Structure of photodetached resonances in relativistic ion beams in a magnetic field

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(Submitted 31 March 1986)
Zh. Eksp. Teor. Fiz. **93**, 762–768 (August 1987)

The influence of cyclotron magnetization of negative ions in relativistic electron beams on the structure of photodetached resonances is investigated. It is found that Larmor rotation of ions in a uniform static magnetic field can suppress the Doppler broadening of the photodetachment spectrum, so that one obtains a sequence of equidistant photodetached cyclotron resonances and a significantly higher photodetachment cross section at the center of the spectrum.

1. Recent experiments on resonant photodetachment of electrons from negative ions (see, e.g., Refs. 1–5) have shed much light on electron–atom interaction in magnetic fields, a problem of interest in plasma physics, astrophysics, and various branches of atomic physics. The influence of the Zeeman effect on the photodetachment process has been studied theoretically and experimentally.[3,6] Moreover, the electron-atom interaction is now known to alter the motion of the electrons normal to the magnetic field, and this process may in turn affect the photodetachment cross section.[2,5,7]

Negative ions in a magnetic field **H** gyrate much more slowly than the electrons due to the large difference in the masses m_i, m_e. When the ions are continuously illuminated by monochromatic radiation, this motion gives rise to a series of equidistant satellites of optical frequency in the rest frame of the ion; the spacing between the satellites is determined by the ion gyrofrequency $\omega_L = eH/m_i c$ (Refs. 8,9). However, in these experiments the ions were irradiated by laser pulses much shorter than the ion cyclotron period, so that the ion gyrations had no effect on the photodetachment spectrum.

In a reference frame moving with the relativistic ion beam at close to the speed of light, $u \sim c$, the frequency of the interacting light is greatly shifted due to the Doppler effect. This makes it possible to use highly coherent tunable cw lasers operating at visible wavelength to investigate narrow photodetachment resonances in the ultraviolet region of the spectrum. We note that in Ref. 10, it was suggested that wavelengths extending into the x-ray region might be attainable by exploiting the relativistic transformation of laser light interacting with a fast ion beam. Unfortunately, the large velocity spread $\Delta u \sim 10^{-3} u(1 - u^2/c^2)$ (Refs. 10,11) of the ion beams produced by modern accelerators causes significant Doppler broadening and greatly decreases the amplitude of the photodetachment resonance.[4,6] In the present paper, we analyze how ions undergoing Larmor gyrations in a magnetic field can be used to eliminate this broadening and obtain a sequence of narrow cyclotron resonances in the photodetachment spectrum and a much higher photodetachment cross section at the center of the spectrum.

2. We will calculate the linear susceptibility and the photodetachment cross section for an ion transition from the ground state n to a state ε in the continuum, which also is also assumed to contain a discrete autodetachment level m. The radiation is assumed to propagate opposite to the ion beam and perpendicular to the static, uniform magnetic field **H'**. In a reference frame moving with the ion beam, the magnetic field is

$$H = H'\gamma, \quad \gamma = (1-\beta^2)^{-1/2}, \quad \beta = u/c, \tag{1}$$

and there is an electric field

$$E = \beta\gamma H', \tag{2}$$

directed normal to the plane of the vectors **u** and **H** (Ref. 12). The laser light frequency ω' in the laboratory frame is related to the frequency ω in the ion rest frame by

$$\omega = \omega'\gamma(1+\beta). \tag{3}$$

We consider the photodetachment process for ions with angular momentum $J_n = 0$ in the ground state, and take $J_\varepsilon = J_m = 1$ in the continuum and autodetachment states. If the wave is linearly polarized and **H** lies in the plane of polarization perpendicular to the direction of the wave vector **k**, only the π-component of the radiation will be present in the spectrum. In this specific case there is thus no Zeeman splitting, and the only effect of the magnetic field on the shape of the photodetachment resonances is through the cyclotron motion of the ions. According to Refs. 13 and 14, the contribution of the static electric field **E** to the broadening of the spectrum is negligible for $\mathbf{E} \perp \mathbf{k}$.

We choose the x, y, z coordinate axes of the comoving frame to lie along the vectors **E**, **u**, and **H**. In this frame the elements of the density matrix corresponding to the transitions m–n, ε–n have the familiar oscillatory behavior

$$\begin{aligned} \rho_{mn} &= r_{mn} \exp\{-i[(\omega-\omega_{mn})t - \mathbf{kr}]\}, \\ \rho_{\varepsilon n} &= r_{\varepsilon n} \exp\{-i[(\omega-\omega_{\varepsilon n})t - \mathbf{kr}]\}. \end{aligned} \tag{4}$$

We now pass to cylindrical coordinates in ion velocity space **v**, writing $\mathbf{v}_\perp$ for the velocity component in the plane normal to **H** and φ for the angle between $\mathbf{v}_\perp$ and **k**. If we neglect the constant acceleration of the ions in the electric field, which has no influence on the spectrum in the present case, we find the equations

$$[\Gamma - i(\Omega - kv_\perp \cos\varphi) - \omega_L \partial/\partial\varphi] r_{mn} = -i(G_{mn} + \delta_{mn} - i\gamma_{mn})\rho_{nn}, \tag{5}$$

$$[-i(\omega - \omega_{\varepsilon n} - kv_\perp \cos\varphi) - \omega_L \partial/\partial\varphi] r_{\varepsilon n} = -i(G_{\varepsilon n}\rho_{nn} + G_{\varepsilon m} r_{mn}), \tag{6}$$

$$\Gamma = \Gamma_{mn} + \gamma_{mm}, \quad \Omega = \omega - \omega_{mn} - \delta_{mm}, \quad G_{jj'} = V_{jj'}/\hbar, \tag{7}$$

$$\gamma_{mj} = \pi\hbar G_{m\varepsilon} G_{\varepsilon j}|_{\varepsilon=\hbar\omega},$$

$$\delta_{mj} = \hbar \sum_l \frac{2G_{ml}G_{lj}\varepsilon_l}{(\hbar\omega)^2 - \varepsilon_l^2} + \frac{1}{\pi} \fint \frac{\gamma_{mj}(\varepsilon)\, d\varepsilon}{\hbar\omega - \varepsilon}, \quad j = m, n \tag{8}$$

 0038-5646/87/080430-03$04.00

for the elements of the density matrix, which determine the polarization of the resonance transition in the presence of an interacting radiation field. Here $V_{jj'}$ denotes a matrix element of the operator describing the interaction with the radiation; $V_{\varepsilon n} = E_0 d_{\varepsilon n}$ and $V_{mn} = E_0 d_{mn}$, where E_0 is the amplitude of the traveling wave and $d_{\varepsilon n}$, d_{mn} are the matrix elements for the electric dipole transitions. The constant Γ_{mn} describes the spontaneous relaxation of the autodetachment state. The parameters γ_{mj} and δ_{mj} allow for interference effects involving the interaction of states m, n with the continuum; the summation in the expression for δ_{mj} includes all the nonresonant states. In deriving and solving Eqs. (5), (6) we use methods similar to those discussed in Refs. 13–16. The population $\rho_{nn}(\mathbf{v})$ of the ground state is assumed independent of time and given by a Maxwellian distribution

$$\rho_{nn}(\mathbf{v})=NF(\mathbf{v}),\quad F(\mathbf{v})=(\pi^{1/2}\Delta u)^{-3}\exp[-v^2/(\Delta u)^2],\quad (9)$$

where $\mathbf{v}$ is the ion velocity relative to the comoving frame, and N is the total number of ions per unit volume.

The solutions of Eqs. (5), (6) for the off-diagonal matrix elements can be expanded in terms of Bessel functions $J_l(\xi)$, where $\xi = kv_\perp/\omega_L$:

$$r_{mn}=-iNF(\mathbf{v})\,(G_{mn}+\delta_{mn}-i\gamma_{mn})\,e^{i\xi\sin\varphi}\sum_{l=-\infty}^{\infty}\frac{e^{-il\varphi}J_l(\xi)}{\Gamma-i(\Omega-l\omega_L)},\quad (10)$$

$$r_{\varepsilon n}=NF(\mathbf{v})\,e^{i\xi\sin\varphi}\sum_{l=-\infty}^{\infty}\left[G_{\varepsilon n}-\frac{iG_{\varepsilon m}(G_{mn}+\delta_{mn}-i\gamma_{mn})}{\Gamma-i(\Omega-l\omega_L)}\right]\times\frac{e^{-il\varphi}J_l(\xi)}{\omega-\omega_{\varepsilon n}-l\omega_L}.\quad (11)$$

The infinite sequence of resonances centered at $\Omega = l\omega_L$ is due to the cyclotron rotation of the ions in the magnetic field.

The elements r_{mn}, $r_{\varepsilon n}$ of the density matrix determine the linear susceptibility of the medium,

$$\chi=\frac{2\hbar}{|E_0|^2}\left\langle G_{nm}r_{mn}+\int G_{n\varepsilon}r_{\varepsilon n}\,d\varepsilon\right\rangle_{\mathbf{v}}.\quad (12)$$

We use the relation

$$[-i(\omega-\omega_{\varepsilon n}-l\omega_L)]^{-1}\approx\pi\delta(\omega-\omega_{\varepsilon n})+\mathscr{P}i/(\omega-\omega_{\varepsilon n})\quad (13)$$

to integrate over the energy ε of the continuum state; here $\mathscr{P}$ denotes principal value. After averaging over the velocities, we get the expression

$$\chi=\sum_{l=-\infty}^{\infty}e^{-\eta}I_l(\eta)\chi_l,\qquad \eta=\frac{(k\Delta u)^2}{2\omega_L^2},\quad (14)$$

for χ in terms of the modified Bessel function $I_l(\eta)$, where

$$\chi_l=\tilde{\chi}_0\left[1+Q\frac{(q_{mn}-i)^2}{(1+iq_{nn})(1-ix_l)}\right],\quad \tilde{\chi}_0=-\frac{2i\hbar N}{|E_0|^2}(\gamma_{nn}+i\delta_{nn}),$$

$$x_l=(\Omega-l\omega_L)/\Gamma,\quad Q=\gamma_{mn}^2/\gamma_{nn}\Gamma,\quad (15)$$

$$q_{mn}=(G_{mn}+\delta_{mn})/\gamma_{mn},\quad q_{nn}=\delta_{nn}/\gamma_{nn},$$

and the parameters γ_{nn}, δ_{nn} for the linear susceptibility χ_l corresponding to a specific cyclotron resonance with $l = \Omega/\omega_l$ are defined as in (8). The dimensionless parameter Q characterizes the overlap of the wave functions of the continuum states to which transitions from levels m, n can occur; its value is $Q = 1$ in the limit $\Gamma_{mn} \to 0$. The square of the dimensionless quantity q_{mn} is proportional to the probability for a transition to the autodetachment state divided by the probability for a transition into a band of width γ_{mm} in the continuum.

It follows from (14) that the formula for the resonant light absorption coefficient α has the same structure:

$$\alpha=4\pi k\,\mathrm{Im}\,\chi=\sum_{l=-\infty}e^{-\eta}I_l(\eta)\alpha_l,$$

$$\alpha_l=\tilde{\alpha}_0\left(1+Q\frac{q_{mn}^2-1+2q_{mn}x_l}{1+x_l^2}\right),\quad \tilde{\alpha}_0=4\pi^2kN|d_{\varepsilon n}|^2_{\varepsilon=\hbar\omega}.\quad (16)$$

For $\Gamma < \omega_L$, α traces out a sequence of equidistant cyclotron resonance separated by ω_L as the detuning frequency Ω varies. The asymmetry of the absorption contour $\alpha_l(\Omega)$ for the lth resonance is due to interference between the direct photodetachment and autodetachment processes during the transition into the continuum. The shape of the resonances $\alpha_l(x_l)$ is sensitive to the value of q_{mn}. On the whole, the profile of the curve $\alpha_l(x_l)$ is similar to that for an ordinary Feshbach resonance, which was calculated by Fano[17–19] without allowing for the motion of the ions. The relative contribution from each cyclotron resonance to the total photoabsorption is proportional to $I_l(\eta)$.

If $H = 0$ or $\omega_L < \Gamma$, the cyclotron resonances coalesce to form a Doppler-broadened line:

$$\alpha(\Omega)=\tilde{\alpha}_0\{1+Q[(q_{mn}^2-1)\,\mathrm{Re}\,w+2q_{mn}\,\mathrm{Im}\,w]\}\pi^{1/2}\Gamma/k\Delta u,\quad (17)$$

$$w(z)=e^{-z^2}\left(1+\frac{2i}{\pi^{1/2}}\int_0^z e^{t^2}dt\right),\quad z=\frac{\Omega+i\Gamma}{k\Delta u}.$$

In the limit of large Doppler broadening ($\Gamma \ll k\Delta u$), we have

$$w=(1+2i\Omega/\pi^{1/2}k\Delta u)\exp[-\Omega^2/(k\Delta u)^2].\quad (18)$$

For $\omega_L > \Gamma$, the contour (17) is the envelope of a sequence of split cyclotron resonances. In this case the Larmor gyration of the ions cancels the Doppler broadening of the photodetachment resonance.

To calculate the photodetachment cross section

$$\sigma=\frac{16\hbar k}{|E_0|^2}\langle\gamma_{nn}\rho_{nn}+\gamma_{mm}\rho_{mm}+2\,\mathrm{Re}(\gamma_{mn}r_{mn})\rangle_{\mathbf{v}}\quad (19)$$

one must solve the equation

$$(2\Gamma+ikv_\perp\cos\varphi-\omega_L\partial/\partial\varphi)\rho_{mm}=2\,\mathrm{Im}[(G_{mn}+\delta_{mn}-i\gamma_{mn})r_{nm}]\quad (20)$$

for the population of the autodetachment state. Recalling Eq. (10) for $r_{mn}(\mathbf{v})$, we get

$$\rho_{mm}=NF(\mathbf{v})\frac{\gamma_{mn}^2(1+q_{mn}^2)}{\Gamma^2}\times\mathrm{Re}\sum_{l=-\infty}^{\infty}\frac{J_l(\xi)\exp[-i(\xi\sin\varphi-l\varphi)]}{1+ix_l},\quad (21)$$

$$\sigma=\sum_{l=-\infty}^{\infty}e^{-\eta}I_l(\eta)\sigma_l,$$

where

$$\sigma_l=\tilde{\sigma}_0\left[1+Q\frac{\theta(1+q_{mn}^2)-2+2q_{mn}x_l}{1+x_l^2}\right],$$

$$\tilde{\sigma}_0=\frac{\tilde{\alpha}_0}{N},\quad \theta=\frac{\gamma_{mm}}{\Gamma}. \tag{22}$$

The parameter θ ranges from 0 to 1 depending on the laser light intensity. For $\Gamma_{mn}\ll\gamma_{mm}$, we have $\theta\approx 1$ and the cyclotron resonances $\sigma_l(x_l)$ in the photodetachment cross section have the same form as the absorption resonances $\alpha_l(x_l)$.

For $\Gamma\ll k\Delta u$, the number of resonances in the Doppler envelope of the photodetachment spectrum is given by the ratio $k\Delta u/\omega_L$. Comparison of expressions (16), (22) with Eq. (17) shows that when $\Gamma\ll\omega_L$, the suppression of the Doppler broadening of the photodetachment resonances due to cyclotron effects is accompanied by an increase in the photodetachment cross section and in the absorption coefficient at the center of the spectrum, by roughly a factor of ω_L/Γ. This increase occurs because the interaction with the continuum, even when a magnetic field is present, does not change the total magnitude of the photoabsorption for transitions to the autodetachment state.[1)]

3. In practice, discrete levels may interact simultaneously with several bands in the continuum. The expressions for the absorption coefficient and photodetachment cross section can be generalized to this case by writing

$$\gamma_{jj'}=\sum_s\gamma_{jj'}^s, \tag{23}$$

where the superscript s labels the continuum bands. Since for atomic ions the Zeeman splitting is roughly m_i/m_e times larger than ω_L, the generalization of the above theory to the case of arbitrary radiation polarization and states with unequal angular momenta reduces to a description of the cyclotron effects for each individual Zeeman component of the spectrum.

The above theory applies without modification to the analysis, e.g., of single-photon detachment of an electron from a negative hydrogen ion that absorbs light during a transition from the $^1S^e$ ground state to the doubly excited $^1P(2)$ state. If the source of the continuous illumination is an Ar^+ laser with wavelength $\lambda'=4880$ Å in the laboratory frame, then by (3) the ion beam velocity must satisfy $\beta\approx 0.95$ for resonance with this transition to occur [for the H^- ion $\lambda=1127$ Å, and $\Gamma_{mn}=1.6\cdot 10^8\ s^{-1}$ (Refs. 4, 21)]. For such a beam, the velocity scatter of the ions will be $\Delta u\sim 10^{-3}\ c\gamma^{-2}\approx 3\cdot 10^6$ cm/s. Hence in this case we have $k\Delta u\approx 2\cdot 10^{12}\ s^{-1}$ and for $\Gamma\sim\Gamma_{mn}$ the condition $\Gamma\ll k\Delta u$ holds. In the laboratory frame the critical magnetic field H_0' for which Doppler broadening of the resonances is eliminated and the cyclotron resonances are separated, must satisfy

$$H'>\Gamma m_i c/\gamma e, \tag{24}$$

i.e., for the above transition $H_0'\approx 10$ kOe. The splitting of the resonances increases with increasing beam velocity.

Cyclotron magnetization of ions in relativistic beams thus provides an effective technique for studying long-lived autodetachment states and for increasing the cross sections for resonant photodetachment.

We are grateful to G. N. Alferov and E. A. Kuznetsov for a helpful discussion of the above results.

[1)] A similar idea in a different guise was suggested in Ref. 20 for increasing the pumping efficiency in masers.

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Translated by A. Mason