Fluctuations of the populations of acceptor levels in zero-gap semiconductors

A. A. Tarasenko and A. A. Chumak

Institute of Physics, Academy of Sciences of the Ukrainian SSR, Kiev
(Submitted 18 February 1986; resubmitted 10 November 1986)
Zh. Eksp. Teor. Fiz. **92**, 1745–1753 (May 1987)

A theory of fluctuations of the acceptor population is developed. It is shown that a characteristic relaxation frequency ν is governed not only by the width of the acceptor level, but also by the acceptor concentration N_A. An increase in N_A reduces the frequency ν and this increases the fluctuations of the population in the low-frequency range. This theory is generalized to the case of degeneracy of electron states at one impurity. The commutation relationships for the electron creation and annihilation operators differ then from the usual Fermi operators, because they allow for the fact that not more than one electron can reside at an impurity.

The interaction of carriers with impurities in zero-gap semiconductors is a subject of considerable topical interest.[1-3] Impurity levels lie within the continuous part of the spectrum so that an electron can always leave a bound state without a change in its energy. The probability of such a transition increases on increase in the density of the final states, i.e., on increase in the effective mass of electrons in the relevant energy band. In HgTe-type semiconductors the mass of a hole m_e is much greater than the mass of an electron m_e, so that acceptor levels are far fewer than donor levels. The energy ϵ_A of binding a hole to an acceptor divided by the width Γ of a level is of the order of $(m_h/m_e)^{3/2}$ (Refs. 1 and 4). The presence of a sharp maximum in the density of states of the conduction band electrons at an energy ε_A gives rise to a nonmonotonic temperature dependence of the conductivity if the Fermi level F_e is close to ε_A (see, for example, Ref. 5).

The capture of carriers by acceptor levels and the release from these levels are random processes, which give rise to additional noise in a sample. Relaxation of fluctuations of the population of acceptor levels δn_A is governed by the level width Γ and by the rate of exchange of particles between states with different energies. If a semiconductor is subjected to a dc voltage, then the fluctuations δn_A give rise to fluctuations of the current. These occur not only because of a change in the number of electrons participating in the electrical conduction process, but also because of a change in the number of charged impurities which scatter free carriers.

DERIVATION OF THE CORRELATION FUNCTION OF FLUCTUATIONS OF THE POPULATION OF ELECTRON STATES NEAR ENERGIES OF ACCEPTOR LEVELS

The populations of one-electron states $|\alpha\rangle$ are described by occupation-number operators $\hat{f}_\alpha = a_\alpha{}^+ a_\alpha$, where a_α^+ and a_α are the particle creation and annihilation operators. We shall assume that the wave functions $|\alpha\rangle$ represent electron states in a field of impurities. The density of the distribution of the particle energies can be expressed in terms of $\hat{f}_\alpha$ and in terms of the imaginary part of the Green function $G_{\alpha\alpha}(\varepsilon)$ as follows:

$$\hat{n}^\varepsilon = \pi^{-1} \sum_\alpha G_{\alpha\alpha}(\varepsilon) \hat{f}_\alpha, \tag{1}$$

where $G_{\alpha\alpha}(\varepsilon) = \pi\delta(\varepsilon - \hat{H})_{\alpha\alpha}$, and $\hat{H}$ is the Hamiltonian of an electron in an impurity field. The time dependence of the number of particles with a given energy is governed by the evolution of the Heisenberg operator $\hat{n}^\varepsilon$. We shall assume that in addition to a strong interaction of electrons with impurities, there is also a weak electron-phonon interaction which we shall allow for (as usual) in the Born approximation. The Hamiltonian is of the form

$$\hat{H}_{ep} = \sum_{\alpha\alpha'\mathbf{q}} V_\mathbf{q} \langle\alpha| e^{i\mathbf{qr}} |\alpha'\rangle (b_\mathbf{q} + b_{-\mathbf{q}}^+) a_\alpha{}^+ a_{\alpha'}, \tag{2}$$

where $b_\mathbf{q}^+$ and $b_\mathbf{q}$ are the phonon operators. Employing the explicit form of Eq. (2), we can readily obtain the following expression for the time derivative of $\hat{n}^\varepsilon$:

$$\partial_t n^\varepsilon = \frac{1}{i\pi\hbar} \sum_{\alpha\alpha'\mathbf{q}} G_{\alpha\alpha}(\varepsilon)\, V_\mathbf{q} (b_\mathbf{q} + b_{-\mathbf{q}}^+) [\langle\alpha| e^{i\mathbf{qr}} |\alpha'\rangle a_\alpha{}^+ a_{\alpha'} - \langle\alpha'| e^{i\mathbf{qr}} |\alpha\rangle a_{\alpha'}{}^+ a_\alpha]. \tag{3}$$

Since we are interested in the time dependence of the population of the acceptor levels, we shall limit our analysis to such energies ε which are within the level width: $|\varepsilon - \varepsilon_A| \lesssim \Gamma$. The right-hand side of Eq. (3) governs the electron-phonon collision integral and a Langevin source of fluctuations $j^L(\varepsilon,t)$. This is of identical form with the right-hand side of Eq. (3), but the equations of motion of the operators a, a^+, b, and b^+ occurring in $j^L(\varepsilon,t)$ are exactly the same as in the case of noninteracting particles.[6] Simple calculations yield the following expression for the spectral function of the Langevin source correlator:

$$\langle j^L(\varepsilon,t) j^L(\varepsilon',t')\rangle_\omega = \frac{1}{\hbar} \sum_{\alpha\alpha'\mathbf{q}} |V_\mathbf{q}|^2 \delta(\varepsilon - \varepsilon_\alpha) |\langle\alpha| e^{i\mathbf{qr}} |\alpha'\rangle|^2 \times [\delta(\varepsilon' - \varepsilon_\alpha) - \delta(\varepsilon' - \varepsilon_{\alpha'})] \{[(N_\mathbf{q}+1)\delta(\varepsilon - \varepsilon_\alpha - \omega_\mathbf{q}) + N_\mathbf{q}\delta(\varepsilon - \varepsilon_{\alpha'} + \omega_\mathbf{q})] f_\alpha (1 - f_{\alpha'}) + [(N_\mathbf{q}+1)\delta(\varepsilon_{\alpha'} - \varepsilon - \omega_\mathbf{q}) + N_\mathbf{q}\delta(\varepsilon_{\alpha'} - \varepsilon + \omega_\mathbf{q})] f_{\alpha'}(1 - f_\alpha)\}, \tag{4}$$

where $\omega_\mathbf{q}$ and $N_\mathbf{q}$ are the energy and number of phonons with a momentum q; ε_α and f_α are the energy and number of electrons in a state $|\alpha\rangle$. Equation (4) is derived in the semiclassical approximation, i.e., on the assumption that $\hbar\omega \ll \varepsilon_A, T$, where T is the temperature in energy units.

We can simplify Eq. (4) by eliminating the α' dependence of the expression in the braces. This can be done because under equilibrium conditions the function f'_α depends only on the energy ε'_α. In fact, if we multiply the expression in the braces by $\delta(\varepsilon_1 - \varepsilon'_\alpha)$ and integrate with respect to ε_1,

 0038-5646/87/050978-05$04.00

we find that we can replace ε'_α with ε_1 in the braces. After this transformation and averaging over the impurity configuration, we obtain

$$\langle j^L(\varepsilon, t)\, j^L(\varepsilon', t')\rangle_\omega$$
$$= \frac{2}{\pi^2\hbar}\sum_{\mathbf{q}} |V_{\mathbf{q}}|^2 \int_{-\infty}^{\infty} d\varepsilon_1 \,\mathrm{Tr}\cdot\{\overline{\hat{G}(\varepsilon)\, e^{i\mathbf{qr}}\hat{G}(\varepsilon_1)\, e^{-i\mathbf{qr}}}\}$$
$$\times [\delta(\varepsilon-\varepsilon') - \delta(\varepsilon_1-\varepsilon')]\,[(N_{\mathbf{q}}+1)\,\delta(\varepsilon-\varepsilon_1-\omega_{\mathbf{q}})$$
$$+ N_{\mathbf{q}}\delta(\varepsilon-\varepsilon_1+\omega_{\mathbf{q}})]\, f_\varepsilon(1-f_{\varepsilon_1}). \qquad (5)$$

The bar above Tr{...} denotes averaging over the impurity configuration. Equation (5) is derived using the equilibrium relationship

$$f_\varepsilon(1-f_{\varepsilon_1})\,(N_{\mathbf{q}}+1) = f_{\varepsilon_1}(1-f_\varepsilon)N_{\mathbf{q}}, \qquad \text{if} \qquad \varepsilon=\varepsilon_1+\omega_{\mathbf{q}}.$$

The electron-phonon collision integral is obtained subject to the same approximations as Eq. (5), by the method of equations of motion for the operators b^+a^+a and ba^+a. We then find

$$\partial_t \delta n^\varepsilon = -\frac{2\pi}{\hbar}\sum_{\alpha\alpha'\mathbf{q}} \delta(\varepsilon-\varepsilon_\alpha)\,|V_{\mathbf{q}}|^2\,|\langle\alpha|e^{i\mathbf{qr}}|\alpha'\rangle|^2$$
$$\times\delta\{[f_\alpha(1-f_{\alpha'})\,(N_{\mathbf{q}}+1) - f_{\alpha'}(1-f_\alpha)N_{\mathbf{q}}]\,\delta(\varepsilon-\varepsilon_{\alpha'}-\omega_{\mathbf{q}})$$
$$+[f_\alpha(1-f_{\alpha'})N_{\mathbf{q}} - f_{\alpha'}(1-f_\alpha)\,(N_{\mathbf{q}}+1)]\,\delta(\varepsilon-\varepsilon_{\alpha'}+\omega_{\mathbf{q}})\} + j^L(\varepsilon, t), \qquad (6)$$

where $\delta n^\varepsilon = \hat{n}^\varepsilon - \langle\hat{n}^\varepsilon\rangle$. The symbol δ in front of the braces means that the functions f_α and $f_{\alpha'}$ should be replaced with $f_\alpha + \delta f_\alpha$ and $f_{\alpha'} + \delta f_{\alpha'}$ and terms in linear in δf should be retained.

We shall simplify the problem by averaging Eq. (6) over a time interval Δt which is considerably greater than $\hbar/\Gamma$, but smaller than the characteristic relaxation times of fluctuations of the acceptor level population. After such averaging we find that $\delta f_\alpha(t)$ no longer depends on the index α, which determines the coordinate of an impurity at which an electron is localized. In fact, in a time Δt an electron can migrate between many impurities since having left one impurity it may be captured by another impurity as a result of a resonant interaction and so on. Since $\delta f_\alpha = \delta f_\varepsilon$, it follows that

$$\delta n^\varepsilon = \sum_\alpha \delta(\varepsilon-\varepsilon_\alpha)\,\delta f_\alpha = g(\varepsilon)\,\delta f_\varepsilon,$$

where

$$g(\varepsilon) = \sum_\alpha \delta(\varepsilon-\varepsilon_\alpha)$$

is the density of the electron states in the conduction band. Subject to the above comments, Eq. (6) becomes

$$\partial_t \delta f_\varepsilon = -\frac{2}{\pi\hbar g(\varepsilon)}\sum_{\mathbf{q}} |V_{\mathbf{q}}|^2 \int_{-\infty}^{\infty} d\varepsilon_1\,\mathrm{Tr}\{\overline{\hat{G}(\varepsilon)\, e^{i\mathbf{qr}}\hat{G}(\varepsilon_1)\, e^{-i\mathbf{qr}}}\}$$
$$\times\,\delta\{[f_\varepsilon(1-f_{\varepsilon_1})\,(N_{\mathbf{q}}+1) - f_{\varepsilon_1}(1-f_\varepsilon)\,N_{\mathbf{q}}]\,\delta(\varepsilon-\varepsilon_1-\omega_{\mathbf{q}})$$
$$+[f_\varepsilon(1-f_{\varepsilon_1})\,N_{\mathbf{q}} - f_{\varepsilon_1}(1-f_\varepsilon)\,(N_{\mathbf{q}}+1)]\,\delta(\varepsilon-\varepsilon_1+\omega_{\mathbf{q}})\}$$
$$+\frac{1}{g(\varepsilon)}\, j^L(\varepsilon, t). \qquad (7)$$

As in Eq. (6), we averaged over the impurity configurations.

The influence of impurities on fluctuations δf_ε is due to the dependence of the density of states $g(\varepsilon)$ and also of Tr{...} on the impurity concentration and on the acceptor level parameters. If the number of impurities is small, so that the wave functions of electrons at neighboring impurities do not overlap, we can average over the impurity configurations on the assumption that

$$\overline{\hat{G}(\varepsilon)\, e^{i\mathbf{qr}}\hat{G}(\varepsilon_1)\, e^{-i\mathbf{qr}}} \approx \overline{\hat{G}(\varepsilon)}\, e^{i\mathbf{qr}}\overline{\hat{G}(\varepsilon_1)}\, e^{-i\mathbf{qr}}$$

and then the kinetics of electrons interacting with phonons in the field of acceptor impurities is determined entirely by the imaginary part of the one-particle Green function averaged over the impurity configurations. The approximations for the latter are known (see, for example, Ref. 7). After such transformations the problem of fluctuations of the electron distribution function reduces to solution of the transport Eq. (7) in which the collisional integral and the Langevin source correlator are renormalized by a strong interaction with the acceptor impurities.

In Eqs. (5) and (7) we can go to the limit by considering the case when there are no impurities. Then, the eigenfunction of the one-electron Hamiltonian are Bloch functions governed by the quasimomentum $\mathbf{p}$ and

$$\langle\mathbf{p}|G(\varepsilon)|\mathbf{p}'\rangle = \delta_{\mathbf{pp}'}\pi\delta(\varepsilon-\varepsilon_{\mathbf{p}}).$$

Using this expression, we readily find that the relaxation term in Eq. (7) reduces to the usual Boltzmann collision integral averaged over the constant-energy surface of electron in the conduction band. The correlation function of Eq. (5) can be simplified in a similar manner.

CALCULATION OF FLUCTUATIONS OF THE ACCEPTOR POPULATION

The solution of Eq. (7) depends strongly on the degree of inelasticity of the electron-phonon collisions, i.e., on the characteristic values of the energy $\omega_{\mathbf{q}}$. We shall now consider the case when $\overline{\omega}_{\mathbf{q}} \gg \Gamma$. This inequality means that an electron of energy $\varepsilon(|\varepsilon-\varepsilon_A| \lesssim \Gamma)$ which has emitted or absorbed a phonon finds itself far outside the limits of the level width. Such electrons are no longer bound to impurities and behave as ordinary band electrons. We shall assume the frequency of resonant scattering to be the largest parameter (compared with $\Gamma/\hbar$ and the momentum relaxation frequency). Then, in a time Δt the electrons outside the limits of a resonance level become thermalized as a result of electron-phonon and electron-electron collisions. Their distribution functions differ then only from the equilibrium function solely because of fluctuations of the acceptor population and because of the corresponding changes in the number of electrons n in the continuous spectrum.

We therefore have

$$\delta f_{\varepsilon_1} = \delta n\,\partial_n f_{\varepsilon_1},$$

where

$$\delta n = -\delta n_A = -\int_{\varepsilon_A-\Delta}^{\varepsilon_A+\Delta} d\varepsilon\,\delta n^\varepsilon$$

and δn_A are the fluctuations of the acceptor populations; the energy Δ is selected in such a way that the interval 2Δ includes all the additional states to pure acceptors ($\Delta \gtrsim \Gamma$). Substituting $\delta f_{\varepsilon 1}$ in Eq. (7), we obtain

$$\partial_t \delta f_\varepsilon = -\frac{1}{\pi g(\varepsilon)} \sum_{\mathbf{p}} G_{\mathbf{p}}^{\varepsilon} \nu_\varepsilon(\mathbf{p})\,(\delta f_\varepsilon + \delta n_A \partial_n f_\varepsilon) + \frac{1}{g(\varepsilon)} j^L(\varepsilon, t), \tag{8}$$

where $G_{\mathbf{p}}^{\varepsilon} \equiv \langle \mathbf{p}|\, \widehat{G(\varepsilon)}\, |\mathbf{p}\rangle$;

$$\nu_\varepsilon(p) = \frac{2\pi}{\hbar} \sum_{\mathbf{p}',\mathbf{q}} |V_{\mathbf{q}}|^2 |\langle \mathbf{p}|e^{i\mathbf{q}\mathbf{r}}|\mathbf{p}'\rangle|^2 [(N_{\mathbf{q}}+1-f_{\mathbf{p}'})\delta(\varepsilon-\varepsilon_{\mathbf{p}'}-\omega_{\mathbf{q}}) + (N_{\mathbf{q}}+f_{\mathbf{p}'})\delta(\varepsilon-\varepsilon_{\mathbf{p}'}+\omega_{\mathbf{q}})].$$

In the derivation of Eq. (8) we have allowed for the fact that the relaxation term on the right-hand side of Eq. (7) vanishes if we assume that δf_ε and $\delta f_{\varepsilon 1}$ are equal to $\delta n \partial_n f_\varepsilon$ and $\delta n \partial_n f_{\varepsilon 1}$, respectively.

If we ignore the Langevin source on the right-hand side of Eq. (8), the above equation describes the process of relaxation of perturbations of the populations of electron states of energy ε.

We shall describe the function $G_{\mathbf{p}}^{\varepsilon}$ by the following approximation[7]:

$$G_{\mathbf{p}}^{\varepsilon} = \mathrm{Im}\,[\varepsilon - \varepsilon_{\mathbf{p}} - A^2/(\varepsilon - \varepsilon_A - i\Gamma)]^{-1}, \tag{9}$$

where $A^2 = N_A \Gamma/\pi g_0(\varepsilon_A)$, N_A is the number of acceptors, and $g_0(\varepsilon)$ is the unperturbed density of electron states:

$$g_0(\varepsilon) = \sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}).$$

Since $G_{\mathbf{p}}^{\varepsilon}$ is independent of the direction of $\mathbf{p}$, the relaxation of δf_ε is governed by the average (over the directions of $\mathbf{p}$) frequency $\nu_\varepsilon(\mathbf{p})$, which from now on we shall denote by $\nu_\varepsilon(\mathbf{p})$. If $A^2 \ll \Gamma\varepsilon_A$, i.e., $N_A \ll 3\pi n/2$, then $G_{\mathbf{p}}^{\varepsilon}$ has a sharp peak at $\varepsilon = \varepsilon_{\mathbf{p}}$ and in Eq. (8) we can sum over $\mathbf{p}$, which gives

$$\partial_t \delta f_\varepsilon = -\frac{g_0(\varepsilon_A)}{g(\varepsilon)} \nu_\varepsilon (\delta f_\varepsilon + \delta n_A \partial_n f_\varepsilon) + \frac{1}{g(\varepsilon)} j^L(\varepsilon, t), \tag{10}$$

where the value of ν_ε corresponds to the momentum $\mathbf{p}$ satisfying the condition $\varepsilon_{\mathbf{p}} = \varepsilon_A$. It follows from Eq. (10) that the relaxation frequency ν of perturbations of the populations of states of energy ε is $\nu_\varepsilon g_0(\varepsilon)/g(\varepsilon)$. If the density of the impurity states is considerably greater than $g_0(\varepsilon)$ $[N_A \gg \pi\Gamma g_0(\varepsilon)]$, $g(\varepsilon)$ is governed by the concentration of impurities using the relationship[8]

$$g(\varepsilon) = N_A \Gamma/\pi[(\varepsilon - \varepsilon_A)^2 + \Gamma^2].$$

In this case we have $g(\varepsilon) \gg g_0(\varepsilon)$ and the characteristic relaxation frequency of the population of the acceptors decreases on increase in their number. We can clarify the physical meaning of this result by representing the relaxation frequency in a somewhat different form. We note that the imaginary part of the denominator of the Green function of Eq. (9) represents decay of an electron state of energy close to $\varepsilon_{\mathbf{p}}$. In other words, the quantity $\mathrm{Im}\, 2A^2\hbar^{-1} \times (\varepsilon - \varepsilon_A - i\Gamma)^{-1}$, which for $\varepsilon = \varepsilon_A$ is equal to

$$2N_A/\pi\hbar g_0 \sim \Gamma g(\varepsilon)/\hbar g_0(\varepsilon)$$

represents the frequency of ν_r of resonance scattering of electrons of energy close to ε_A. We then readily find that the frequency ν is given by

$$\nu = \frac{g_0(\varepsilon)}{g(\varepsilon)} \nu_\varepsilon \sim \frac{\Gamma}{\hbar} \frac{\nu_\varepsilon}{\nu_r}. \tag{11}$$

We recall that Eq. (10) is valid if $\nu_r \gg \nu_\varepsilon$. In this case the relaxation frequency of acceptors is considerably less than $\Gamma/\hbar$. The dependence of ν on Γ, ν_ε, and ν_r described by Eq. (11) is due to the fact, as already pointed out, that electrons leaving localized states (at a frequency $\Gamma/\hbar$) with a high probability (characterized by a frequency ν_r) are captured by other impurities and only a small fraction of them escapes beyond the limits of the level width (at a frequency ν_ε) colliding with phonons or other electrons.

It readily follows from Eq. (10) that the Fourier component $\delta f_\varepsilon(\omega)$ is maximal and is independent of the frequency ω if $\omega \ll \Gamma\nu_\varepsilon/\hbar\nu_r$. At higher frequencies (but still lower than $\Gamma/\hbar$) the value of $\delta f_\varepsilon(\omega)$ decreases on increase in the frequency as ω^{-1}. In the low-frequency range we readily obtain

$$\delta f_\varepsilon(\omega) = -\delta n_A \partial_n f_\varepsilon + j^L(\varepsilon)/g_0(\varepsilon)\nu_\varepsilon. \tag{12}$$

Using the explicit form of the frequency $\nu_\varepsilon(\mathbf{p})$, we find that the correlation function of Eq. (5) can be written in the simpler form

$$\langle j^L(\varepsilon) j^L(\varepsilon')\rangle_\omega = \frac{1}{\pi^2}\delta(\varepsilon - \varepsilon') \sum_{\mathbf{p}} G_{\mathbf{p}}^{\varepsilon} \nu_\varepsilon(\mathbf{p}) f_\varepsilon(1 - f_\varepsilon). \tag{13}$$

Then, in the same approximation as that used in Eq. (12) the correlation function of fluctuations of the acceptor population becomes

$$\langle (\delta n_A)^2\rangle_\omega = \frac{1}{\pi\nu_\varepsilon} f_{\varepsilon_A}(1 - f_{\varepsilon_A}) \int_{\varepsilon_A - \Delta}^{\varepsilon_A + \Delta} d\varepsilon\, g^2(\varepsilon)/g_0(\varepsilon_A)\,(1 + N_A \partial_n f_{\varepsilon_A})^2. \tag{14}$$

In the integration with respect to ϵ we assumed that $T \gg \Gamma$ and took the smoothly varying function outside the integral at the maximum of the function $g^2(\varepsilon)$. After integration with respect to ε, Eq. (14) becomes

$$\langle (\delta n_A)^2\rangle_\omega = N_A^2 f_{\varepsilon A}(1 - f_{\varepsilon A})/2\pi^2 \nu_\varepsilon \Gamma g_0(\varepsilon_A)\,(1 + N_A \partial_n f_{\varepsilon A})^2. \tag{15}$$

Using the above expression for the relaxation frequency ν, we can rewrite Eq. (15) as follows:

$$\langle (\delta n_A)^2\rangle_\omega = 3nTX/4\pi\nu F_e (1 + X)^2, \tag{16}$$

where

$$X = N_A \partial_n f_{\varepsilon_A} = 2(N_A F_e/3nT) f_{\varepsilon_A}(1 - f_{\varepsilon_A}).$$

The Fermi energy F_e is determined by the number of unbound electrons n in accordance with the usual expression $F_e \propto n^{2/3}$. In numerical estimates of the quantity $\langle (\delta n_A)^2\rangle_\omega$ we shall bear in mind that the value of the parameter N_A is limited to the range of validity of our formulas. In particular, the peak of the function $G_{\mathbf{p}}^{\varepsilon}$ is small if $N_A \ll 3\pi n/2$. However, we can easily see that even for comparable values of N_A and $3\pi n/2$, Eq. (15) gives the correct order of magnitude of the value of $\langle (\delta n_A)^2\rangle_\omega$ and Eq. (16) can be regarded generally as exact, although the relaxation frequency differs from the value given by Eq. (11). We find the frequency ν for the general case if we know the explicit form of $\nu_\varepsilon(p)$ and $G_{\mathbf{p}}^{\varepsilon}$ in a wide range of ε. It should be pointed out that the approximate expression for $G(\varepsilon)$ for practically all values of ε and N_A can be found in Ref. 9. It follows from Eq. (16) that

fluctuations of the acceptor population are small if $|\varepsilon_A - F_e| \gg T$. The fluctuations are maximal if the parameters of the system are such that $X = 1$ and the frequency ν is low, which is true at high values of N_A. At the limit of validity of our theory we have $N_A^{max} \sim n$. It is important to point out that at low temperatures we can expect electrons to leave states of energies $|\varepsilon - \varepsilon_A| \lesssim \Gamma$ not because of the electron-phonon interaction, but because of electron-electron collisions. Then the frequency $\nu_\varepsilon(p)$ is governed by the linearized electron-electron collision integral and Eqs. (13)–(16) remain valid.

We shall now consider the case when the characteristic phonon energy is low compared with the level width: $\overline{\omega}_q \ll \Gamma$. This inequality is obeyed only in the interaction with acoustic phonons and can be reduced to the following expression: $sp_A \ll \Gamma$ (s is the velocity of sound and p_A is the momentum of a band electron of energy ε_A). In this case Eq. (7) simplifies greatly after expansion of the relaxation contribution in terms of a small parameter ω_q. Simple transformations give

$$\partial_t \delta f_\varepsilon = \frac{1}{g(\varepsilon)} \partial_\varepsilon L_\varepsilon (\partial_\varepsilon + \beta_\varepsilon) \delta f_\varepsilon + \frac{1}{g(\varepsilon)} j^L(\varepsilon, t), \qquad (17)$$

where $\beta_\varepsilon = (1 - 2f_\varepsilon)/T$;

$$L_\varepsilon = \frac{2}{\pi\hbar} \sum_q |V_q|^2 \omega_q^2 N_q \operatorname{Sp} \{\overline{\hat{G}(\varepsilon) e^{iqr} \hat{G}(\varepsilon) e^{-iqr}}\}.$$

Similar parameters can be used also to describe the correlation function of Eq. (5):

$$\langle j^L(\varepsilon, t) j^L(\varepsilon', t') \rangle_\omega = -\frac{1}{\pi} \partial_\varepsilon L_\varepsilon (\partial_\varepsilon + \beta_\varepsilon) f_\varepsilon (1 - f_\varepsilon) \delta(\varepsilon - \varepsilon')$$
$$= \frac{1}{\pi} \partial_\varepsilon \partial_{\varepsilon'} L_\varepsilon f_\varepsilon (1 - f_\varepsilon) \delta(\varepsilon - \varepsilon'). \qquad (18)$$

It follows from Eq. (17) that the process of energy relaxation is a form of diffusion. If we assume that the characteristic parameter of the change δf_ε is the temperature T (i.e., if in estimates we assume that $\partial_\varepsilon \sim 1/T$), we find that $L_\varepsilon / g(\varepsilon) T^2$ can be regarded as a characteristic value of the electron energy relaxation frequency. Since we are interested in the relaxation frequency of the populations of the acceptor levels, the latter will be clearly T/Γ times greater than the value just given because electrons escape outside the level width when their energy changes by a value of the order of Γ. Adopting the same approximations as before, we obtain

$$\operatorname{Tr} \overline{\{\hat{G}(\varepsilon) e^{iqr} \hat{G}(\varepsilon) e^{-iqr}\}} \approx \pi^2 \sum_p \delta(\varepsilon - \varepsilon_p) \delta(\varepsilon_p - \varepsilon_{p-q}).$$

Then, the coefficient L_ε is of the order of $g_0(\varepsilon) \nu_\varepsilon \overline{\omega_q^2}$, where ν_ε has the same meaning as before, i.e., it determines the frequency of collisions of band electrons with phonons. We then find that the characteristic frequency of the relaxation of the acceptor populations is of the order of

$$\nu = \nu_\varepsilon \frac{g_0(\varepsilon_A)}{g(\varepsilon_A)} \frac{\overline{\omega_q^2}}{T\Gamma} \sim \frac{\nu_\varepsilon}{\nu_r} \frac{\overline{\omega_q^2}}{\hbar T}.$$

As expected, this frequency is considerably less than in the case of a narrow level (the difference is a factor of $T\Gamma/\overline{\omega_q^2}$). The exact solution of Eq. (17) is easy and in the low-frequency case it becomes

$$\delta f_\varepsilon = \frac{2F_e \partial_\varepsilon f_e}{3n(1+X)} \int_0^\infty d\varepsilon' g(\varepsilon') \partial_{\varepsilon'} f_{\varepsilon'} \int_{\varepsilon'}^{\varepsilon} \frac{d\varepsilon_1}{L_{\varepsilon_1} \partial_{\varepsilon_1} f_{\varepsilon_1}} \int_0^{\varepsilon_1} j^L(\varepsilon_2) d\varepsilon_2. \qquad (19)$$

We can determine the integration constants from the conservation of the total number of electrons ($\int \delta f_\varepsilon g_\varepsilon d\varepsilon = 0$), which is equivalent to the condition $\delta n = -\delta n_A$, used earlier, and from the boundary condition $(\partial_\varepsilon + \beta_\varepsilon)\delta f_\varepsilon = 0$ in the limit $\varepsilon \to \infty$. Equation (19) is derived using also the identity

$$\exp\left\{-\int_{\varepsilon'}^{\varepsilon} \beta_{\varepsilon_1} d\varepsilon_1\right\} = \partial_\varepsilon f_\varepsilon / \partial_{\varepsilon'} f_{\varepsilon'}.$$

Equations (18) and (19) yield the following expression for the correlation function of the fluctuations of the acceptor populations:

$$\langle (\delta n_A)^2 \rangle_\omega = \frac{3n}{\pi} \frac{TN_A}{F_e L_{F_e}} f_{\varepsilon_A}(1 - f_{\varepsilon_A}) \frac{X}{(1+X)^2} \operatorname{ch} \frac{\varepsilon_A - F_e}{T}. \qquad (20)$$

The function $f_{\varepsilon A}(1 - f_{\varepsilon A}) \operatorname{ch}[(\varepsilon_A - F_e)/T]$ depends weakly on ε_A. If $\varepsilon_A = F_e$, it amounts to 1/4. Comparing at this point Eqs. (20) and (16), we can see that the coefficient L_{F_e}/TN_A, which apart from the factor π, is equal to

$$\nu_\varepsilon g_0(\varepsilon_A) \overline{\omega_q}^2 / g(\varepsilon_A) T\Gamma$$

and plays the role of the relaxation frequency of the acceptor populations. This conclusion supports the earlier quantitative estimates of this quantity.

The above theory can be applied in the case when the state of an electron at an impurity is nondegenerate. However as shown in Ref. 1, the lowest level of a localized electron is quadruply degenerate. A strong Coulomb repulsion between electrons at the same site can be allowed for automatically by assuming that the existence of such states is forbidden. In other words, we shall assume that not more than one electron may be located at one site. This condition changes the commutation relationships between the particle creation and annihilation operators for one site. If the index α denotes the coordinate of an impurity and the number of a degenerate state is identified by an index i, then instead of the usual Fermi transposition relationships, we now have

$$a_{\alpha_i} a_{\alpha_i}^+ + \sum_{i'} a_{\alpha_{i'}}^+ a_{\alpha_{i'}} = 1. \qquad (21)$$

Then, all the above formulas can be generalized to the degenerate case by replacing N_A everywhere with $4N_A$. Moreover, the average occupation number of a state α_i with an energy ε differs from that in the case of the Fermi distribution and is given by

$$f_\varepsilon^i = \left[1 + 4 \exp\left(\frac{\varepsilon - F_e}{T}\right)\right]^{-1}.$$

Consequently, in all the above formulas we have to replace f_ε with f_ε^i and the factors $1 - f_\varepsilon$ with $1 - 4f_\varepsilon^i \equiv 1 - \Sigma_i f_\varepsilon^i$.

NUMERICAL ESTIMATES

We shall now estimate the influence of fluctuations of the acceptor populations on the general noise in a semiconductor. We shall consider the case when the levels are due to the presence of mercury vacancies in HgTe. The region of

localization of the impurity potential is of the same order as the lattice constant. The wave function of the lower level has similar typical dimensions. Therefore, the wave functions of electrons at the nearby impurities do not overlap and do not contribute to the process of conduction in such a semiconductor. A fluctuation δn_A alters a current (in the presence of a given electric field $\mathbf{E}$) flowing in the direction of $\mathbf{E}$ and the change in the current is

$$\delta \mathbf{j} = \delta n_A \partial_{n_A} \sigma \mathbf{E}, \tag{22}$$

where σ is the electrical conductivity. We can estimate $\delta_{nA}\sigma$ if we know the electron momentum relaxation frequency ν_p. Since the energy ε_A is low ($\varepsilon_A = 2.2$ meV for mercury vacancies in HgTe), it follows that for $F_e \sim \varepsilon_A$ the main electron scattering mechanism is the interaction with ionized impurities. In this case we have

$$\sigma \propto n F_e^{3/2}/N_I \propto n^2/N_I,$$

where N_I is the number of ionized impurities. The change in the conductivity σ is

$$\delta\sigma = -\delta n_A \sigma (2+n/N_I)/n.$$

We shall assume that $n \propto N_I$. We then obtain $\partial_{nA}\sigma = -3\sigma/n$. We shall compare the correlation function of the current fluctuations given by Eq. (22) with the thermal noise S_ω given by the Nyquist formula $\sigma T/\pi$. Using Eqs. (11) and (16) and allowing for the quadruple degeneracy of the impurity levels, we readily find that

$$\langle (\partial_{n_A} \sigma \mathbf{E} \delta n_A)^2 \rangle_\omega S_\omega^{-1} \approx \frac{18}{\pi} \frac{N_A}{n} \frac{(e\mathbf{E})^2}{m_e \nu_\varepsilon \nu_p \Gamma} \frac{X}{(1+X)^2}.$$

If we substitute in this formula the values $X = 1$, $n = N_A$, $\nu_p = 10^{12}$ sec^{-1}, $\nu_\varepsilon = 10^{11}$ sec^{-1} and $\Gamma = 10^{-2}\varepsilon_A$, we find that the population fluctuations make the same contribution to the noise as the thermal fluctuations if the electric field is 5 V/cm. At low temperatures even moderate fields $\mathbf{E}$ can heat the electron gas. If electron-electron collisions then control the formation of the symmetric part of the distribution function f_ε, the form of f_ε will be the same as before, except that now T should be replaced with T_e.

We conclude by noting that it is clear from the above theory that the characteristic relaxation time of perturbations of the acceptor populations is not equal to the electron lifetime at an impurity $\hbar/\Gamma$ and in the case of strong resonant scattering it is much greater than this quantity. An increase in the lifetime of the population perturbations increases the fluctuations $\langle (\delta n_A)^2 \rangle_\omega$ in the low-frequency range and these in turn can enhance greatly the noise in a sample in which an electric current is flowing. Then, Eqs. (13) and (18) cease to be exact although the initial formulas (4) and (7) remain unchanged. A similar situation occurs also in the case of nonequilibrium electron–phonon systems which are described by the Boltzmann equation (see, for example, Ref. 10).

[1]N. N. Ablyazov, B. L. Gel'mont, M. É. Raĭkh, and A. L. Éfros, Zh. Eksp. Teor. Fiz. **87**, 646 (1984) [Sov. Phys. JETP **60**, 370 (1984)].
[2]M. I. D'yakonov and A. V. Khaetskiĭ, Zh. Eksp. Teor. Fiz. **86**, 1843 (1984) [Sov. Phys. JETP **59**, 1072 (1984)].
[3]N. N. Ablyazov and M. É. Raĭkh, Fiz. Tekh. Poluprovodn. **18**, 883 (1984) [Sov. Phys. Semicond. **18**, 550 (1984)].
[4]B. L. Gel'mont, V. I. Ivanov-Omskiĭ, and I.M. Tsidil'kovskiĭ, Usp. Fiz. Nauk **120**, 337 (1976) [Sov. Phys. Usp. **19**, 879 (1976)].
[5]C. Verie, F. Raymond, F. Bailly, I. Vacquie, G. Weill, A. Kozacki, and J. Rioux, Proc. Intern. Conf. on Physics of Semiconductors, Edinburgh, 1978, publ. by Institute of Physics, London (1979), p. 241.
[6]P. M. Tomchuk and A. A. Chumak, Preprint No. 9 (in Russian), Institute of Physics, Academy of Sciences of the Ukrainian SSR, Kiev (1971); Fiz. Tverd. Tela (Leningrad) **15**, 1011 (1973) [Sov. Phys. Solid State **15**, 697 (1973)].
[7]P. Gosar, Proc. Intern. Conf. on Physics of Semiconductors, Edinburgh, 1978, publ. by Institute of Physics, London (1979), p. 269.
[8]A. Mauger and J. Friedel, Phys. Rev. B **12**, 2412 (1975).
[9]G. Bastard, Phys. Status Solidi B **80**, 641 (1977).
[10]S. V. Gantsevich, V. L. Gurevich, and R. Katilius, Riv. Nuovo Cimento **2**, No. 5, 1 (1979).

Translated by A. Tybulewicz