Current-voltage characteristics of mesoscopic semiconductor contacts

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Resonance tunneling of electrons via impurity states is the conduction mechanism in short semiconductor contacts. Inelastic processes are negligible at low temperatures. An analysis is made of the cases when the current passes through channels with one or two resonance impurities. The voltage scale of fluctuations of the random quantity $J(V)$ is equal to the width of the corresponding level. An increase in temperature T does not smooth out the fluctuations due to two-impurity channels. Regions with a negative differential conductance should appear at sufficiently high voltages.

1. INTRODUCTION

Strong fluctuations of the conductance of small-area contacts are observed because of the random distribution of impurities in a contact. The associated mesoscopic phenomena are discussed in a recent review[1] (see also the bibliography cited in this review). An example of such phenomena is a strong nonlinearity of the current-voltage characteristics of such contacts. The characteristics of metal contacts had been investigated before[2] and it was shown there that they become nonlinear at voltages inversely proportional to the transit time and also that they have regions with a negative differential resistance. An experimental study of nonlinear current-voltage characteristics of mesoscopic semiconductor contacts was reported in Ref. 3. Impurities with resonance levels whose energy is close to the chemical potential are important in the conductance of semiconductor contacts. At low temperatures the length of a thermal jump may become greater than the length L of the contact and then inelastic processes are unimportant. The conductance is then dominated by channels with an almost periodic distribution of impurities.[4,5] The effective number of impurities in a channel may vary, depending on the length of the contact and on the impurity concentration. We shall consider the case of a low impurity concentration and short contacts, when one or two impurities are located in a channel. The case of a large number of impurities does not differ qualitatively from the case of two impurities. However, the current-voltage characteristics in the cases of one or two impurities in a channel are very different. The case of one resonance impurity has been discussed extensively in the literature.[6,7] In this case the current through a contact is a monotonic function of the applied voltage since the contribution to the current made by one resonance reaches saturation on increase of the voltage, but the number of resonance impurities increases. In the case of two resonance impurities in a channel the tunneling probability is maximal when the energies of these impurities are equal. An increase in the voltage alters the relative energies of the impurities and a channel conductance may decrease. This may give rise to a negative differential conductance. The effective number of channels depends on the width of the contact and on the impurity concentration. We shall consider the case of sufficiently wide contacts when the number of channels is much greater than unity. The mesoscopic effects are then small and general formulas can be derived for them. In the case of narrow contacts one channel is the most important and then the contact conductance depends on the actual disposition of the impurities. In this case the above results are only qualitatively valid.

2. ONE IMPURITY IN A CHANNEL

We shall consider a contact of area S and of length $2L$ which has one resonance impurity of energy ε_i. The Schrödinger equation for an electron in the field of this impurity is

$$\left(i\frac{\partial}{\partial t}-\varepsilon_p\right)\psi_p=T_p\psi_i,\quad \left(i\frac{\partial}{\partial t}-\varepsilon_i\right)\psi_i=\sum_p T_p^*\psi_p+\sum_k T_k^*\psi_k,\quad \times\left(i\frac{\partial}{\partial t}-\varepsilon_k\right)\psi_k=T_k\psi_i, \tag{1}$$

where ψ_p is the amplitude of the electronic state p to the left of the contact; ψ_k is the amplitude to the right of the contact; ψ_i is the amplitude at the impurity; T_p is the matrix element of the Hamiltonian between the states p and i. In the cases discussed below the nonresonance tunneling is negligible. Therefore, the corresponding terms are omitted from the system (1). Solution of the system (1) gives the general formula for the probability of resonance tunneling per unit time (from a state p to a state k):

$$w_{pk}=\frac{2\pi}{\hbar}\frac{|T_p|^2|T_k|^2}{(\varepsilon-\varepsilon_i)^2+\Gamma^2}\delta(\varepsilon_p-\varepsilon_k), \tag{2}$$

where $\Gamma=\Gamma_l+\Gamma_r$,

$$\Gamma_l=\pi\sum_p|T_p|^2\delta(\varepsilon_i-\varepsilon_p),\qquad \Gamma_r=\pi\sum_k|T_k|^2\delta(\varepsilon_i-\varepsilon_k). \tag{3}$$

The conductance at absolute zero is found by summing Eq. (2) over all final states and over all initial states of energy equal to the Fermi value ε:

$$g_i(\varepsilon)=2e^2\sum_{k,p}\delta(\varepsilon-\varepsilon_p)w_{pk}=\frac{4e^2}{\pi\hbar}\frac{\Gamma_l\Gamma_r}{(\varepsilon-\varepsilon_i)^2+\Gamma^2}. \tag{4}$$

The conductance has a sharp maximum amounting to $e^2/\pi\hbar$ if $\varepsilon=\varepsilon_i$ and $\Gamma_l=\Gamma_r$. We shall measure the coordinate of the impurity z_i from the plane where $\Gamma_l=\Gamma_r=\Gamma_1$. In the case of a symmetric barrier this plane passes through the middle of the barrier. In the case of impurities located near this plane, we have

$$\Gamma_{l,r}=\Gamma_1 \exp(\mp 2\varkappa z_i), \tag{5}$$

where $\varkappa$ is the reciprocal radius of the impurity state obeying the relationship $\hbar^2\varkappa^2/2m = \varepsilon_i$. The quantity Γ_1 is exponentially small. The majority of the results is independent of the explicit form of Γ_1. The values of Γ_l and Γ_r can be found for specific potentials of the barrier and impurity using the formulas in Eq. (3). We shall calculate the matrix element $T_p = \langle p|H|i\rangle$. The state $|p\rangle$ should be orthogonal to the impurity state $|i\rangle$ and it can be taken in the form $|p\rangle = |p'\rangle - |i\rangle\langle i|p'\rangle$, where $|p'\rangle$ is the eigenstate of the Hamiltonian without an impurity and the wave function of this state $\varphi_p(\mathbf{r})$ decays exponentially inside the barrier. Apart from exponentially small terms, we have

$$T_p=\langle p'|H-\varepsilon_i|i\rangle=\langle p|V|i\rangle=\varphi_p(\mathbf{r})\hbar^2(2\pi\varkappa)^{1/2}/m.$$

The last equation applies to a small-radius potential. In the case of rectangular barriers, we have

$$\Gamma_l=\frac{2p\varkappa}{p^2+\varkappa^2}\varepsilon_i\frac{\exp[-2\varkappa(L+z_i)]}{\varkappa(L+z_i)}, \tag{6}$$

where p is the modulus of the wave vector of the state $|p\rangle$. If $z_i \ll L$, we have Eq. (5).

If there are several impurities in a contact and they are sufficiently far apart, the conductance is given by

$$g_1(\varepsilon)=\sum_i g_i(\varepsilon).$$

The above quantity is self-averaging if the number of impurities is large and it can be calculated by averaging over the positions and energies of the impurities:

$$\bar{g}_1=\pi e^2\rho S\Gamma_1/\hbar\varkappa, \tag{7}$$

where ρ is the number of impurities per unit volume and per unit energy interval. It therefore follows that the number

$$N_1=\pi^2\rho S\Gamma_1/\varkappa \tag{8}$$

represents the effective number of one-impurity channels. This number is equal to the number of impurities which are found in a layer of thickness $\sim\varkappa^{-1}$ near the $z = 0$ plane and which have energies in the interval $\sim\Gamma_1$ near the Fermi energy.

In the case of semiconductor contacts we can have an experimental situation in which the chemical potential of electrons is altered by the electric field applied across the contact. The contact conductance then fluctuates near its average value described by Eq. (7). These fluctuations are characterized by a correlation function

$$K_g(\varepsilon_1,\varepsilon_2)=\overline{g(\varepsilon_1)g(\varepsilon_2)}-\overline{g(\varepsilon_1)}\,\overline{g(\varepsilon_2)}. \tag{9}$$

Here, the bar denotes averaging over ε_1 for a fixed value of the difference $\varepsilon_1 - \varepsilon_2$. If the averaging is carried out over a wide range of energies ε_1, then K_g for a contact with a large number N_1 of channels is self-averaging and depends weakly on the actual distribution of the impurities. Averaging of K_g over the distribution of the impurities and their energies gives

$$K_g(\varepsilon_1-\varepsilon_2)=\frac{e^4}{2\hbar^2}\frac{\rho S\Gamma_1}{\varkappa}\frac{1}{(1+\Delta^2)^{1/2}[1+(1+\Delta^2)^{1/2}]}, \tag{10}$$

where $\Delta = (\varepsilon_1 - \varepsilon_2)/4\Gamma_1$. If $\Delta = 0$, then K_g is equal to the variance of the conductances. Its value is N_1 times less than the square of the average conductance. The characteristic energy scale in which there is a change in the conductance is equal to the width Γ_1 of the level of an impurity located at the middle of the contact.

A finite temperature gives rise to inelastic processes, but broadening of the distribution function of electrons at the edges of a contact is now more important. The total current through this contact is

$$J=\frac{1}{e}\int g(\varepsilon)[f(\varepsilon-eV/2)-f(\varepsilon+eV/2)]d\varepsilon, \tag{11}$$

where $f(\varepsilon)$ is the Fermi distribution function. For a low voltage, the conductance is

$$g(\varepsilon,T)=\frac{1}{4T}\int g(\varepsilon')\frac{d\varepsilon'}{\mathrm{ch}^2[(\varepsilon'-\varepsilon)/2T]}. \tag{12}$$

The average value of the conductance is independent of temperature. However, the correlation function depends strongly on temperature. At finite temperatures, it follows from Eq. (12) that

$$K_g(\varepsilon_1-\varepsilon_2,T)$$
$$=\frac{1}{16T^2}\iint\frac{K_g(\varepsilon_1'-\varepsilon_2',0)}{\mathrm{ch}^2[(\varepsilon_1'-\varepsilon_1)/2T]\mathrm{ch}^2[(\varepsilon_2'-\varepsilon_2)/2T]}d\varepsilon_1'\,d\varepsilon_2'. \tag{13}$$

If $T\ll\Gamma_1$, Eq. (13) reduces to the low-temperature limit described by Eq. (10). In the opposite limiting case of high temperatures $T\gg\Gamma_1$, we have

$$K_g(\varepsilon_1-\varepsilon_2,T)$$
$$=2\frac{e^4}{\hbar^2}\frac{\rho S\Gamma_1^2}{\varkappa T} \tag{14}$$
$$\times\frac{[(\varepsilon_1-\varepsilon_2)/2T]\mathrm{ch}[(\varepsilon_1-\varepsilon_2)/2T]-\mathrm{sh}[(\varepsilon_1-\varepsilon_2)/2T]}{\mathrm{sh}^3[(\varepsilon_1-\varepsilon_2)/2T]}.$$

The variance governing the fluctuation amplitude is now a factor of $4\Gamma_1/3T$ smaller than at the absolute zero. The energy scale of the conductance fluctuations is equal to the absolute temperature.

The differential conductance at nonzero voltage is equal to the derivative of Eq. (11) with respect to the voltage:

$$g(V,\varepsilon)=\frac{1}{2}[g(\varepsilon+eV/2)+g(\varepsilon-eV/2)]. \tag{15}$$

The correlation function of the conductances measured under different voltages is

$$\tilde{K}_g(V_1,V_2)=\overline{g(V_1,\varepsilon)g(V_2,\varepsilon)}-\overline{g(V_1,\varepsilon)}\,\overline{g(V_2,\varepsilon)}$$
$$=\frac{1}{2}\{K_g(e(V_1-V_2)/2)+K_g(e(V_1+V_2)/2)\}. \tag{16}$$

The correlation function $\tilde{K}_g$ is obtained for fixed values of V_1 and V_2 by averaging over the chemical potential. It is clear from Eq. (15) that averaging over ε can be replaced by averaging over V_1 for a fixed difference $V_1 - V_2$. We then obtain

$$\tilde{K}_g(V_1-V_2)={}^1/_2K_g(e(V_1-V_2)/2), \qquad (17)$$

where K_g is given by Eqs. (10) and (14). Therefore, the differential conductance considered as a function of the voltage is a random quantity. The relative amplitude of fluctuations is equal to $N_1^{-1/2}$ at low temperatures $T\ll\Gamma_1$ and to $(\Gamma_1/N_1T)^{1/2}$ at high temperatures $T\gg\Gamma_1$. The characteristic scale of the voltage fluctuations is Γ_1/e and T/e at low and high temperatures, respectively. We shall show that channels with two or more impurities may give rise to large fluctuations of the conductance.

3. TWO IMPURITIES IN A CHANNEL

The case of a low impurity concentration when the overlap of the electron states can be ignored was discussed above. An increase in the impurity concentration increases the importance of the electron-tunneling channels involving two resonance impurities. Then, the current is dominated by those channels in which one impurity is located closer to the left edge of the contact and the other closer to the right edge. The Schrödinger equation for an electron in the field of two such impurities is

$$\left(i\frac{\partial}{\partial t}-\varepsilon_p\right)\psi_p=T_p\psi_i, \quad \left(i\frac{\partial}{\partial t}-\varepsilon_i\right)\psi_i=\sum_p T_p^*\psi_p+H_{ij}\psi_j,$$

$$\left(i\frac{\partial}{\partial t}-\varepsilon_j\right)\psi_j=H_{ji}\psi_i+\sum_k T_k^*\psi_k, \quad \left(i\frac{\partial}{\partial t}-\varepsilon_k\right)\psi_k=T_k\psi_j, \qquad (18)$$

where H_{ij} is the matrix element of the Hamiltonian between the states of the ith and jth impurities. It decreases exponentially on increase in the distance r_{ij} between the impurities. In the case of small-radius impurities, we find by analogy with Eq. (6) that

$$H_{ij}=\varepsilon\frac{\exp(-\varkappa r_{ij})}{\varkappa r_{ij}}. \qquad (19)$$

The formulas for the tunneling probability and for the conductance are then obtained by analogy with Eqs. (2)–(4). We thus obtain

$$g_{ij}(\varepsilon)=\frac{4e^2}{\pi\hbar}\frac{\Gamma_l\Gamma_r|H_{ij}|^2}{|(\varepsilon-\varepsilon_i+i\Gamma_l)(\varepsilon-\varepsilon_j+i\Gamma_r)-|H_{ij}|^2|^2}. \qquad (20)$$

The maximum value of the conductance is $e^2/\pi\hbar$, as in the case of one impurity. This value is obtained when

$$\varepsilon_i=\varepsilon_j=\varepsilon, \quad |H_{ij}|^2=\Gamma_l\Gamma_r=\Gamma_2^2.$$

In the case of a rectangular barrier this condition is satisfied when the distance between impurities is equal to half the length of the barrier. We then have

$$\Gamma_2^2=\frac{4p\varkappa}{p^2+\varkappa^2}\frac{\varepsilon^2}{(\varkappa L)^2}\exp(-2\varkappa L)=\frac{2\varepsilon}{\varkappa L}\Gamma_1. \qquad (21)$$

For a symmetric distribution of the impurities the width of a two-impurity resonance is of the order of Γ_2 and it is much greater than Γ_1, which is equal to the width of the narrowest and strongest one-impurity resonance. In the case of an asymmetric position of the impurities the width of a two-impurity resonance is of the order of the width of the narrowest level. The contribution of two-impurity channels to the average conductance of a contact is found by averaging Eq. (20) over the positions of impurities and their energies, and it is given by

$$\bar{g}_2=e^2N_2/\pi\hbar, \quad N_2=8\pi^3L^3S\rho^2\Gamma_2^2/\varkappa, \qquad (22)$$

where N_2 is the number of two-impurity channels.

The contribution of two-impurity channels becomes comparable with the one-impurity contribution if

$$\rho\gtrsim\varkappa/L^2\varepsilon. \qquad (23)$$

At lower values of the impurity concentration the two-impurity channels make a small contribution to the average conductance, but can make a major contribution to mesoscopic fluctuations of the current-voltage characteristic. It is important to note that the impurity energies in Eq. (20) depend on the voltage across the investigated contact:

$$\varepsilon_i=\varepsilon_i^{(0)}+eVz_i/L. \qquad (24)$$

When the voltages are V_1 and V_2, the correlation function of the currents is

$$K_J(V_1, V_2)=\overline{J(V_1)J(V_2)}-\overline{J(V_1)}\,\overline{J(V_2)}. \qquad (25)$$

The averaging in Eq. (25) is carried out over the chemical potential or over the voltage $V=(V_1+V_2)/2$ for a fixed difference V_1-V_2. If the number N_2 of channels is large, the quantity K_J is self-averaging and it can be calculated by averaging over the positions of impurities and their energies. The current $J(V)$ in Eq. (25) can be calculated using Eqs. (11), (20), and (24). We shall consider the case when either the temperature $T\gg\Gamma_2$ or the voltage $eV\gg\Gamma_2$ is high. The impurities with energies close to one another, $|\varepsilon_i-\varepsilon_j|\sim\Gamma_2$, are different from the chemical potential by an amount of the order of T or eV and then play the dominant role. Averaging of Eq. (25) over the sum of the impurity energies and over the difference between them should be carried out independently and Eq. (25) should be factorized to give

$$K_J(V_1, V_2)=\frac{32\pi^3e^2}{\hbar^2}\frac{\rho^2SL\Gamma_2^3}{\varkappa^3}F\left(\frac{e(V_1-V_2)}{8\Gamma_2}\right)\Phi(V_1, V_2, T), \qquad (26)$$

$$\Phi=\int[f(\varepsilon-eV_1/2)-f(\varepsilon+eV_1/2)]$$
$$\times[f(\varepsilon-eV_2/2)-f(\varepsilon+eV_2/2)]d\varepsilon,$$

$$\Phi=\begin{cases} e^2V_1V_2/6T, & V_1, V_2\ll T \\ eV(\operatorname{cth}(eV/2T)-2T/eV), & |V_1-V_2|\ll V, \end{cases}$$

$$\frac{32\pi^3e^2}{\hbar^2}\frac{\rho^2SL\Gamma_2^3}{\varkappa^3}F\left(\frac{e(V_1-V_2)}{8\Gamma_2}\right)=\frac{1}{e^2}\int\overline{g(\varepsilon)g(\varepsilon+\varepsilon')\,d\varepsilon'}, \qquad (27)$$

$$F(\alpha)=\int_0^\infty\left\{(1+\alpha^2x^2+x^4)^{1/2}-(1+\alpha^2x^2)^{1/2}\right.$$
$$\left.-\frac{1}{\alpha x}\ln\frac{(1+x^4)^{1/2}[(1+\alpha^2x^2)^{1/2}+\alpha x]}{(1+\alpha^2x^2+x^4)^{1/2}+\alpha x}\right\}\frac{dx}{x^4},$$

$$F(\alpha)=\begin{cases} 0.618-0.212\alpha^2+|\alpha|^3/6, & \alpha\ll1 \\ \dfrac{1}{2\alpha}\ln(2.6\alpha), & \alpha\gg1 \end{cases}. \qquad (28)$$

In the case of two-dimensional contacts of the kind used in field-effect transistors, Eqs. (10) and (14) are not affected if ρ is understood to be the number of impurities per unit area and per unit energy interval, and S denotes the width of the contact. The formulas (26) and (28) are then modified somewhat to

$$K_J(V_1, V_2)=\frac{8\pi^2e^2}{\hbar^2}\frac{\rho^2S\Gamma_2^3}{\varkappa^2}\left(\frac{L}{\varkappa}\right)^{1/2} \times F\left(\frac{e(V_1-V_2)}{8\Gamma_2}\right)\Phi(V_1, V_2, T), \qquad (26')$$

$$F(\alpha)=\begin{cases} 3{,}79-0{,}75\alpha^2+\dfrac{\alpha^3}{12|\ln\alpha|^{1/2}}, & a\ll 1\\ 2\pi^{1/2}\dfrac{\ln\alpha}{\alpha}, & \alpha\gg 1 \end{cases} \qquad (28')$$

The correlation function of the conductances is obtained from Eq. (26) by differentiation of

$$K_g(V_1, V_2)=\frac{\partial^2}{\partial V_1\,\partial V_2}K_J(V_1, V_2). \qquad (29)$$

At high voltages when $eV\gg\Gamma_2$, we have to differentiate the function F:

$$K_g=-\frac{\pi^3e^4}{2\hbar^2}\frac{\rho^2SL\Gamma_2}{\varkappa^3}F''\left(\frac{e(V_1-V_2)}{8\Gamma_2}\right)\Phi(V_1, V_2, T). \qquad (30)$$

If $V_1=V_2$, then $F''=-0.424$ and Eq. (30) gives the expression for the variance of the differential conductance. For a constant voltage the function $\Phi(V, V, T)$ rises quadratically if $eV\ll T$ and linearly if $eV\gg T$. Therefore, at sufficiently high values of V the variance may become greater than the square of the average value of the differential conductance given by Eqs. (22) and (7). This means that the current-voltage characteristic should have regions with a negative differential conductance. When the average conductance is dominated by two-impurity channels, the relevant condition is

$$eV/\Gamma_2\gtrsim\rho^2\Gamma_2^2SL^4\varkappa L\sim N_2(\varkappa L)^2. \qquad (31)$$

As pointed out in the Introduction, the physical reason for this effect is that the conductance of a two-impurity channel may decrease on increase in the voltage under off-resonance conditions. At some voltage the number of these channels may be accidentally greater than the number of channels with a growing conductance. An increase in the voltage by $\Delta V<\Gamma_2/e$ produces a relative change in the current amounting to

$$\Delta J/J=\Delta V/V\pm e\Delta V/\Gamma_2(N(V))^{1/2},$$

where $N(V)=eVN_2/\Gamma_2$ is the effective number of channels at a voltage $V>\Gamma_2/e$. The first term is associated with an increase in the average number of channels and the second with a change in the conductance of a single channel. When the condition (31) is satisfied, the second term is larger than the first and the current-voltage characteristics should have regions with a negative slope.

4. GENERATION OF HARMONICS

The nonlinearity of the current-voltage characteristic of a mesoscopic contact should result in generation of harmonics if the contact is part of an alternating-current circuit. Let us assume that the voltage across the contact is a periodic function of time and its frequency is $\omega\ll\Gamma_2/\hbar$. The current through the contact is determined by the instantaneous voltage and the amplitude of the nth harmonic is

$$J_n=\omega\int J(U_0\cos\omega t)e^{i\omega nt}\,dt.$$

The average of J_n for many contacts vanishes if $n\geqslant 2$. The average intensity of a harmonic differs from zero and can be expressed in terms of the correlation function of the currents

$$P_n=\overline{|J_n|^2}=\iint K_J(U_0\cos\varphi, U_0\cos\varphi')e^{in(\varphi-\varphi')}\,d\varphi\,d\varphi'. \qquad (32)$$

In the case of an individual contact the intensity of a harmonic agrees with Eq. (32) only in respect of the order of magnitude. A quantitative comparison with Eq. (32) can be made by averaging the intensity of the harmonic over the chemical potential or over the applied additional static voltage. In the case of a contact with a large number of channels such averages depend weakly on the actual distribution of impurities.

We shall now consider the contribution of one-impurity channels to the generation of harmonics. In the case of such channels the correlation function of the currents is expressed in terms of the correlation function of the conductances [Eq. (16)] with the aid of Eq. (29) and it is given by

$$K_J(V_1, V_2)=4\frac{e^2}{\hbar^2}\frac{\rho S\Gamma_1^2}{\varkappa}\left[\varphi\left(\frac{V_1+V_2}{2}\right)-\varphi\left(\frac{V_1-V_2}{2}\right)\right], \qquad (33)$$

where

$$\varphi(V)=[16\Gamma_1^2+(eV)^2]^{1/2}-4\Gamma_1\ln\{1+[1+(eV)^2/16\Gamma_1^2]^{1/2}\}, \quad T\ll\Gamma_1.$$

$$\varphi(V)=eV\,\mathrm{cth}\,(eV/2T), \quad T\gg\Gamma_1.$$

If the amplitude of the alternating voltage is small, Eq. (33) can be expanded in a series. Consequently, the intensity of the nth harmonic is given by

$$P_n^{(1)}=\frac{\pi^2}{2^{4n-6}(n!)^2}\frac{e^2}{\hbar^2}\frac{\rho S\Gamma_1^2}{\varkappa}U_0^{2n}\frac{\partial^{2n}}{\partial V^{2n}}\varphi(V). \qquad (34)$$

Equation (34) is valid in the case of odd harmonics. It follows from Eq. (33) that the amplitude of the even harmonics vanishes. This is due to the fact that the impurities located near the middle of a symmetric contact are important and the energies of these impurities depend weakly on the applied voltage. Even harmonics appear in the next order of the small parameter $(\varkappa L)^{-1}$.

If the amplitude of an alternating voltage is large, so that $eU_0\gg\Gamma_1$, T, the intensity of odd harmonics is

$$P_n^{(1)}=\frac{256}{4n^2-1}\frac{e^3U_0}{\hbar^2}\frac{\rho S\Gamma_1^2}{\varkappa}. \qquad (35)$$

This is valid in the case of harmonics characterized by the inequalities $n<eU_0/\Gamma_1$, eU_0/T. In the case of higher harmonics the intensity falls exponentially on increase of n.

The contribution of two-impurity channels to harmonic generation is described by Eqs. (32) and (26). Under a low voltage such that $eU_0\ll\Gamma_2$ and at high temperatures $T\gg\Gamma_2$, we find from Eqs. (27) and (28) that

$$P_2^{(2)} = 10{,}8\,\frac{e^4U_0^2}{\hbar^2}\,\frac{\rho^2SL\Gamma_2^3}{\varkappa^3T},$$

$$P_n^{(2)} = \frac{2\pi^3}{45}\,\frac{64n^2+1125}{(4n^2-25)\,(4n^2-9)\,(4n^2-1)}\,\frac{e^5U_0^3}{\hbar^2}\,\frac{\rho^2SL\Gamma_2^2}{\varkappa^3T}. \tag{36}$$

A comparison of Eqs. (34) and (36) demonstrates that the contribution of two-impurity channels falls more slowly on increase in the temperature or in the harmonic number than does the contribution of one-impurity channels. Therefore, two-impurity channels dominate harmonic generation even when they make only a small contribution to the total conductance of a contact.

5. CONCLUSIONS

As in the case of metal contacts, the scale of voltages typical of nonlinear current-voltage characteristics is inversely proportional to the transit time. This time is equal to the lifetime of a resonance level.

Our analysis deals with contacts of length less than the length of a thermal jump associated with inelastic processes. An increase in the temperature or voltage of a contact enhances the role of inelastic processes. The mesoscopic effects then become weaker but do not disappear completely.[8]

The temperature convenient for the observation of mesoscopic effects is of the order of the width of a resonance level. This width depends exponentially on the parameter $\varkappa L$. This parameter will not be very high if the chemical potential is close to the bottom of the conduction band. This can be achieved in field-effect transistors when the channel is close to the transition to the metallic state. An irregular dependence of the conductance on the voltage applied to the substrate and gate of a field-effect transistor was reported in Ref. 9. This effect may be attributed to an irregular dependence of the mesoscopic conductance on the chemical potential. It would be interesting to repeat this experiment at low temperatures so that the length of a thermal jump is comparable with the length of the contact and to determine the current-voltage characteristic. The conductance of one-dimensional contacts with just one channel was determined in the experiments described in Ref. 3. In this case the mesoscopic effects were very strong and they could not be described quantitatively by the theory presented above. However, a qualitative explanation can still be provided. Sharp peaks of the conductance considered as a function of the gate voltage correspond to resonance tunneling. Wider peaks may correspond to channels with two resonance impurities. The current-voltage characteristic near such a peak exhibits a negative differential conductance region. This can be explained by the fact that an increase in the voltage increases the difference between the energies of two impurity levels and reduces the channel conductance. Harmonic generation was also observed in these experiments.

Harmonic generation was reported too in Ref. 10. However, in this case (and also in the cases described in Refs. 3 and 9) the length of a contact was greater than the thermal jump length, so that an allowance for inelastic processes was essential in a quantitative comparison of the theory and experiment. Temperature-independent resonance tunneling without inelastic processes was reported in Ref. 11. However, as in Ref. 3, the contacts were not wide enough. In contacts of this kind only one channel contributes to the current at each energy. Therefore, the mesoscopic effects are very strong, depend on the actual distribution of the individual impurity, and are not described by a correlation function. It would be interesting to carry out such an experiment using wide contacts and to determine the current-voltage characteristic.

Investigations of mesoscopic effects in contacts with Lifshitz-Kirpichenkov channels were started after the discussion of this problem with L. G. Aslamazov. Unfortunately, his tragic death prevented him from participating in the work reported above.

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