Interference contribution of the current correlator. The fluctuation spectrum

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The long-time asymptotic behavior of the unequal-time current correlator in disordered conductors is considered. It is shown that scattering of electrons by systems with a well-defined discrete spectrum (defects with an internal degree of freedom, tunneling states of atoms, and optical phonons) leads to oscillations of the current correlator in time. In this case the other processes determining the relaxation of the electron phase do not lead to damping of the oscillations in time, but only affect their amplitude.

1. INTRODUCTION

Recently, the question of fluctuations of the electrical conductance of small (but macroscopic) samples has been discussed intensively. In Refs. 1–3 and other papers it has been shown that the conductivity of samples of identical chemical composition and geometry fluctuates strongly from sample to sample. The appearance of these fluctuations is due to quantum effects—in particular, to the interference that arises between an electron state and the time-reversed state as a consequence of the fact that the two states are scattered by the same impurities.

We shall be interested in another aspect of this problem: We shall consider large samples and shall study the interference contribution to the correlator of the current densities $\mathbf{j}(\mathbf{R}, t)$ at different times:

$$s_{ik}(t)=\langle j_i(\mathbf{R}, t)j_k(0, 0)\rangle-\langle j_i\rangle^2\delta_{ik}. \tag{1.1}$$

The angular brackets here denote averaging over the configurations of the scatterers, The time dependence (of the correlator s) of interest to us arises from the presence in the ensemble of scatterers of intrinsic degrees of freedom that can exchange energy with the electrons.

The problem of the quantum contribution to the low-frequency noise was considered recently in Refs. 4 and 5. In the latter the quantum contribution to the shot noise in the case of purely elastic scattering of electrons by impurities was considered. The appearance of the time dependence of the current correlator in such a situation is due to the fact that one and the same electron can pass along segments of trajectories with self-intersection at different times. In the framework of the calculations performed in Refs. 4 and 5 the above-indicated contribution is associated with matrix elements of the current-density operator $\hat{\mathbf{j}}$ that are off-diagonal in the electron states.

In the case of interest to us the correlation is due to the scattering of two different electrons by the same scatterers (see Fig. 1a). We shall see that in a number of important cases the contribution of inelastic scattering to the time-dependent part of the current correlator turns out to be dominant. In particular, it attenuates slowly in time and can be distinguished experimentally by its time dependence.

We shall consider a Fermi gas of electrons interacting with impurities and inelastic scatterers. The latter will be in the form of both acoustic phonons and scatterers with a well-defined discrete spectrum (defects with an internal degree of freedom, tunneling states of atoms, or optical phonons with small dispersion and damping). We shall see that the presence of scatterers of the second kind leads to a new qualitative effect in the fluctuations—to oscillations of s in time with a characteristic frequency ω_0, where $\hbar\omega_0$ is the energy of excitation of the scatterer. Here it is important that scatterers of the first type, which determine the phase-relaxation time τ_φ, do not lead to damping of the oscillations but only affect their amplitude. The damping of the oscillations is determined entirely by the intrinsic damping γ of the scatterers or (in the case of optical phonons) by their dispersion.

In the next section we shall elucidate the qualitative physics of this phenomenon, and in Sec. 3 we shall give the quantitative calculation.

2. QUALITATIVE PICTURE[1)]

In the correlator s(t) there appears the product of the probability $W^{(1)}$ of a transition of the first electron from the point r_1 to the point r_2 in a certain time t_0, with the probability $W^{(2)}$ of an analogous transition of a second electron in the time t_0+t. Each of these probabilities can be written in the form

$$W^{(k)}=\sum_{i,j}A_i^{(k)}A_j^{*(k)},$$

where $A_i^{(k)}$ is the amplitude of the transition of the k th electron along the ith trajectory.

The product $W^{(1)}W^{(2)}$ should be averaged over the configurations of the impurities and also over the ensemble of the inelastic scatterers, and then the product of the average probabilities should be subtracted from the result obtained.

We shall assume that the inelastic scatterers give rise to a random potential field $U(\mathbf{r},t)$ acting on the electrons; it will be necessary to average the answer over all realizations of this field. For the following it will be important that this field is a rapid function of the spatial coordinate $\mathbf{r}$ and a slow function of the time t. The first property is a consequence of the fact that in a collision the momentum transfer δp is of the order of the initial electron momentum p, which is much greater than the mean free path l:

$$pl\gg\hbar. \tag{2.1}$$

The second property is a consequence of the condition that the scattering be quasi-elastic:

$$\delta\varepsilon=\hbar\omega_0\ll\varepsilon;\ T, \tag{2.2}$$

where ε is the electron energy and T is the temperature.

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The product $W^{(1)}W^{(2)}$ is a sum over trajectories of products of type $A_i^{(1)}A_j^{*(1)}A_l^{(2)}A_m^{*(2)}$. Most of these terms disappear in the averaging, since for each of the trajectories the phase advance of the quantity A_i is not correlated with the advance of the phases on the other trajectories. The only exception is provided by those combinations of trajectories on which the advances of the phase cancel in the principal approximation—namely, those closed segments of trajectory that pass in mutually opposite directions. Here there are two possibilities. The first ($i=j, l=m$) appears in the product of the average probabilities and is unimportant for the correlation. The second ($i=m, j=l$) corresponds to the passage by different electrons along the closed segment of trajectory in opposite directions. In fact, if the energies of the first and second electrons are equal ($\varepsilon_1=\varepsilon_2$) and the field does not depend on t, the phase advances for the states 1 and 2 should be equal and interference of these states should appear. This interference is manifested in the correlator s to an extent determined by the smallness of the energy difference $|\varepsilon_1-\varepsilon_2|\lesssim\hbar/\tau_\varphi$ and by the slowness of the time dependence of U. It is this interference contribution that we shall consider.

We now take into account the time dependence of the potential $U(\mathbf{r},t)$. Let the total time of the motion along the closed part of the trajectory be t_0, and let the coordinate of the intersection point be $\mathbf{r}_0$. We parametrize the motion along the trajectory by the time t_1 required to reach the point $\mathbf{r}$ of the trajectory in the counterclockwise direction if at the time $t=0$ the electron wave emerged from the point $\mathbf{r}_0$. Then the contribution of the field U to the phase advance in the passage of the first electron along the trajectory segment of interest to us is

$$\hbar^{-1}\int_0^{t_0} dt_1\, U[\mathbf{r}(t_1), t_1].$$

Moving in the opposite direction and starting at time t from the point $\mathbf{r}_0$, the second electron arrives at the point $\mathbf{r}$ at time t_0+t-t_1. As a result the phase difference will be equal to

$$\Delta\varphi(U)=\hbar^{-1}(\varepsilon_1-\varepsilon_2)t_0+\Delta_1\varphi(U), \tag{2.3}$$

where the first term is the trivial phase advance due to the difference in the energies of the electrons, and

$$\Delta_1\varphi(U)=\hbar^{-1}\int_0^{t_0} dt_1\{U[\mathbf{r}(t_1), t+t_0-t_1]-U[\mathbf{r}(t_1), t_1]\}. \tag{2.4}$$

Next it is necessary to average $\exp[i\Delta_1\varphi(U)]$ over all realizations of the random potential U. Since in quasi-elastic scattering the transfer of energy occurs in small portions and is equally probable in both directions, we shall assume that the distribution of $\Delta_1\varphi(U)$ is Gaussian,[2)] i.e.,

$$\langle\exp[i\Delta_1\varphi(U)]\rangle_U=\exp\{-\tfrac{1}{2}\langle[\Delta_1\varphi(U)]^2\rangle_U\}. \tag{2.5}$$

Thus, the problem reduces to the averaging of products of the type

$$U[\mathbf{r}(t_1),\ T_i(t_1)]U[\mathbf{r}(t_2),\ T_j(t_2)] \tag{2.6}$$

($T_i(t_1)=t_1; T_j(t_1)=t+t_0-t_1$), appearing in the expression for $(\Delta_1\varphi)^2$. We now take into account that the potential $U(\mathbf{r}, t)$ depends very rapidly on the first argument. By virtue of this, the potentials at different points of the trajectory are practically uncorrelated. Therefore, the average of products of the type (2.6) is proportional to $\delta[\mathbf{r}(t_1)-\mathbf{r}(t_2)]$, which, in turn, is proportional to $\delta(t_1-t_2)$. As a result we have

$$\langle[\Delta_1\varphi(U)]^2\rangle_U=\beta\hbar^{-2}\int_0^{t_0} dt_1\{\langle U^2[\mathbf{r}(t_1), t_0+t-t_1]\rangle_U$$
$$-2\langle U[\mathbf{r}(t_1), t_0+t-t_1]U[t_1, \mathbf{r}(t_1)]\rangle_U+\langle U^2[\mathbf{r}(t_1), t_1]\rangle_U\}, \tag{2.7}$$

where β is a dimensional constant whose value we shall establish below.

In order to illustrate the phenomenon, we shall be interested in large times t,substantially exceeding the characteristic phase-relaxation time τ_φ due to other channels of phase relaxation. Here we should assume that the time t_0 is much shorter than τ_φ; otherwise, interference cannot be manifested. We shall also assume that the characteristic frequency ω_0 of the scatterer, and also the damping γ of the scatterer, is much smaller than τ_φ^{-1}. Then in the time arguments of U we can neglect that terms t_1 and t_0-t_1 in comparison with t. With these assumptions the two equal-time terms in (2.7) give the same contribution, and the unequal-time term is proportional to $\cos\omega_0 t\cdot\exp[-\gamma|t|]$. It remains to estimate the quantity $\hbar^{-2}\beta\langle U^2(\mathbf{r}, t)\rangle_U$. It does not depend on the time (since in the averaging we should sample all possible configurations of the fields $U(t)$) and, from the meaning of the derivation, is proportional to the probability of scattering of the electron by one quasi-elastic scatterer; if we take into account that it has the dimensions of inverse time, it becomes clear that in order of magnitude it is τ_0^{-1}. As a result, the average (2.5) can be represented in the form

$$\langle\exp[i\Delta_1\varphi(U)]\rangle_U=\exp\{-(t/\tau_0)[1-\cos\omega_0 t\exp(-\gamma|t|)]\}, \tag{2.8}$$

where τ_0 is the mean free time of the electron between scat-

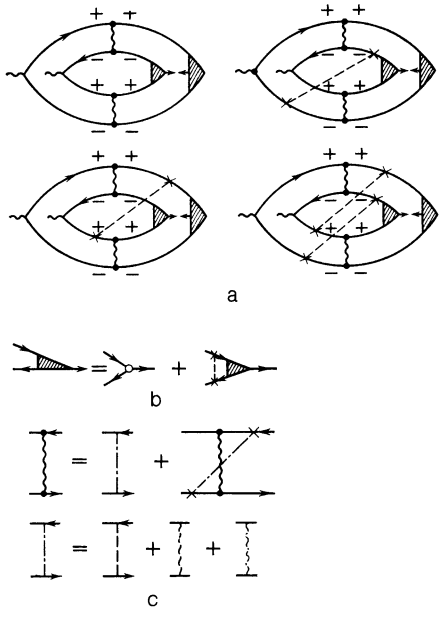

FIG. 1. The signs + and − denote the Green functions G_R and G_A, respectively.

terings by defects possessing a discrete spectrum. The quantity

$$\nu_0(t)=\tau_0^{-1}[1-\exp(-\gamma|t|)\cos\omega_0 t] \tag{2.9}$$

is the contribution of these scatterers to the frequency of the phase relaxation. We note that ν_0 oscillates as a function of the time of observation of the correlation. The physical reason for the oscillations of this quantity is that in the presence of scatterers with a fixed energy their action on the electrons at different times is correlated. The source of the time dependence of the correlator is the transitions of scatterers from one energy state to another. In fact, if the electrons of interest to us are scattered at different times by scatterers in the same state, the interference contribution is not small. This situation obtains over time intervals that are multiples of the period $2\pi/\omega_0$ of the oscillations of the scatterer. The electrons here play the role only of an indicator of how much the states of the scatterers have changed in the time t of observation of the correlator. For phenomena of this type the authors of Ref. 6 introduced what is in our view an appropriate name—"correlation via the medium."

In order to calculate the current-density fluctuations, it is necessary to sum the contributions of all the electrons. This is done in the usual way (see, e.g., Refs. 1 and 7), and this calculation will be performed in the next section. Here we shall give the answer for the two-dimensional case in the conditions that were discussed above:

$$\tau,\ \hbar/T\ll\tau_\varphi\ll t,\ \omega_0^{-1}.$$

It has the form

$$s_{\omega\omega}(t)=\frac{3}{4}\frac{e^4E^2}{(2\pi)^4\hbar^2}[\alpha(t)-\alpha(\infty)], \tag{2.10}$$

where E is the electric-field intensity, and

$$\alpha(t)=\frac{\hbar\nu(t)}{T}\ln\frac{1}{\nu(t)\tau},\quad \nu(t)=\frac{1}{\tau_\varphi}+\nu_0(t). \tag{2.11}$$

In the case when there are scatterers with different frequencies ω_s, or one scatterer has several energy states, in the expression for $\nu_0(t)$ (2.9) the sum

$$\sum_s \tau_s^{-1}(1-\exp(-\gamma_s|t|)\cos\omega_s t)$$

should appear. Thus, we can judge the nature of the spectrum of the low-energy excitations of the conductor from the spectrum of the current fluctuations.

Calculations of the fluctuations of the total current for a given spatially uniform distribution of the electric field under the same conditions leads to the estimate

$$\Sigma\approx\frac{e^4E^2L_T^2}{\hbar^2}\ln\frac{\nu(\infty)}{\nu(t)},\quad L_T^2=\frac{D\hbar}{T}. \tag{2.12}$$

The processes considered in Ref. 5 give a contribution, proportional to E^2, of order

$$\frac{e^4E^2L_T^2}{\hbar^2}\begin{cases}\ln\tau_\varphi/t, & t\ll\tau_\varphi,\\ \tau_\varphi/t, & t\gg\tau_\varphi.\end{cases} \tag{2.13}$$

Thus, the contribution considered by us turns out to be more important, at least at large observation times.

We now compare the contribution of interest to us with that considered in Ref. 6, which is due to fluctuations of the phonon degrees of freedom. The characteristic magnitude of the ratio of these contributions is of order $(\hbar/pl)^2N_c$, where N_c is the number of fluctuating phonon modes. In the case when the phonons are equilibrium phonons, and the temperature fluctuates, $N_c\sim(T/\hbar s)^3V_0$ (s is the sound velocity and V_0 is the volume of the system), and for samples that are not too small the contribution of interest to us is the the much greater. But if the phonons are not in equilibrium, then, as shown in Ref. 6, the characteristic number of fluctuating modes can be substantially smaller. We note that the spectrum of the fluctuations in this case[6] has dispersion at the characteristic frequency of the nonequilibrium phonons.

3. CALCULATION OF THE CURRENT FLUCTUATIONS

For the calculation of $s(t)$ it is necessary to take into account all the two-loop graphs in which the electron loops are joined together by two cooperons (i.e., by sums of fan diagrams).[1] Since we shall be interested in the single-point but unequal-time correlator s, the calculation must be carried out for finite values of the wave vector $\mathbf{Q}$ corresponding to the Fourier transformation in the coordinate difference $\mathbf{R}_1-\mathbf{R}_2$ (in the region $Ql\ll1$, where l is the mean free path of the electrons). A check on the correctness of the calculation should be provided by fulfillmentof the identity

$$Q_iQ_ks_{ik}(\mathbf{Q},\ \Omega_c\to0)=0 \tag{3.1}$$

($s(\mathbf{Q},\Omega_c)$ is the Fourier transform of $s(R,t)$), which is in fact a form of the law of conservation of the number of electrons. One can verify that for (3.1) to be fulfilled it is necessary to take into account a sufficiently large number of graphs and to perform a subtraction that we shall discuss in the Appendix.

The graphs depicted in Fig. 1a automatically satisfy the identity (3.1), and therefore, for simplicity of the account, we shall first discuss their contribution to the correlator s and then analyze the diagrams that require subtractions. The left vertices of these graphs correspond to operators of the interaction with the electric field (we use the Lorentz gauge and $\mathbf{E}=-c^{-1}\dot{\mathbf{A}}$). The right vertices correspond to current-density operators, and the so-called correlation momentum $\mathbf{Q}$ and frequency Ω_c (corresponding to the Fourier transformations with respect to $\mathbf{R}_1-\mathbf{R}_2$ and t) flow across from the external electron line to the internal line; it is assumed that all the phenomena of interest to us are due to the region Ql, $\Omega_c\tau\ll1$. Because this condition is fulfilled it turns out to be necessary, for finite values of Ω_c and Q, to take into account the ladder graphs in the right vertices (Fig. 1b). As a result, the last three graphs give, in order of magnitude, the same contribution as the first graph. As usual, we shall calculate s for a finite frequency Ω of the external field, and then let $\Omega\to0$. For definiteness we shall be interested in times $t\gg\tau_\varepsilon$, where τ_ε is the energy-relaxation time. In this case only the fan graphs are important, and graphs of the diffusion type (see Ref. 1) can be disregarded. In addition, we shall assume that the strongest scattering mechanism is scattering by elastic impurities. It is assumed that besides this scattering there is scattering by acoustic phonons and scattererswith a fixed frequency ω_0 ($\hbar\omega_0\ll T$). For the corresponding relaxation times in respect of these processes the inequalities

$$\tau_{im}\ll\tau_{ac},\ \tau_0,\ \omega_0^{-1} \tag{3.2}$$

are fulfilled. We note that smallness of the product $\tau_{im}\omega_0$ is required only for representation of the results in the simplest analytical form. The oscillations should exist for an arbitrary magnitude of this parameter.

The wavy lines on the graphs denote "cooperons," in the calculation of which we should take into account all three scattering mechanisms—impurity scattering (the dashed lines), scattering by acoustic phonons (the wavy dashed line), and scattering by quasi-elastic scatterers (the wavy dashed-dotted line) (Fig. 1c).

The calculation of the graphs of Fig. 1a by means of the Matsubara technique leads to the expression

$$s_{ik}^{(1)}(\mathbf{Q},\Omega_c)=\frac{(2e^2D\tau)^2}{(2\pi)^3}\int d\varepsilon_1\,d\varepsilon_2\frac{\partial n_0}{\partial\varepsilon_1}\frac{\partial n_0}{\partial\varepsilon_2}(dq)\,d\omega \times I_{ik}(\mathbf{Q},\Omega_c)F(\varepsilon_1-\varepsilon_2,\mathbf{Q}+\mathbf{q},\Omega_c+\omega)F(\varepsilon_2-\varepsilon_1,\omega,\mathbf{q}), \tag{3.3}$$

where n_0 is the Fermi function,

$$I_{ik}=E_iE_k-Q_iE_k(\mathbf{E},\mathbf{Q})/(Q^2+i\Omega_c/D)-E_iQ_k(\mathbf{E},\mathbf{Q})/(Q^2-i\Omega_c/D) +Q_iQ_k(\mathbf{E},\mathbf{Q})^2/(Q^4+\Omega_c^2/D^2), \tag{3.4}$$

D is the diffusion coefficient, and F is the sum of the fan graphs, reduced in the appropriate manner to a dimensionless sum. A calculation analogous to that given in Ref. 8 shows that the quantity F in the time representation satisfies the integral equation

$$(1+\Delta)F(\varepsilon_1-\varepsilon_2,\mathbf{Q},t) =1+\int_0^\infty dt_1\exp(-t_1/\tau)F(\varepsilon_1-\varepsilon_2,\mathbf{Q},t_1-t)\left[\frac{1}{\tau_{ac}}K_{ac}(t+t_1) +\frac{1}{\tau_{im}}+\frac{1}{\tau_0}\exp(-\gamma|t+t_1|)\cos\omega_0(t+t_1)\right], \tag{3.5}$$

where

$$K_{ac}(t)=\frac{\sin 2pst}{pst}-\left[\frac{\sin pst}{pst}\right]^2,$$
$$\tau^{-1}=\tau_{ac}^{-1}+\tau_{im}^{-1}+\tau_0^{-1},\quad \Delta=DQ^2\tau+i(\varepsilon_1-\varepsilon_2)\tau. \tag{3.6}$$

After going over to the time representation and performing the integration over the frequency ω, we can represent the expression (3.3) in the form

$$s_{ik}(\mathbf{Q},t)=\int d\varepsilon_1\,d\varepsilon_2(dq)\int_{-\infty}^{\infty}dt_1F(\varepsilon_2-\varepsilon_1,\mathbf{Q}+\mathbf{q},t-t_1) \times F(\mathbf{q},\varepsilon_1-\varepsilon_2,t_1-t)A_{ik}(t_1), \tag{3.7}$$

where the function $A_{ik}(\varepsilon_1,\varepsilon_2,\mathbf{Q})$ incorporates all the factors except the cooperon factors. The time dependence of the function A_{ik} is determined by the pole characte of the function I_{ik}. However, one can verify that this time dependence has a δ-function character. In fact, the characteristic value of the product DQ^2 in the calculation of the single-point correlator turns out to be of the order of $\tau_\varphi^{-1}\sim\tau_{ac}^{-1}$. But we are interested in the behavior at large times $\tau\gg\tau_\varphi,\tau_\varepsilon$. Therefore, the quantity $i\Omega_c/D$ can be replaced by $i\delta$, i.e., we can take into account only the passage round the pole. As a result, the time dependence of $s(t)$ is determined by the integral over $\mathbf{q}$ of a product of two cooperons. To determine this time dependence we must solve Eq. (3.5). This can be done in the same way as in Ref. 8. (Henceforth, for definiteness, we shall assume the samples to be quasi-two-dimensional.) The solution looks simplest for $t\gg 1/ps$, when the kernel K_{as} can be neglected. (In this situation the acoustic phonons are taken into account only in the self-energy insertions of the Green functions; physically, this means that the correlation arising from the scattering of electrons by acoustic phonons is already damped on this time scale.) If, moreover, we assume that $\omega_0\tau_{im}\ll 1$, then, with allowance for the inequalities (3.2), we obtain

$$F(t,\mathbf{Q},\varepsilon_1-\varepsilon_2)=\int_0^\infty dx\exp\left\{-x\left(\Delta+\frac{\tau}{\tau_{ac}}+\frac{\tau}{\tau_0}\right)+\frac{1}{\omega_0\tau_0} \times[\exp(-\gamma|t|)\sin\omega_0t+\exp(-\gamma|t-\tau x|)\sin(\omega_0\tau x-\omega_0t)]\right\}. \tag{3.8}$$

In the case of extremely low frequencies ω_0,

$$\omega_0\ll\tau_{ac}^{-1},\ \tau_0^{-1}, \tag{3.9}$$

from (3.8) we have

$$F(t,\mathbf{Q},\varepsilon_1-\varepsilon_2)=[\Delta+\tau\nu(t)], \tag{3.10}$$

where

$$\nu(t)=\tau_{ac}^{-1}+\tau_0^{-1}(1-\exp(-\gamma|t|)\cos\omega_0t).$$

It is now necessary to subtract from the unequal-time correlator the contribution of the diagrams in which the inelastic interaction has been taken into account only in the self-energy insertions and the electron loops are connected only by Green functions describing the elastic scattering (such graphs are proportional to $\delta(\omega)$ and are a time average of our expression). Their value can be obtained from the expression (3.10) by letting γ or $t\to\infty$. It is this limit which gives the contribution of the diagrams indicated above. Such processes appear in the product of the average currents, and their contributions should be subtracted in the correlator. But if the condition $\omega_0\tau_0\gg 1$ is fulfilled, the expression (3.8) can be expanded in a series in $1/\omega_0\tau_0$. This regime corresponds to the fact that the electron phase is shifted by an amount of order unity during one collision with a scatterer with a fixed frequency. In this case the answer could be obtained directly by means of perturbation theory and the time-dependent term would have order of smallness $(\omega_0\tau_0)^{-1}$.

We turn now to the graphs depicted in Fig. 2a, in which both cooperons are on the same side of the vertex (to these, of course, we must add the mirror-image graphs in which the cooperons are on the lower line). The necessity of taking these graphs into account was first pointed out in Ref. 9.

Up to now, in the calculation of the current correlator it has been assumed that a contribution to the dissipative current is given only by terms containing a product of an advanced and a retarded Green function in each electron loop (as in the calculation of the current density averaged over the scatterers); i.e., in the exact expression for the current density (before the averaging over the positions of the impurities)

$$\int \frac{d\varepsilon}{2\pi i}\Big\{\Omega\frac{\partial n_0}{\partial\varepsilon}G_R(\varepsilon+\Omega)G_A(\varepsilon)-n(\varepsilon+\Omega)G_A(\varepsilon+\Omega)G_A(\varepsilon) +n(\varepsilon)G_R(\varepsilon+\Omega)G_R(\varepsilon)\Big\} \qquad (3.11)$$

only the first term has been taken into account. (The coordinate arguments, vertices, and all the remaining integrations have been left out for simplicity.) Here it has been assumed that the last two terms, as usual, are cancelled by the diamagnetic current. Our assertion is that in the calculation of the correlator this is not so. If only the first term is taken into account, the identity (3.1) will not be fulfilled (and this, of course, contradicts the law of conservation of electrons). We shall discuss first the difference between the current density averaged over the positions of the impurities, and the product of two densities (3.11) which are subsequently averaged over the positions of the impurities. The first question has been discussed repeatedly in the literature. The answer reduces to the following: In the calculation of any loop, expressions of the type

$$\int d\varepsilon\, d\varepsilon_p\, n_0(\varepsilon+\Omega)G_A(\varepsilon+\Omega,\varepsilon_p)G_A(\varepsilon,\varepsilon_p)$$

arise. In the leading approximation in $(pl/\hbar)\gg 1$ all the limits of integration can be assumed to be infinite, and all the Green functions depend only on the combination $\varepsilon-\varepsilon_p$. Therefore, after the change to the new variable $x=\varepsilon_p-\varepsilon$ the internal integral ceases to depend on the variable ε, but has both poles in the same complex half-plane. The external integral diverges logarithmically at the lower limit of integration. Thus, from a mathematical point of view, an indeterminacy of the type $0\cdot\infty$ arises (here the difference of the arguments of the Fermi functions in the second and third terms of (3.11) is unimportant, since the term

$$\Omega\frac{\partial n_0}{\partial\varepsilon}G_A(\varepsilon)G_A(\varepsilon+\Omega)$$

does not have a divergence in ε). The indeterminacy is revealed in the manner required by the gauge invariance; these terms cancel the diamagnetic current. This procedure can be formally deduced, e.g., by means of the f-sum rule.[10] In the calculation of the correlator of the current densities, retarded and advanced Green functions appear in the integrals over all the energies ε_p (see Fig. 2b); from a mathematical point of view, these graphs are infinite and it is necessary to perform a subtraction. The assertion that we shall prove in the Appendix is that in the second term in formula (3.11) it is necessary to represent $n(\varepsilon+\Omega)=n(\varepsilon)+\Omega\partial n/\partial\varepsilon$, after which terms with $n(\varepsilon)$ (part of the second and third terms in (3.11)) are cancelled by the diamagnetic current before the averaging over the positions of the impurities. Thus, in the calculation of the current calculator in the Lorentz gauge a contribution to the dissipative current is given by the terms

$$\int\frac{d\varepsilon}{2\pi i}\Omega\frac{\partial n_0}{\partial\varepsilon}[G^R(\varepsilon+\Omega)G^A(\varepsilon)-G^A(\varepsilon+\Omega)G^A(\varepsilon)]. \qquad (3.12)$$

The expression (3.12) must be multiplied by the current density from the second loop and averaged over the positions of the impurities. After this, terms of the type

$$G^R(\varepsilon_1+\Omega)G^A(\varepsilon_1)G^A(\varepsilon_2+\Omega)G^A(\varepsilon_2)$$

give the same contributions to $s(t)$ as do the graphs depicted in Fig. 2a. The extra graphs that have arisen from the second term in the expression (3.12) are depicted in Fig. 2b (plus

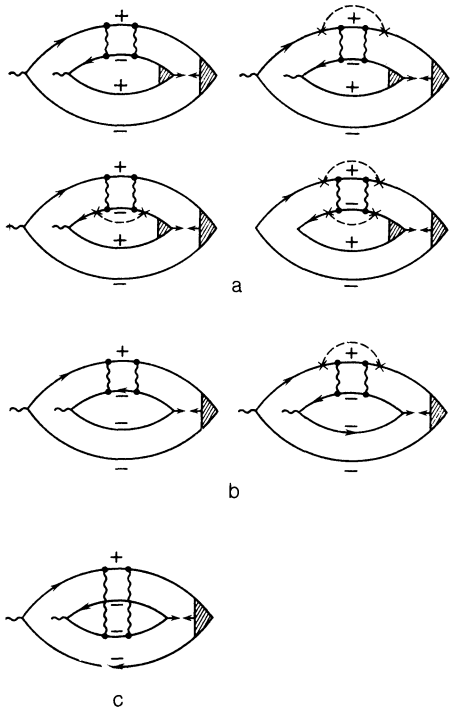

FIG. 2. The signs + and − denote the Green functions G_R and G_A, respectively.

the mirror-image graphs in which the cooperons are situated in the lower part of the loop; the external loop consists of only advanced functions, and the internal loop consists of retarded and advanced functions). Besides these diagrams there are also diagrams of the type depicted in Fig. 2c. The fan-diagram sum corresponding to these graphs has pole expression of the form

$$\left[DQ^2-i(\varepsilon_1+\Omega_1-\varepsilon_2-\Omega_2)+\frac{1}{\tau_{ac}}\right]^{-1},$$

and, consequently, the current has a singularity in the lower half-plane in the frequency Ω_2 of the external field. This, of course, contradicts causality, and therefore such diagrams cancel each other in the sum. The sum of the diagrams depicted in Figs. 2a and 2b (compare with Ref. 3) satisfies the identity (3.1). It is equal to

$$s_{ik}^{(2)}(\mathbf{Q},\Omega_c)=\frac{2(e^2D\tau)^2}{(2\pi)^3}\int d\varepsilon_1\, d\varepsilon_2\,(dq)\,d\omega\,\frac{\partial n_0}{\partial\varepsilon_1}\frac{\partial n_0}{\partial\varepsilon_2}$$
$$\times\Big\{\Big[\frac{Q_iQ_k(\mathbf{E},\mathbf{Q})^2}{Q^4+\Omega_c^2/D^2}-\frac{Q_iE_k(\mathbf{E},\mathbf{Q})}{Q^2-i\Omega_c/D}\Big]F(\varepsilon_1-\varepsilon_2,\omega+\Omega_c,\mathbf{q}+\mathbf{Q})$$
$$\times F(\varepsilon_1-\varepsilon_2,-\omega,\mathbf{q})+\text{c.c.}\Big\}. \qquad (3.13)$$

The expression (3.13) has the same form as (3.3), and is analyzed further in the same manner. Summing the contributions $s^{(1)}$ and $s^{(2)}$, using the expression (3.10) for F, and integrating over ε_1, ε_2, and Q, we arrive at the expression (2.10) for the single-point correlator. The fluctuations of the total current are expressed in terms of $s(t,Q=0)$. In this case only the first term in formula (3.4) turns out to be important; as a result, the fluctuations of the total current for a given spatially uniform distribution of the electric field are determined in order of magnitude by the formula (2.12).

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APPENDIX

In order to justify the subtraction formulated in Sec. 3, it is necessary to prove that before the averaging over the positions of the impurities the contribution to the current density from the term proportional to

$$\int \frac{d\varepsilon_1}{2\pi i} n(\varepsilon)\,[G_R(\varepsilon+\Omega)G_R(\varepsilon)-G_A(\varepsilon+\Omega)G_A(\varepsilon)],$$

is cancelled, in the zeroth approximation in Ω, by the diamagnetic current, and the term linear in Ω is equal to zero. The question of interest to us also exists in the simplest problem, in which electrons are scattered only by elastic impurities. Therefore, for simplicity we shall discuss just this case. We shall find it convenient to work in the basis of the exact wavefunctions $\varphi_n(x)$, to assume that the vector potential $\mathbf{A}$ depends only on the coordinates (or on the wave vector $\mathbf{q}$), and to let $q\to 0$ only in the final answers:

$$\hat{H}(x)\varphi_n(x)=E_n\varphi_n(x),\quad \hat{H}(x)=\hat{H}_0+U_{imp}(x). \tag{A.1}$$

Here U_{imp} is the impurity potential. The exact (with respect to the impurities) Green function can be represented in the form

$$G_E^R(x_2,x_1)=\sum_n \frac{\varphi_n(x_1)\varphi_n^*(x_2)}{E-E_n+i\delta}, \tag{A.2}$$

and the expression for the current density is

$$j_\alpha(x_1)=\frac{1}{c}\int d^3x_2\,A_\beta(x_2)\,\Pi_{\alpha\beta}(x_1,x_2),$$

$$\Pi_{\alpha\beta}(x_1,x_2)=\sum_{m,n}\int\frac{d\varepsilon}{2\pi i}n(\varepsilon)\left[\frac{j_{mn}^{\beta}(x_1)j_{nm}^{*\alpha}(x_2)}{(\varepsilon+\Omega-E_n+i\delta)(\varepsilon-E_n+i\delta)}-\frac{j_{nm}^{\beta*}(x_1)j_{mn}^{\alpha}(x_2)}{(\varepsilon-E_n-i\delta)(\varepsilon+\Omega-E_m-i\delta)}\right],$$

$$j_{mn}^{\alpha}(x)=\frac{e}{2m}[\varphi_m^*(x)\hat{p}_\alpha\varphi_n(x)-\varphi_n(x)\hat{p}_\alpha\varphi_m^*(x)], \tag{A.3}$$

where $\hat{p}_\alpha$ is the momentum operator. Taking into account only the pole contributions, using the continuity equation, and taking into account the invariance of the theory under time reversal, according to which for each quantity $\rho_{mn}(x)=\varphi_m(x)\varphi_n^*(x)$ we can find a quantity ρ_{mn}^* equal to it (to within an unimportant phase factor), we can represent the expression for $\Pi_{\alpha\beta}$ in the form

$$\Pi_{\alpha\beta}(\mathbf{q}_1,\mathbf{q}_2,\Omega)=-\frac{2e^2}{q_\alpha^{(1)}q_\beta^{(2)}}\sum_{m,n}n_0(E_m)\frac{\omega_{mn}^2}{\omega_{mn}+\Omega+i\delta}\rho_{mn}(\mathbf{q}_1)\rho_{nm}(\mathbf{q}_2). \tag{A.4}$$

Now we must prove that $\Pi_{\alpha\beta}$ $(\Omega=0)$ is cancelled by the diamagnetic current. The proof consists in generalizing the f-sum rule to the nonuniform case. It is based on the direct calculation of the commutator of the density matrix $\hat{\rho}$ with the Hamiltonian and the use of the continuity equation:

$$\sum_m n_0(E_m)\,\omega_{nm}\rho_{mn}(\mathbf{q}_1)\rho_{nm}(\mathbf{q}_2)=\sum_m\frac{1}{2}n_0(E_m)\langle m|[[\hat{H},\hat{\rho}],\hat{\rho}]|m\rangle=\frac{q_\alpha^{(1)}q_\alpha^{(2)}}{2m}\sum_m\rho_{mm}(\mathbf{q}_1-\mathbf{q}_2)\,n_0(E_m). \tag{A.5}$$

Taking this equality into account, we can convince ourselves that $\Pi_{\alpha\beta}$ $(\Omega=0)$ is cancelled exactly by the diamagnetic current.

We shall discuss the term linear in the frequency of the external field. It is nonzero and gives a current density equal to

$$j_\alpha(\mathbf{q}_1,\Omega)=e^2\int\frac{d^3q}{(2\pi)^3\,iq_\alpha^{(1)}q_\beta^{(2)}}E^\beta(\mathbf{q}_2,\Omega)N(\mathbf{q}_2-\mathbf{q}_1), \tag{A.6}$$

where E is the intensity of the electric field and N is the electron density:

$$N(\mathbf{q})=2\sum_m\rho_{mm}(\mathbf{q})\,n_0(E_m).$$

In order that its physical nature should become clear, we shall calculate divj and go over to the gauge $\mathbf{A}=0$, $e\mathbf{E}=-i\mathbf{q}U$ (U is the potential of the field). Going over to the coordinate representation, we obtain

$$\frac{\partial}{\partial t}N(x,t)=-|e|\hbar^{-1}U(x,t)N(x,t), \tag{A.7}$$

i.e., the current (A.6) is nondissipative and arises because of the fact that the gas of free fermions overflows from one point to another in order that its energy in the external field be a minimum. At this point it is necessary to recall that our system as a whole is electrically neutral: The ions of the crystal lattice are arranged in such a way as to compensate the electron charge. The field created by these ions induces a current that compensates the current (A.6).

[1)] The idea of qualitative interpretations of this type is due to A. L. Shelankov.

[2)] The confirmation of this assumption is one of the tasks of the quantitative calculation given below.

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