The anomalous temperature dependence effect of the magnon spectrum of ferromagnetic metals

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We predict a new effect of an anomalous temperature dependence of the magnon spectrum of ferromagnetic metals with collectivized electrons. This effect is caused by the influence of the dynamics of the crystal lattice on the properties of the spin fluctuations and by the anisotropy of the electron and magnon spectra, and manifests itself in differences in the magnon rigidity coefficient in various frequency and temperature ranges. The anomalous frequency and temperature dependence of the magnon-spectrum rigidity reveals the cause of the difference between the results of experimental studies of the magnon spectrum obtained from magnetic measurements and from inelastic neutron scattering.

1. INTRODUCTION

A phenomenological approach to the theory of conducting magnetics with collectivized electrons leads to the following temperature dependence of the magnon spectrum at low temperatures:[1–3]

$$\omega(\mathbf{k},\ T)=\omega(\mathbf{k},\ 0)[1-AT^2+BT^{5/2}+CT^4]. \tag{1.1}$$

The term $\propto T^2$ is determined here by the Fermi excitations of the electrons, and the terms with $T^{5/2}$ and T^4 are caused, respectively, by spin fluctuations and by the interaction between the magnons and the lattice vibrations. The theory of the calculation of the coefficients A, B, C using various models and approximations has been discussed in a number of papers (see Ref. 4 and the citations there). It is important that in such a theory the temperature dependence (1.1) with constant coefficients A, B, C occurs in the entire low-temperature range $\varkappa T < \hbar\omega_{\max}$ ($\hbar\omega_{\max}$ is the maximum magnon energy and $\varkappa$ the Boltzmann constant).

The present paper is devoted to a new effect of an anomalous temperature dependence of the magnon spectrum of ferromagnetic metals, which manifests itself in an anomalous change of the coefficient B at temperatures and magnon frequencies which are determined by the dynamics of the crystal lattice and of the electron fluid in the metals. It is caused by the dynamical screening by the lattice of the long-range Coulomb interaction between the electrons and also by the effect of an anisotropic momentum dependence of the electron energy and an anisotropic wave vector dependence of the magnon frequency. In this case an estimate of the widths of the temperature range ΔT and of the frequency range $\Delta\omega$ in which the coefficient B changes gives $\Delta T \sim \hbar\omega_{\max}/\varkappa$ and $\Delta\omega \sim \omega_{\max}$.

Following Ref. 4 we use the following representation of the density matrix:

$$\hat\rho(t)=\int(d\omega/2\pi)\exp(-i\omega t)\hat\rho(\omega)$$

and of the matrix elements of the energy operator

$$\hat\varepsilon(t)=\int(d\omega/2\pi)\exp(-i\omega t)\hat\varepsilon(\omega)$$

of the quasi-particles:

$$\left\langle\sigma,\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right|\hat\rho(\omega)\left|\sigma',\mathbf{p}-\frac{\hbar\mathbf{k}}{2}\right\rangle$$

$$=(2\pi)^4\delta(k)\delta_{\sigma\sigma'}n^\sigma(\mathbf{p})+\delta\rho^{\sigma\sigma'}(\mathbf{p},k), \tag{1.2}$$

$$\left\langle\sigma,\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right|\hat\varepsilon(\omega)\left|\sigma',\mathbf{p}-\frac{\hbar\mathbf{k}}{2}\right\rangle$$

$$=(2\pi)^4\delta(k)\delta_{\sigma\sigma'}\varepsilon^\sigma(\mathbf{p})+\delta\varepsilon^{\sigma\sigma'}(\mathbf{p},k), \tag{1.3}$$

where $n^\sigma(\mathbf{p})=n_F[\varepsilon^\sigma(\mathbf{p})]\equiv n(\mathbf{p})+\sigma s(\mathbf{p})$ is the equilibrium Fermi distribution function which depends on the energy $\varepsilon^\sigma(\mathbf{p})=\varepsilon(\mathbf{p})-\sigma\hbar\Omega_0/2$ of quasi-particles with momentum $\mathbf{p}$ and spin component $\sigma=\pm1$ and which neglects fluctuations, $k=(\omega,\mathbf{k})$. The energy of the spin splitting

$$\hbar\Omega_0=-2\Psi\int d\tau[n^+(\mathbf{p})-n^-(\mathbf{p})]=-4\Psi S \tag{1.4}$$

is here determined by the spontaneous spin density S, where Ψ is the exchange interaction constant, $d\tau=d\mathbf{p}/(2\pi\hbar)^3$.

For the matrix elements of the nonequilibrium self-consistent electron potential $\delta\varepsilon^{\sigma\sigma'}(\mathbf{p},k)$ we use the following model functional dependence:

$$\begin{gathered}\delta\varepsilon^{\sigma\sigma'}(k)=2\delta_{\sigma\sigma'}\phi(\mathbf{k})\delta n(k)+2(\hat\sigma)_{\sigma\sigma'}\Psi(\mathbf{k})\delta\mathbf{s}(k)\\+i\delta_{\sigma\sigma'}N_i[4\pi Ze^2/\mathbf{k}^2+\lambda(\mathbf{k})](\mathbf{k}\mathbf{u}(k)),\\ \delta_{\sigma\sigma'}\delta n(k)+(\hat{\boldsymbol\sigma})_{\sigma\sigma'}\delta\mathbf{s}(k)=\int d\tau\,\delta\rho^{\sigma\sigma'}(\mathbf{p},k),\end{gathered} \tag{1.5}$$

where $\hat{\boldsymbol\sigma}$ is the Pauli matrix, $\phi(\mathbf{k})=4\pi e^2/\mathbf{k}^2+\varphi(\mathbf{k})$, $\mathbf{u}(\mathbf{k})$ is the Fourier component of the displacement of the crystal ions. Z and N_i are their charge and density. The first and second terms on the right-hand side of (1.5) describe the way the electron energy depends on their non-equilibrium distributions. The last term is caused by the interaction between the electrons and the longitudinal lattice vibrations; we confine ourselves to them for the sake of simplicity. The functions $\Psi(\mathbf{k})$, $\varphi(\mathbf{k})$ characterize the short-range electron-electron interaction, and $\lambda(\mathbf{k})$ characterizes the deformation interaction between the particles and the crystal lattice. We shall in what follows use in the long-wavelength approximation

$$\Psi(\mathbf{k})=\Psi+k_ik_j\Psi_{ij}''/2+k_ik_jk_lk_m\Psi_{ijlm}^{\mathrm{IV}}/24, \tag{1.6}$$

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where $\Psi = \Psi(0)$, and the k_i are components of the wave vector.

Bearing in mind that $\mathbf{u}(k)$ satisfies the equation of motion for the longitudinal lattice vibrations

$$(\omega^2-\omega_{Li}^2-c_i^2\mathbf{k}^2)\mathbf{u}(k)=-\frac{2i}{M}\left[\frac{4\pi Ze^2}{\mathbf{k}^2}+\lambda(\mathbf{k})\right]\mathbf{k}\delta n(k) \quad (1.7)$$

and using that equation to eliminate $\mathbf{u}(k)$ we write Eq. (1.5) in the form

$$\delta\varepsilon^{\sigma\sigma'}(k)=2\delta_{\sigma\sigma'}\phi_{eff}(k)\delta n(k)+2(\hat{\sigma})_{\sigma\sigma'}\Psi(\mathbf{k})\delta s(k). \quad (1.8)$$

The function

$$\phi_{eff}(k)=\phi(\mathbf{k})+\frac{\omega_{Li}^2}{\omega^2-\omega_{Li}^2-c_i^2\mathbf{k}^2}\frac{4\pi e^2}{\mathbf{k}^2}\left[1+\frac{\lambda(\mathbf{k})\mathbf{k}^2}{4\pi Ze^2}\right]^2 \quad (1.9)$$

which appears here characterizes the dynamic electron-electron interaction and, in particular, describes the effects of the dynamic screening of such an interaction by the crystal lattice. Here the $\omega_{Li}^2 + c_i^2\mathbf{k}^2$ are the diagonal elements of the dynamic matrix which respond to the lattice vibrations without taking into account the influence of the electron fluid which we shall assume, restricting our consideration to an isotropic metal, not to depend on the direction of the wavevector $\mathbf{k}$ ω_{Li} $(4\pi Z^2N_ie^2/M)^{1/2}$ and M are the ion plasma Langmuir frequency and mass.

The frequency dependence of the function (1.9) due to the dynamics of the crystal lattice is the cause of the new effect in the temperature dependence of the magnon spectrum considered by us which, as we shall show below, manifests itself in an anomalous dependence of the coefficient B in Eq. (1.1) on the temperature and the magnon frequency.

To construct a theory of the temperature dependence of the magnon spectrum we restrict ourselves in what follows to considering low-frequency and long-wavelength fluctuations. This will mean that the frequency in our considerations is small compared to the maximum magnon frequency $\omega_{\max}$ and the wavevector small compared to the maximum wavevector $k_{\max}$. Under those conditions the following quantities can be considered to be small:

$$c_i^2\mathbf{k}^2/\omega_{Li}^2, \quad 2|\lambda(\mathbf{k})/Z\phi(\mathbf{k})|,$$
$$|\varphi(\mathbf{k})/\phi(\mathbf{k})|, \quad |\Psi(\mathbf{k})/\phi(\mathbf{k})|\ll 1.$$

Using these inequalities we can write Eq. (1.9) in the following asymptotic form:

$$\phi_{eff}(k)=\phi_{eff}(0)-\frac{4\pi e^2}{\mathbf{k}^2}\left[\frac{\omega^2}{\omega_{Li}^2}\theta(\omega^2-c_0^2\mathbf{k}^2)-\left(\frac{\omega^2}{\omega_{Li}^2}+1\right)\theta(\omega^2-\omega_{Li}^2)\right], \quad (1.10)$$

where $\theta(x) = 1$ when $x>0$ and $\theta(x) = 0$ when $x<0$. In the low-frequency limit $\omega^2<c_0^2\mathbf{k}^2$ and under the additional assumption $c_0^2\mathbf{k}^2<\omega_{Li}^2$, it follows from (1.10), that the long-wavelength Coulomb interaction is thus completely screened by the crystalline ion lattice which in this limit is adiabatically, without any lag, displaced, following the electrons. In that case

$$\phi_{eff}(k)\rightarrow\phi_{eff}(0)=\varphi(0)-\frac{2\lambda(0)}{Z}+\frac{Mc_i^2}{Z^2N_i}\equiv\frac{Mc_0^2}{Z^2N_i}. \quad (1.11)$$

In the intermediate frequency range $c_0^2k^2<\omega^2<\omega_{Li}^2$, where, according to (1.10), $\phi_{\rm eff}(k)\approx -(4\pi e^2/\mathbf{k}^2)(\omega^2/\omega_{Li}^2)$ there occurs a partial screening of the long-range Coulomb interaction of the electrons. Finally, in the high-frequency limit $\omega^2>\omega_{Li}^2$, when the lattice is practically undeformed, there is no dynamical screening: $\phi_{\rm eff}(k)\approx 4\pi e^2/\mathbf{k}^2$. We shall show in sections 2 and 3 that this frequency dependence of $\phi_{\rm eff}(k)$ caused by the dynamics of the crystal lattice is the cause of the anomalous change in the coefficient B near a temperature $\sim\hbar\Omega/\varkappa$ and the frequency Ω of the intersection of the magnon and the sound modes. It is just for the low-frequency (cold) magnons with frequencies $\omega(\mathbf{k},0)<\varkappa T/\hbar$ that the coefficient B changes steeply with increasing temperature near $T\sim\hbar\Omega/\varkappa$ from a value B_1 to B_1+B_2 where the quantities B_1 and B_2 are of the same order. In the low-temperature limit $\varkappa T<\hbar\omega(\mathbf{k},0)$ a similar change in the coefficient B by an amount B_2 occurs with an increase of magnon frequency near $\omega(\mathbf{k},0)\sim\Omega$. Another reason, established below, for an anomalous change in the coefficient B is caused by singularities in the effective magnon-electron interaction without electron spin flip (Sec. 2). Such singularities lead under conditions of an anisotropic momentum dependence of the electron energy and an anisotropic wavevector dependence of the magnon frequency to an anomalous change in the coefficient B near a temperature $T\sim\hbar\omega(\mathbf{k},0)/\varkappa$ which we consider in Sec. 4 using a simple model for the band structure.

2. GENERAL FORMULAE DESCRIBING THE MAGNON SPECTRUM

To determine the magnon spectrum with fluctuations taken into account we use a nonlinear equation of motion

$$[\hbar\omega+\varepsilon^{\sigma'}(\mathbf{p}-\hbar\mathbf{k}/2)-\varepsilon^{\sigma}(\mathbf{p}+\hbar\mathbf{k}/2)]\delta\rho^{\sigma\sigma'}(\mathbf{p},k)$$
$$=\sum_{\sigma''}\int(dk')[\delta\varepsilon^{\sigma\sigma''}(k')\delta\rho^{\sigma''\sigma}(\mathbf{p}-\hbar\mathbf{k}'/2,k-k')-\delta\rho^{\sigma\sigma''}(\mathbf{p}+\hbar\mathbf{k}'/2,k-k')\delta\varepsilon^{\sigma''\sigma'}(k')] \quad (2.1)$$

for the electron density matrix (1.2). Following the dynamical approach of our Ref. 4 we use (1.8) and (2.1) to eliminate the variables $\delta\rho^{\sigma\sigma'}(\mathbf{p},k)$ and $\delta\varepsilon^{\sigma\sigma'}(k)$ which characterize the fluctuations in the density matrix, the charge and the longitudinal component of the spin density, and also $\mathbf{u}(k)$—the amplitude of the crystal lattice vibrations. As a result we get the following dynamical equation

$$D_+(k)\delta s^+(k)=\int(dk')(dk'')T(k,k',k'')\delta s^+(k-k')\times\delta s^+(k'-k'')\delta s^-(k'') \quad (2.2)$$

for the transverse Fourier components of the spin density $\delta s^{\pm}=\delta s^x\pm i\delta s^y$ (the z-axis is taken along the spontaneous spin density S). Here

$$D_+(k)=1-\Psi(\mathbf{k})\Pi^{-+}(k) \quad (2.3)$$

is the well known magnon dispersion function, neglecting fluctuations, $(dk)=d\omega d\mathbf{k}/(2\pi)^4$,

$$\Pi^{\sigma\sigma'}(k)=2\int d\tau\,\Pi^{\sigma\sigma'}(\mathbf{p},k)=2\int d\tau\frac{n^{\sigma'}(\mathbf{p}-\hbar\mathbf{k}/2)-n^{\sigma}(\mathbf{p}+\hbar\mathbf{k}/2)}{\hbar\omega+\varepsilon^{\sigma'}(\mathbf{p}-\hbar\mathbf{k}/2)-\varepsilon_{\sigma}(\mathbf{p}+\hbar\mathbf{k}/2)}. \quad (2.4)$$

The kernel $T(k,k',k'')$ characterizes the interaction of the transverse spin fluctuations and differs from the one given in

Ref. 4 through the substitution $\phi(\mathbf{k}) \to \phi_{\text{eff}}(k)$ corresponding to taking into account the effects of the crystal lattice dynamics.

Averaging the dynamic Eq. (2.2) we are led to the magnon dispersion equation

$$D_+(k) - \int \frac{d\mathbf{k}'}{(2\pi)^3} T(\mathbf{k},\mathbf{k}')\,(\delta s^+ \delta s^-)_{\mathbf{k}} = 0, \tag{2.5}$$

which takes into account the effects of the fluctuations, where

$$T(\mathbf{k},\mathbf{k}') = [T(k,0,-k') + T(k,k-k',-k')]_{\omega=\omega(\mathbf{k}),\,\omega'=\omega(\mathbf{k}')}, \tag{2.6}$$

$\omega(\mathbf{k})$ is the magnon frequency, if we neglect fluctuations,

$$(\delta s^+ \delta s^-)_{\mathbf{k}} \approx S \operatorname{cth}[\hbar\omega(\mathbf{k})/2\varkappa T] \tag{2.7}$$

is the phase density of the fluctuations for magnons of wavevector $\mathbf{k}$.

Thermodynamic considerations[4] show that the value $T(k,0,-k')$ of the kernel T must be taken in the sense of a limit

$$T(k,0,-k') = \lim_{\mathbf{k}''\to 0} \lim_{\omega''\to 0} T(k,k'',-k').$$

As we are interested in the fluctuation temperature dependence of the magnon spectrum, we shall in what follows neglect in (2.5) the change with temperature of the distribution function $n^\sigma(\mathbf{p})$ and of the energy $\varepsilon^\sigma(\mathbf{p})$ of the electrons which is caused by the Fermi excitations which lead to corrections $\propto AT^2$ to the magnon frequency.

For what follows it is necessary to expand the quantities $D_+(k)$ and $T(\mathbf{k}\mathbf{m}\ \mathbf{k}')$ in (2.5) in power series in $\mathbf{k}$ and $\mathbf{k}'$. Restricting our consideration of the magnon spectrum to the quadratic approximation ($\omega(\mathbf{k},T) \propto \mathbf{k}^2$) we retain in the expansion of $D_+(k)$ terms $\propto \mathbf{k}^2$:

$$D_+(k) = [\omega(\mathbf{k}) - \omega]/\Omega_0, \quad \omega(\mathbf{k}) = \alpha_{ij} k_i k_j, \tag{2.8}$$

where $\alpha_{ij} = \alpha_{ij}^{(B)} + \alpha_{ij}^{(H)}$ is the magnon rigidity coefficient,

$$\alpha_{ij}^{(B)} = \frac{2\Psi}{\Omega_0} \int d\tau v_i v_j \left[\frac{2s(\mathbf{p})}{\hbar\Omega_0} + \frac{\partial n}{\partial\varepsilon}\right], \quad \alpha_{ij}^{(H)} = -\frac{\Omega_0}{2\Psi} \Psi_{ij}'', \tag{2.9}$$

$\mathbf{v} = \partial\varepsilon(\mathbf{p})/\partial\mathbf{p}$ is the electron velocity. In the expansion of the function $T(\mathbf{k},\mathbf{k}')$ we retain the terms $\propto (\mathbf{k}\cdot\mathbf{k}')$ and $\mathbf{k}^2\cdot\mathbf{k}^1$. [2] As a result we get

$$T(\mathbf{k},\mathbf{k}') = \frac{8\Psi^2}{\hbar^2\Omega_0^2}\left[\frac{2\alpha_{ij}k_i k_j'}{\Omega_0} + \tau(\mathbf{k},\mathbf{k}')\right]. \tag{2.10}$$

The first term on the right-hand side of (2.10) $\propto \alpha_{ij} k_i k_j'$ being bilinear in $\mathbf{k}$ and $\mathbf{k}'$ does not contribute to the temperature dependence of the magnon spectrum. The function

$$\tau(\mathbf{k},\mathbf{k}') = (t_{ijlm}^{(0)} + t_{ijlm}^{(1)}) k_i k_j k_l' k_m' + \tau^{(2)}(\mathbf{k},\mathbf{k}') + \tau^{(3)}(\mathbf{k},\mathbf{k}') + \tau^{(4)}(\mathbf{k},\mathbf{k}') + \tau^{(5)}(\mathbf{k},\mathbf{k}') \tag{2.11}$$

characterizes the fluctuation effects in the magnon spectrum where the functions $\tau^{(2)}(\mathbf{k},\mathbf{k}')$ and $\tau^{(4)}(\mathbf{k},\mathbf{k}')$ contain $\phi_{\text{eff}}(k-k')$ and take into account the effects of the crystal lattice dynamics.

3. EFFECT OF THE CHANGE IN THE $T^{5/2}$ LAW IN THE ISOTROPIC MODEL

In this section we demonstrate that qualitative change in the magnon-spectrum temperature dependence which is determined by the dispersion of the effective electron interaction $\phi_{\text{eff}}(k)$ caused by the electron-lattice coupling. We shall then assume that the electron energy ε depends on the modulus of $\mathbf{p}$ and neglecting the contribution from the thermal fluctuations the expression for the magnon frequency has the form $\omega = \alpha\mathbf{k}^2$, where $\alpha = \alpha_B + \alpha_H$,

$$\alpha_B = \frac{2\Psi}{3\Omega_0}\int d\tau \mathbf{v}^2\left[\frac{2s(\mathbf{p})}{\hbar\Omega_0} + \frac{\partial n}{\partial\varepsilon}\right], \quad \alpha_H = -\frac{\Omega_0}{6}\frac{\Psi_{ii}''}{\Psi}. \tag{3.1}$$

In this case, using the asymptotic expression (1.10) we get for the terms on the right-hand side of Eq. (2.11)

$$t_{ijlm}^{(0)} k_i k_j k_l' k_m' = \left[\delta_H + \delta_1 + \delta_2 + \frac{(2d+\alpha)\alpha - 2\alpha_H^2}{\Omega_0}\right]\frac{\mathbf{k}^2\mathbf{k}'^2}{\Omega_0} + 2\left(\delta_H + \delta_1 + \frac{\delta_3}{2} - \frac{2\alpha_H^2}{\Omega_0}\right)\frac{(\mathbf{k}\mathbf{k}')^2}{\Omega_0}, \tag{3.2}$$

$$t_{ijlm}^{(1)} k_i k_j k_l' k_m' = \frac{\mathbf{k}^2\mathbf{k}'^2}{(1-\Psi\Pi_n)\Omega_0^2}\left\{(a_1+\alpha)^2 - \frac{c_0^2}{c^2}\frac{[(1-\Psi\Pi_n)a_2 + \Psi\Pi_s(a_1+\alpha)]^2}{\Psi\Pi}\right\} \equiv t_1\mathbf{k}^2\mathbf{k}'^2, \tag{3.3}$$

$$\tau^{(2)}(\mathbf{k},\mathbf{k}') = (\mathbf{k}\mathbf{k}')^2\{t_2 + \tau_2[\theta(\mathbf{k}^2 - k_1^2) + \theta(k_1^2 - \mathbf{k}^2)\theta(\mathbf{k}'^2 - k_1^2)]\}, \tag{3.4}$$

$$\tau^{(3)}(\mathbf{k},\mathbf{k}') = -\Psi\Pi_n\frac{\alpha^2}{\Omega_0^2}\mathbf{k}^2\mathbf{k}'^2 + \frac{2\alpha}{\Omega_0^2}(\Psi\Pi_n\alpha + \alpha_B - 2\alpha_H - a_1)(\mathbf{k}\mathbf{k}')^2, \tag{3.5}$$

$$t_2 = \frac{1}{(1-\Psi\Pi_n)\Omega_0^2}\left\{(a_1 - \alpha\Psi\Pi_n)^2 - \frac{c_0^2}{c^2}\frac{[(1-\Psi\Pi_n)a_2 + \Psi\Pi_s(a_1-\alpha)]^2}{\Psi\Pi}\right\} \tag{3.6}$$

$$\tau_2 = \left(\frac{c_0^2}{c^2} - 1\right)\frac{[(1-\Psi\Pi_n)a_2 + \Psi\Pi_s(a_1-\alpha)]^2}{(1-\Psi\Pi_n)\Psi\Pi\Omega_0^2}. \tag{3.7}$$

We have used in this case the notation

$$d = -\frac{2\Psi}{3\Omega_0}\int d\tau \mathbf{v}^2\frac{\partial n}{\partial\varepsilon}, \quad a_1 = \frac{\hbar\Psi}{3}\int d\tau \mathbf{v}^2\frac{\partial^2 s}{\partial\varepsilon^2} + \alpha_B - 2\alpha_H,$$

$$a_2 = -\hbar\Psi\int\frac{d\tau}{m}\left[\frac{2s(\mathbf{p})}{\hbar\Omega_0} + \frac{\partial n}{\partial\varepsilon}\right], \quad \delta_H = \frac{\Omega_0}{60\Psi}\Psi_{iijj}^{\text{IV}},$$

$$\delta_1 = \frac{8\Psi}{15\Omega_0^3}\int d\tau \mathbf{v}^4\left[\frac{2s(\mathbf{p})}{\hbar\Omega_0} + \frac{\partial n}{\partial\varepsilon} - \frac{\hbar^2\Omega_0^2}{12}\frac{\partial^3 n}{\partial\varepsilon^3}\right],$$

$$\delta_2 = \frac{\hbar^2\Psi}{\Omega_0}\int\frac{d\tau}{m^2}\left[\frac{2s(\mathbf{p})}{\hbar\Omega_0} + \frac{\partial n}{\partial\varepsilon}\right], \tag{3.8}$$

$$\delta_3 = -\frac{2\hbar^2\Psi}{3\Omega_0}\int\frac{d\tau}{m}\mathbf{v}^2\frac{\partial}{\partial\varepsilon}\left[\frac{2s(\mathbf{p})}{\hbar\Omega_0} + \frac{\partial n}{\partial\varepsilon}\right],$$

$$\Pi_{n,s} = \int d\tau\,\partial(n^+ \pm n^-)/\partial\varepsilon, \quad \Pi = \Pi_n - \Psi(\Pi_n^2 - \Pi_s^2),$$

where

$$m^{-1}(\mathbf{p}) = \partial^2\varepsilon(\mathbf{p})/\partial\mathbf{p}^2, \quad c^2 = c_0^2 - Z^2 N_i(1-\Psi\Pi_n)/M\Pi$$

is the square of the longitudinal sound speed.

After that we find, in accordance with Eq. (2.6) and taking into account the anti-symmetry of the functions $\tau^{(4)}$,

$\tau^{(5)}$ under the substitution $\mathbf{k}\leftrightarrow\mathbf{k}'$, in the isotropic case considered by us that $\tau^{(4)} = \tau^{(5)} = 0$.

Substituting Eqs. (2.11), (3.2)–(3.5) into Eq. (2.5) we get for the magnon spectrum the expression

$$\omega(\mathbf{k},T)=\omega(\mathbf{k},0)-\frac{\Omega_0\mathbf{k}^2}{6S}\int\frac{d\mathbf{k}}{(2\pi)^3}\mathbf{k}'^2\,\mathrm{cth}\frac{\hbar\alpha\mathbf{k}'^2}{2\varkappa T}\times\{\tau_1+\tau_2[\theta(\mathbf{k}^2-\mathbf{k}_1^2)+\theta(k_1^2-\mathbf{k}^2)\theta(\mathbf{k}'^2-k_1^2)]\},\quad (3.9)$$

where

$$\tau_1=(6-\Psi\Pi_n)\frac{\alpha^2}{\Omega_0^2}+\frac{2\alpha(3d+\alpha_B-2\alpha_H-a_1)}{\Omega_0^2}-10\frac{\alpha_H^2}{\Omega_0^2}+5\frac{\delta_H+\delta_1}{\Omega_0}+\frac{3\delta_2+\delta_3}{\Omega_0}+3t_1+t_2\quad (3.10)$$

and the integration over $\mathbf{k}'$ is taken up to the value $k_{\max}$. The magnon frequency $\omega(\mathbf{k},0)$ is determined here by Eq. (2.5) and takes into account fluctuation effects at $T=0$ which we do not discuss in what follows as we are interested in the temperature dependence of the magnon spectrum.

The integral terms on the right-hand side of (3.9) describe the effect of the spin fluctuations on the magnon spectrum. When evaluating the integral which occurs here we retain only the main terms corresponding to the term $\propto BT^{5/2}$ in Eq. (1.1). However, the presence of θ-functions on the right-hand side of (3.9) indicates that the coefficient B in our considerations is a function, firstly, of the magnon wavevector $\mathbf{k}$ (or frequency) and, secondly, due to the presence of $\theta(\mathbf{k}'^2\to k_1^2)$ a function of the temperature. Using the notation $\Omega = c^2/\alpha$ for the frequency where the sound and magnon modes intersect we can, using (3.9) write down the following asymptotic formula which characterizes the functional dependence $B[\omega(\mathbf{k}),T]$:

$$\begin{aligned}&B\approx B_1 \quad \text{if} \quad \hbar\Omega\gg\varkappa T,\ \hbar\omega(\mathbf{k}),\\ &B\approx B_1+B_2 \quad \text{if} \quad \varkappa T\gg\hbar\Omega\gg\hbar\omega(\mathbf{k}),\\ &B\approx B_1+B_2 \quad \text{if} \quad \omega(\mathbf{k})\gg\Omega.\end{aligned}\quad (3.11)$$

Here

$$B_{1,2}=-\frac{\pi}{16}\zeta\left(\frac{5}{2}\right)\frac{\Omega_0}{\alpha S}\left(\frac{\varkappa}{\pi\hbar\alpha}\right)^{5/2}\tau_{1,2},\quad (3.12)$$

where $\zeta(x)$ is Riemann's zeta function.

We see thus from Eqs. (3.11) and (3.12) that there is an anomalous $B(\omega,T)$ dependence which qualitatively distinguishes the effect of spin fluctuations in the case of a ferromagnetic with mobile electrons from the corresponding effect in a Heisenberg ferromagnet.[5] In particular, for relatively low-frequency magnons when, $\omega(\mathbf{k})\ll\Omega$, the function $B(\omega,T)$ changes when the temperature increases at $\varkappa T\ll\hbar\Omega$ from a value B_1 to a value B_1+B_2. On the other hand, in the range of relatively low temperatures, when $\varkappa T=\hbar\Omega$, Eqs. (3.11) describe an anomalous frequency dependence corresponding for increasing frequency to a change in the function $B(\omega,T)$ from a value B_1 to a value B_1+B_2 for $\omega(\mathbf{k})\sim\Omega$.

At the same time we must note that according Eq. (3.7) under conditions when the ion contribution c_0 to the sound speed c appreciably exceeds the contribution from the mobile electrons and when the difference between c^2 and c_0^2 is thus small, the quantity B_2 turns out to be negligibly small which allows us to see it as the condition for a small manifestation of the anomalous $B(\omega,T)$ dependence. The limit $c^2=c_0^2$ formally corresponds to the theory of the temperature dependence of the magnon spectrum[4] in which the lattice dynamics is neglected.

The way the coefficients B_1 and B_2 depend on the sound velocity c allows us also to confirm that the sound-speed change connected, for instance, with the invar anomalies and with structural phase transitions, may show up in changes in the coefficient B of Eq. (1.1) determining the fluctuation temperature dependence of the magnon spectrum.

We apply the formulae obtained here to the case of a weak ferromagnet ($2S\ll ZN_i$) in the model with the simplest dispersion law for the electron energy $\varepsilon(\mathbf{p})=\mathbf{p}^2/2m$. Using the estimates

$$\begin{gathered}a_2\sim a_1\frac{\hbar\Omega_0}{\varepsilon_F}\ll a_1,\quad \delta_H\sim\delta_1\sim\delta_2\sim\delta_3\sim\frac{\alpha^2}{\Omega_0}\ll\frac{\alpha d}{\Omega_0},\\ a_1=-2\alpha,\quad \alpha_B=\frac{\hbar^2\Omega_0}{12p_F^2},\quad d=-\frac{16\varepsilon_F^2}{\hbar^2\Omega_0^2}\alpha_B,\\ \Pi_n=-\nu\left[1-\frac{\hbar^2\Omega_0^2}{32\varepsilon_F^2}\right],\quad \Pi_s=-\frac{\hbar\Omega_0}{4\varepsilon_F}\nu,\quad \Pi=-\frac{\nu}{12}\frac{\hbar^2\Omega_0^2}{\varepsilon_F^2},\\ \frac{\alpha_H}{\alpha_B}=\frac{2\Psi_{ii}''p_F^2}{\Psi\hbar^2}\equiv\bar{\Psi},\end{gathered}\quad (3.13)$$

where $\nu=mp_F/\pi^2\hbar^3$ is the electron density of states at the Fermi surface, $\varepsilon_F=p_F^2/2m$ is the Fermi energy, we get using Eqs. (3.7), (3.10), (3.12)

$$B_1=-36\pi^{1/2}\zeta\left(\frac{5}{2}\right)\left(\frac{\varkappa}{\hbar\omega_{max}}\right)^{5/2}\frac{\omega_{max}}{\Omega_0}\left[1-\frac{1}{6(1+\bar{\Psi})}-\frac{3}{4}\frac{c_0^2}{c^2}\right],\quad (3.14)$$

$$B_2=\frac{81}{4}\pi^{1/2}\zeta\left(\frac{5}{2}\right)\left(\frac{\varkappa}{\hbar\omega_{max}}\right)^{5/2}\frac{\omega_{max}}{\Omega_0}\left(1-\frac{c_0^2}{c^2}\right).\quad (3.15)$$

Here

$$\omega_{max}=\omega(\mathbf{k})\big|_{|\mathbf{k}|=k_{max}}=(1+\bar{\Psi})(\Omega_0/48)(\hbar\Omega_0/\varepsilon_F)^2,$$

and the quantity $k_{\max}\approx\Omega_0 m/p_F$ is determined by the occurrence of collisionless magnon damping caused by their decay into Fermi excitations.

We emphasize here that the occurrence of anomalies in the magnon spectrum caused by the crystal lattice must be expected in those metals where the contribution of the itinerant electrons to the sound speed is not small ($c^2-c_0^2\sim c^2$). Invar alloys with an anomalous temperature dependence of the elastic moduli belong, for instance, to such metals.

4. EFFECT OF A CHANGE IN THE $T^{5/2}$ LAW DUE TO AN ANISOTROPIC DISPERSION OF THE ELECTRON ENERGY

A change in the $T^{5/2}$ law can, according to our theory, also be caused by anisotropic dependences of the electron energy on their quasi-momentum and of the magnon frequency on the wave vector. This effect which is new as compared to what is expounded in the previous section is, according to Eq. (2.11), caused by the vanishing of the terms $\tau^{(4)}$ and $\tau^{(5)}$ in the isotropic model. Such terms are non-vanishing when $\mathbf{k}^2>\mathbf{k}'^2$ and lead to a singularity in the function $\tau(\mathbf{k},\mathbf{k}')$ close to $\mathbf{k}^2=\mathbf{k}'^2$ which is connected with the scattering of magnons by electrons. In other words, this means that the anisotropy of the electron and magnon dispersion leads to an additional anisotropic contribution to the coefficient B which changes suddenly in the vicinity of $\varkappa T \sim \hbar\omega(\mathbf{k})$ by an amount $B_a(\mathbf{k}) \sim B$.

To illustrate this effect we consider the limit of a weak ferromagnet with an anisotropic electron dispersion law. We use here the simple electron structure model of a metal with parabolic bands

$$\varepsilon_s(\mathbf{p}) = \mathbf{p}^2/2m_s, \quad \varepsilon_q(\mathbf{p}) = [\mathbf{p}\mathbf{e}_q]^2/2m_c, \quad q=1,\ldots,q_0, \quad (4.1)$$

which corresponds to a Fermi surface in the shape of spheres connected by cylindrical sections (the unit vectors $\mathbf{e}_q$ are directed along the axes of the cylinders).

In the model considered the function $\tau(\mathbf{k},\mathbf{k}')$ which determines the fluctuation temperature dependence of the magnon spectrum is given by Eq. (2.11) where

$$t_{ijlm}^{(0)} k_i k_j k_l' k_m' = \frac{2\alpha k'^2}{\Omega_0^2}\Big(d_c \sum_q [\mathbf{k}\mathbf{e}_q]^2 + d_s \mathbf{k}^2\Big), \quad (4.2)$$

$$t_{ijlm}^{(1)} k_i k_j k_l' k_m' = \frac{1-\Phi_{eff}(0)\Pi_n}{D(0)} \frac{\alpha^2 \mathbf{k}^2 \mathbf{k}'^2}{\Omega_0^2}, \quad (4.3)$$

$$\tau^{(2)}(\mathbf{k},\mathbf{k}') = \frac{1-\Phi_{eff}(k-k')\Pi_n}{D(k-k')} \frac{\alpha^2}{\Omega_0^2}\Big\{ (2-\Psi\nu_s)(\mathbf{k}\mathbf{k}') - \Psi\nu_c \sum_q \frac{\mathbf{k}\mathbf{k}' - (\mathbf{k}\mathbf{e}_q)(\mathbf{k}'\mathbf{e}_q)}{[\mathbf{k}'\mathbf{e}_q]^2} \mathbf{k}'^2 \Big\}^2, \quad (4.4)$$

$$\tau^{(4)}(\mathbf{k},\mathbf{k}') = \theta(\mathbf{k}^2 - \mathbf{k}'^2) \frac{\alpha d_c}{\Omega_0^2} \mathbf{k}^2 \mathbf{k}'^2 \sum_q \Big\{ \frac{[\mathbf{k}'\mathbf{e}_q]^2}{\mathbf{k}'^2} - \frac{[\mathbf{k}\mathbf{e}_q]^2}{\mathbf{k}^2} \Big\}, \quad (4.5)$$

$$\tau^{(5)}(\mathbf{k},\mathbf{k}') = \theta(\mathbf{k}^2 - \mathbf{k}'^2) \frac{1-\Phi_{eff}(k-k')\Pi_n}{D(k-k')} \Psi\nu_c \frac{\alpha^2}{\Omega_0^2} \times \Big\{ 2(2-\Psi\nu_s)(\mathbf{k}\mathbf{k}') - \Psi\nu_c \sum_q [\mathbf{k}\mathbf{k}' - (\mathbf{k}\mathbf{e}_q)(\mathbf{k}'\mathbf{e}_q)] \Big(\frac{\mathbf{k}'^2}{[\mathbf{k}'\mathbf{e}_q]^2} + \frac{\mathbf{k}^2}{[\mathbf{k}\mathbf{e}_q]^2} \Big) \times \sum_{q'} \Big(\frac{\mathbf{k}'^2}{[\mathbf{k}'\mathbf{e}_q]^2} - \frac{\mathbf{k}^2}{[\mathbf{k}\mathbf{e}_q]^2} \Big) [\mathbf{k}\mathbf{k}' - (\mathbf{k}\mathbf{e}_q)(\mathbf{k}'\mathbf{e}_q)] \Big\}. \quad (4.6)$$

Here

$$D(k-k') = 1 - \Psi\Pi_n - \Phi_{eff}(k-k')\Pi, \quad \omega = \alpha\mathbf{k}^2, \quad \omega' = \alpha\mathbf{k}'^2.$$

We have assumed here that

$$\tau^{(3)}(\mathbf{k},\mathbf{k}') = 0, \quad \alpha_{ij} k_i k_j = \alpha \mathbf{k}^2 = (\alpha_H + \alpha_s)\mathbf{k}^2,$$

$$\alpha_s = -\frac{\Psi\nu_s \hbar^2 \Omega_0}{24 m_s \varepsilon_F}, \quad d_s = \frac{\Psi n_s}{\Omega_0 m_s}, \quad d_c = \frac{\Psi n_c}{\Omega_0 m_c}, \quad (4.7)$$

$\nu_c = n_c/\varepsilon_F$ and $\nu_s = 3n_s/2\varepsilon_F$ are the electron densities of state on the cylindrical and spherical sections of the Fermi surface, n_c and n_s are the number densities of the electrons filling the corresponding bands, and the quantities Ψ and $\hbar\Omega_0$ are connected through the equation of state $(\hbar\Omega_0/\varepsilon_F)^2 = 96(1+\Psi\nu)/\Psi\nu_s$ where ν is the total number of electron states on the Fermi surface. We have here in Eqs. (4.2)–(4.6) neglected terms $\propto \alpha^2/\Omega_0^2$ which are small compared to terms $\sim (\alpha^2/\Omega_0^2)(\varepsilon_F/\hbar\Omega_0)^2 \sim \alpha d_{s,c}/\Omega_0^2$.

One can simplify the expression for the function $\tau(\mathbf{k},\mathbf{k}')$ considerably when the electrons of the cylindrical sections of the Fermi surface turn out to be "light": $m_c \ll m_s$ and $n_c \gtrsim n_s$. In that case we arrive, using Eqs. (2.11), (4.2)–(4.6) and neglecting terms containing $t_{ijlm}^{(1)}, \tau^{(2)}, \tau^{(5)}$, which are smaller than the terms with $t_{ijlm}^{(0)}$ and $\tau^{(4)}$ by a factor $n_c m_s/n_s m_s \gg 1$, at the following expression for this function:

$$\tau(\mathbf{k},\mathbf{k}') = \frac{\alpha d_c}{\Omega_0^2} \mathbf{k}^2 \mathbf{k}'^2 \sum_q \Big\{ 2\frac{[\mathbf{k}\mathbf{e}_q]^2}{\mathbf{k}^2} + \theta(\mathbf{k}^2 - \mathbf{k}'^2) \times \Big(\frac{[\mathbf{k}'\mathbf{e}_q]^2}{\mathbf{k}'^2} - \frac{[\mathbf{k}\mathbf{e}_q]^2}{\mathbf{k}^2} \Big) \Big\}. \quad (4.8)$$

Substituting (4.8) into the dispersion Eq. (2.5) we find the magnon spectrum in the model considered here of a metal with an anisotropic electron dispersion law:

$$\omega(\mathbf{k},T) = \omega(\mathbf{k},0) - \frac{\alpha d_c}{S} \sum_q \int \frac{d\mathbf{k}'}{(2\pi)^3} \mathbf{k}'^2 \operatorname{cth}\Big(\frac{\hbar\alpha\mathbf{k}'^2}{2\varkappa T} \Big) \times \Big\{ \frac{[\mathbf{k}\mathbf{e}_q]^2}{\mathbf{k}^2} + \theta(\mathbf{k}^2 - \mathbf{k}'^2) \Big(\frac{1}{3} - \frac{[\mathbf{k}\mathbf{e}_q]^2}{2\mathbf{k}^2} \Big) \Big\}. \quad (4.9)$$

Performing the integration in (4.9) and retaining only the main terms $\sim T^{5/2}$ we arrive at the following expression for the coefficient:

$$B = \bar{B} + 2B_a(\mathbf{k}) \text{ при } \hbar\omega(\mathbf{k}) < \varkappa T,$$
$$B = \bar{B} + B_a(\mathbf{k}) \text{ при } \hbar\omega(\mathbf{k}) > \varkappa T, \quad (4.10)$$

where the quantities

$$\bar{B} = 6\pi^{1/2}\zeta\Big(\frac{5}{2}\Big) q_0 \frac{\nu_c m_s \alpha \bar{\omega}_{max}}{\nu m_c \alpha_s \Omega_0} \Big(\frac{\varkappa}{\hbar\bar{\omega}_{max}} \Big)^{5/2}, \quad (4.11)$$

$$B_a(k) = \bar{B} \frac{1}{2q_0} \sum_q \Big(\frac{3}{2} \frac{[\mathbf{k}\mathbf{e}_q]^2}{\mathbf{k}^2} - 1 \Big) \quad (4.12)$$

are of the same order of magnitude, $\bar{\omega}_{max} = 2\alpha\Omega_0^2 \varepsilon_F/m_s$. It follows from (4.11), (4.12) that in this case the quantity $B_a(\mathbf{k})$ vanishes when averaged over the directions of the vector $\mathbf{k}$. We note that there is no anisotropic part $B_a(\mathbf{k})$ of the coefficient B in the case of cubic symmetry of the ferromagnet, when the anisotropy of the electron energy dispersion is determined by three mutually orthogonal vectors $\mathbf{e}_q$.

On the other hand, under the conditions when the quantity (4.12) occurs the sudden change in the coefficient (4.10) in the quantity $B_a(\mathbf{e})$ in the vicinity of $\omega(k) \sim \varkappa T/\hbar$ corresponds to an anomalous temperature dependence of the magnon spectrum of ferromagnetic metals with an anisotropic electron energy dispersion. Comparing the coefficients (4.10)–(4.12) with the values (3.14), (3.15) obtained in the preceding section for an isotropic metal model we find that the presence of "light" electrons corresponding to the cylindrical sections of the Fermi surface leads to an appreciable increase—by a factor $m_s/m_c \gg 1$—in the quantity $B(\mathbf{k},T)$ in metals with an anisotropic electron dispersion law.

In conclusion we emphasize that the anomaly considered by us of the temperature dependence of the magnon spectrum of ferromagnetic metals caused by the effects of the dynamics of the crystal lattice and the anisotropies in the electron and magnon energies correspond to a difference in the magnon rigidity in different temperature and frequency ranges. Such a difference allows us, in particular, to see the cause of the discrepancy of the results of studies of the magnon spectrum using magnetic measurements[6] and using various methods of inelastic scattering of thermal neutrons.[7]

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