Excitation of plasma waves by an electromagnetic wave packet

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Nonlinear excitation of longitudinal Langmuir waves in a plasma by a short electromagnetic wave packet is considered. The possibility of accelerating particles by using fast plasma waves excited by a short laser pulse in a low-density plasma is discussed.

The feasibility of Čerenkov and transition radiation from a packet of electromagnetic waves was first discussed more than twenty years ago in Refs. 1 and 2. Čerenkov radiation from a femtosecond laser pulse was relatively recently recorded experimentally in a nonlinear electro-optical medium,[3] and the pertinent theory is presented in Ref. 4.

No Čerenkov radiation is possible in an isotropic plasma, since the phase velocity of the transverse waves exceeds the speed of light. A wave packet, however, can emit longitudinal plasma waves. This question has attracted attention relatively recently in view of the development of new particle-acceleration methods.[5,6] A computer experiment[5,6] has shown that the efficiency with which plasma waves are excited by a short wave packet can be quite high.

The present paper is devoted to the theory of excitation of plasma Langmuir waves by a packet of electromagnetic radiation. We obtain the amplitude of the excited waves and the energy loss due to radiation. The conditions under which the packet can be spread out by dispersion and diffraction are formulated. The possibility is discussed of using plasma waves excited by a laser pulse to accelerate electrons, as well as in diagnostic methods.

Conceivably, a phenomenon similar to the emission of plasmons by photons, considered in the present paper, is feasible also in other nonlinear material media.

1. ONE-DIMENSIONAL CASE: BASIC EQUATIONS

We consider first emission of plasma waves in the one-dimensional case. This approximation is justified if the length of the packet (in the direction of propagation) is much smaller than its cross section. This condition indeed holds in experiments with femtosecond laser pulses. A pulse of duration 10^{-14} s is 3 μm long, whereas its transverse dimension, determined by the focusing system, is usually tens or even hundreds of microns.

We describe the wave packet by using a system consisting of the Maxwell equations and the hydrodynamic equations for the electrons, disregarding the thermal motion and collisions of the latter.[7] The plasma ions are assumed fixed. From these equations we obtain for the electron momentum component $p_\perp$, the electric field intensity E, and the magnetic induction B, all perpendicular to the X axis along which the packet propagates,

$$\frac{\partial p_\perp}{\partial t}+v_\parallel\frac{\partial p_\perp}{\partial x}=eE-\frac{e}{c}v_\parallel B, \tag{1.1}$$

$$\frac{\partial E}{\partial x}=-\frac{1}{c}\frac{\partial B}{\partial t}, \tag{1.2}$$

$$\frac{\partial B}{\partial x}=-\frac{1}{c}\frac{\partial E}{\partial t}+\frac{4\pi e}{c}nv_\perp, \tag{1.3}$$

where $v_\perp$ and $v_\parallel$ are the transverse and longitudinal components of the electron velocity and are connected with $p_\perp$ by the relation $p_\perp = mv_\perp[1-(v_\perp^2+v_\parallel^2)/c^2]^{-1/2}$, and n is the electron density. If the condition $v_\perp \gg v_\parallel$ is met, Eqs. (1.1)–(1.3) lead in the weakly relativistic case $((v_\perp^2/c^2)<1)$ to the following equation for $v_\perp$:

$$c^2\frac{\partial^2 v_\perp}{\partial x^2}-\frac{\partial^2 v_\perp}{\partial t^2}-\omega_p^2 v_\perp =-\frac{1}{2}\frac{\partial^2 v_\perp^3}{\partial x^2}+\frac{1}{2c^2}\frac{\partial^2 v_\perp^3}{\partial t^2}+\frac{\omega_p^2}{n_0}\delta n v_\perp, \tag{1.4}$$

where it is assumed that the electron density is equal to $n_0+\delta n$, n_0 is the density unperturbed by the packet, δn is the density perturbation due to the packet, and $\omega_p=(4\pi e^2 n_0/m)^{1/2}$ is the plasma frequency.

The terms in the right-hand side of (1.4) take into account nonlinear effects due to the relativistic transverse motion of the electrons, and also to the density perturbation δn produced by the packet. This perturbation is due to the electron motion along the packet propagation direction, and the corresponding system of equations takes the form

$$\frac{\partial v_\parallel}{\partial t}+v_\parallel\frac{\partial v_\parallel}{\partial x}=-\frac{e}{m}\frac{\partial \varphi}{\partial x}+\frac{e}{mc}v_\perp B, \tag{1.5}$$

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nv_\parallel)=0, \tag{1.6}$$

$$\frac{\partial^2\varphi}{\partial x^2}=-4\pi e(n-n_0), \tag{1.7}$$

where φ is the charge-separation potential. In the weakly relativistic limit we obtain from (1.5)–(1.7) and (1.1) for small perturbations of the electron density.

$$\frac{\partial^2\delta n}{\partial t^2}+\omega_p^2\delta n=\frac{n_0}{2}\frac{\partial^2 v_\perp^2}{\partial x^2}. \tag{1.8}$$

The system (1.4) and (1.8) determines the interrelated transverse and longitudinal motions of the electron in the wave packet.

The collisionless absorption, due to nonlinear forces, of the packet energy by a plasma is considered in a number of papers, (see, e.g., Ref. 8). We consider this question as applied to our statement of the problem.

In the nondissipative case, the total packet energy is, of course, conserved. This follows from the equations for the energy density, which follow from Eqs. (1.1)–(1.3) and (1.5)–(1.7)

$$\frac{c}{4\pi}\frac{\partial}{\partial x}(EB)+\frac{1}{8\pi}\frac{\partial}{\partial t}(E^2+B^2)=-env_\perp E, \tag{1.9}$$

$$\frac{\partial}{\partial x}\left(\frac{mnv_\parallel^3}{2}\right)+\frac{\partial}{\partial t}\left[\frac{mnv_\parallel^2}{2}+\frac{1}{8\pi}\left(\frac{\partial\varphi}{\partial x}\right)^2\right]=-nv_\parallel v_\perp\frac{\partial p_\perp}{\partial x}. \tag{1.10}$$

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Adding (1.9) to (1.10) and using the continuity equation (1.6), we obtain on the right-hand side

$$-\frac{\partial}{\partial t}(nv_\perp p_\perp)-\frac{\partial}{\partial x}(nv_\perp p_\perp v_\parallel).$$

Thus, with allowance for the transverse and longitudinal particle motions, and also for the transverse and longitudinal fields, the equation for the total energy density is reduced to a divergent form, so that the energy-conservation law holds. Individually, however, the transverse and longitudinal energies are not conserved

If we take the packet energy to mean only its transverse part, we can speak of energy loss. In our approximation, in which the density perturbations are small and relativistic effects are weak, we find from (2.9) that the transverse-energy density is

$$W_\perp=(E^2+B^2)/8\pi+mnv_\perp{}^2/2,$$

and its change per unit time amounts to

$$\delta W_\perp=mv_\perp{}^2(\partial\delta n/\partial t)/2.$$

In the same approximation, it follows from (1.10) that the change of the longitudinal-energy density is given by

$$\delta W_\parallel=-mn_0v_\parallel(\partial v_\perp{}^2/\partial x)/2.$$

The total energy of a packet localized in space is then conserved, since

$$\int dx(\delta W_\perp+\delta W_\parallel)=0.$$

2. EXCITATION OF PLASMA WAVES

Consider the excitation of plasma waves by a one-dimensional packet having a specified permanent form. The conditions under which this approximation is valid will be discussed below.

Assume that the transverse electron velocity is

$$v_\perp(x,t)=\frac{1}{2}[a(\xi)e^{-i\omega t+ikx}+a^*(\xi)e^{i\omega t-ikx}], \qquad (2.1)$$

where ω and k are respectively the high carrier frequency and the wave number, which are connected by the dispersion relation $\omega^2=k^2c^2+\omega_p^2$ (Ref. 7), and a is a slowly varying envelope that depends on the combination of variables $\xi=x-v_g t$, where v_g is the group velocity.

Substituting (2.1) in (1.8) we find that the packet produces high- and low-frequency density perturbations. The high-frequency perturbations (δn_2) occur at the second harmonic and if the plasma is transparent (for $2\omega>\omega_p$) they are localized only the region of space where the packet is present:

$$\delta n_2=\frac{n_0k}{8\omega^2}\left\{\exp(-2i\omega t+2ikx)\left[ka^2\left(1+\frac{\omega_p{}^2}{4\omega^2}\right)\right.\right.$$
$$\left.\left.-i\left(1-\frac{kv_g}{\omega}\right)\frac{da^2}{d\xi}\right]+\text{c.c.}\right\}. \qquad (2.2)$$

The low-frequency perturbation δn_0 is determined by the space-time variation of the envelope. Under the condition that as $x\to\infty$ there are no such perturbations ahead of the packet, we get from (1.8)

$$\frac{\delta n_0}{n_0}=\frac{1}{4v_g{}^2}\left\{|a(\xi)|^2-k_p\int_\infty^\xi d\xi'|a(\xi')|^2\sin k_p(\xi-\xi')\right\}, \qquad (2.3)$$

where $k_p=\omega_p/v_g$. According to (1.7), the perturbations of the potential are given by

$$\varphi=-\frac{mk_p}{4e}\int_\xi^\infty d\xi'|a(\xi')|^2\sin k_p(\xi-\xi'). \qquad (2.4)$$

Specifying the envelope $a(\xi)$, we can easily find with the aid of (2.3) and (2.4) the density and potential perturbations. By way of the simplest example, we consider a packet of rectangular shape:

$$|a(\xi)|^2=a_0{}^2[\theta(\xi+L/2)-\theta(\xi-L/2)],$$

where θ is the Heaviside unit step function. From (2.3) we get

$$\frac{\delta n_0}{n_0}=\frac{a_0{}^2}{4v_g{}^2}\begin{cases}1-2\sin^2\left[k_p\left(\dfrac{\xi}{2}-\dfrac{L}{4}\right)\right] & \text{if } \dfrac{L}{2}>\xi>-\dfrac{L}{2}\\[2ex] 2\sin k_p\xi\sin\dfrac{k_pL}{2} & \text{if } \xi<-\dfrac{L}{2}\end{cases} \qquad (2.5)$$

It can be seen from (2.5) that even inside the packet there exists, besides the terms that duplicate the form of the envelope, also a periodic perturbation of the density. What remains behind the packet is only this oscillating perturbation, and its amplitude depends on k_pL. The maximum perturbations occur at $k_pL=\pi(2l+1)$, where $l=0,1,2,\ldots$.

The low-frequency perturbations of the density of the potential can therefore be plasma waves of length $\lambda_p=2\pi v_g/\omega_p$ and can exist outside the region within which the transverse field of the packet is localized. This makes emission of plasma waves by the packet meaningful.

The plasma-wave potential far behind the packet can be obtained from (2.4) without specifying the actual shape of the envelope

$$\varphi(\xi)=\varphi_0\sin(k_p\xi+\psi), \qquad (2.6)$$

where $\varphi_0=(mk_p/4e)R$,

$$R=\left[\left(\int_{-\infty}^{\infty}d\xi|a(\xi)|^2\cos k_p\xi\right)^2+\left(\int_{-\infty}^{\infty}d\xi|a(\xi)|^2\sin k_p\xi\right)^2\right]^{1/2}, \qquad (2.7)$$

$$\operatorname{tg}\psi=\left(\int_{-\infty}^{\infty}d\xi|a(\xi)|^2\sin k_p\xi\right)\left(\int_{-\infty}^{\infty}d\xi|a(\xi)|^2\cos k_p\xi\right)^{-1} \qquad (2.7')$$

If the packet dimension L is small compared with the length of the excited plasma wave, then

$$\varphi_0=\frac{mk_p}{4e}\int_{-\infty}^{\infty}d\xi|a(\xi)|^2. \qquad (2.8)$$

The order of magnitude of the integral in (2.8) is $Lv_E{}^2$, where v_E is the amplitude of the velocity of the oscillatory motion of the electrons in the transverse field. It can be seen that in the case of a small packet the electron energy in a longitudinal wave is approximately Lk_p times smaller than the average energy of the transverse oscillatory motion (called high-frequency potential) $mv_E{}^2/4$. If the packet dimensions are commensurate with the wavelength λ_p, the potential energy of an electron in a plasma wave is of the same order as the high-frequency potential.

We consider one more example that permits a better understanding of the physical excitation of plasma waves, viz., a packet of Gaussian form

$$|a(\xi)|^2 = a_0^2 \exp[-\xi^2/2L^2],$$

where L is the length of the packet. From (2.3) we get

$$\frac{\delta n_0}{n_0} = \frac{a_0^2}{4v_g^2}\Big[\exp(-\xi^2/2L) - \frac{i(2\pi)^{1/2}}{4} k_p L \exp(-k_p^2 L^2/2) \cdot \Big\{\exp(-ik_p\xi)\Big[1-\Phi\Big(\frac{\xi}{2^{1/2}L} + \frac{i}{2^{1/2}k_p\xi}\Big)\Big] - \text{c.c.}\Big\}\Big], \quad (2.9)$$

where Φ is the probability integral. Figure 1 shows the function (2.9) at $k_p L = 1$. It can be seen that at this ratio of the plasma-wave and packet lengths the electron density increases in the region of the leading front and falls off at the trailing edge of the packet. This density distribution is due to the joint action exerted on the electrons by the high-frequency field and the charge-separation field. The high-frequency pressure forces on the leading and trailing edges of the packet are equal in size and opposite in direction. The charge separation field, on the contrary, is directed in the same direction at the packet location, and accelerates the electrons in the negative direction. This gives rise to a significant perturbation of the density behind the packet and to formation of a plasma wave.

3. ENERGY LOSS. DISTORTION OF PACKET SHAPE

The assumption that the shape of the packet remains unchanged is valid if the time during which energy is lost to plasma-wave emission and the packet shape changes is much longer than the plasma period (or takes place over a length greatly exceeding the plasma wavelength).

We consider first the energy loss and find the conditions under which it can be neglected. The expression given in Sec. 1 for $\delta W_\perp$ allows us to find the total energy loss per unit time:

$$\delta W = \int_{-\infty}^{\infty} d\xi\, \delta n(\xi) \frac{dv_\perp^2(\xi)}{d\xi} \frac{mv_g}{2}.$$

With the aid of relations (1.8), (2.1), (2.3), and (2.6) this equation is transformed into

$$\delta W = \frac{mv_g^2}{2n_0}\Big\{\delta n^2(\xi\to-\infty) + \frac{1}{k_p^2}\Big(\frac{d\delta n}{d\xi}\Big)^2_{\xi\to-\infty}\Big\} = \frac{mn_0k_p^2}{32v_g}R^2. \quad (3.1)$$

Using the expression for the amplitude of the electric field in the plasma wave ($E_0 = k_p\varphi_0$) we can represent Eq. (3.1) in the form $\delta W = (E_0^2/8\pi)v_g$. A packet propagating at the group velocity leaves a trail in the form of a plasma wave having an energy density $E_0^2/8\pi$.

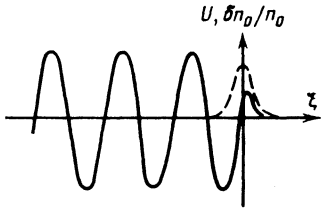

FIG. 1. High-frequency potential $U = m|a(\xi)|^2/4$ (dashed line) and perturbation of the electron density $\delta n_0/n_0$ (solid) as functions of the coordinate ξ, for a Gaussian packet at $k_p L = 1$.

We obtain the characteristic time during which the packet energy

$$W_0 = \frac{m^2\omega^2}{8\pi e^2}\int_{-\infty}^{\infty} d\xi |a(\xi)|^2$$

is altered by the emission of plasma waves by dividing W_0 by the (3.1):

$$T_{\text{rad}} = \frac{16v_g^3\omega^2}{\omega_p^4 R^2}\int_{-\infty}^{\infty} d\xi |a(\xi)|^2.$$

This leads to the following for a small packet:

$$T_{\text{rad}}\,\omega_p = 8\Big(\frac{\omega}{\omega_p}\Big)^4 \frac{n_0 m v_g^3}{W_0\omega_p}. \quad (3.2)$$

Note that the distance traversed by the packet before its energy is decreased by radiation of plasma waves is equal approximately to $T_{\text{rad}} v_g \sim T_{\text{rad}} c$.

The condition $T_{\text{rad}}\omega_p \gg 1$ under which the radiation can be neglected is met if the right-hand side of (3.2) is large. This leads to an constraint on the packet energy:

$$W_0 \ll \frac{8n_0 m v_g^2}{k_p}\Big(\frac{\omega}{\omega_p}\Big)^4. \quad (3.3)$$

In addition to losing energy, the packet changes shape. This is due not only to spreading by the usual linear dispersion, but also to nonlinear effects. To take the packet shape change into account, we assume that the envelope in Eq. (2.1) is not only a function of the variable ξ, but also a slowly varying function of the time. Substituting (2.1) in (1.4) and using (2.2) and (2.3), we obtain the equation for the envelope:

$$2i\omega\frac{\partial a}{\partial t} + \frac{c^2\omega_p^2}{\omega^2}\frac{\partial^2 a}{\partial \xi^2} = \frac{5\omega_p^4}{32\omega^4}\frac{\omega^2}{c^2} a|a|^2 - \frac{\omega_p^3}{4v_g^3} a \int_{\infty}^{\xi} d\xi' |a(\xi')|^2 \sin k_p(\xi-\xi'). \quad (3.4)$$

Note that the first term in the right-hand side of (3.4) is the result of allowance for weak relativistic effects, and also for the high-frequency (second-harmonic) and low-frequency perturbations of the electron density.

The packet spreading due to linear dispersion can be easily shown, by comparing the first and second terms in the left-hand side of (3.4), to take place in a characteristic time T_L and over a distance L_L:

$$T_L \sim \frac{2\omega^3}{c^2\omega_p^2}L^2, \quad L_L \sim 2\Big(\frac{\omega}{\omega_p}\Big)^2\frac{\omega L^2}{c}. \quad (3.5)$$

The time of the nonlinear distortion of the packet shape depends on the ratio of the plasma wavelength λ_p to the packet dimension L. For small packets the integral in the right-hand side of (3.4) is approximately equal to $k_p L^2 a_0^2$, where a_0 is the maximum of the envelope. From a comparison of the linear terms it follows that the last is $(L\omega/c)^2$ times larger than the first and therefore determines the time and length of the nonlinear distortion of the packet:

$$T_{\text{nl}} \sim 8\frac{c^2}{a_0^2}\frac{1}{k_p^2L^2}\frac{\omega}{\omega_p^2}, \quad L_{\text{nl}} \sim 8\frac{c^2}{a_0^2}\frac{1}{k_p^3L^2}\frac{\omega}{\omega_p}. \quad (3.6)$$

Comparing (3.2) and (3.6) we conclude that the most-substantial nonlinear effect is the change of the packet shape. To corroborate this conclusion we point out that to obtain the energy loss (3.1) it is necessary to retain in the left-hand side of (3.4) the small discarded term proportional to $\partial^3 a/\partial\xi^3$.

Let us formulate finally the conditions under which it is valid to assume a packet with given properties. If the packet energy is low ($W_0 \ll 2LN_0mc^2(k_pL)^{-4}$), the principal role is played by the spreading of the packet on account of linear dispersion. For this spreading to be negligible, the packet must be larger than $(c/\omega_p)(\omega_p/\omega)^{3/2}$. If the packet energy is high (the opposite inequality holds), the change of the packet shape by nonlinear effects is decisive, and this change can be neglected by satisfying the inequality $W_0 \ll Ln_0mc^2(\omega/\omega_p)^3(k_pL)^{-2}$.

Thus, the packet spreads faster than it loses energy. It follows therefore that as the packet propagates and its initial shape is distorted, the excitation of plasma waves continues and the amplitude of the radiated wave changes and depends on the ratio of the packet size to λ_p.

4. THREE-DIMENSIONAL PACKET

We consider in this section a more realistic problem—emission of plasma waves by a three-dimensional cylindrical-symmetry wave packet. We assume that the high-frequency pressure (or high-frequency potential) connected with the packet is characterized by longitudinal ($L_\parallel$) and transverse ($L_\perp$) scales. Besides the previously discussed longitudinal shape distortion, such a packet diffuses also in the transverse direction by diffraction. If the diffractive broadening extends to distances greatly exceeding the plasma wavelength, it can be neglected. This condition leads to the following constraint on the transverse dimension of the packet:

$$L_\perp \gg (1/k_p)(\omega_p/\omega)^{1/2}. \tag{4.1}$$

For a sufficiently short packet ($L_\perp \gg L_\parallel$) the distortion of the packet shape can therefore be neglected under the conditions discussed above, in which L must be taken to mean $L_\parallel$, and also if the inequality (4.1) holds.

To determine the electron-density perturbations produced by the packet one can use the hydrodynamic equations for a plasma in a high-frequency electromagnetic field.[9] Neglecting the ion displacements, we obtain in the linear approximation

$$\partial^2\delta n/\partial t^2 + \omega_p^2\delta n = (n_0/m)\Delta U, \tag{4.2}$$

where $U = m\,\overline{v_1^2}/2$ is the high-frequency potential and the superior bar denotes an average over the high-frequency motion of the electrons. In contrast to (1.8), Eq. (4.2) describes only slow averaged perturbations of the electron density.

We assume that U depends on the variable $\xi = x - v_g t$ that characterizes the longitudinal structure of the packet, and also on the transverse variable ρ. To be specific we consider a packet with a Gaussian dependence on this variable:

$$U(\xi,\rho) = V(\xi)\exp[-\rho^2/2L_\perp^2]. \tag{4.3}$$

From (4.2) we get

$$\delta n(\xi,\rho) = \frac{n_0}{mv_g^2}\exp\left(\frac{-\rho^2}{2L_\perp^2}\right)\left\{V(\xi) - k_p\int_\infty^\xi d\xi'\,V(\xi')\right.$$

$$\left.\cdot\sin k_p(\xi-\xi')\left[1-(k_pL)^{-2}\left(\frac{\rho^2}{L_\perp^2}-1\right)\right]\right\}. \tag{4.4}$$

The expression for the potential follows from the Poisson equation

$$\varphi(\xi,\rho) = \frac{k_p}{e}\exp\left(\frac{-\rho^2}{2L_\perp^2}\right)\int_\infty^\xi d\xi'\,V(\xi')\sin k_p(\xi-\xi'). \tag{4.5}$$

In addition to the one-dimensional equation (2.4), in which the high-frequency potential V should be taken to mean the quantity $m|a|^2/4$, Eq. (4.5) contains a factor that determines the radial dependence. In particular, the potential far behind the packet is of the form

$$\varphi(\xi,\rho) = \frac{mk_p}{4e}R\exp\left(\frac{-\rho^2}{2L_\perp^2}\right)\sin(k_p\xi+\psi), \tag{4.6}$$

where R and ψ are defined by Eqs. (2.7) and (2.7′) in which $|a(\xi)|^2$ must be replaced by $4V(\xi)/m$. Figure 2 shows lines of constant values of the function (4.3), and also the equipotentials for a packet that has a Gaussian dependence also on the variable ξ.

As already noted, the oscillations of the electron density and of the potential set in not behind the packet, but right after its leading front passes. This explains the result of the two-dimensional numerical simulation of the passage of the front of a laser beam through a low-density plasma,[10] where a pattern of equipotentials similar to that of Fig. 2 was observed.

5. CONCLUSION

The plasma-wave "wake" produced behind the packet can be recorded by using Raman scattering of a probing wave of frequency ω_0 propagating at a small angle ϑ to the direction of the packet propagation. The dispersion laws for the probing, scattered, and Langmuir waves lead to a connection between the frequencies and the incidence angle (which is practically equal to the scattering angle):

$$\vartheta^2 \approx (\omega_p^2/\omega^2\omega_0^2)(\omega_0^2-\omega^2\pm\omega_p\omega_0), \tag{5.1}$$

where ω is the packet carrier frequency, and the $\pm$ signs correspond to the anti-Stokes and Stokes scattered radiation with frequencies $\omega_0 \pm \omega_p$. Given the frequencies, Eq. (5.1) determines the incident and scattering angles of the probing radiation. In particular, for $\vartheta = 0$ Eq. (5.1) leads to the connection $\omega_0 = \omega \pm \omega_p/2$, between the frequency of the probing radiation and the carrier frequency of the packet.

In a low-density plasma the longitudinal wave excited by the packet has a velocity close to that of light. At sufficiently high amplitude, it can be used to accelerate charged particles.[5] The acceleration mechanism is the following. The

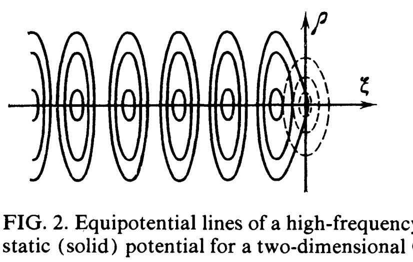

FIG. 2. Equipotential lines of a high-frequency (dashed) and an electrostatic (solid) potential for a two-dimensional Gaussian packet.

particle is injected into the plasma, in the pocket-propagation direction, with an initial velocity v_g and an energy $\varepsilon_0 = mc^2\gamma$, where

$$\gamma=[1-(v_g/c)^2]^{-1/2}=\omega/\omega_p\gg 1.$$

A particle is accelerated if it lands in a plasma-wave region in which the electric field is directed along the wave motion. The maximum energy that such a resonant particle can acquire before it reaches that plasma-wave region in which the field reverses sign and begins to decelerate the particle is equal to

$$\Delta\varepsilon=\varepsilon_0\gamma(eE_L/m\omega_p c)[1+(1+2mk_p/eE_L\gamma)^{1/2}].$$

In this case the particle negotiates, together with the wave, a path $L_a = \Delta\varepsilon/eE_L$. It is assumed that the plasma dimensions are also close to L_a. We estimate now the field intensity in a plasma wave produced by a laser pulse of duration 10^{-13} s and a frequency $\omega = 2\cdot 10^{14}\ \mathrm{s}^{-1}$ (the wavelength is $10\,\mu$m). In a plasma of density $n_0 = 10^{17}\ \mathrm{cm}^{-3}$, used in experiments on laser acceleration of articles,[11] the plasma wavelength is $\sim 100\,\mu$m. According to (2.8), the longitudinal-field intensity is approximately equal to

$$E_L\approx 4\pi n_0 eLv_E^2/4c^2. \tag{5.2}$$

At an intensity 10^{15} W/cm^2 this yields $E_L\approx 8\cdot 10^6$ V/cm. We indicate for comparison that in experiments in which the plasma wave is excited by a beat wave produced by a two-frequency laser[11] the field intensity is $(3\text{–}10)\cdot 10^6$ V/cm. A beat wave, however, can excite a plasma wave only under a resonance condition in which the difference between the frequencies of the two laser beams is equal to the plasma frequency. Since the plasma is actually inhomogeneous, this condition is met only in narrow regions of space, making difficult the use of a plasma wave for particle acceleration. On the contrary, when a plasma wave is excited by a short laser pulse, the inhomogeneity of the plasma does not limit the size of the excitation region, and influences mainly the length of the excited plasma wave. If it is assumed in the foregoing example that the laser beam is focused in the same way as in Ref. 11, the pulse energy is $4\cdot 10^{-2}$ J. Acceleration calls for an initial energy of 5 MeV. As a result of the acceleration, which is completed within a distance $L_a\approx 0.6$ cm, the electron energy increases to 10 MeV. It follows then from the expressions above that the pulse-shape change, which is determined by linear dispersion and diffraction, is effected over a distance of 10 cm and is insignificant.

The model considered by us is restricted by a number of assumptions. One is that the ions are immobile. Consideration of the ion motion introduces a possibility of parametric instabilities of the plasma wave.[12] the growth rate of the most rapidly developing instability is approximately $\omega_p(m/m_i)^{1/3}$, where m_i is the ion mass. It can therefore be assumed that over distances $\sim(c/\omega_p)(m_i/m)^{1/3}$ behind the packet the available time is insufficient for instability to develop.

Allowance for the thermal motion of the particles leads to two effects: first, damping of the Langmuir wave, which will apparently be small because the phase velocity of the wave is close to the speed of light, and second, to radial, spreading of the "wake" of the plasma waves, which becomes substantial over a distances $\sim L_1^2\omega_p c/v_{Te}^2$ behind the packet, where v_{Te} is the electron thermal velocity.

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