The spectrum of electron-nuclear γ transitions in the nucleus of a multiply-charged atomic ion

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The nuclear emission or absorption spectrum of an atom contains a set of electron satellites that are due to the change in the state of atomic electrons. It is shown that, in the case of neutral atoms and multiply-charged ions, the satellites may be due to different mechanisms. In the former case (loose electron shell), the satellites are mainly due to the "shaking" of the electron shell when the nucleus interacts with a γ ray. In the latter case (rigid electron shell), the γ ray interacts directly with the electrons. The second mechanism is significant for dipole nuclear transitions and predominates for γ-ray energies $\lesssim 4z^*$ keV, where z^* is the effective charge of the nucleus. It is unrelated to the usual selection rules or the well-known hierarchy of intensities based on the multipolarity of electron transitions. This substantially enriches the satellite spectrum and brings into the analysis phenomena such as transitions between fine structure and hyperfine structure components, 0–0 transitions, and transitions without a change in the electron configuration. The relative satellite intensities are determined by the small parameter $\mu_p^{2\lambda}$, where λ is the multipolarity of the nuclear transition, $\mu_p \simeq 1/2z$ is the relative mass of the proton, and z is the charge of the core. In the spectrum of the plasma source, the electron satellites corresponding to γ-ray emission and absorption lines are not overlapped by the Doppler profile of the γ-ray line.

1. INTRODUCTION

This is a continuation of our study[1,2] of dynamic effects arising in the interaction between the electron shell of a multicharged atomic ion and nucleons in the nucleus. In our previous papers,[1,2] we determined the positions and intensities of the electron satellites of the γ-ray emission spectrum of the nucleus in a neutral or weakly-charged atom. We also estimated the effect of the electron shell on the rate of decay of a metastable nucleus, and showed that this effect was negligible[2] in low-charge systems. In the case of multicharged ions, on the other hand, the geometric and energy parameters of the electron shell, which determine the nature of its interaction with the nucleus, are different: the orders of magnitude change and new channels become available for electron-nuclear processes. In neutral atoms under standard experimental conditions, strong satellites are overlapped by the Doppler profile of the γ-ray emission line; the techniques of intra-Doppler spectroscopy were proposed in Refs. 1 and 2 as a way of observing these satellites. A more favorable situation for the observation of the satellites prevails in plasmas containing multicharged ions: the separation between the lower electron energy levels of ions can be much greater than the Doppler shift of the γ-ray line.

As we pass from the neutral atom to the ion, we find that, in addition to the usual decay channels available to an excited nuclear state, there is also, for example, the creation of an electron-positron pair (during the de-excitation of the nucleus) with the capture of the new electron into one of the available bound states. One can readily imagine the situation where the process becomes energetically possible only when a strongly-bound electron is removed in the initial state.

Theoretical analyses are complicated by the variety of channels available to electron-nuclear processes in ions, and by possible interference between them. Moreover, the analysis of many such processes must be based on a rigorous relativistic approach, and the transition to the nonrelativistic limit must be properly justified. Systematic analyses of the decay and excitation of the system in this complex situation are best carried out by means of a theoretical formalism providing a unified approach to all significant processes. A convenient route in this sense is the energy approach, in which the total probability of spontaneous decay and of excitation of the system is related to the imaginary part Im E of the energy of the atom + field system. For radiative decays, this appears as a delay of interaction and self-interaction, and is calculated by electrodynamic perturbation theory.[3] Perturbation-theory corrections for Im E are represented by sums over virtual states. In the lowest orders, the individual terms in these sums are additive contributions of different physical channels to the total decay probability.[4] Effects due to interference between channels can be isolated in the higher orders. Only radiative decays appear in the lowest, second order of electrodynamic perturbation theory.

2. MODEL AND FORMULATION OF THE PROBLEM

Most excited nuclear states are multiparticle in character. The first excited states with one or two nucleons or vacancies above an even-even core are the only exceptions.[5] They are convenient for theoretical analysis because they can be described by the single-particle model. Generalization of the formulas to the many-particle problem does not lead to qualitatively different results because the dynamic (radial) parts of the nuclear matrix elements do not appear in the expressions for the relative intensities of the electron satellites and the main nuclear line.

As in Ref. 2, we consider a simple model consisting of three particles: a rigid nuclear core and a proton and electron outside the core. The masses of the three particles are $\mu_c M$, $\mu_p M$, and $\mu_e M$, where M is the mass of the entire atom and $\mu_c + \mu_p + \mu_e = 1$. The position coordinates of these parti-

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cles will be denoted by r_c, r_p, and r_e, respectively. The charge of the nuclear core will be denoted by z, and z^* will represent the effective charge of the Coulomb field for an optically active electron in an atom or ion.

It is not our aim in this paper to perform an accurate calculation of the intensities of electron satellites. We shall confine our attention to qualitative aspects of the process, so that we shall be able to use the nonrelativistic approximation, which gives the correct order-of-magnitude results even for highly-charged ions. We shall need electrodynamics only to justify the formulas of the energy approach, which, as we have already noted, simplifies the analysis of multichannel decay and excitation of the system.

We shall calculate the imaginary part of the excited-state energy. The exact electrodynamic solution is not known even in the case of the two-particle system. We shall follow the quasipotential method[6] and introduce the bare interaction between the particles, which can be looked upon as the zero-order approximation of some formally exact perturbation theory. It will be sufficient to take this interaction as the potential in the nonrelativistic Schrödinger equation, i.e.,

$$V_{NR}(r_c,\ r_p,\ r_e)=v(r_{pc})-ze^2/r_{ec}-e^2/r_{pe}, \tag{1}$$

where $v(r_{pc})$ simulates the interaction between the proton and the core (nuclear and Coulomb). The imaginary part of the energy of a three-particle state Φ_I in the lowest (second) perturbation-theory order is

$$\begin{aligned}\operatorname{Im} E=e^2 \operatorname{Im} i \lim_{\gamma\to 0} \iint dx_1\, dx_2 \exp[\gamma(t_1+t_2)]\\ \cdot\{D(r_{c1}t_1, r_{c2}t_2)\langle\Phi_I|T(\hat{j}_{cv}(x_1)\hat{j}_{cv}(x_2))|\Phi_I\rangle\\ +D(r_{p1}t_1, r_{p2}t_2)\langle\Phi_I|T(\hat{j}_{pv}(x_1)\hat{j}_{pv}(x_2))|\Phi_I\rangle\\ +D(r_{e1}t_1, r_{e2}t_2)\langle\Phi_I|T(\hat{j}_{ev}(x_1)\hat{j}_{ev}(x_2))|\Phi_I\rangle\},\end{aligned} \tag{2}$$

where $D(r_1t_1, r_2t_2)$ is the photon propagator that depends on the coordinates of two points in three-dimensional space, the angle brackets contain the current matrix elements, $\hat{j}_{cv}, \hat{j}_{pv}, \hat{j}_{ev}$ are the four-dimensional components of current operators for the particles in the core, the protons, and the electrons, the symbol $x=(r_c, r_p, r_e, t)$ includes the position coordinates of the three particles and the time (the same for all three particles), and γ is the adiabatic index. The following exact electrodynamic expression will be used for the photon propagator:[7]

$$D(r_1t_1, r_2t_2)=-\frac{1}{8\pi^2 r_{12}}\int_{-\infty}^{\infty} d\omega \exp(i\omega t_{12}+i|\omega|r_{12}) \tag{3}$$

(Lorentz gauge). Equations (1) and (2) imply summation over the polarizations of the photon. Equation (2) describes single-photon processes.

Higher-order electrodynamic corrections to Im E will not be considered. Moreover, the state functions of the system in (2) will be taken to be the nonrelativistic solutions of the Schrödinger equation with the Hamiltonian

$$H=-\frac{\hbar^2}{2M}\left(\frac{\Delta_{rc}}{\mu_c}+\frac{\Delta_{rp}}{\mu_p}+\frac{\Delta_{re}}{\mu_e}\right)+V_{NR}(r_c, r_p, r_e). \tag{4}$$

Here and henceforth, $\Delta_{rc}, \Delta_{rp}, \Delta_{re}$ are the Laplacians and $\nabla_{rc}, \nabla_{rp}, \nabla_{re}$ are the gradients with respect to the variables r_c, r_p, r_e.

The nonrelativistic expression for the current operator of a particle a is

$$\hat{j}_{av}=\left\{-\frac{iz_a}{2M\mu_a}(\hat{\Phi}_a^*\nabla_{ra}\hat{\Phi}_a-\hat{\Phi}_a\nabla_{ra}\hat{\Phi}_a^*);\ iz_a\hat{\Phi}_a^*\Phi_a\right\} \tag{5}$$

which is a four-dimensional vector ($\nu=1,2,3,4$), z_a is the particle charge, and $\hat{\Phi}_c$, $\hat{\Phi}_p$, and $\hat{\Phi}_e$ are the second-quantization operators for the field of the core particles, the proton field, and the electron field. We now substitute (3) and (5) in (2), and integrate with respect to time t and frequency ω. The result will be written in the form of the sum of core, proton, and electron contributions:

$$\begin{aligned}\operatorname{Im} E&=\operatorname{Im} E_c+\operatorname{Im} E_p+\operatorname{Im} E_e,\\ \operatorname{Im} E_a&=-\frac{z_a^2}{4\pi}\sum_F \iint dr_{c1}\, dr_{c2} \iint dr_{p1}\, dr_{p2}\\ &\cdot\iint dr_{e1}\, dr_{e2}\Phi_I^*(1)\Phi_F^*(2)\hat{T}_a(1,2)\Phi_F(1)\Phi_I(2),\\ \hat{T}_a(1,2)&=\frac{\sin\tilde{\omega}_{IF}r_{a12}}{r_{a12}}\left[\frac{1}{M\mu_a}(\nabla_{ra1}, \nabla_{ra2})+1\right],\end{aligned} \tag{6}$$

where $r_{a12}=|r_{a1}-r_{a2}|$, the sum over F is evaluated over the final states of the system, i.e., in second order of the electrodynamic perturbation theory, and the total level width is the sum of partial contributions due to radiative decays to particular final states of the system. These contributions are proportional to the probabilities of the corresponding transitions and $\tilde{\omega}_{IF}$ is the total transition energy, including the change in the kinetic energy of the ion as a whole (recoil energy). Transitions involving the excitation (de-excitation) of the electron shell correspond to the set of red (blue) satellites of the main line. The expressions given by (6) can be deduced in the usual way, using amplitudes with nonrelativistic wave functions. The form of the operator in (6) is determined by the gauge of the propagator (3).

Each of the contributions to Im E_a the individual interaction of the particle a with a photon of energy $\tilde{\omega}_{IF}$. The change in the state of the remaining particles is due to the "shake" of the system on recoil. The neutral atom can be modeled[2] by two strongly-bound particles (nuclear core and proton) and a weakly-bound electron. The energy of the photon is close to that of the nuclear transition. The cross section describing the interaction between this photon and the almost-free electron is small and the contribution of E_e is neglected. The picture becomes qualitatively different as we pass to multicharged ions with an increasing ($\sim z^{*2}$) electron binding energy. Firstly, for the strongly-bound electron, the recoil on emission (absorption) of the photon by the core-proton subsystem is smaller, i.e., the "shake" of the electron shell has a smaller effect. The contribution of E_c+E_p to the intensity of the electron satellites falls as $1/z^{*2}$. Secondly, the cross section for the direct interaction between the high-energy photon and the bound electron increases, i.e., the contribution of Im E_e increases. It becomes the dominant contribution for $\omega_{IF}\lesssim 4z^*$ keV. When this condition is satisfied, the photon wavelength is equal to or greater than the radius of the electron orbital.

3. APPROXIMATIONS

It will be convenient in the ensuing analysis to separate out the motion of the center of mass. In the nonrelativistic limit, this can be done by changing the variables as follows:[2]

$$R=\mu_c r_c+\mu_p r_p+\mu_e r_e, \quad R_p=r_p-r_c, \quad R_e=r_e-r_c. \tag{7}$$

The inverse transformation $r_a = R + R_{a\ \text{in}}$, where R is the coordinate of the center of mass and $R_{a\ \text{in}}$ represents the internal state of the system, is defined by

$$\begin{gathered} R_{c\ in}=-\mu_p R_p-\mu_e R_e, \quad R_{p\ in}=(\mu_c+\mu_c) R_p-\mu_e R_e, \\ R_{e\ in}=(\mu_p+\mu_c) R_e-\mu_p R_p, \\ \nabla_{rc}=\mu_c\nabla-\nabla_p-\nabla_e, \quad \nabla_{rp}=\mu_p\nabla+\nabla_p, \quad \nabla_{re}=\mu_e\nabla+\nabla_e. \end{gathered} \tag{8}$$

where (and henceforth) $\nabla, \nabla_p, \nabla_e$ represent the gradients with respect to the new variables R, R_p, and R_e, respectively. The change of variables (7) reduces the three-particle problem to the problem of the relative motion of two particles, namely, a heavy particle of mass $M\mu_c\mu_p/(\mu_c+\mu_p)$ with charge ze and a light particle with mass $M\mu_c\mu_e/(\mu_c+\mu_e)$ and charge $-e$. As before, these particles will be referred to as the proton and the electron although, actually, they correspond to certain collective combinations of the coordinates of real particles. The motion of the electron and the proton can then be described by the nonrelativistic Schrödinger equation with the potential

$$v(R_p)-ze^2/R_e-e^2/R_{pe}-\hbar^2(\nabla_p, \nabla_e)/2M\mu_c. \tag{9}$$

The first two terms in this expression represent the external field and the other two the effective interaction between the two particles, i.e., the Coulomb interaction, and the recoil. The motion of the center of mass in the initial and final states is described by plane waves. The electron-proton state functions in the potential (9) can be found numerically almost exactly. The next approximation relies on the fact that the proton-electron interaction in (9) can be taken into account by perturbation theory. In the zero-order approximation of this perturbation theory, the dependence of Φ_I, Φ_F on the variables R, R_p, and R_e can be factored:

$$\Phi_A(R, R_p, R_e)=\Phi_A(R)\Phi_{Ap}(R_p)\Phi_{Ae}(R_e), \tag{10}$$

where Φ_A are plane waves, Φ_{Ap} is the state function of the proton in the potential $v(R_p)$, and Φ_{Ae} is the nonrelativistic Coulomb function. However, composite electron-proton single-photon transitions have an effect even in the zero-order approximation. This is so because each of the operators $\hat{T}_c$, $\hat{T}_p$, and $\hat{T}_e$ in (6) contains a combination of all the variables R, R_p, and R_e. It will be shown below that the contribution of the zero order to the satellite intensity is $\sim\mu_p^2$. The contribution of the proton-electron interaction to the satellite intensity is significant only in second-order perturbation theory in this interaction, and, when compared with the main contribution, is additionally of order $1/z^{*2}$ (for the Coulomb part) and μ_e^2 (for the recoil interaction). We shall neglect these contributions. Thus, the main effect of the appearance of the electron satellites of nuclear transitions is purely kinetic in character: it constitutes a shift of the center of mass of the system relative to the proton or electron orbital when the γ-ray is emitted.

The sum over the final states in (6) includes an integral with respect to the momentum of the center of mass after the emission of the γ-ray. To evaluate this integral, it is convenient to replace the quotient $\sin(\omega r)/r$ in (6) with the Fourier integral, as follows:

$$\frac{\sin\omega r}{r}=\frac{1}{4\pi\omega}\int d\mathbf{k}\,\delta(|k|-\omega)e^{i\mathbf{kr}}. \tag{11}$$

This replacement factors the dependence of the integrand on the coordinates of the center of mass and, at the same time, enables us to evaluate the integral with respect to these coordinates. The final result is

$$\begin{aligned} \operatorname{Im} E_a = &-\frac{z_a^2}{4\pi}\sum_{F_c F_p}\int\!\!\int d\mathbf{R}_{p1}\cdot d\mathbf{R}_{p2}\int\!\!\int d\mathbf{R}_{e1}\cdot d\mathbf{R}_{e2} \\ &\cdot\frac{\sin\omega_{IF}R_{a\,in12}}{R_{a\,in12}}\Phi_{Ie}^*(R_{e1})\Phi_{Ip}^*(R_{p1})\Phi_{Fe}^*(R_{e2}) \\ &\cdot\Phi_{Fp}^*(R_{p2})\left[\frac{1}{M^2\mu_a{}^2}(\nabla_{ra1}, \nabla_{ra2})+1\right] \\ &\cdot\Phi_{Fe}(R_{e1})\Phi_{Fp}(R_{p1})\Phi_{Ie}(R_{e2})\Phi_{Ip}(R_{p2}), \end{aligned} \tag{12}$$

where ω_{IF} is the transition energy without the recoil energy of the ion as a whole and ∇_{ra} is given by (8) in terms of ∇, ∇_p, and ∇_e. Terms containing ∇ do not contribute, which can be readily verified by assuming that, in the initial state, the system as a whole is at rest. The matrix elements of the gradients in (11) are replaced by the combination

$$\langle a_i|\nabla_a|a_f\rangle=-M\mu_a\omega_{if}\langle a_i|R_a|a_f\rangle. \tag{13}$$

Expansion of the operator $\sin(\omega_{IF}R_{a\ \text{in}\ 12})/R_{a\ \text{in}\ 12}$ in terms of spherical harmonics generates the multipole expansion of the decay probability. The other well-known variant is the power-type expansion. The two expansions are virtually identical in most problems, and produce a clear reduction in the successive contributions with increasing multipolarity. This situation occurs in the case of Im E_c and Im E_p. The parameters of the power-type expansion are now the combinations $\omega_{\text{IF}}R_p$, $\omega_{\text{IF}}R_e\mu_e$. If we retain only the quadratic terms in each of these parameters in the integrand for E_c and E_p, we can isolate the effects due to the pure dipole proton transition, the pure dipole electron transition, and the mixed dipole electron + dipole proton transition. The result is reproduced in Coulomb units, convenient for numerical calculations (for energy, 1 c.u. $= z^2$ a.u.).

For the pure dipole proton transition

$$\begin{aligned} &\operatorname{Im}[E_c(dp)+E_p(dp)] \\ &\quad=-z^{-1}(\tilde{\alpha}\omega_{IF})^3(1+z^2\mu_p{}^2)P(I, F)^2\langle p1, p2\rangle_1 \end{aligned} \tag{14}$$

and for the dipole electron + dipole proton transition

$$\begin{aligned} &\operatorname{Im}[E_c(dp-de)+E_p(dp-de)] \\ &\quad=-\frac{\mu e^2}{15z}(\tilde{\alpha}\omega)^5\{(6+11\mu_p{}^2z^2)\langle p1, p2\rangle_1 \\ &\cdot\langle e1, e2\rangle_1+(2+7\mu_p{}^2z^2)\langle p1, e2\rangle_1\langle p2, e1\rangle_1\}P^2(I,F)\mathscr{E}^2(I,F), \end{aligned} \tag{15}$$

where $\tilde{\alpha}=\alpha z$. The radial integrals are

$$P(IF)=\int drr^3P_{Ip}(r)R_{Fp}(r), \quad \mathscr{E}(IF)=\int drr^3R_{Ie}(r)R_{Fe}(r), \tag{16}$$

and R_{Ap}, R_{Ae} are the radial parts of the proton and electron state functions. The angular parts of the matrix elements are

$$\begin{aligned} \langle a1, b2\rangle_l= &\int\!\!\int d\Omega_1\, d\Omega_2 Y_{aI}^*(\Omega_1)Y_{bF}^*(\Omega_2) \\ &\times P_l(\cos(a1, b2))Y_{aF}(\Omega_1)Y_{bI}(\Omega_2), \end{aligned} \tag{17}$$

where Y_{aA} is the angular part of the state function of particle

a, $P_l(x)$ is the Legendre polynomial, and $\cos(a1,b2) = (\mathbf{R}_{a1},\mathbf{R}_{b2})/R_{a1}R_{b2}$. The electron $\langle e1,e2\rangle_l$ and mixed $\langle p1,e2\rangle_l$ electron-proton angular matrix elements can be determined in a similar way. All the integrals in (17) with respect to the angular variables and the sums over the angular momentum components of the final states of the quasiparticles can be evaluated analytically. The technique for doing this is standard.[8]

We now turn to the contribution of the direct interaction between the photon and an electron, Im E_e. In the "potential" $\sin(\omega R_{e\,\mathrm{in}\,12})/R_{e\,\mathrm{in}\,12}$, we identify the two parameters

$$k_e = R_e(\mu_p+\mu_c)\omega_{IF}, \quad k_p = R_p\omega_{IF}.$$

In our previous paper,[2] we considered that, in a significant part of the integration region (for a neutral atom),

$$k_e \gg k_p, \quad k_e \gg 1. \tag{18}$$

The second condition determines the rapid oscillations of the potential and the smallness of the contribution of Im E_e. For multicharged ions, the second condition is invalid, but the first condition holds for all real systems. Let us expand the potential in powers of the parameter k_p, and retain only the quadratic terms, i.e., the contribution of the composite dipole proton and electron transitions:

$$\frac{\sin(\tilde{\alpha}\omega R_{e\,in12})}{R_{e\,in12}} \approx \mu_p^2(\tilde{\alpha}\omega)^3\left(\frac{\pi}{2}\right)^{1/2} \times\left\{\frac{J_{3/2}(\tilde{\alpha}\omega R_{e12})}{(\tilde{\alpha}\omega R_{e12})^{3/2}}\left[(\mathbf{R}_{p1}\cdot\mathbf{R}_{p2}) + 6\frac{(\mathbf{R}_{e1}\cdot\mathbf{R}_{p2})(\mathbf{R}_{p1}\cdot\mathbf{R}_{e2})}{R_{e12}^2}\right] - \frac{J_{1/2}(\tilde{\alpha}\omega R_{e12})}{(\tilde{\alpha}\omega R_{e12})^{1/2}}\frac{(\mathbf{R}_{e1}\cdot\mathbf{R}_{p2})(\mathbf{R}_{e2}\cdot\mathbf{R}_{p1})}{R_{e12}^2}\right\}, \tag{19}$$

where J_ν are Bessel functions that depend on the electron coordinates. We note that, in contrast to the contributions of $\mathrm{Im}(E_c + E_p)$, when $\tilde{\alpha}\omega > 1$, we do not have for Im E_e a parameter defining the hierarchy of contributions according to the multipolarity of electron transitions. Moreover, the selection rules, valid for pure electron transitions, do not work. This enables us to include in our analysis, on equal terms with the others, the transitions between fine and hyperfine structure components, and also the *ns–n's* and 0–0 transitions for systems with two or more electrons.

Part of the intensity of dipole electron satellites is due to the contributions E_p, E_c [see (15)]. The ratios of the contributions of these lines are given by

$$\frac{\mathrm{Im}\,E_e/\mathrm{Im}[E_c(\lambda p-de)+E_p(\lambda p-de)]}{\sim\begin{cases}(\mu_p/\mu_e)^{2\lambda}, & \tilde{\alpha}\omega\approx 1,\\ (\mu_p/\mu_e)^{2\lambda}/(\tilde{\alpha}\omega)^m, & \tilde{\alpha}\omega > 1,\end{cases}} \tag{20}$$

where λ is the multipolarity of the nuclear transition, the exponent m depends on the transition, and the γ-ray energy is related to $\tilde{\alpha}\omega$ by

$$E_\gamma[\mathrm{keV}] \approx 4z(\tilde{\alpha}\omega). \tag{21}$$

When $\tilde{\alpha}\omega < 1$, the power-type dependence of satellite intensity on the multipolarity q of electron transitions reappears:

$$\mathrm{Im}\,E_e/\mathrm{Im}[E_c(\lambda p)+E_p(\lambda p)] \sim (\mu_p)^{2\lambda}(\tilde{\alpha}\omega)^{2q}. \tag{22}$$

However, when $q = 0$, we have Im $E_e \to 0$ for $\tilde{\alpha}\omega \to 0$ because the radial parts of the electron functions of the initial and final states are orthogonal.

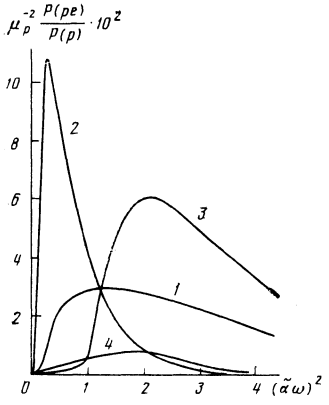

FIG. 1. Contribution of Im E_e to the relative intensity of a satellite. $P(pe)$ is the satellite intensity, $P(p)$ is the intensity of the nuclear line, and μ_p is the relative mass of the proton ($\mu_p \simeq 1/2z$). 1—$1s$–$2p_{3/2}$ transition, 2—$2s$–$2p_{3/2}$ transition, 3—$2p_{1/2}$–$2p_{3/2}$ transition, 4—$1s$–$2s$ transition. The line intensities are related to the calculated imaginary parts of the energy (12), (14), (15) by $P = 2\,\mathrm{Im}\,E/\hbar$. The energy of the γ-ray is related to the parameters $\tilde{\alpha}\omega$ by (21).

4. RESULTS OF CALCULATIONS

We shall now estimate the contribution of Im E_e to the relative intensity of electron satellites in the one-electron ion in the case of the $1s$–$2s$ (monopole), $1s$–$2p_{3/2}$, $2s$–$2p_{3/2}$ (dipole), and $2p_{1/2}$–$2p_{3/2}$ (quadrupole) transitions. Figure 1 shows the calculated quantities as functions of the parameter $\tilde{\alpha}\omega$, related to the γ-ray energy by (21). The entire dependence on the charge of the nucleus is contained in this parameter. Inclusion of relativistic effects leads mostly to a shift of the graphs toward higher energies. The numerical difference between the intensities of different satellites is mainly due to the difference between the radial electron integrals, determined by the overlap of the wave functions. For transitions to the $2p_{1/2}$ state, the satellite intensity is lower by a factor of two as compared with $2p_{3/2}$ (in accordance with the statistical weights of the states).

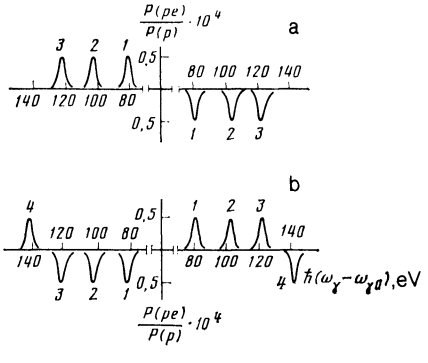

FIG. 2. Position of some of the close electron satellites of emission and absorption lines (in the positive and negative directions of the abscissa axis, respectively) for the ion FeXIX in low-lying states of the ground electron configuration $2s2p^4$ (a) and the excited electron configuration $2s2p^5$ (b) relative to the nuclear γ-transition with $\hbar\omega_{0\gamma} = 14.41$ keV in the isotope $^{57}_{26}$Fe. The figure shows lines accompanied by electron transitions: 1—$2s^22p^4\ ^1S_0$–$2s2p^5\ ^3P_1$; 2—$2s^22p^4\ ^3P_1$–$2s2p^5\ ^3P_2$; 3—$2s^22p^4\ ^3P_2$–$2s2p^5\ ^3P_1$; 4—$2s2p^5\ ^3P_1$–$2p^6\ ^1S_0$. The relative intensities of these lines are $P(pe)/P(p) \simeq 5\times 10^{-5}$, and the Doppler broadening is $\delta\hbar\omega_D \simeq 5$ eV.

TABLE I. L-shell energy levels of the ion FeXIX, measured from the ground state $2s^22p^4\ {}^3P_2$ (Ref. 9).

Configuration	$2s^22p^4$				$2s2p^5$				$2p^6$
States	${}^3P_0$	${}^3P_1$	${}^1D_2$	${}^1S_0$	${}^3P_2$	${}^3P_1$	${}^3P_0$	${}^1P_1$	${}^1S_0$
E, eV	9.4	11.1	20.9	40.3	114.4	122.0	127.7	157.1	264.6

Let us now estimate the ratio of the shift of the electron satellite to the Doppler linewidth of the γ-ray line in thermalized plasma. Suppose that the K-shell of the plasma ions is significantly disturbed. The mean kinetic energy of the ions in the plasma (Coulomb units) is approximately $E_i/10 \simeq 1/20$, where E_i is the $1s$-electron binding energy. The corresponding Doppler shift is $\delta\hbar\omega_D \simeq \tilde{\alpha}\omega/(10M)^{1/2}$. Suppose that $\tilde{\alpha}\omega = 1$, in which case

$$\delta\hbar\omega_D \approx 1/(10M)^{1/2} \text{ c.u.} \approx 1/200\ z^{1/2} \text{ c.u.} \approx 0.15\ z^{3/2} \text{ eV}. \quad (23)$$

Let us compare this with the electron transition energies. For $z = 10$–50,

$$E(1s-2p_{3/2}) = 10^3 - 2.6\cdot 10^4 \text{ eV},$$

$$E(2s-2p_{3/2}) \approx E(2p_{1/2}-2p_{3/2}) = 0.4-300 \text{ eV}.$$

The splitting of the $2s_{1/2}$–$2p_{1/2}$ levels (Lamb shift) is 0.01–5 eV. Comparisons show that the transition energies involving no change in the principal quantum number are of the order of the Doppler shift. In the one-electron ions, this splitting is wholly due to relativistic effects, which explains why they are so small.

The situation is much more favorable for many-electron systems in which there is additional splitting due to the interaction between the electrons. Here, the spectrum of resolved satellites is much richer. The Table lists the L-shell energy levels of the oxygen-like ion FeXIX ($z = 26$). At a sufficient distance from the Doppler profile of the main line, there are a large number of electron satellites due to the 2–2 transitions. These are the $2s^n 2p^{6-n} LSJ$–$2s^n 2p^{6n} L'S'J'$ transitions without change in the electron configuration and transitions with a change in the state of one electron ($2s$–$2p$ transitions). The estimate obtained above for the one-electron problem remains in force: the only change is in the angular coefficients (by not more than an order of magnitude), and the correction to the interaction between the electrons has the additional small parameter $1/z$. As an example, consider the well-known nuclear transition in ${}^{57}_{26}$Fe with the emission of the 14.41-keV γ-ray. The transition half-life is $T_{1/2} = 9.77\times 10^{-8}$ s and the recoil energy of the nucleus is 1.96×10^{-6} keV. For this transition, $\tilde{\alpha}\omega = 0.27$. According to Fig. 1, the strongest electron satellites in this region are those associated with the $2s$–$2p$ transitions. Transitions without a change in the electron configuration are quadrupole transitions and are weak for $\tilde{\alpha}\omega < 1$. Figure 2 shows the disposition of some of the electron satellites relative to the nuclear line $\hbar\omega_{0\gamma}$. The Doppler widths, which, for these parameters, amount to $\delta\hbar\omega_D \simeq 5$ eV, are indicated qualitatively. The relative intensities of these satellites are approximately 5×10^{-5}. Satellites associated with the 1–2 transitions lie at about 6 keV from $\omega_{0\gamma}$, but their intensity is somewhat lower.

We are now in a position to summarize our calculations: (1) as we pass from neutral atoms to ions, the spectrum of electron satellites due to nuclear dipole transitions becomes much richer and the relative intensity of satellites for lines with transition energies $E_\gamma \lesssim 4z$ keV may reach μ_p^2 ($\mu_p \simeq 1/2z$ and λ is the multipolarity of the nuclear transition) and (2) the emission and absorption spectra can contain observable electron-nuclear lines that are not overlapped as a result of Doppler broadening.

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