in which electron interaction can be taken into account exactly (when states within only one Landau level are taken into account). The complete isotropy of the corresponding Hamiltonian explains the paradoxical result[4,8]—the absence of interaction in the ground state of the exciton gas in a symmetric neutral system in a strong magnetic field.

As was shown in Refs. 9 and 10, symmetry breaking in the Hamiltonian gives rise to interaction between excitons. The corresponding symmetry breaking was a result of the nonequivalence of electrons and holes. The general cause lies in the fact that a Hamiltonian not commuting with the operator $\hat{S}_-$, which coincides with the creation operator of an exciton with zero momentum, necessarily gives rise to interaction between excitons. In the present paper it is shown that, owing to intervalley scattering and nonsymmetry of the spectrum, which are small corrections (the former $\sim (a/l_H)^3$, the latter $\sim (a/l_H)^2$, where a is the interatomic distance), a charge-density-wave-type state with a wave vector Q equal to the distance between valleys, and symmetric with respect to both valleys is realized when the intervalley scattering is predominant. When the corrections to nonsymmetry predominate, a state with preferable filling of one valley is realized. The temperature of the corresponding phase transition is determined by the Coulomb interaction and is of the order $T_c \sim (e^2/\varepsilon l_H)\nu(2-\nu)$, where ε is the dielectric constant; T_c reaches a maximum at $\nu = 1$. We emphasize that the results obtained about the symmetry of the Hamiltonian are valid with exponential accuracy $\exp(-l_H^2/a^2)$. Silicon-based two-dimensional structures with surface orientation (110) should have this kind of behavior.

It is also shown that in the case of GaAs-based two-dimensional structures a closely related situation for electrons may be realized, owing to the smallness of the g factor and of the effective mass. Here two spin subbands of the Landau level play the role of the two valleys, and the spin-orbit interaction (when the absence of inversion is taken into account) causes wave functions with different spins to be not symmetric. In this case the Hamiltonian commutes only with the operator $\hat{S}_z$. Now the Coulomb interaction plays the role of the intervalley interaction, and the frequency of the spin resonance the role of the asymmetry of $\varepsilon(\mathbf{k})$.

When a situation similar to a ferromagnet with "easy-plane" anisotropy takes place, gapless collective excitations occur in the system, and for small momenta we have $\omega \approx cp$ $(c \approx (e^2/\varepsilon\hbar \times (Ql_H)^{-3/2})$. If, predominant filling of one valley is energywise favored, a gap is present in the excitation spectrum.

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