Coulomb interaction of carriers and two-dimensional metal insulator junctions on germanium surfaces

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It is established that the conducting germanium layer adjacent to crystal cleavage surfaces and to bicrystal intergrowth surfaces is an ideal model of a two-dimensional conductor with strong electron-electron interaction. It is observed that when the temperature is lowered a two-dimensional metal with such properties loses its electric conductivity in accordance with the universal law $\sigma(T_1) - \sigma(T) \approx (e^2/4h)\ln(T_1/T)$, and after reaching the minimum metallic conductivity $\sigma_{\min} = e^2/h$ it is transformed into an insulator with a "Coulomb gap." The values of $\sigma(T)$ measured in the regions of both quasimetallic and nonmetallic conductivity agree qualitatively with the predictions of the scaling theory of localization.

Ever since I. E. Tamm deduced for the first time, about fifty years ago, the existence of special electron states localized on crystal surfaces,[1] it became obvious that the subsurface layers can have properties that distinguish them substantially from those of the crystal itself; and that conditions are possible in which layers with metallic properties are produced near a fresh surface of an insulator.[2] Phenomena of this type were observed a quarter-century later in experiment, near the intergrowth boundaries of germanium bicrystals[3,4] and on crystal cleavage surfaces.[5] The properties of these layers, and particularly the connections between the electric conductivity $\sigma_\square$ and the surface structure of germanium, were the subject of Refs. 6–14.

Conducting layers in germanium have recently become the object of numerous investigations in view of the increasing interest in transport phenomena in few-dimensional systems. It has been established, in particular, that the subsurface layers in germanium have the properties of a two-dimensional metal[15,16] and that the conductivity of this metal cannot be less than the value $\sigma_{\min} \approx (e^2/h) \approx 4\cdot 10^{-5}\Omega^{-1}$ predicted theoretically[17] for two-dimensional media (e is the electron charge and h the Planck constant). In the present study we have investigated in detail the electronic phenomena in the region quasimetallic and hopping conduction, and also the laws governing the two-dimensional metal–insulator transition in germanium subsurface layers.

§1. INVESTIGATED OBJECTS

We have experimented with germanium single crystals cleaved in liquid helium and with germanium bicrystals. These experimental conditions were chosen because they ensured maximum purity of the surface at minimum cost, which becomes very high when special high-vacuum apparatus is used.

The (111) cleavage surfaces were obtained by splitting the crystals along the cleavage plane; the resultant surfaces were smooth and had mirror brilliance. Electron-microscope investigations have shown that such surfaces have steps that extend along the cleavage line and are spaced more than tens of microns apart. It was established that the surface conductivity σ_s immediately after the cleavage in liquid helium is vanishingly small and is not discernible against the bulk conductance $\Sigma \approx 10^{-9}\Omega^{-1}$ of the crystals. The values of σ_s however, can be increased several orders by intermediate heatings in helium vapor.

We used a heating procedure that ensured good reproducibility of the surface properties. The germanium samples were placed in liquid-helium vapor at $T \approx 5$ K and a heater raised their temperature in a time $t = 20$–50 s to some rather high value T_i subsequently maintained at the specified level for various time intervals, from several minutes to several hours, depending on the desired resultant surface properties. After the heating, the samples were immersed in liquid helium and thier properties were measured.

Figure 1 shows by way of example one plot of σ_s (4.2 K) $= f(T_i)$. This typical and well-reproducible curve was obtained after heating six different surfaces, but under identical conditions, in which the temperature T_i, was monotonically increased from one intermediate heating step to another, and the heating time remained practically unchanged at ~ 2 min. Just as the results obtained under different heat-treating conditions, the data shown in Fig. 1 serve as "initial" sample properties as soon as a surface state stimulated by each annealing was subsequently "frozen" and kept unchanged in liquid helium during the long time needed to perform the entire research program.

Each annealing produced thus in fact a new "sample." For each of them we measured the Hall coefficient R_H to-

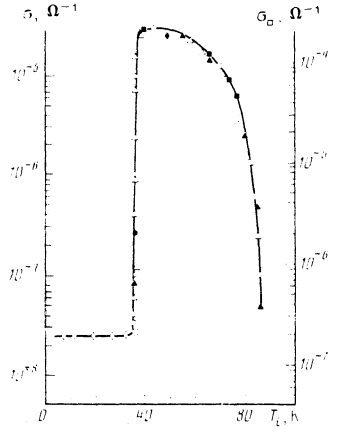

FIG. 1. Surface electric conductivity $\sigma_\square$ (4.2 K) vs temperature T_i of intermediate annealing in He vapor.

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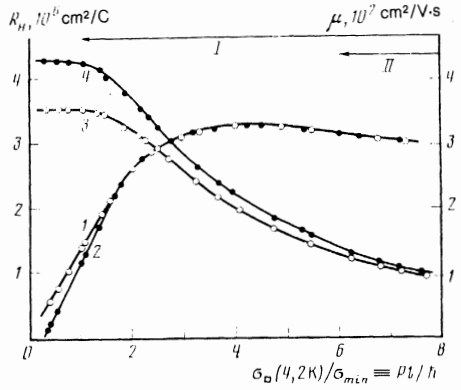

FIG. 2. Connections between the conductivity $\sigma_\square$ (4.2 K) and the values of the Hall coefficient R_H (curves 3 and 4) and of the mobility μ (curves 1 and 2), measured on one of the cleavage surfaces of Ge at the following temperatures: curves 1.3 – 4.2 K; 2.4 – 1.1 K region in which $\sigma \propto$ lin T; II—region of $\sigma_\square$ where the "rule of two" is satisfied).

gether with σ_s at $T = 4.2$ K, and calculated the mobility $\mu = R_H \sigma_\square$. The connection between these quantities, obtained on one of the cleavage surfaces at $T = 4.2$, is illustrated in Fig. 2 and shows how many closely spaced values of σ can be obtained in the interval $0,1\sigma_{\min} \leqslant \sigma_\square \leqslant 10\sigma_{\min}$ of interest to us.

As for the cause of the resultant high surface conductivity in germanium, it can be touched upon here only most superficially. As seen from the data of Fig. 1, σ_s increases after being heated to $T_i = 30$–50 K and drops steeply at $T_i > 80$ K. There is no doubt that the conductivity drop at high T_i is due to surface contamination, since helium vapor contains at $T \approx 80$ K a rather significant admixture of foreign gases. More unexpected were the data of Ref. 18, which showed that the increase of σ_s at T–30 = 50 K is also due to impurity adsorption, for after annealing in a very deep vacuum ($p \ll 10^{-12}$ Torr) the electric conductivity remains just as low as immediately after the cleavage. On the whole, the transition to metallic conductivity in germanium subsurface layers is complicated and a discussion of the concomitant processes[19] is way beyond the scope of the present paper.

1.1 "Internal" surfaces

Interfaces grown in the interior of perfect crystals have the advantage that they are protected from contact with the ambient by the surrounding source material in which the number of impurities is strictly monitored and can be set at the lowest level $\sim 10^{11}$ cm^{-1}. Being rid of the liability of freshly cleaved surfaces, the "internal" surfaces acquire such features as stability and compactness, so that they can be investigated under external conditions—at infralow temperatures and in superstrong magnetic fields.

The "internal" surfaces are grown by the Czochralski method on a "double" seed.[3,4] The seed is obtained by cutting an ordinary crystal seed and rotating one part through an angle θ relative to the other. When the crystal is grown on this seed, the interface produced in the ingot shows atomic half-planes tracks produced by disparity of the crystal-lattice angles in different parts of the ingot. The distances between the half-plane depend, naturally, on the angle θ and change greatly with increase of the latter. At a misorientation angle $\theta = 20°$–$30°$ the number of intersections on the intergrowth surface is $\sim 10^7$; the density of the broken bonds on the intergrowth surface is then comparable with the total number of surface atoms. The properties of the intergrowth surfaces are similar in this case to the properties of the cleavage surface; the conductivity of bicrystals with angles $\theta = 20°$–$30°$ is isotropic and of metallic character, while the carriers are almost uniformly distributed in the conducting layer.

At smaller misorientation angles, $\theta < 12°$, the distance between the chains of edge atoms increases, the overlap of the carrier wave function decreases, and the conducting plane breaks up into strips that conduct better close to chains of edge atoms and much worse between them. With further decrease of the misorientation angle, $\theta \approx 7°$, the conductivity along the rows of the edge atoms also decreases, apparently as a result of the greater isolation of the individual bands and the tendency of each band to change to conditions of quasi-one-dimensional conductivity.

We have investigated bicrystals with inclination angles $6 \leqslant \theta \leqslant 25°$. The samples were cut from the ingots in such a way that the current could flow parallel or perpendicular to the rows of edge atoms. The samples were $\sim 2 \times 2 \times 7$ mm parallelepipeds with conducting-surface area $\sim 2 \times 7$ mm. The contacts were made of fused-in indium and were resistive for all the investigated subsurface layers. The conductivity $\sigma_\parallel$ was measured for samples in which the chains of edge atoms extended from one current contact to the other, while $\sigma_\perp$ was measured for samples in which the rows of edge atoms were perpendicular to the line joining the current contacts. Since the resistance of the germanium crystal itself was very high, the leakage currents in the bulk could be neglected in the entire experimentally investigated temperature range.

We investigated several batches of bicrystals grown under different conditions from germanium having different properties. Figure 3 shows by way of example the dependence of the conductivities $\sigma_\parallel$ and $\sigma_\perp$ at $T = 4.2$ K on the angle θ for one of the better batches having high values of μ; the dashed lines show results for a different batch with lower carrier mobility. It can be seen from these data that the "initial" properties of the investigated bicrystals also span a large range of surface conductivity. It was this which al-

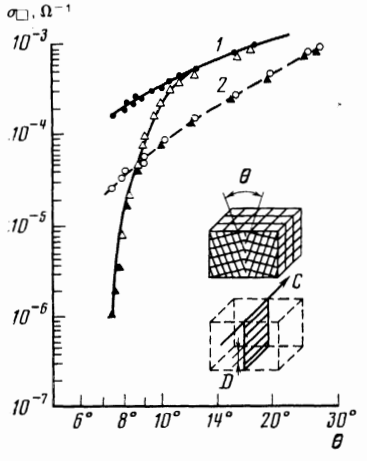

FIG. 3. Dependence of the conductivities at $T = 4.2$ on the "mismatch" angle θ in Ge bicrystals: ○, ● – $\sigma_\parallel$; ▽, ▲ – $\sigma_\perp$. Curve 1 pertains to bicrystals with high mobility, curve 2—with lower mobility (inset—intergrowth surface as interface between two single crystals; C—axis of relative rotation of the halves of the seed; D—distance between rows of edge atoms).

lowed us to study the electronic processes occurring on both sides of a metal-insulator junction.

§2. CURRENT CARRIERS IN GERMANIUM SUBSURFACE LAYERS

Measurements of the Hall coefficient have shown that the conduction in the subsurface layers in germanium is effected by holes, irrespective of the type of crystal conductivity and of the impurity density in the crystals. The Hall coefficient R_H remains constant in a wide range of the measuring-current density J and of the magnetic field strength H. The maximum hole density is $p_s \approx 1 \cdot 10^{13}$ cm^{-2} at the cleavage surfaces and double this value in the bicrystals. The onset of so high a hole density cannot be attributed to impurities in the crystals, since the number of foreign atoms in the conducting layer is small ($< 10^{11}$ cm^{-2}) and their influence can be neglected.

Investigations have shown that the electronic processes on the free cleaved germanium surface are qualitatively and quantitatively similar to phenomena observed on the bicrystal intergrowth surface. This suggests that the decisive role is played in both cases by the transfer of the electrons to the broken bonds, regardless of whether they are produced directly when the bicrystal is grown or on the surface after cleavage of the single crystal and subsequent annealing. Since the high hole density in the subsurface germanium layers is preserved down to the lowest temperatures achieved by us, it is natural to assume that the energy levels of the electrons on the unoccupied bonds on the germanium surface lie lower than the valence-electron levels in the interior of the crystal.

Under these conditions, an electron reaching the surface releases a certain energy E_s and is pinned there on broken bonds. The surface is then negatively charged, and in the layers adjacent to the surface there appear free holes that are retained near the surface by its electric field. The electron transfer process continues until the negative charge density accumulated on the surface is so high that, owing to repulsion, further transfer of electrons can no longer be offset by energy release through pinning to the surface.

Calculation by the Thomas-Fermi method[13,20] shows that at the surface-charge density $p_s \approx 10^{13}$ cm^{-2} determined by us the energy released on pinning of one electron on the surface is $E_s \approx 0.1$ eV and that the hole layer is ~ 30 Å thick. With this spatial confinement, the difference between the ground energy level and the next excited one is so large that the hole can move in the subsurface layers only parallel to the surface. The carrier motion under these conditions can be regarded as taking place in a two-dimensional medium.

2.1 Hole mobility in subsurface layers

The dependence of the hole mobility on the density shows highly pronounced manifestations of close similarities as well as differences between the two investigated systems with two-dimensional hole conductivity. These results, obtained from measurements of μ at $T = 4.2$ K in subsurface layers of more than 30 cleaved surfaces of crystals and intergrowth surfaces of bicrystals, are shown in Fig. 4. Curves 1–3 of this figure pertain to three cleaved surfaces on which the hole surface density and hole mobility were varied by the

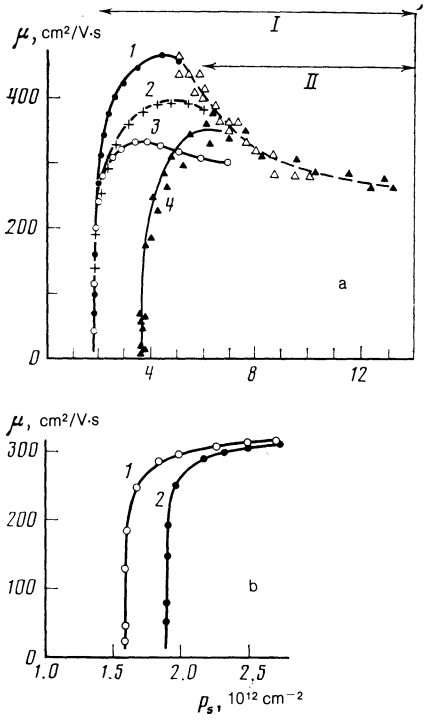

FIG. 4. a) Hole mobility vs density at $T = 4.2$ K: curves 1 to 3—cleavage surfaces; 4—Ge bicrystals. b) Dependence of hole mobility on density for cleavage surface at temperatures 1.1 K (curve 1) and 4.2 K (curve 2).

intermediate-annealing procedure. The remaining results in Fig. 4 pertain to thirty different surfaces. Each light triangle corresponds to different cleaved surfaces, and a dark triangle to different intergrowth surfaces. It can be seen from these data that the relation $\mu = f(p_s)$ is similar in form for different germanium subsurface layers: the hole mobility goes through a maximum at a density $p_s \approx 5 \cdot 10^{12}$ cm^{-2} and decreases slightly with increase of p_s. This result is typical of strongly degenerate conductors. Obviously, in the region $6 \cdot 10^{12} \leqslant p_s \leqslant 12 \cdot 10^{12}$ cm^{-2} both investigated systems can be regarded as metallic media in which the carriers are scattered by charged centers N_i whose number is approximately equal to the carrier density p_s.

At low values of p_s the hole mobility decreases strongly. This is due to strong localization of the carriers at the critical density p_c. On the other hand, the density interval

$$p_c \leqslant p_s \leqslant 5 \cdot 10^{12} \text{ cm}^{-2}$$

refers to the "intermediate" region and cannot be reached by simple models. The two systems of conducting holes differ only in their critical densities p_c. As seen from Fig. 4, in the case of bicrystals p_c is double the corresponding value on cleaved surfaces. These data suggest that bicrystal intergrowth surfaces are similar in some way to two cleavage surfaces in immediate contact with each other.

§3. TWO-DIMENSIONAL CONDUCTIVITY OF METALLIC TYPE

The semiclassical approach to the problem of carrier transport suggests the existence of a distinct boundary between insulators and metals, set by the minimal metallic conductivity specified by the condition $Pl \approx h$, and P is the electron momentum on the Fermi level and l is the mean free path. In the two-dimensional case this value is determined

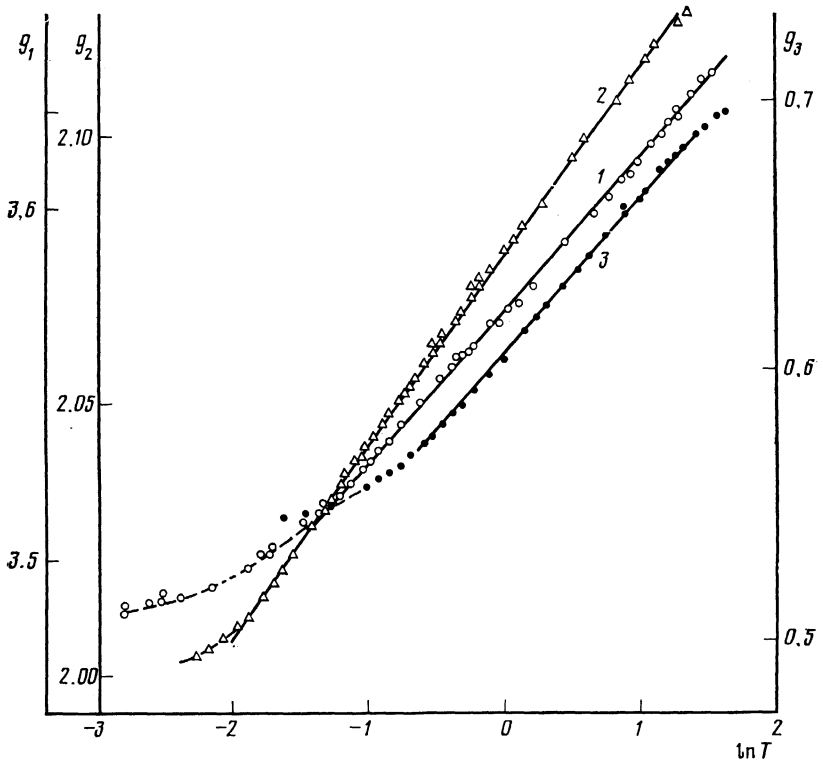

FIG. 5. Temperature dependences of dimensionless conductivity $g = (\hbar/e^2)\sigma_\square$ for several Ge bicrystals.

by fundamental constants, the electron charge and the Planck constant, and is independent of the band structure of the material and of the mechanisms of carrier scattering in it.[17]

An alternate point of view, to which much attention is paid lately, is that a two-dimensional metal is impossible in principle and that as the temperature is lowered two-dimensional conductors (regardless of the initial electric conductivity) must inevitably turn into insulators. To answer this question, experimental investigations of two-dimensional electric conductivity were performed on a large number of samples in a rather wide range of temperatures, from 10 to 1.1 K on cleavage surfaces and down to 0.02 in bicrystals. The results have shown that the quantity $\sigma_{\min}$ introduced by Mott is indeed a very important property of two-dimensional electric conductivity and determines correctly the singular point near which the conduction mechanism change type. However, the properties of two-dimensional conductors, in both the quasimetallic and nonmetallic conductivity regions, turned out to be much more complicated and interesting than predicted by the semiclassical theory.

Typical results of the measurements in the region of quasimetallic conductivity are given in Fig. 5 and show that, in a rather large temperature range, the conductivity of the intergrowth surfaces of germanium decrease logarithmically with decrease of temperature. The $\sigma(T)$ dependence of germanium-crystal cleavage surfaces are similar.

Investigations of the cleavage surfaces have shown that the conductivity change $\Delta\sigma_\square = \sigma_\square(T_1) - \sigma_\square(T)$ in the temperature interval from T_1 to T can be approximated for all the investigated samples by the relation

$$\Delta\sigma \approx 1\cdot 10^{-5} \ln(T_1/T), \qquad (1)$$

which shows that the decrease of the metallic conductivity in the subsurface layers of germanium proceeds with a rate close to $\frac{1}{4}\sigma_{\min}$ in an interval ΔT in which $\ln(T_1/T) = 1$. (The decrease of $\sigma_\square$ in certain bicrystals is faster by a factor 1.5–2.)

The experimental logarithmic decrease of the conductivity can be due to various effects: 1) "weak" localization[21–23] and 2) "interaction effects."[24–26] Indeed, when noninteracting electrons move on looplike self-intersecting trajectories interference takes place between "conjugate" waves traveling along the loops in opposite directions. This increases the electron-scattering probability and slows down their diffusion at low T, while the necessary correction to the electric conductivity can be written in the form

$$\Delta\sigma = \left(\frac{Ce^2}{2\pi^2\hbar}\right)\ln(T_1/T) \quad [\Omega^{-1}], \qquad (2)$$

which agrees qualtitatively with experiment. A similar dependence, however, is obtained also from allowance for "interaction" effects.

As shown in Ref. 24, if the interacting electrons diffuse they have a probability of meeting again after a characteristic time $t_\varepsilon \sim \hbar/\Delta\varepsilon$ determined by the energy $\Delta\varepsilon$ transferred in the interaction. During this time, the wave functions of the electrons remain coherent, but the interactions are subject to interference that leads to restructuring of the energy spectrum in the energy interval $\Delta\varepsilon$. In the analysis of the conductivity $\Delta\sigma$ allowance for the effects[24] yields the same logarithmic dependence (2) that that follows from the "weak" localization theory.[22]

The constant C depends, of course, on the initial premises of the theory, "interaction" or "localization." In both cases, however, it can have a value $C \approx 1$ at which the calculation and measurement results are equal. To determine which of the described mechanisms makes the main contribution to the temperature-dependent correction to the electric conductivity in the surface layers of germanium, we in-

vestigated the temperature dependence of the Hall coefficient, since the effects observed in Refs. 22 and 25 make significantly different contributions to R_H. In fact, under conditions of "weak" localization $\sigma = f(T)$ is decreased by the decrease of the carrier mobility rather than density. The Hall coefficient is then independent of T. In those cases when electron—electron interaction is important, the Hall constant should contain an additional term ΔR_H that depends on T.

Our measurements show that the main contribution to the temperature-dependent correction (1) is made by the decrease of the hole density and not of their mobility. This is seen even from a simple comparison of the values of μ and R_H at two measurement temperatures $T_1 = 1.1$ K and $T_2 = 4.2$ K, shown in Fig. 2. More detailed investigations of the $R_H(T)$ dependence have shown that when the temperature is lowered the Hall coefficient increases in proportion to $\ln T$, in full agreement with the "interaction" theory, while the change $\Delta R_H = R_H(T_1) - R_H(T)$, of the Hall coefficient in a pronounced metallic conductivity region follows the theoretically predicted "rule of two":

$$\Delta R_H/R_H = -2\Delta\sigma/\sigma, \qquad (3)$$

as can be seen from the data of Fig. 6; relation (3), however, is valid only in a small region.

It can be seen from the data of Fig. 6 that the connection between ΔR_H and $\Delta\sigma_\square$ is described by relation (3) only at low values of the product $R_H|\Delta\sigma/\sigma|$, when the quantities R_H and $\sigma_\square$ belong to the region of definitely metallic conductivity (this region is numbered II in Figs. 2 and 4 to distinguish it from the larger region I in which relation (2) holds). It is seen from the results shown in Fig. 6 that as the surface conductivity $\sigma_\square$ decreases and approaches $\sigma_{\min}$ the slope of the experimental curve $\Delta R_H = f(R_H|\Delta\sigma/\sigma|)$ deviates gradually from the value called for by the "rule of two" and tends to "unity." This disparity between the theory[25] and the measured values of R_H and $\sigma_\square$ in the intermediate conductivity region $\sigma_{\min} \sigma_\square 5\sigma_{\min}$, can hardly be ascribed to an increasing contribution of localization effects,[22] since the slope of the $\Delta\sigma_\square \sim \ln T$ curves remains practically unchanged in this region of $\sigma_\square$. The experimentally obtained constant in front of $\ln(T_1/T)$ is close in value to the universal $\sim e^2/2\pi^2\hbar$ obtained theoretically for a two-dimensional metal under conditions when localization effects can be neglected.[26]

Obviously, the deviations from the "rule of two," observed in the interval $\sigma_{\min} \leqslant \sigma_\square \leqslant 5\sigma_{\min}$, indicates not a decreased role of the Coulomb interaction in the region $\sigma_\square \approx \sigma_{\min}$, but attests more likely to a transition from the conditions of "weak" disorder, on which the calculations of Ref. 25 are based, to the conditions of the "strong disorder" realized in the conductivity region $\sigma_\square \to \sigma_{\min}$.

In the entire region of quasimetallic conductivity, the quantitiy $\Delta R_H = R_H(T_1) - R_H(T)$ increases linearly with increase of the product of the corresponding quantities $R_H(T_1)$ and $R_H(T)$. This means that in a fixed temperature interval, i.e., at $\Delta T = \text{const}$, the hole density decreases by approximately the same amount in samples with different initial carrier density. (Our experiments yielded $\Delta p_s \approx 3\cdot 10^{11}$ cm^{-2} at $\Delta T = 3.3$ K.)

The observed decrease of the hole density at low T serves as a "harbinger of a Coulomb gap." Another manifestation of this effect is the shift of the critical hole density towards lower values of p_c, as seen from the $\mu = f(p_s)$ plots in Fig. 4b. As for the idea that a two-dimensional conductor cannot be a real metal, it is difficult to reconcile this statement[22] with our present data.

As seen from Fig. 5, the logarithmic decrease of the

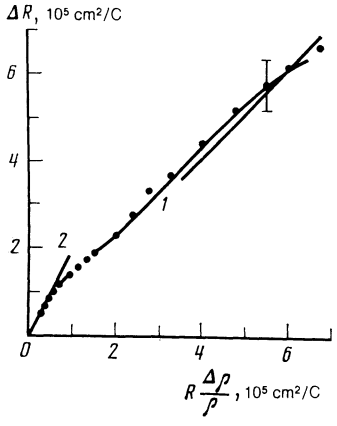

FIG. 6. Temperature correction $\Delta R = R_1 - R_2$ for Hall coefficient vs $R_2(\Delta\rho/\rho)$, where $R_1 = R_H(1.1\text{ K})$; $R_2 = R_H(4.2\text{ K})$ on cleavage surfaces of Ge crystals, and ρ is the reciprocal of the surface conductivity $\sigma_\square$.

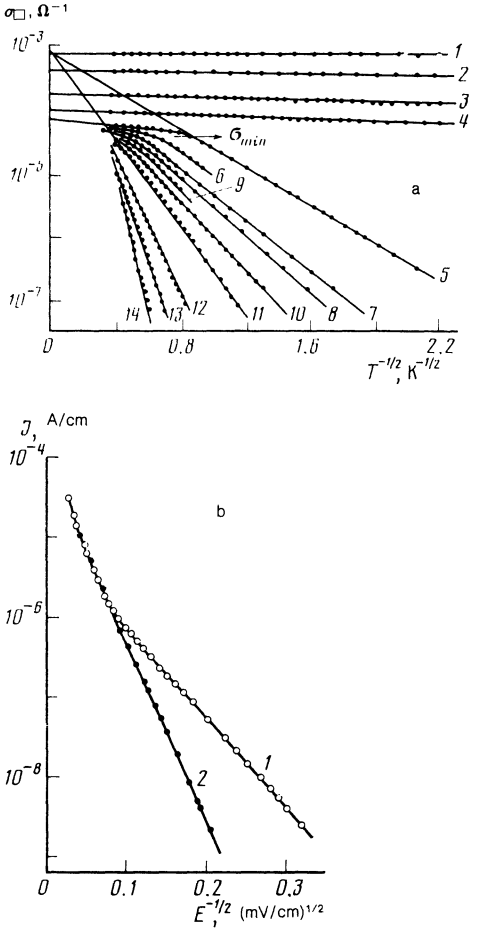

FIG. 7. a) Temperature dependences of the conductivities $\sigma_\parallel$ and $\sigma^\perp$ in Ge bicrystals. Curves 1–4, 6, and 8–11 pertain to $\sigma_\parallel$, curves 1, 2, 5, 7, and 12–14 pertain to $\sigma_\perp$; b) dependences of current density on the reciprocal field strength in samples No. 5. (curve 1) and No. 7 (curve 2).

quasimetallic conductivity gives way in the upshot, when the temperature is lowered, to a weaker dependence, and tends to saturation at the very lowest T. This behavior of $\sigma = f(T)$ cannot be attributed solely to the influence of overheating of the samples themselves or of their electron subsystem, since no correlation can be traced in our experiments between the power (released in the sample) and the value of T_c at which the logarithmic dependence on T is replaced by a weaker one. For example, the data of Fig. 5 show that under identical experimental conditions and at the same measurement-current density the logarithmic $\sigma(T)$ dependence of the sample with lower conductivity ($g_2 = 2.1$) extends to substantially lower temperatures ($T_c \sim 0.15$ K) than that of the sample with higher conductivity, $g_1 = 3.6$, for which a deviation from the logarithmic dependence is observed already at $T_c \sim 0.3$ K. (T_c remains constant here when the measuring current is changed severalfold.)

One cannot exclude the possiblity that a transtion to a temperature-independent metallic conductivity at the very lowest temperatures is an inherent property of the two-dimensional system itself. This assumption seems even more probable if the measured $\sigma = f(T)$ are compared for samples with different $\sigma_\square$. As seen from the data of Fig. 7, samples with close values of $\sigma_\square$ show substantially different $\sigma(T)$ dependences if their initial conductivities ($\sigma_\square$ at $T = 1$ K) turn out to be on opposite sides of the threshold value e^2/h. If the sample conductivity $\sigma_\square$ (1 K) exceeds e^2/h, the samples behave like a metal and hardly change with decrease of T, but if $\sigma_\square$ (1 K) $< e^2/h$ they become insulators with an exponentially decreasing electric conductivity.

§4. TRANSITION FROM QUASIMETALLIC TO THERMALLY ACTIVATED CONDUCTIVITY

At values $\sigma_\square < \sigma_{\min}$ the transport processes become thermally activated and have a complicated character at increased temperatures. At sufficiently low temperatures, $T < 1$ K, one can neglect in conducting germanium layers the contribution of processes connected with thermal transfer of the carriers from localized ones into states lying above the percolation threshold. At $T < 1$ K the electric conductivity is effected by hopping from one localized state to another, and the probability of hopping over a certain distance R is proportional to

$$\exp(-R/R_0)\exp(-W_a/kT), \tag{4}$$

where R_0 is the characteristic length of the wave function, k is the Boltzmann constant, and W_a is the activation energy.

In those cases when the main contribution to the activation energy is made by the Coulomb interaction between an electron and the hole it leaves,[27] W_a can be assumed to equal $e^2/\varkappa R$. The length of the hop corresponding to the maximum conductivity is then

$$R_m = (e^2 R_0/\varkappa kT)^{1/2}, \tag{5}$$

and the dependence of the electric conductivity on the temperature is described by the relation

$$\sigma(T) = \sigma_0 \exp[-(T_0/T)^{1/2}], \tag{6}$$

called the Efros-Shklovskiĭ law, where σ_0 is the pre-exponential factor, $\varkappa$ is the dielectric constant, and T_0 is the characteristic temperature (equal to $6{,}4e^2/\varkappa kR_0$ in the two-dimensional case).

The same relation (6) can be obtained, however, even without invoking the "Coulomb gap" concept,[27] by recognizing that germanium bicrystals have noticeable tendency to acquire quasi-one-dimensional properties in the nonmetallic-conductivity region. The Mott mechanism of hopping conduction[17] leads in this case to an activation energy $W_a = 1/N_0 R$, where N_0 is the density of the states, approximately equal to $m/\pi^2\hbar^2 n$, m is the effective carrier mass, and n the linear carrier density. Optimization of relation (4) under conditions of quasi-one-dimensional motion leads also to relation (6), but with a different characteristic temperature $T_0 = 4/kN_0R_0$.

There exists, finally, a third electric conduction mechanism that can lead to a relation similar to (6), but only in a very narrow range of $\sigma_\square$ in the immediate vicinity of the metal-insulator transition. Inasmuch as near this transition the envelope of the electron wave functions and the lengths of their hops are very large, the main contribution to the electric conductivity can be made under these conditions by hops that are shorter than optimal. Such a process, considered by Zvyagin,[28] led to a relation $\sigma = f(T)$ in the form (6), but with smaller values of the characteristic temperature T_0 and of the pre-exponential factor σ_0. From among the hopping-conduction mechanisms considered by us, this model is the only one in which σ_0 is a constant. (According to Ref. 28, $\sigma_0 \equiv \sigma_{\min}$ under conditions of "nonoptimal" hops).

To derive the relations described above, the measured surface conductivities of germanium cleaved surfaces and in bicrystals were plotted with $\log\sigma$ and $T^{-1/2}$ as the coordinates. In the region $\sigma_\square < \sigma_{\min}$ of interest to us these plots turned out to be straight lines that differed only in slope and in the values of σ_0 obtained by extrapolating these lines.

Most bicrystals (with conductivity $\sigma_\square < \sigma_{\min}$) are described by relation (6) with a pre-exponential factor $\sigma_0 \sim 10^{-3}\ \Omega^{-1}$, as seen from the data shown in Fig. 7. (Exceptions are samples 12–14 with $\sigma_\square$ (4.2 K) $\ll \sigma_{\min}$. We assume that in these samples the two-dimensional conducting channel is partly disrupted.

In the case of cleavage surfaces, the $\log\sigma = f(T^{-1/2})$ plots form two families of straight lines, as shown in Ref. 29. One of them ("insulator") has the same value $\sigma_0 \sim 10^{-3}\ \Omega^{-1}$, and the second ("intermediate between insulator and metal") has $\sigma_0^* \sim 4 \cdot 10^{-5}\ \Omega^{-1}$ and much lower values of the characteristic temperature T_0^*. The latter dependence was observed on cleaved surfaces in a very narrow range of temperatures and only at values of $\sigma_\square$ not much smaller than $\sigma_{\min}$; this dependence is most likely governed by the contribution of the "non-optimal" hops.[28] It is important that no such relation is observed at all in the case of bicrystals. This result is obviously due to the fact that the ingrowth surfaces of bicrystals with angles $\theta < 10°$ contain macroscopic quasi-one-dimensional inhomogeneities elongated along the rows of edge atoms. The contribution of the size effects limits the lengths of the hops in the bicrystals and makes the contribution of the "nonoptimal" hops[28] negligible.

The dominant role in the conductivity of layers adjacent to the cleavage and ingrowth surfaces of germanium crystals is played by a process that can be described by the

relation (6) with characteristic temperatures T_0 in the interval

$$10 \leqslant T \leqslant 100 \text{ K}, \tag{7}$$

and with a preexponential factor close to

$$\sigma_0 \approx (2-3)\frac{e^2}{\hbar} \lesssim 10^{-3}\Omega^{-1} \tag{8}$$

Obviously, this process does not depend on the peculiarities of the investigated systems and is to a certain degree universal. We have assumed that it is due to the Coulomb interaction.[27] To verify this assumption, the described investigations of $\sigma = f(T)$ were supplemented by measurements of $\sigma_\square$ in strong electric fields. Under conditions when the main contribution is made by Coulomb interaction, the carrier activation energy is $W_a \approx e^2\varkappa R$. On application of an electric field E the value of

$$W_a = e^2/\varkappa R - eER \tag{9}$$

decreases gradually, and tends to zero in sufficiently strong electric fields. In the upshot, the hops reach distances R_E given by

$$e^2/\varkappa R_E - eER_E \approx 0. \tag{10}$$

An electron can move in such strong electric fields in the direction of the field and emit phonons after each jump transition. The current is then independent of T but is determined only by the electric field strength

$$J \sim \exp\left[-(E_0/E)^{1/2}\right], \tag{11}$$

where $E_0 = e/\varkappa R_0^2$ is the characteristic electric field. The value of E_0, just as the characteristic temperature T_0, depends on the parameter R_0 and the two should be in one-to-one correspondence.

The investigations of $\sigma = f(T,E)$ were carried out at 20 mK $\leqslant T \leqslant$ 1 K on Ge bicrystals with inclination angles $\theta = 8$–$9°$. Their electric coductivity at $T = 4.2$ K was quasimetallic, and the loss of this conductivity was quite abrupt, as seen from curves 5 and 7 of Fig. 7, obtained under conditions when the current was linear in the applied voltage. Further investigations have shown that the linear connection between the field E and the current J in the investigated samples is observed only in a narrow electric-field interval. When the temperature is lowered this interval decreases greatly and is replaced by a region in which σ depends on E exponentially.[31] With further increase of the electric field intensity the $\sigma = f(T,E)$ plots become ever more gently sloping and eventually merge into one smooth curve that does not depend on T. At the lowest temperatures, $0.05 \leqslant T \leqslant 0.10$ K, the current density J (in fields $E > 1$ mV/cm) dpends only on the electric field strength. In this region of t, as seen from the data of Fig. 7b, the measured $J = f(T,E)$ are well approximated by the relation $J \sim \exp[-(E_0/E)^{1/2}]$, predicted theoretically by Shklovskiĭ.[30]

The table shows a comparison of the values of T_0 and E_0 obtained from the slopes of the experimental curves (shown in Fig. 7), and the values of the characteristic length of the wave function, calculated from the relations $R_0(1) = 6.4e^2/\varkappa T_0$ and $R_0(2) = (e/\varkappa E_0)^{1/2}$ with the empirical values of T_0 and E_0. (It is assumed in the table, to be definite, that $\varkappa = \text{const} = 16$.) From the tabulated data it can be seen that the ratio of $R_0(1)$ and $R_0(2)$ does not exceed 1.5, which can be regarded as a fair agreement. These data offer convincing evidence in favor of the "Coulomb gap" model on which their calculations are based. On the whole, too, the entire aggregate of the investigation results (for both metallic and nonmetallic conductivity) leads to the conclusion that the conducting subsurface layers in germanium constitute a splendid model of a two-dimensional conductor with strong Coulomb interaction.

§5. EXPERIMENTAL VERIFICATION OF THE SCALING THEORY OF LOCALIZATION

The results on the relations $\sigma = f(T)$ on germanium cleaved crystal surfaces and bicrystals were used for a comparison with the scaling theory of localization[11] and to verify the latter.

It is known that the theory of one-parameter scaling is based on the assumption that there exists a universal function

$$\beta = \partial \ln g/\partial \ln L = \beta(g),$$

that depends only on the dimensionality d of the space; here $g = \hbar e^{-2}\sigma L^{d-2}$ is the dimensionless conductivity, $\hbar$ Planck's constant, e the electron charge, σ the electric conductivity, and L the linear dimension of the system. To check on the scale invariance hypothesis, it is convenient to use the results of measurements of the surface conductivity $\sigma_\square$, since a simple relation obtains under conditions of two-dimensional conductivity between the measured quantity $\sigma_\square$ and the only parameter of the theory[22]

$$g^{(2)} = \hbar e^{-2}\sigma_\square \approx 4 \text{ k}\Omega \cdot \sigma_\square. \tag{12}$$

The question of experimental verification of the theory of Ref. 22 for the case of a two-dimensional medium was considered in Ref. 32, where it was shown that the empirical function

$$\beta_e = -\Delta \ln g/\Delta \ln T, \tag{13}$$

which is easily deduced from the measured values of $\sigma_\square$, proportional to the scaling function $\beta(g)$ under the conditions usually realized in experiment, viz., when the phase relaxation of the wave function is determined by inelastic-scattering processes, and the role of L can be assumed by the diffusion length L_{in} determined by these inelastic precesses.

TABLE I

№	$\sigma_\square$ (4,2 K)	T_0, K	E_0, V/cm	$R_0(1)$, cm	$R_0(2)$, cm
1	$5.5\cdot10^{-5}$	11.5	0.7	$6\cdot10^{-5}$	$9\cdot10^{-5}$
2	$4.0\cdot10^{-5}$	22	2.5	$3.7\cdot10^{-5}$	$6\cdot10^{-5}$

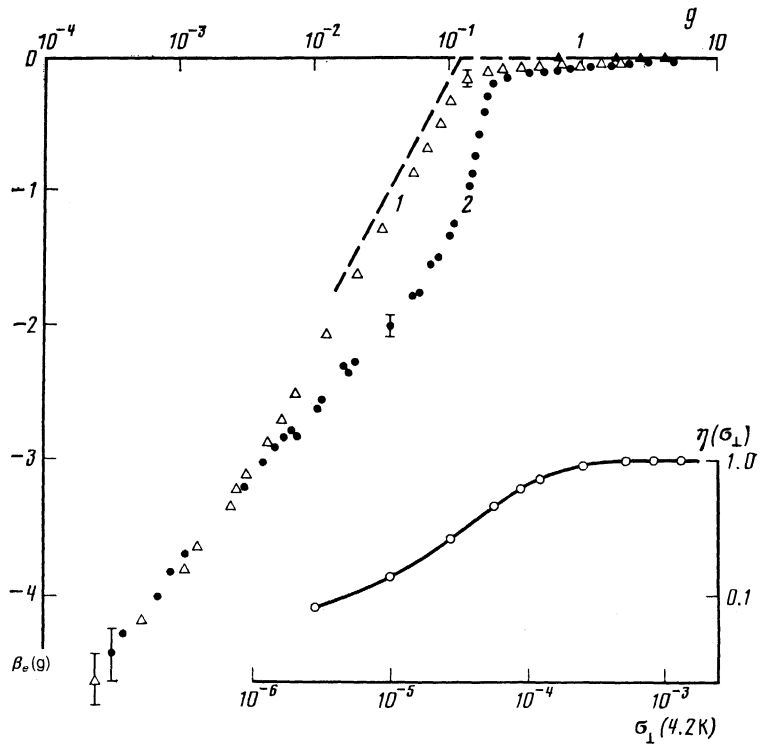

FIG. 8. Plot of $\beta_e = -\Delta \ln g/\Delta \ln T$, obtained by measuring the electric conductivity: △—for Ge cleavage surface at $T = 1.1$–1.5 K; ●—for Ge bicrystals in the region $0.3 \leqslant T \leqslant 5$ K (▲—for $T < T_c$). The dashed line is the function $\beta(g)$ from Ref. 33; $\eta = \sigma_\perp/\sigma_\parallel$ is the anisotropy coefficient of the electric conductivity of bicrystals at $T = 4.2$ K.

Usually L_{in} has a power-law dependence on the temperature and it can be assumed that $L_{in} \propto T^{-\gamma}$, where γ is a numerical coefficient.

Thus, in all cases when it is permissible to replace the system dimension L by a quantity proportional to $T^{-\gamma}$, the theoretical and empirical functions in a two-dimensional medium should be connected by the simple relation

$$\beta_e = \gamma\beta(g), \tag{14}$$

and analysis of the empirical relation $\beta_e = f(g)$ can determine whether the one-parameter-scaling hypothesis is valid, provided a universal empirical $\beta_e(g)$ dependence is found.

Attempts to find a $\beta_e(g)$ dependence were undertaken in Refs. 32 and 33, but led to contradictory results: in the case of inversion layers on an Si–SiO_2 interface the function β_e was not universal,[32] while for the two-dimensional systems considered in Ref. 33 the relations $\beta_e = f(g)$ turned out to be in good agreement with one another and with results of numerical calculations.[34]

In the present paper we determined the functions $\beta_e = -\Delta \ln g/\Delta \ln T$ from the meaurements of the electric conductivity in the subsurface layers of germanium adjacent to the crystal cleavage surfaces and to the bicrystal intergrowth boundaries. The obtained $\beta_e = f(g)$ plots are shown in Fig. 8. It can be seen that in the region of small as well as large values of g the functions $\beta_e(g)$ do not depend on the distinctive features of the investigated systems and have a universal form that agrees with the results of the scaling theory of localization. At small g, in the region of hopping conduction, the slopes of the linear section of the $\beta_e(\ln g)$ curves in Fig. 8 are close to $\frac{1}{2}$; this accords with the theoretical asymptote if the inelastic length L_{in} is taken to be the length of the hop and it is assumed that $\gamma = \frac{1}{2}$. At large g, the conductivity has a logarithmic dependence on the temperature, in agreement with calculations of the localization and correlation corrections to the conductivity of two-dimensional systems. In the intermediate region $g \approx 0.1$, however, which corresponds to the transition from a logarithmic temperature dependence of the conductivity to an exponential one, the curves 1 and 2 of Fig. 8 do not coincide. The plot of β_{e1} obtained by measuring $\sigma(T)$ on germanium cleavage surfaces agrees qualitatively with $\beta(b)$ numerically calculated for the Anderson model,[34] and represented by the dashed line in Fig. 8.

As for the β_{e2} dependence obtained by measuring the conductivity of bicrystals, it can be seen from Fig. 8 that β_{e2} drops quite steeply at $g \approx 0.1$, and then decreases smoothly enough with further decrease of g. The observed peculiarities of the function β_{e2} are obviously due to the variety of structures of bicrystal intergrowth surfaces. As already noted, however, the intergrowth surfaces in germanium bicrystals contain macroscopic inhomogeneities that stretch along the chains of the edge atoms. Owing to the contribution of size effects, these longitudinal inhomogeneities can substantially influence the function β_{e2} in the region $g \approx 0.1$ if the length L_{in} becomes comparable in this case with the characteristic width of the percolation channels. The function $\overline{\beta} = \partial \ln \bar{g}/\partial \ln L_{in}$ for a model two-dimensional system (made up of conducting bands) was calculated by Zvyagin[35] by a localization-correction method developed in Refs. 36 and 37. This calculation has shown that in the transition region of the g values the $\overline{\beta}(g)$ curve for the model system of bands agrees qualitatively with the β_{e2} dependence obtained from measurements of $\sigma(T)$ of bicrystals. This correspondence between the functions $\beta(g)$ and β_{e2} allows us to assume that in germanium bicrystals the transition from metallic to activation conduction is due to the lowering of the effective dimensionality of the conducting system and to a transition from top-dimensional to quasi-one-dimensional conductivity at values $g \approx 0.1$.

With further decrease of g, in the hopping-conduction region, the effective dimensionality of the bicrystal surface can again increase because of "two-dimensionalization" of the trajectories as formed by successive carrier hops. It is known that in one-dimensional systems the topological conduction regime[38] lead to a steeper decrease of $\sigma = f(T)$ than in the case of two-dimensional systems. As a result the optimal trajectories may turn out to be not along the one-dimensional channels, but two-dimensional trajectories, even though they pass over regions with large values of T_0 in the relation $\sigma = \sigma_0 \exp[-T_0/T)^{1/2}]$.

Thus, the experimental data on the conductivities of crystal cleavage surfaces and bicrystal intergrowth surfaces can on the whole be qualitatively described within the framework of a one-parameter scaling localization theory. The observed singularities of the function β_e in the region $g \sim 0.1$ do not contradict the conclusions of this theory and can be attributed to a change of the effective dimensionality of the system (2) in this region of g.

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