Reflection of a monochromatic signal from moving polarization of a resonant medium

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An analysis is presented of the problem of a monochromatic laser signal reflecting from a moving polarization region in a resonant three-level medium. The frequency dependences of the reflection coefficients are determined for coherent and noncoherent interactions of fields allowing for the Doppler frequency shift. It is shown that the strongest reflection occurs when the frequencies of the incident or reflected waves are in resonance with a transition frequency. The conditions under which significant reflection occurs are identified.

Investigations of the interaction of laser pulses with multilevel media are currently attracting much interest. In many cases the central problems are the conversion of radiation from one frequency range to another, parametric interaction of laser pulses, and coherent effects in multilevel media.[1–3] The ability to control propagation of ultrashort pulses of one frequency in resonant media by radiation of a different frequency has also been studied.[4,5] The use of multilevel media also extends the range of applications of the well-known effects. For example, stimulated and multipulse echo phenomena are observed experimentally in multilevel media and have a number of advantages compared with the photon echo in a two-level medium.[3]

A theoretical description of the nonlinear interaction and propagation of pulses in resonant media is normally developed using reduced equations for the envelopes of the fields. This ignores the possible manifestations of reflected waves.

Reflection of a laser signal in a three-level medium with a cascade configuration of transitions from a coherent (2π) pulse propagating in resonance with a lower transition in the medium was investigated theoretically by Rupasov.[6] However, analytical results were obtained only for the case when the Doppler frequency shift of radiation reflected from a moving polarization region can be neglected compared with the initial frequency detuning. In experiments on resonant media it has been possible to generate pulses with propagation velocities v two or three orders of magnitude smaller than the velocity of light c. Since the reflection effects are the strongest in the vicinity of a resonance, it follows that the Doppler frequency shift can alter the reflection behavior. When a small signal is reflected from a moving polarization region induced by a laser pulse, the sign of the frequency detuning of the reflected signal may be reversed and consequently the spatial and temporal dispersion effects of the medium may play an important role. These are the topics which are treated below.

1. REFLECTION FROM A SATURATION PULSE

We shall consider the problem of reflection in a three-level medium of a small signal from a laser pulse which saturates a resonant transition (Fig. 1). We shall assume that in the initial state before the interaction with the radiation the medium is at the lowest level. A laser pulse, which is in resonance with the 1–2 transition, equalizes the populations of the lowest and middle levels of the medium and propagates with a velocity less than the velocity of light: $v = c(1 + N_0/2I)^{-1}$, where N_0 is the density of the resonating particles and I is the photon density in the pulse.[7] This type of propagation occurs in the case of pulses of duration τ_p less than the longitudinal relaxation time T_1 and greater than the transverse relaxation time in the medium, T_2: $T_2 < \tau_p T_1$. Before arrival of the saturation pulse the medium is transparent to radiation with a frequency close to the 2–3 transition (Fig. 1). In the saturation region level 2 is populated and the small-signal absorption coefficient and refractive index of the medium are different from the initial values. A change in the properties of the medium occurs at the moving boundary of the saturation region and a probe signal can be reflected. The size of the transition region is[7]

$$l \sim \frac{1}{\sigma N_0}, \qquad \sigma = \frac{4\pi}{c}\frac{\mu_{12}^2}{h}\omega_{12}T_2,$$

where σ is the absorption cross section for the 1–2 transition, and μ_{12} and ω_{12} are the dipole moment and the frequency of the transition.

We shall consider reflection of a signal E, which is in resonance with the upper cascade transition 2–3, by a saturation pulse. The system of equations describing the propagation of the signal E is

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P}{\partial t^2},$$
$$\frac{\partial^2 P}{\partial t^2} + 2\gamma\frac{\partial P}{\partial t} + \omega_{23}^2 P = 2\mu_{23}^2\frac{\omega_{23}}{h}N(x,t)E. \tag{1}$$

The carrier frequency ω_0 of this signal is close to the transition frequency ω_{23}; μ_{23} is the dipole moment of the transi-

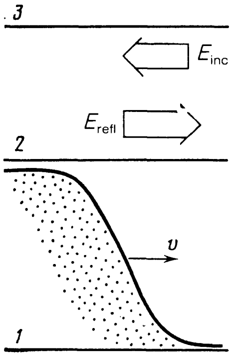

FIG. 1. Diagram of levels in a resonant medium and the fields of a probe signal ($E_{\rm inc}$, $E_{\rm refl}$) and of a saturation pulse (1–2 transition).

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tion; $N(x, t)$ is the density of particles in level 2; P is the polarizability of the medium; γ is the damping rate. The signal is assumed to be small, so there is no saturation of the 2–3 transition. The problem of reflection simplifies significantly if the size l of the transition region where saturation occurs is less than the wavelength of the small signal $\lambda = 2\pi c/\omega_0$. This condition is satisfied if $4\pi\mu_{12}{}^2 N_0 T_2/\hbar \gg \omega_0/\omega_{21}$. In this case, we have $N(x,t) = 1/2 N_0 \theta(vt - x)$, where $\theta(\xi)$ is the theta function. We shall also assume that the saturation wave velocity is much less than the velocity of light, $v/c \ll 1$. The frequency of the reflected signal is then close to the frequency of the 2–3 transition.

In the system of coordinates moving with the front of the saturation pulse, $\xi = x - vt$ and $\tau = t$, the system of equations (1) becomes

$$\frac{\partial^2 E}{\partial \xi^2} + \frac{2v/c^2}{1-v^2/c^2}\frac{\partial^2 E}{\partial\xi\,\partial\tau} - \frac{1/c^2}{1-v^2/c^2}\frac{\partial^2 E}{\partial \tau^2} = \frac{4\pi}{c^2}\frac{1}{1-v^2/c^2}\left(\frac{\partial}{\partial\tau} - v\frac{\partial}{\partial\xi}\right)^2 P, \tag{2}$$

$$\left(\frac{\partial}{\partial\tau} - v\frac{\partial}{\partial\xi}\right)^2 P + 2\gamma\left(\frac{\partial}{\partial\tau} - v\frac{\partial}{\partial\xi}\right)P + \omega_{23}{}^2 P = \frac{\mu_{23}{}^2}{\hbar}\omega_{23}N_0\theta(-\xi)E. \tag{3}$$

The dependence on τ can be chosen to be exponential: $\propto \exp[-i\omega(1+v/c)\tau]$. If $\xi > 0$, the polarizability vanishes and the solutions for the field E are traveling waves:

$$E = E_{\text{inc}}\exp\left[-i\omega\left(t + \frac{x}{c}\right)\right] + E_{\text{refl}}\exp\left[-i\omega\frac{1+v/c}{1-v/c}\left(t - \frac{x}{c}\right)\right].$$

The first term describes the field of a proble signal of frequency ω propagating in the direction opposite the saturation pulse and the second is the field of the reflected signal which is frequency-shifted because of the Doppler effect. If $\xi < 0$ (in the saturation pulse region) the solutions can be found in the form $\exp[-i\omega(1+v/c)\tau - i\alpha\xi]$, where α is found from the cubic equation

$$\left(\alpha - \frac{\omega}{c}\right)\left(\alpha + \frac{\omega}{c}\frac{1+v/c}{1-v/c}\right)\left[\alpha - \frac{\omega(1+v/c) - \omega_{23} + i\gamma}{v}\right] = \frac{2\pi}{c^2}\omega^2\frac{\mu_{23}{}^2 N_0}{\hbar v}. \tag{4}$$

An allowance is made for the fact that the frequency shift due to reflection is much less than the carrier frequency of the signal: $\omega \gg \omega v/c$. Each root of Eq. (4) describes different possible waves in the region behind the front of the saturation pulse. If the right-hand side of Eq. (4) is small, the root $\alpha_1 \approx \omega/c$ corresponds to the solution $E_1 \propto \exp[-i\omega(t + x/c)]$ describing the propagation of the transmitted wave, and the root

$$\alpha_2 \approx -\frac{\omega}{c}\frac{1+v/c}{1-v/c}$$

corresponds to the solution representing the reflected component

$$E_2 \propto \exp\left[-i\omega\frac{1+v/c}{1-v/c}\left(t - \frac{x}{c}\right)\right].$$

The root of Eq. (4)

$$\alpha_3 \approx [\omega(1+v/c) - \omega_{23} + i\gamma]/v$$

governs the solution

$$E_3 \propto \exp\left[-i\omega_{23}t - i\frac{\omega}{c}x - i\frac{\omega-\omega_{23}}{v}x + \frac{\gamma}{v}(x - vt)\right],$$

which appears at the natural frequency of the 2–3 transition because of the transient process of establishment of the polarization of the medium.

In determining the reflection coefficient of the small signal from the saturation pulse we must bear in mind that the field in the saturation region is a combination of the transmitted wave and a transient polarization wave (the reflected wave is absent because the reflection by the trailing edge of the saturation region is weak). If $\xi > 0$, the field is a combination of the incident and reflected waves. The boundary conditions allow for the discontinuity of the field and its derivative and for the vanishing of the polarization at the leading edge of the saturation region. Using Eqs. (2) and (3), we find that these boundary conditions for $\xi = 0$ are

$$\frac{E_1}{\alpha_1 v - [\omega(1+v/c) - \omega_{23} + i\gamma]} + \frac{E_3}{\alpha_3 v - [\omega(1+v/c) - \omega_{23} + i\gamma]} = 0,$$
$$E_1 + E_3 = E_{\text{inc}} + E_{\text{refl}}, \tag{5}$$
$$\alpha_1 E_1 + \alpha_3 E_3 = E_{\text{inc}}\frac{\omega}{c} - E_{\text{refl}}\frac{\omega}{c}\frac{1+v/c}{1-v/c}.$$

The system of conditions (5) yields the following expression for the amplitude reflection coefficient

$$R = \frac{E_{\text{refl}}}{E_{\text{inc}}} = \left(\frac{\omega}{c}\frac{1+v/c}{1-v/c} + \alpha_2\right)\left(\frac{\omega}{c} - \alpha_2\right)^{-1}. \tag{6}$$

The amplitude of the reflection coefficient is similar to the Fresnel form. It is determined by the ratio of the difference between the wave vectors of the reflected wave in the unsaturated and saturated regions of the medium to the sum of the wave vectors of the incident wave (in the unsaturated region) and of the reflected wave (in the saturated region). The pure Fresnel reflection corresponds to the case when the Doppler shift is small compared with the initial detuning of the signal frequency from the frequency of the resonance transition:

$$\omega v/c \ll |\omega - \omega_{23}|, \gamma.$$

Then,

$$\alpha_2 = -\frac{\omega}{c}\frac{1+v/c}{1-v/c} + \frac{\pi\omega}{c}\frac{\mu_{23}{}^2 N_0}{\hbar(\omega - \omega_{23} + i\gamma)}$$

and the maximum of the reflection coefficient is attained at the radiation frequency close to the resonance transition frequency:

$$R_{\max} \approx i\pi\mu^2{}_{23}N_0/2\hbar\gamma.$$

If the Doppler frequency shift of the radiation is large, $\omega v/c \gg \gamma$ (this situation is most likely under experimental conditions), the range of frequencies corresponding to the maximum reflection is concentrated in the vicinity of the resonance of the reflected signal with the transition frequency:

$$\omega_{\text{refl}} = \omega\frac{1+v/c}{1-v/c} \approx \omega_{23}.$$

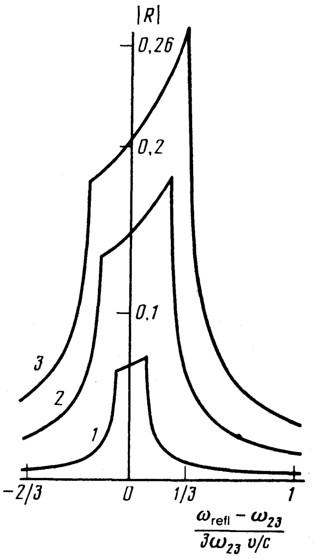

FIG. 2. Modulus of the reflection coefficient of the amplitude of a probe signal $R = E_{ref}/E_{inc}$. The abscissa gives the detuning of the reflected signal from a resonance with the 2–3 transition in the form of a dimensionless combination with the Doppler frequency shift. The parameter $\pi\mu_{23}{}^2N_0(\hbar\omega v/c)^{-1}$ has the following values: 1) $1/30\sqrt{3}$; 2) $1/6\sqrt{3}$; 3) $1/3\sqrt{3}$.

Figure 2 shows how the amplitude of the reflection coefficient depends on the frequency of the incident signal when $\omega v/c \gg \gamma$ and the parameter $\pi\mu_{23}{}^2N_0(\hbar\omega v/c)^{-1}$ has a range of values. The maximum value of the reflection coefficient can be estimated from the expression

$$R_{max} \approx \frac{1}{2}\left(\frac{\pi\mu_{23}{}^2N_0}{\hbar\omega v/c}\right)^{1/2},$$

and the frequency interval where the reflection is significant is given by

$$\Delta\omega \approx \left(\omega \frac{v}{c} \frac{\pi\mu_{23}{}^2N_0}{\hbar}\right)^{1/2}.$$

It thus follows that the spatial and temporal dispersion associated with the change in the frequency due to reflection from a moving resonant saturation region shifts the frequency of the maximum of the reflection coefficient and deforms the profile of the frequency dependence of the reflection coefficient.

2. REFLECTION FROM A SELF-INDUCED TRANSPARENCY PULSE

We shall now consider the reflection of a small signal from a 2π pulse traveling in a three-level medium at a velocity much less than that of light. In the case of coherent propagation the pulse length must be less than the relaxation time of the medium. The polarization of the medium is then found from the Schrödinger equation for the amplitudes of the populations of the levels and is a function of the amplitudes and phases of the fields at all the earlier times i.e., it has a phase memory.

We shall assume that the carrier frequency of the 2π pulse is identical with the frequency ω_{23} of the 1–2 transition and that the frequency of the probe pulse is close to the frequency ω_{23} of the 2–3 transition. The amplitude of the population of the second level is determined by the parameters of the 2π pulse:

$$a_2 = \mathrm{ch}^{-1}\left(\frac{t - x/v}{\tau}\right),$$

where the velocity v and the pulse length τ are related by[8]

$$c/v - 1 = 2\pi N\omega_{12}\mu_{12}{}^2\tau^2/\hbar.$$

If we assume that the conditions for the resonance approximation are satisfied by the incident and reflected signals

$$|\omega - \omega_{23}|,\ \omega_{23}v/c \ll ||\omega_{12}| - |\omega_{23}||,$$

we find that the Maxwell and material equations yield (see, for example, Ref. 6)

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \pm i\frac{4\pi}{c^2}\frac{\omega_{23}{}^2\mu_{23}{}^2N}{\hbar}\exp(-i\omega_{23}t)\,\mathrm{ch}^{-1}\left(\frac{t - x/v}{\tau}\right) \times \int_{-\infty}^{t} E\exp(i\omega_{23}t')\,\mathrm{ch}^{-1}\left(\frac{t' - x/v}{\tau}\right)dt'. \tag{7}$$

The upper sign corresponds to the case of a Λ configuration for the interaction of the fields with the medium, where the energy of the level 2 is greater than the energy of the level 3, and the lower sign corresponds to the cascade interaction. In a moving system of coordinates

$$\xi = \frac{t - x/v}{\tau}, \quad z = \frac{x}{v\tau}$$

Eq. (7) becomes

$$\left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2 E}{\partial \xi^2} - 2\frac{\partial^2 E}{\partial \xi\,\partial z} + \frac{\partial^2 E}{\partial z^2} = \pm i\frac{4\pi N\mu_{23}{}^2\omega_{23}{}^2v^2\tau^3}{\hbar c^2} \times \frac{\exp(-i\omega_{23}\tau\xi)}{\mathrm{ch}\,\xi}\int_{-\infty}^{\xi}\frac{E\exp(i\omega_{23}\tau\xi')}{\mathrm{ch}\,\xi'}d\xi'. \tag{8}$$

We shall now separate the rapidly oscillating dependence of the field E on ξ and z:

$$E(\xi, z) = \varepsilon(\xi)\exp(-i\omega_{23}\tau\xi - iqz). \tag{9}$$

The parameter q depends on the frequency of the probe signal and on the boundary conditions at $\xi \to \pm\infty$. The asymptotic form of the solutions of Eq. (8) in the limit $\xi \to \pm\infty$ is given by

$$E \propto \exp[-i\omega_0(1 \pm v/c)(t \mp x/c)],$$

where the upper sign corresponds to a wave traveling in the direction of motion of the 2π pulse and the lower sign corresponds to a wave traveling in the opposite direction; ω_0 is the signal frequency in the system of coordinates of a self-induced transparency pulse.

We can distinguish two different cases in the cascade and Λ interaction schemes: reflection in the case of parallel (concurrent) and antiparallel (head-on) interactions with a 2π pulse. If a probe wave catches up with this pulse, the frequencies of the incident ($\xi \to +\infty$) and transmitted ($\xi \to -\infty$) signals are equal $\omega = \omega_0(1 + v/c)$, and the frequency of the reflected ($\xi \to +\infty$) signal is given by $\omega_{refl} = \omega_0(1 - v/c)$. The parameter q is then $q = \omega\tau(1 - v/c)$. In the case of antiparallel incidence on a 2π pulse the frequencies of the incident ($\xi\to-\infty$) and transmitted ($\xi\to+\infty$) signals are $\omega+\omega_0(1-v/c)$, and the frequency of the reflected ($\xi\to-\infty$) signal is $\omega_{0Tp}=\omega_0(1+v/c)$. The parameter q for the antiparallel interaction case is $q=\omega\tau(1+v/c)$.

Substitution of Eq. (9) reduces Eq. (8) to

$$\left\{\frac{\partial}{\partial\xi}-i\left[\omega_{23}\tau-\omega_0\tau\left(1-\frac{v}{c}\right)\right]\right\}$$
$$\times\left\{\frac{\partial}{\partial\xi}-i\left[\omega_{23}\tau-\omega_0\tau\left(1+\frac{v}{c}\right)\right]\right\}\varepsilon$$
$$=\pm i\frac{2(v/c)\,\omega_{23}\tau\varkappa^2}{\operatorname{ch}\xi}\int_{-\infty}^{\xi}\frac{\varepsilon(\xi')}{\operatorname{ch}\xi'}d\xi'. \qquad (10)$$

Here, $\varkappa^2=\mu_{23}{}^2\omega_{23}/\mu_{12}{}^2\omega_{12}$ and the relationship between the velocity and length of a 2π pulse is utilized. Since the problem of reflection of a probe signal by a 2π pulse in a three-level medium is mathematically similar in all possible cases, we shall carry out calculations for one of them and give only the final results for the others.

We shall consider the cascade interaction scheme in the case of parallel propagation of a small signal. This corresponds to the lower sign in Eq. (10) and the frequencies of the incident ($\xi\to+\infty$) and reflected ($\xi\to+\infty$) waves are then $\omega_0(1+v/c)$ and $\omega_0(1-v/c)$, respectively. A complete analysis was carried out by numerical integration of Eq. (10) and analytical results were obtained in those special cases when the initial detuning from a resonance is large, $|\omega_{23}-\omega|\gg\omega_0 v/c$ (Ref. 6), or in the case of a large Doppler shift: $\omega_{23}\tau v/c\gg\max[1,\varkappa]$. Since the reflection in the $|\omega_{23}-\omega|\gg\omega v/c$ case is weak, it is most interesting to consider the reflection near resonances of the incident or reflected signals with the transition frequency ω_{23}. We shall assume that the frequency of the reflected signal is close to ω_{23}:

$$|\omega_0(1-v/c)-\omega_{23}|\tau=|\delta_{\text{refl}}|\ll 2\Delta \text{ and } \Delta=\omega_{23}\tau v/c>\max\,[1,\,\varkappa].$$

Under these conditions the field of the reflected signal can be determined using perturbation theory. We shall find first the transmission of radiation without allowance for the reflection and calculate the reflected signal in the first order of perturbation theory. The equation for the incident and transmitted waves follows from Eq. (10):

$$\left(\frac{\partial}{\partial\xi}+i\delta\right)\varepsilon_0=\frac{\varkappa^2}{\operatorname{ch}\xi}\int_{-\infty}^{\xi}\frac{\varepsilon_0}{\operatorname{ch}\xi'}d\xi', \qquad (11)$$

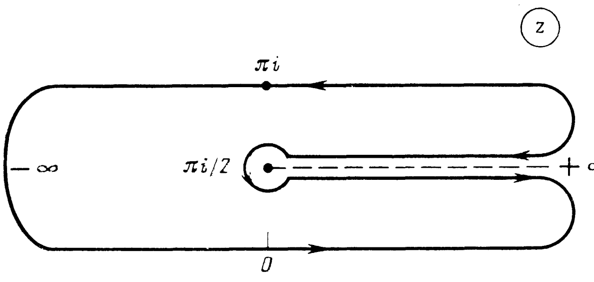

FIG. 3. The integral along the edges of a cut (contour C) is equal to that along the $z=\pi i$ axis and the real axis.

where $\delta=\delta_{\text{refl}}+2\Delta$. The substitution of

$$u_0=\int_{-\infty}^{\xi}\frac{\varepsilon_0}{\operatorname{ch}\xi'}d\xi'$$

reduces this equation to the hypergeometric form, which gives the solution

$$\varepsilon_0=\exp(-i\delta\xi)F(\varkappa i,\,-\varkappa i,\,(1-i\delta)/2,\,(1+\operatorname{th}\xi)/2).$$

An analysis of the asymptotes of the solution ε_0 in the case ($\xi\to\pm\infty$) yields the following expression for the transmission coefficient

$$T=\left|\frac{\varepsilon_0(-\infty)}{\varepsilon_0(+\infty)}\right|^2=\frac{\operatorname{ch}^2(\pi\delta/2)}{\operatorname{ch}^2(\pi\delta/2)+\operatorname{sh}^2\varkappa\pi}. \qquad (12)$$

Equations (10) and (11) then yield an equation describing the reflected signal ε ($|\varepsilon|\ll\varepsilon_0|$):

$$\left(\frac{\partial}{\partial\xi}+i\delta_{\text{refl}}\right)\varepsilon=-\frac{\varkappa^2}{\operatorname{ch}\xi}\int_{-\infty}^{\xi}\frac{\varepsilon}{\operatorname{ch}\xi'}d\xi'-\frac{1}{2i\Delta}\left(\frac{\partial}{\partial\xi}+i\delta\right)^2\varepsilon_0. \qquad (13)$$

The solution of this equation in the limit ($\xi\to+\infty$) gives the amplitude of the reflected wave:

$$\varepsilon(+\infty)=\frac{\varkappa^2 i}{2\Delta(1-i\delta)}\int_{-\infty}^{\infty}d\xi\,e^{-2i\xi\Delta}F\left(\varkappa,-\varkappa,\frac{1-i\delta_{0Tp}}{2},\frac{1-\operatorname{th}\xi}{2}\right)$$
$$\times\frac{\partial}{\partial\xi}\left[F\left(1+\varkappa i,1-\varkappa i,\frac{3-i\delta}{2},\frac{1+\operatorname{th}\xi}{2}\right)\operatorname{ch}^{-2}\xi\right]. \qquad (14)$$

We can solve Eq. (14) by considering an integral over the contour shown in Fig. 3. If the amplitude of the reflection coefficient is described by $R=\varepsilon(+\infty)/\varepsilon_0(+\infty)$, we obtain

$$R=\frac{\varkappa^2 i}{2\Delta(1-i\delta)}\frac{e^{\pi\Delta}}{e^{2\pi\Delta}-1}\int_C dz\,e^{-2i\Delta z}F\left(\varkappa,-\varkappa,\frac{1-i\delta_{\text{refl}}}{2},\frac{1-\operatorname{cth}z}{2}\right)\frac{\partial}{\partial z}\left[F\left(1+\varkappa i,1-\varkappa i,\frac{3-i\delta}{2},\frac{1+\operatorname{cth}z}{2}\right)\operatorname{sh}^{-2}z\right],$$

where the integration is carried out along the upper and lower edges of the cut $z=\pi i/2$ (Fig. 3) going round the pole in the positive direction. Assuming that $\Delta=\omega_{23}\tau v/c\gg 1$ and using the expansion of the hypergeometric functions about the point at infinity,[9] we finally obtain

$$|R|=2^{1/2}\pi\varkappa\Delta^{\varkappa}\exp\left(-\pi\Delta+\frac{\pi\varkappa}{2}\right)\left|(1-\varkappa i)\Gamma\left(\frac{1-i\delta_{\text{refl}}}{2}\right)\Gamma(2\varkappa)\right/$$
$$\left.\Gamma(2+\varkappa-\varkappa i)\Gamma(\varkappa)\Gamma\left(\frac{1-i\delta_{\text{refl}}}{2}+\varkappa\right)\right|. \qquad (15)$$

It follows from Eq. (15) that the maximum of $|R|$ considered as a function of δ_{refl} is obtained for $\delta_{\text{refl}}=0$, i.e., the reflected signal is in resonance with the transition frequency. A similar analysis shows that in the vicinity of a resonance with the incident signal (when the frequency of the incident wave is close to the frequency ω_{23}) no singularity appears in the reflection coefficient for this interaction scheme. Figure 4 shows the results of a numerical integration of Eq. (8), when the reflected signal can be large and perturbation theory cannot be applied. The maximum of the reflection coeffi-

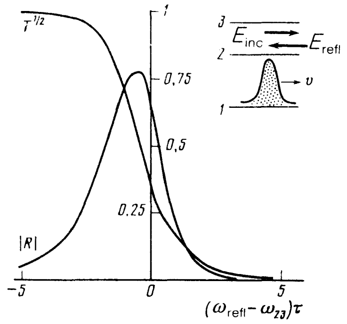

FIG. 4. Reflection from a parallel (concurrent) bleaching pulse and transmission of the amplitude of a probe signal across this pulse; $\Delta = 10$, $\varkappa = 8$.

cient is attained, as in the analytical treatment, when the frequency of the reflected signal coincides with the frequency of the 2–3 transition.

The frequency dependence of the reflection coefficient of the signal is completely different when the interaction is antiparallel in the cascade scheme. In this case a strong reflection occurs when the frequency of the incident signal is close to the transition frequency. The expressions for the transmission coefficient T and for the amplitude of the reflection coefficient R are then

$$T = |\varepsilon_0(+\infty)/\varepsilon_0(-\infty)|^2 = 1 - \frac{\sin^2 \varkappa\pi}{\mathrm{ch}^2(\pi\delta/2)},$$

$$|R| = \frac{\varepsilon(-\infty)}{\varepsilon_0(-\infty)} = 2\pi\Delta^{\varkappa} e^{-\pi\Delta} \left| \Gamma\left(\frac{1-i\delta}{2}\right)\Gamma(2\varkappa) \Big/ \Gamma^2(\varkappa)\Gamma\left(\frac{1-i\delta}{2}+\varkappa\right)\right|. \qquad (16)$$

When $\varkappa$ is an integer, the transmission coefficient could be unity and the reflection coefficient exhibits a resonance as a function of the frequency detuning (the square of the reflection coefficient can be expressed in terms of the product $\varkappa$ of Lorentzian profiles). The reflection maximum occurs when the frequency of the opposite signal coincides with the frequency of the 2–3 transition. Figure 5 shows how $|R|$ and $T^{1/2}$ depend on the frequency of the incident signal for two values of $\varkappa$ at which the conditions for application of perturbation theory are not satisfied. Such dependences are in qualitative agreement with the expressions in Eq. (16).

In contrast to the cascade interaction of fields, if we have a Λ configuration of the levels, amplification effects play an important role in reflection. This is due to the fact that in the region of a 2π pulse the upper level is populated and the medium amplifies a small signal. It is known[10,11] that if $\varkappa > 1/2$, then the Stokes signal is trapped in the inversion region and a strong transfer of energy from the 2π pulse to the Stokes wave takes place. Nevertheless, in the case of short propagation paths, when the time available is insufficient for the transformation into a Stokes wave, we can consider the problem of reflection of a probe wave at a frequency close to ω_{23}.

In the case of parallel propagation of a small signal the transmission coefficient in the Λ configuration has resonance peaks for specific ratios of the oscillator strengths of the resonant transitions (if $\varkappa = n + 1/2$, where n is an integer):

$$T = \mathrm{ch}^2\frac{\pi\delta}{2} \Big/ \left(\mathrm{ch}^2\frac{\pi\delta}{2} - \sin^2\varkappa\pi\right).$$

The dependence of the reflection coefficient on the signal frequency then has two peaks at the resonance frequencies of the incident and reflected signals. Near resonances these dependences are as follows:

$$|R(\omega_{\mathrm{refl}} \approx \omega_{23})| = \frac{\pi\Delta^{\varkappa}\varkappa(1+\varkappa)\exp(^3/_2\pi\varkappa - \Delta\pi)}{|\Gamma(2+\varkappa-\varkappa i)|\,\mathrm{ch}^{1/2}(\pi\delta_{\mathrm{refl}}/2)} \times \left|\Gamma\left(\frac{1-i\delta}{2}\right)\Gamma(2\varkappa) \Big/ \Gamma(\varkappa)\Gamma\left(\frac{1-i\delta}{2}+\varkappa\right)\right|,$$

$$|R(\omega \approx \omega_{23})| = T(\varkappa, \delta)$$

$$2\pi\Delta^{\varkappa}e^{-\pi\Delta}\left|\Gamma\left(\frac{1-i\delta}{2}\right)\Gamma(2\varkappa) \Big/ \Gamma^2(\varkappa)\Gamma\left(\frac{1-i\delta}{2}+\varkappa\right)\right|. \qquad (17)$$

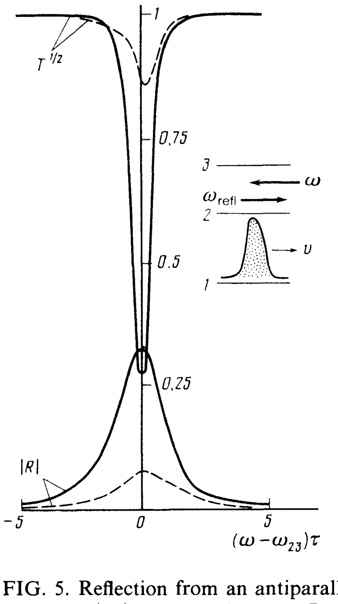

FIG. 5. Reflection from an antiparallel pulse ($\Delta = 10$). The reflection coefficient $|R|$ corresponds to $\varkappa = 7$ and $\varkappa = 7.5$ in the case of the dashed and continuous curves; the reverse is true of the transmission coefficient $T^{1/2}$.

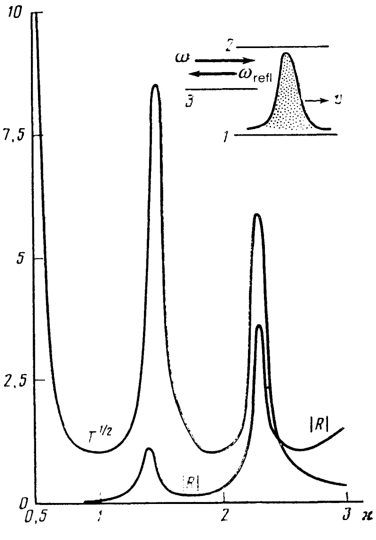

FIG. 6. Coefficients of reflection and transmission from a parallel (concurrent) pulse in an amplifying medium plotted as a function of the oscillator strengths of the transitions; $\Delta = 3$, $(\omega - \omega_{23})\tau = 3$.

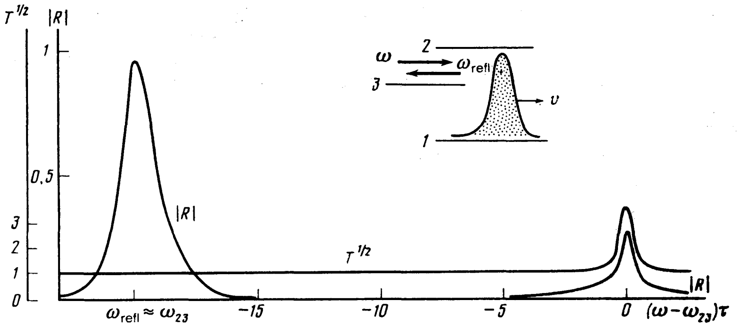

FIG. 7. Reflection from a parallel (concurrent) pulse in an amplifying medium; $\Delta = 10$, $\varkappa = 7$.

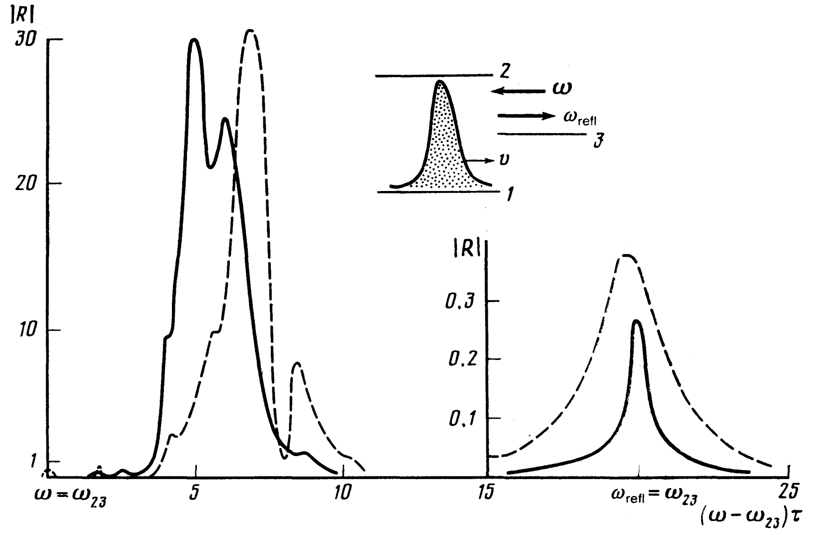

FIG. 8. Reflection coefficient $|R|$ for the antiparallel incidence on a pulse in an amplifying medium ($\Delta = 10$): the continuous curve corresponds to $\varkappa = 7$ and the dashed curve to $\varkappa = 7.5$.

Figures 6 and 7 give the dependences of the reflection and transmission coefficients on the ratio of the oscillator strengths and on the frequency of the incident signal in the case when the conditions for the application of perturbation theory are not obeyed. The resonance-life dependence on the ratio of the oscillator strengths is manifested by both the reflection and transmission coefficients. When $\varkappa$ is small ($\varkappa \sim 1$), the amplitudes of the reflection coefficient peaks are of the same order of magnitude at both resonance frequencies. As $\varkappa$ increases, the peak corresponding to the resonance of the reflected signal with ω_{23} rises faster.

Coherent interaction effects arise particularly strikingly in the case of antiparallel reflection of a signal in the Λ configuration (Fig. 8). The results obtained by numerical integration demonstrate that in addition to a peak $\omega_{\text{refl}} = \omega_{23}$, the frequency dependence of the reflection coefficient exhibits a whole series of additional extrema whose number is proportional to $\varkappa$. It was shown in Ref. 11 that when a 2π pulse propagates in a medium with the Λ configuration the radiation is transformed into a Stokes signal and the number of unstable modes is governed by the integral part of the parameter $\varkappa - 1/2$. Each mode represents the field of one of the coupled states of the Stokes signal interacting with the 2π pump pulse. These modes differ in the number of zeros and, consequently, in frequency. The resonant peaks of the reflection coefficient appear when the frequency of the incident signal is close to the frequency of an unstable mode. If $\varkappa = 7$, there are six unstable modes and the dependence $|R(\omega)|$ exhibits six peaks. Similarly, if $\varkappa = 7.5$, there are seven unstable modes and seven peaks in the frequency dependence of the reflection coefficient. It should also be noted that in the Λ configuration scheme the reflection coefficient can reach values greater than unity.

It follows from the above analysis that reflection of a small signal from a moving polarization region induced by a laser pulse is influenced significantly by the spatial and temporal dispersion of the resonant medium. The amplitude of the reflected signals depends on the parameters of the medium and on the direction of propagation of the interacting fields. The media most suitable for experimental observation of these effects are proposed in Ref. 12, where an analysis is made of the feasibility of significantly reducing the pulse length in the case of coherent stimulated Raman scattering in a resonant three-level medium.

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