Self-wave structures and metastability of current states in metals

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It is shown theoretically that in a metal irradiated by a large-amplitude radio wave there can exist low-frequency induced magnetic-field self-wave structures. A self-wave constitutes a moving domain wall that divides the sample into two regions in which the induced field **h** is parallel and antiparallel to the external field $\mathbf{h}_0$. The velocity and direction of such a switchover wave are determined by the magnitude and sign of the field h_0. It is proved that the current state with an induced field **h** antiparallel to the field $\mathbf{h}_0$ is metastable. This conclusion confirms the principle of minimum entropy production for a metal in a current state. The results of the paper explain the available experimental facts.

The recently initiated active experimental investigations of nonlinear electrodynamic properties of metals continue to this day to provide nontrivial result. Although many nonlinear effects have been theoretically explained, many experimental facts have not even found a physical interpretation. These include the spatially inhomogeneous magnetic-field structure induced by a radio wave of large amplitude and observed by Dolgopolov and Chuprov,[1] the metastability of current states,[2] and nonlinear-wave propagation accompanied by reversal of the magnetic moment of the sample.[2] The theoretical investigation reported here shows that the above experimental facts have a common cause and can be explained in the context of self-wave (SW) theory.

In contrast to other wave processes, self-waves have unique properties: their characteristics depend only on the properties of the medium, are independent of the initial conditions, and are independent far from the edges of the boundary conditions and of size of the samples. A particular case of self-waves are the waves in which the system is switched over from a high-energy to low-energy state. In this case the energy consumed in maintaining the waves is not recovered. The mathematical model used to describe such waves is a parabolic equation of the Komogorov-Petrovskiĭ-Piskunov (KPP) type.[3]

Self-wave theory is used to explain various processes in nonequilibrium media (see, e.g., Refs. 4 and 5). Note that in the case of weak spatial dispersion the distribution of an electromagnetic field in a metal can be described by a diffusion equation (of parabolic type). It is therefore natural to expect the appearance of electromagnetic self-waves in metals under certain conditions. We show here theoretically, for the first time ever, that low frequency self-waves of the induced-field structure are produced in a metallic plate irradiated by a radio wave of high amplitude and are describable by an equation of the KPP type.

1. Let a monochromatic wave of frequency ω and amplitude $\mathscr{H}$ be normally incident on the boundary of a metal and let its polarization be such that the vector of its magnetic component is collinear with the external magnetic field $\mathbf{h}_0$. The frequency and amplitude of the incident waves are such that a quasistatic anomalous skin effect is produced:

$$\delta\sim\{c^2\nu[1-\exp(-2\pi\gamma)]/3\pi\omega\sigma_0\}^{1/3},\qquad(1)$$
$$\omega\ll\nu,\ \delta\ll l,\ R,\ \gamma=R/l,\ R=cp_F/e2\mathscr{H}.$$

Here e, p_f, ν, and l are respectively the charge, Fermi momentum, relaxation frequency, and mean free path of the electron, δ is the skin-layer depth at the frequency ω of the incident wave, σ_0 is the static conductivity of the unbounded metal, R is the curvature radius of the electron trajectory in a field $2\mathscr{H}$, and c is the speed of light.

It is known[6] that at amplitudes

$$2\mathscr{H}\gtrsim h=8cp_F\delta/el^2\qquad(2)$$

a magnetodynamic nonlinearity effect appears in metals and its mechanism is connected with the influence exerted on the conduction electron dynamics by the proper magnetic field of the current excited in the sample. Note that in this case the electronic system is in slight disequilibrium even in the strong-nonlinearity regime: the addition eEl to the electron energy as a result of the disequilibrium is always small compared with the Fermi energy. The magnetodynamic nonlinearity leads to the onset of a current state (CS), meaning appearance of an intrinsic magnetic moment in the irradiated sample. The theory describing the excitation of a constant and homogeneous magnetic field h was developed in Refs. 7–10. An analysis was made there of the dynamics of the dependence of the induced field h on the external field h_0 with change of the amplitude $\mathscr{H}$, and it was proved that at amplitudes higher than a certain critical value $\mathscr{H}_{cr}$ the function $h(h_0)$ becomes multiply valued in some regions. In other words, there exists an interval of external fields h_0, within which the sample is a multistable system. The actual state of the metallic sample depends on the prior history, i.e., on the state from which the transition takes place, on the character of the perturbation that causes the transition, and on the dynamics of the transition itself (see Refs. 1, 2, 6).

A metal in which a CS is excited is similar in a certain sense to a system with several metastable states in thermodynamics. Each such state corresponds to a relative minimum of the free energy and therefore, being stable to small fluctuations, is unstable to fluctuations of finite amplitude. The lifetime of the system in the metastable state is determined by the probability of producing such a fluctuation. An absolutely stable state corresponds to an absolute minimum of the free energy.

No general principle similar to the free-energy-minimum principle was established for essentially nonlinear nonequilibrium systems. Attempts to use the principle of mini-

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mum entropy production as the general principle cannot be regarded as sufficiently well founded, since it was proved only for linear nonequilibrium systems.[11] To determine which of the states is absolutely stable it is necessary to analyze in detail the complete set of equations. In this case, as shown in Ref. 12, a large class of initial conditions for an equation of the KPP type leads to solutions that tend asymptotically (as $t \to +\infty$) to SW systems. This fact makes it possible to detect metastability of states on the basis of an analysis of the SW solutions.

2. Assume that the magnetic field h induced in the sample outside the limits of the high-frequency (hf) skin layer is not constant and not homogeneous. The characteristic times of variation of this field will be assumed long compared with the period $2\pi/\omega$ of the incident wave. Since the electromagnetic field in the metal contains in this case two substantially different time scales, it is natural to expect two characteristic space scales to appear. Added to the scale δ over which the hf fields attenuate is a scale $\tilde{\delta} \gg \delta$ due to the presence of the low-frequency (lf) time dependence.

The nonlinear interaction between the magnetic fields, which excites the current state in the metal and gives rise to the self-wave structures, takes place in the skin layer δ. It is quite obvious that a current localized in this depth is a surface current from the standpoint of the low-frequency problem. Therefore the effect of magnetodynamic nonlinearity can be described in terms of a nonlinear boundary conditions for the lf fields. This effective boundary condition is derived in the Appendix.

Consider bilateral excitation, symmetric with respect to the magnetic field, of a compensated-metal plate ($-d/2 \leqslant x \leqslant d/2$, $-\infty \leqslant y,z \leqslant +\infty$) by a hf wave. We direct the z axis parallel to the vector of the constant and homogeneous external magnetic field $\mathbf{h}_0$ (see Fig. 1).

We seek the electric and total magnetic fields in the metal in the form

$$\mathbf{E}=\{E_x(x,y,t),\ E_y(x,y,t),\ 0\},\quad \mathbf{H}=\{0,\ 0,\ H(x,y,t)\}. \tag{3}$$

In this geometry, the Maxwell equations are

$$-\frac{\partial H(x,y,t)}{\partial x}=\frac{4\pi}{c}j_y(x,y,t),\qquad \frac{\partial H(x,y,t)}{\partial y}=\frac{4\pi}{c}j_x(x,y,t),$$
$$\frac{\partial E_y(x,y,t)}{\partial x}-\frac{\partial E_x(x,y,t)}{\partial y}=-\frac{1}{c}\frac{\partial H(x,y,t)}{\partial t}. \tag{4}$$

To find the low-frequency field induced outside the hf skin layer we must supplement the Maxwell equations (4) with effective boundary conditions on both sides of the plate ($x=\pm d/2$). Assuming the plate to be thick enough ($d \gg \delta$), we can formulate the boundary conditions independent of one another. According to (A.13) we have

$$E_y(\pm d/2,y,t)=\mp 2\mathscr{H}\frac{\omega\delta}{c}\Phi\{\varkappa(\pm d/2,y,t)\},$$
$$\varkappa(x,y,t)=H(x,y,t)/2\mathscr{H}. \tag{5}$$

The function $\Phi(\varkappa)$ defined by Eq. (A.12) can be easily written in explicit form by using a simplified model that does not take into account the inessential time dependence of the conductivity of the trapped electrons. In this model, $\Phi(\varkappa)$ is described by the expression

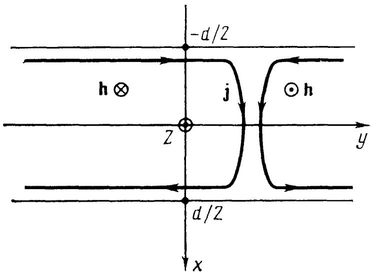

FIG. 1.

$$\Phi(\varkappa,a)=\frac{(1-a^2)^{1/2}\operatorname{sign}\varkappa}{\pi[\exp(b/|\varkappa|)-1]+\arccos(-a\operatorname{sign}\varkappa)}-\varkappa+a, \tag{6}$$

where $a=h_0/2\mathscr{H}$ and $b=(8R\delta)^{1/2}/l=(h/2\mathscr{H})^{1/2}$ is the nonlinearity parameter. We assume here for simplicity that the metal boundary is diffuse. A nondiffuse reflection of the electrons from the sample surface can be accounted for in a manner similar to that used in Ref. 10. It will be shown below that the feasibility of self-wave structures is determined by the most general properties of the function $\Phi(\varkappa)$. Therefore allowance for the non-diffuse reflection does not lead to qualitatively new effects.

We confine ourselves to finding smooth distributions of the low-frequency fields, i.e.,

$$R \ll l \ll \tilde{\delta}. \tag{7}$$

The inequalities (7) make it possible to use the local connection between the density of the current and the electric field

$$j_{x,y}(x,y,t)=\frac{\sigma_0\gamma^2}{\varkappa^2(x,y,t)}E_{x,y}(x,y,t). \tag{8}$$

We substitute the current density (8) in the Maxwell equations (4) and eliminate from the latter the electric field. With allowance for the boundary condition (5) we obtain a complete system that determines the distribution of the magnetic field in the interior of the plate:

$$\frac{4\pi}{c^2}\sigma_0\gamma^2\frac{\partial\varkappa}{\partial t}=\frac{\partial}{\partial x}\left(\varkappa^2\frac{\partial\varkappa}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varkappa^2\frac{\partial\varkappa}{\partial y}\right), \tag{9}$$

$$\varkappa^2\frac{\partial\varkappa}{\partial x}\bigg|_{x=\pm d/2}=\pm\frac{\gamma^2R}{2\delta^2}\Phi\left\{\varkappa\left(\pm\frac{d}{2},y,t\right)\right\}. \tag{10}$$

Recall that outside the hf skin layer δ, where Eqs. (9) and (10) are valid, the quantity $\varkappa$ is the sum of the external magnetic field h_0 and the induced low-frequency field $h(x,y,t)$ in units of $2\mathscr{H}$.

The system (9), (10) has static homogeneous solutions implicitly specified by the equality

$$\Phi(\varkappa,a)=0. \tag{11}$$

These solutions $\varkappa(a)$ were obtained in Ref. 9. It follows from that reference that at amplitudes $\mathscr{H}$ lower than the critical value $\mathscr{H}_{cr}$ the function $\Phi(\varkappa)$ decreases monotonically at all values of the external parameter a. This corresponds to a unique $\varkappa(a)$ dependence, i.e., to absence of hysteresis of the induced field as a function of the external one. At amplitudes $\mathscr{H}>\mathscr{H}_{cr}$ the function $\Phi(\varkappa)$ becomes N-shaped (Fig. 2). In this situation we get a range of external fields $-a^*<a<a^*$ in which Eq. (11) has three solutions $\varkappa_1<\varkappa_2<\varkappa_3$ at a fixed

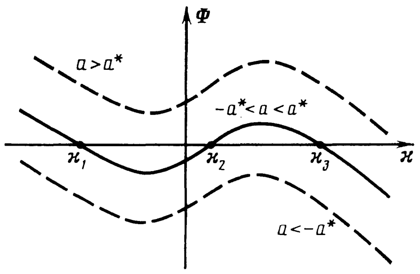

FIG. 2.

a. In other words, the $\varkappa(a)$ dependence becomes multiply valued, corresponding to onset of hysteresis of the induced field.

Let us test the static homogeneous solutions ($i = 1,2,3$) Eq. (11) for stability to perturbations of the form

$$\Delta\varkappa \propto \exp(\lambda t + iqy)\,\mathrm{ch}(kx). \qquad (12)$$

Substitution of $\varkappa_i + \Delta\varkappa$ in the linearized Eq. (7) yields the dispersion law

$$\lambda = (c^2\varkappa_i^2/4\pi\sigma_0\gamma^2)(k^2 - q^2). \qquad (13)$$

Allowance for the boundary condition (10) leads to a "selection rule" for the wave number k:

$$\frac{kd}{2}\,\mathrm{th}\left(\frac{kd}{2}\right) = \frac{\gamma^2 Rd}{4\delta^2}\,\Phi'(\varkappa_i). \qquad (14)$$

where the prime denotes the derivative with respect to the argument.

At $\Phi' < 0$ Eq. (14) has only imaginary solutions. According to (13), this means damping, with time, of the fluctuations (12) near the homogeneous solutions $\varkappa_1$ and $\varkappa_3$ (see Fig. 2). At $\Phi' > 0$ Eq. (4) has, beside the imaginary solutions, one real solution that corresponds at $|q| < |k|$ to growth of fluctuations, long-wave in y, near the homogeneous solution $\varkappa_2$. Thus, the branches of the multiple-valued functions $\varkappa(a)$ that correspond at fixed a to the minimum and maximum values of the induced field are stable, while the inner branch is unstable to infinitely small lf electromagnetic perturbations. Note that the stability criterion $\Phi' < 0$ is the same for perturbations that are both symmetric and antisymmetric with respect to the magnetic field.

The physical meaning of the stability condition $\Phi' > 0$ is quite lucid. The fluctuating lf electric field produces in the hf skin layer a current that contains, besides a smoothly modulated hf component also a purely lf component (rectification effect). According to (10), the magnetic field $\Delta H = 2\mathscr{H}\Delta\varkappa$ produced in this case by the rectified current in the transition region is connected with the electric field by the relation

$$E_y(\pm d/2) \propto \mp\Phi'(\varkappa_i)\Delta H(\pm d/2).$$

It follows hence that the direction of the x-projection of the Poynting vector on the effective boundary is uniquely connected with the sign of the derivative Φ'. If $\Phi' > 0$ the Poynting vector is directed into the interior of the plate at both boundaries, and at $\Phi' < 0$ it is directed outward. The fact that at $\Phi' > 0$ energy transfer is possible from the hf skin layer to the interior of the plate leads to instability and allows us to expect formation of self-wave structures that are due to competition between the transfer of the electromagnetic energy from the hf skin layer and its dissipation over the entire thickness of the plate. Indeed, it will be shown below that the static homogeneous distributions of the induced magnetic field are not the only solutions of the problem (9) and (10).

It is impossible in the general case to obtain inhomogeneous solutions of the system (9) and (10). We confine ourselves therefore to a limiting case when the plate thickness d is much less than the spatial scale $\tilde{\delta}$ of variation of the fields. Expanding in (9) in powers of $x/\tilde{\delta}$ and using the boundary conditions (10), we obtain an equation that describes the magnetic field distribution and is independent, in the principal approximation in $d/\tilde{\delta}$, of the transverse coordinate:

$$\frac{\partial\varkappa}{\partial\tau} = \frac{\partial}{\partial\eta}\left(\varkappa^2\frac{\partial\varkappa}{\partial\eta}\right) + \Phi(\varkappa). \qquad (15)$$

Here η is a dimensionless coordinate in the direction of the hf current, τ is the dimensionless time:

$$\eta = y/\tilde{\delta}, \quad \tilde{\delta} = (\delta/\gamma)(d/R)^{1/2}, \quad \tau = \delta\omega t/d. \qquad (16)$$

We seek the self-wave solutions of Eq. (15) in the form

$$\varkappa = \varkappa[(y - Vt)/\tilde{\delta}].$$

Putting $\xi = (y - Vt)/\tilde{\delta}$ in (15), we obtain the following ordinary nonlinear differential equation:

$$\frac{d}{d\xi}\left(\varkappa^2\frac{d\varkappa}{d\xi}\right) + \frac{VR\tilde{\delta}\gamma^2}{\omega\delta^3}\frac{d\varkappa}{d\xi} + \Phi(\varkappa) = 0. \qquad (17)$$

A standard analysis of this equation (see, e.g., Ref. 13) makes it possible to find a large class of self-wave solutions. All but one of them, however, are unstable to fluctuations of the form $\Delta\varkappa(\xi)\exp(\lambda t)$. The only stable solution is of the kink type, corresponding to a self-wave of the induced magnetic field

$$h = 2\mathscr{H}\varkappa[(y - Vt)/\tilde{\delta}], \qquad (18)$$

which is a domain wall of width $\tilde{\delta}$. Induced-field values corresponding to the homogeneous solutions $\varkappa_1(a)$ and $\varkappa_3(a)$ are realized on both sides of the domain wall. The domain-wall velocity V is determined by the magnitude and direction of the external magnetic field h_0. To be able to determine the direction of the velocity V and to estimate its magnitude, it suffices to multiply Eq. (17) by $\varkappa^2 d\varkappa/d\xi$ and integrate it with respect to ξ from $-\infty$ to $+\infty$. Recognizing that $d\varkappa/d\xi = 0$ for kink-type solutions at $\xi = \pm\infty$, we get

$$V = -\frac{\omega\delta^2\gamma}{(Rd)^{1/2}}\,\frac{\int_{-\infty}^{\infty} d\xi\,\frac{d\varkappa}{d\xi}\varkappa^2\Phi(\varkappa)}{\int_{-\infty}^{\infty} d\xi\,\varkappa^2\left(\frac{d\varkappa}{d\xi}\right)^2} \sim \frac{\omega\delta^2\gamma^{-1}}{(Rd)^{1/2}}\,a\,\mathrm{sign}\,\varkappa(\xi\to-\infty). \qquad (19)$$

In the absence of an external magnetic field h_0, the domain-wall velocity vanishes and the induced field in the sample has a static inhomogeneous distribution whose values on opposite side of the wall are of equal magnitude and opposite direction. In a nonzero external field, the domain-wall motion is such that the region in which the induced field is parallel to the external wall crowds out the region in which these fields are antiparallel.

3. The fact that the sample is bounded in the direction the domain-wall motion means that the SW solutions in a zero field h_0 describe a transient process—a switchover wave. Perturbations of finite amplitude change the sample to a state in which its intrinsic magnetic moment is parallel to the external magnetic field. Thus, the state in which the magnetic moment is antiparallel to the external field is metastable. This conclusion agrees with the principle of minimum entropy production. It is known (see, e.g., Ref. 14) that the rate of entropy production in an isothermal process is described by the expression:

$$\dot{S} \propto \left\langle \int dx\, jE \right\rangle \propto \mathrm{Re}\, Z_\omega(h_0). \tag{20}$$

The angle brackets denote here averaging over the period $2\pi/\omega$ of the incident wave (see the Appendix), and Z_ω is the surface impedance of the metal at the frequency ω. Figure 3 shows schematically the dependence, obtained in Ref. 15, of the real part of the impedance Z_ω on the external field h_0 for a metal located in a current state. The arrows on the curves indicate the direction of the change of h_0, and the markings on the curves indicate the homogeneous state to which they pertain. It can be seen from (20) that the entropy production is a minimum in a state with an induced field $\varkappa = \varkappa_3 > 0$ if $h_0 > 0$, and in a state with $\varkappa = \varkappa_1 < 0$ if $h_0 < 0$. The principle of minimum entropy production is valid for a metal in a current state.

4. It is undoubtedly of interest to ascertain the possibility of existence of self-wave structures of an induced magnetic field at an arbitrary plate thickness. Progress in the answer to this question can apparently be made only be making some minor simplification, viz., replace the quantity $\varkappa$ and its order-of-magnitude estimate in (8) by unity. This means neglect of the influence of the inhomogeneity of the induced magnetic field on the surface. The system (9) and (10), which describes the distribution of the total magnetic field outside the hf skin layer, takes then the form

$$\frac{4\pi}{c^2}\sigma_0\gamma^2\frac{\partial\varkappa}{\partial t} = \frac{\partial^2\varkappa}{\partial x^2} + \frac{\partial^2\varkappa}{\partial y^2}, \tag{21}$$

$$\left.\frac{\partial\varkappa}{\partial x}\right|_{x=\pm d/2} = \pm\frac{\gamma^2 R}{2\delta^2}\Phi\{\varkappa(\pm d/2, y, t)\}. \tag{22}$$

From the definition (6) of the function $\Phi(\varkappa,a)$ it is seen that in the case of strong nonlinearity $(b = h/2\mathscr{H})^{1/2} \ll 1)$ we have

$$\Phi(\varkappa, a) = -\varkappa + a + \begin{cases} -\dfrac{(1-a^2)^{1/2}}{\arccos a}, & \varkappa < 0, \\ \dfrac{(1-a^2)^{1/2}}{\arccos(-a)}, & \varkappa > 0. \end{cases} \tag{23}$$

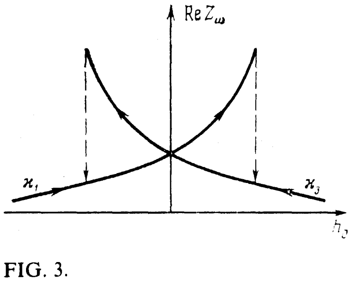

FIG. 3.

The system (21) and (22) with the function $\Phi(\varkappa,a)$ in the form (23) is mathematically equivalent to a problem investigated in Ref. 16. The authors of that paper proved the existence of SW solutions at arbitrary plate thickness and obtained an implicit analytic expression for the domain-wall velocity (Eq. (22) of Ref. 16 at $\varkappa(\xi\to\infty) > 0$):

$$\frac{p}{\pi}\int_0^\infty \frac{d\nu}{r}\frac{\mathrm{th}\, r}{B + r\,\mathrm{th}\, r} = 1 - 2\Theta_*, \qquad r = (\nu^2 + p^2)^{1/2}. \tag{24}$$

In our case we have

$$p = \frac{4\pi}{c^2}\sigma_0\gamma^2\frac{d}{2}V, \qquad B = \frac{\gamma^2 Rd}{4\delta^2},$$

$$\Theta_* = \left[-a + \frac{(1-a^2)^{1/2}}{\arccos a}\right] \Big/ \left[\frac{(1-a^2)^{1/2}}{\arccos(-a)} + \frac{(1-a^2)^{1/2}}{\arccos a}\right]. \tag{25}$$

In a thin plate ($B \ll 1$) the parameter $p \approx B^{1/2}(1 - 2\Theta_*)$, i.e.,

$$|V| \approx \frac{\omega\delta}{2B^{1/2}}(1 - 2\Theta_*) = \omega\delta^2\gamma^{-1}(Rd)^{-1/2}(1 - 2\Theta_*). \tag{26}$$

This agrees with the result (19) above, thus confirming to some degree that the approximation made in the present section is admissible. In a thick plate ($B \gg 1$) we have $p \approx B(1 - 2\Theta_*)$, i.e.,

$$|V| \approx {}^1\!/_2\,\omega\delta(1 - 2\Theta_*). \tag{27}$$

Note that the stability (metastability) criterion is the same for plates of any thickness.

5. The dissipative structure (zero-velocity self-waves) of the induced field in a sample in a current state was first observed in experiment in Ref. 1. It was noted there that in an experiment with one sample the transition to an inhomogeneous distribution of the induced field takes place at different values of the incident-wave amplitude. This confirms the metastability of the current state. The feasibility of using a metallic sample in which a current state is excited as a memory element was investigated in Ref. 2. A pulsed magnetic field was applied to one end of a long sample, and a signal was picked-off the other end. In full accord with our results, the signal proceeded to the receiving coil only if the magnetic moment of the sample at the initial state was antiparallel to the external magnetic field. The transition, effected by the self-wave, of a sample of length L from a homogeneous metastable into a stable one occurs within a time $T = L/V$. Let us estimate the switchover time T under conditions of the experiment of Ref. 2. A bismuth sample of conductivity $\sigma_0 \sim 3\cdot 10^{17}$ s^{-1}, length $L = 1$ cm, and thickness $d = 0.05$ cm is placed in an external constant magnetic field $h_0 = 0.15$ Oe. The current state is excited by an electromagnetic wave of frequency $\omega = 1.6\cdot 10^7$ Hz and amplitude $\mathscr{H} = 17$ Oe. The electron mean free path is $l = 0.1$ cm. Since the parameter $B \approx 8.3\cdot 10^2$ in this situation, we calculate the self-wave velocity V using expression (27) and expanding the function $1 - 2\Theta_*$ in powers of the small $a = h_0/2\mathscr{H}$:

$$V \approx {}^1\!/_2\,\omega\delta a. \tag{28}$$

Substituting in (28) and (1) the numerical values of all the quantities, we obtain a self-wave velocity $V \approx 2.7\cdot 10^2$ cm/s, and hence a switchover time $T \approx 3.7\cdot 10^{-3}$ s. Thus, the results of the proposed theory agree well with the experimental facts.

Note that allowance for the inhomogeneity of the external magnetic field and of the sample (e.g., on account of the impurities) permits certain features of the experimental observation of self-wave structures of the induced field to be explained. It was shown in a cycle of studies (see Ref. 17 and the literature cited therein), in which the influence of various inhomogeneities on the solution of the nonlinear equation was analyzed, that domains and domain walls are pinned by the inhomogeneities. In the case of an inhomogeneous external field (but smooth compared with the scale δ) the domain wall is localized in a region where $h_0 = 0$. A situation with several such regions is possible, and the metallic plate is in a multidomain state. A domain wall can also be pinned by inhomogeneities much smaller than the wall thickness $\tilde{\delta}$.

In measurements of the intrinsic magnetic moment averaged over the sample, the influence of the inhomogeneity is manifested in the following manner. By varying the electric field h_0 we land in the metastability region of the current state. The self-wave produced, which in the absence of impurities would change the sample from a metastable to a stable homogeneous state, is pinned. The intrinsic magnetic moment averaged over the sample assumes an intermediate value. Further change of the external field tears the domain wall away from the impurity, and the detached self-wave ultimately brings the sample to a homogeneous stable state. Thus, the presence of inhomogeneities explains the existence of additional hysteresis jumps of the intrinsic magnetic moments of the sample as functions of the external field, and also the onset of inhomogeneous structures of the induced field at $h_0 \neq 0$ (Ref. 1).

APPENDIX

Derivation of the effective boundary condition

Consider a semi-infinite metal in an external constant and homogeneous magnetic field $\mathbf{h}_0$ parallel to its surface. We direct the x axis into the interior of the metal perpendicular to its boundary $x = 0$, and the z axis along the vector $\mathbf{h}_0$. The net magnetic field on the metal surface is

$$H(0,\ y,\ t) = 2\mathscr{H} \cos \omega t + h_0. \tag{A.1}$$

To derive the effective boundary condition we determine the distribution of the electromagnetic field in the region $x \sim \delta$ and takes its asymptotic value at $\delta \ll x \ll \tilde{\delta}$.

Note that the inhomgeneity of the fields along the coordinate y is due only the low-frequency time dependence, and the corresponding spatial scale is $\tilde{\delta} \gg \delta$. Therefore the current density calculated in the principal approximation in $\delta/\tilde{\delta}$ in the skin layer δ does not depend explicitly on y. This dependence is contained in the current only via the y-dependence of the electric and magnetic fields, and we can use for the current density the expression obtained in Ref. 18.

The conductivity operator is nonlocal, and it is convenient to write the connection of the current with the electric field for the cosine Fourier transform

$$\begin{gathered} j_y(k, y, t) = 2\int_0^\infty dx\, j_y(x, y, t) \cos kx, \\ \mathscr{E}_y(k, y, t) = 2\int_0^\infty dx\, E_y(x, y, t) \cos kx. \end{gathered} \tag{A.2}$$

According to Ref. 18, the action of the conductivity operator on the long- and short-wave components of the electric field is different, and we represent the field $\mathscr{E}_y(k,y,t)$ in the form

$$\mathscr{E}_y(k,y,t) = \mathscr{E}_y^{\rm hf}(k,y,t) + \mathscr{E}_y^{\rm lf}(k,y,t). \tag{A.3}$$

The hf component of the electric field $\mathscr{E}_h^{\rm hf}(k,y,t)$ is localized at a distance δ from the metal boundary, and its Fourier transform is a maximum at $k \sim \delta^{-1}$. The low-frequency electric field present in the skin layer δ varies over much larger distances $\Delta x \sim \tilde{\delta}$, and its Fourier components are of the order of $k \sim \tilde{\delta}^{-1}$.

Following Refs. 18 and 19, we represent the current density in the form

$$\begin{gathered} j_y(k, y, t) = \frac{\sigma_0}{kl}[1 - e^{-2\pi\gamma}]^{-1} S(k, y, t) \mathscr{E}_y^{\rm hf}(k,y,t) \\ + \frac{\sigma_0}{kl} R E_y^{\rm lf}(0) + \Theta_- \frac{\sigma_0}{kl} \operatorname{cth} \nu T_3 \cos kx_0 J_1(kx_0)\, x_0 E_y^{\rm lf}(0), \end{gathered} \tag{A.4}$$

$$S(k,\ y,\ t) = \Theta_+ + \alpha(y,\ t) F(kx_0) \Theta_-,$$

$$\alpha = \frac{1 - \exp(-2\pi\gamma)}{1 - \exp(-2\nu T_3)}, \qquad \Theta_\pm = \Theta\left(\pm \frac{2\mathscr{H}\cos\omega t + h_0}{H_{\rm eff}(y,t)}\right),$$

where x_0 is the x-coordinate of the surface on which the resultant magnetic field vanishes, $H_{\rm eff}$ is the resultant magnetic field in the transition region (on the effective boundary in the case of the low-frequency problem), and J_1 is a Bessel function.

The first term in (A.4) describes the response to the high-frequency electric field. The factor $(kl)^{-1}$ in this term is a reflection of the usual conductivity spatial dispersion observed in the anomalous skin effect. The function $\Theta_\pm$ takes into account the fact that different electron groups make the main contribution to the conductivity at different instants of time. In those instants when

$$(2\mathscr{H} \cos \omega t + h_0)/H_{\rm eff}(y,\ t) > 0, \tag{A.5}$$

the spatial distribution of the resultant magnetic field is of constant sign and the effective electrons move along trajectories that are close to Larmor circles. In the case of the opposite inequality

$$(2\mathscr{H}\cos\omega t + h_0)/H_{\rm eff}(y,t) < 0, \tag{A.6}$$

however, a new group appears in the metal, viz., captured electrons. Their trajectories wind around the surface $x = x_0(y,t)$ on which the resultant magnetic field vanishes. The period $2T_3$ of the captured electrons is much shorter than the period $2\pi\gamma/\nu$ of the Larmor particles. As a result, during the period $2\pi/\omega$ of the hf wave the conductivity in the skin layer δ changes by a factor $\alpha \gg 1$. It is just this time dependence of the conductivity of the hf skin layer which rectifies the current and leads to excitation of an induced magnetic field at $x > \delta$.

The function $F(kx_0)$, which makes more precise the captured-particle conductivity dispersion, tends to $(x_0/\delta)^2$ as $kx_0 \to 0$. The next terms of the current (A.4) describe the response to the low-frequey field $\mathscr{E}_y^{\rm lf}(k,y,t)$, the first of them being the contribution of the surface electrons, and the second that of the captured particles. Since both the surface and the captured electrons exist in a space region that is narrow compared with δ, their current is proportional to the value of the low-frequency field at $x=0$.

The first of the Maxwell equations (4) takes in the k-representation the form

$$2h_0+4\mathscr{H}\cos\omega t-kH(k,y,t)=\frac{4\pi}{c}j_y(k,y,t), \qquad (A.7)$$

where $H(k,y,t)$ is the sine Fourier transform of the resultant magnetic field. We divide Eq. (A.7) with a current density (A.4) by $S(k,y,t)$ and average the result over the "fast" time (over the period of the hf wave):

$$2h_0\left\langle\frac{1}{S(k,y,t)}\right\rangle+4\mathscr{H}\left\langle\frac{\cos\omega t}{S(k,y,t)}\right\rangle-k\left\langle\frac{H(k,y,t)}{S(k,y,t)}\right\rangle$$
$$=\frac{4\pi}{c}\frac{\sigma_0}{kl}R\langle S^{-1}(k,y,t)\rangle E_y^{ef}(0)$$
$$+\frac{4\pi}{c}\frac{\sigma_0}{kl}E_y^{ef}(0)\ \langle\Theta_-S^{-1}(k,y,t)\,\mathrm{cth}\,\nu T_3\cos kx_0J_1(kx_0)x_0\rangle. \qquad (A.8)$$

The angle brackets denote here averaging over the "fast" time:

$$\langle\ldots\rangle=\frac{\omega}{2\pi}\int_t^{t+2\pi/\omega}dt'(\ldots). \qquad (A.9)$$

Assuming satisfaction of the inequality

$$R^2/l\delta\ll 1, \qquad (A.10)$$

we can neglect the first term in the right-hand side of (A.8). Equation (A.8) as $k\to 0$ (letting k go to zero corresponds to using the asymptotic forms of the fields at $x\gg\delta$) takes now the form

$$2\mathscr{H}\left\langle\frac{\cos\omega t}{\mu S}\right\rangle-H_{\text{eff}}+h_0=C\frac{4\pi}{c}\sigma_0E_y^{lf}(0), \qquad (A.11)$$

where

$$S\equiv S(k=0,y,t),\qquad \mu=\langle S^{-1}\rangle,$$

$$C=\mu^{-1}\left\langle\Theta_-S^{-1}\frac{x_0^2}{l}\,\mathrm{cth}\,\nu T_3\right\rangle\approx\delta^2/R.$$

Omitting the "lf" superscript of the electric field and introducing

$$\Phi=\left\langle\frac{\cos\omega t}{\mu S}\right\rangle-\varkappa+a, \qquad (A.12)$$

we rewrite (A.11) in the form

$$E_y(0,y,t)=2\mathscr{H}(\omega\delta/c)\Phi\{\varkappa(0,y,t)\}. \qquad (A.13)$$

Here a and $\varkappa$ are respectively the external field h_0 normalized to $2\mathscr{H}$ and the resultant magnetic field outside the skin layer δ (on the effective boundary for the lf problem):

$$a=h_0/2\mathscr{H},\ \varkappa=H_{\text{eff}}/2\mathscr{H}.$$

Expression (A.13) is the sought-for boundary condition.

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