Critical fields of V/Si superlattices

L. I. Glazman, I.M. Dmitrenko, V. L. Tovazhnyanskiĭ, N. Ya. Fogel',
and V. G. Cherkasova

V. I. Lenin Polytechnic Institute, Kharkov
(Submitted 1 September 1986)
Zh. Eksp. Teor. Fiz. **92**, 1461–1470 (April 1987)

An experimental investigation was made of superconducting properties of metal-semiconductor superlattices formed by consecutive condensation of vanadium ($d = 220$Å) and silicon ($s = 10$, 30, 40, 60, and 240 Å) layers. The temperature dependence of the parallel critical field was affected strongly by the thickness of the silicon layers. In the case of superlattices with $s = 30$–60 Å the three-dimensional behavior of $H_{c\|}$ near T_c changed to two-dimensional at low temperatures. The critical parameters of the superlattices were compared with the corresponding parameters of their components. A theoretical model of a superlattice with a weak Josephson coupling between metal layers of finite thickness was considered. This model was used to find three independent methods for determination of the extrapolation length l, representing the force coupling the metal layers. The values of l calculated from the experimental data by all three methods agreed to within 15%. Estimates were obtained of the influence of the imperfection of the superstructure of these superlattices on the behavior of $H_{c\|}(T)$.

Synthetic superconducting superlattices represent convenient objects for the investigation of the size effects in superconductivity. Their properties, considered as a function of the superlattice parameters (superconducting layer thickness and superlattice period), can be close to the properties of three-dimensional anisotropic superconductors and to the properties of two-dimensional films. Moreover, the transition from the three-dimensional ($3D$) to the two-dimensional ($2D$) behavior is observed in the same sample as temperature is varied.[1-3] This transition is known as the crossover and it is exhibited not only by synthetic superlattices, but also by intercalated superconducting compounds.[4,5] Synthetic superlattices have an advantage over layer dichalcogenides of transition metals because we can readily set the desired thicknesses of the insulating layers and vary the coupling between the layers as well as the thickness of metal layers. Moreover, the existing technology of fabrication of synthetic superlattices can ensure their regularity to a higher degree than in the case of superconducting layered compounds intercalated with large organic molecules.

We shall report an investigation of critical magnetic fields of superconducting superlattices formed from vanadium and silicon layers. To the best of our knowledge, superlattices made of these materials have not yet been investigated. The coupling between the layers was varied by altering the thickness of the semiconductor. In contrast to the work familiar to us, we compared the properties of superlattices with the properties of single layers of vanadium and of a V/Si sandwich prepared in the course of the same evaporation cycle, i.e., it was possible to compare the behavior of a superlattice with the behavior of its components.

In addition to this experimental investigation, we carried out a theoretical analysis of a superlattice model with Josephson coupling between the layers, which gives more precise results than those obtained in Ref. 6. We considered various methods for the determination of the coupling force from the temperature dependence of the parallel critical field $H_{c\|}(T)$. Various characteristic parts of this dependence made it possible to suggest three independent methods for the determination of the coupling parameter (extrapolation length l). All these methods gave similar values of l for the same sample. We also obtained the influence of the imperfection of the superlattice structure on the behavior of $H_{c\|}(T)$.

SAMPLES AND EXPERIMENTAL METHOD

We investigated metal-semiconductor superlattices formed by consecutive evaporation of vanadium and silicon layers from different sources in a vacuum chamber in which the residual gas pressure was 1.3×10^{-6}–2.7×10^{-7} Torr. Vanadium and silicon were deposited by electron-beam evaporation using two guns of different power. A superlattice was deposited on previously annealed ($T = 800$°C) fluorophlogopite substrates kept at 100°C. The advantage of these synthetic mica (fluorophlogopite) substrates was their insensitivity to heating in a wide range of temperatures up to ~ 1000°C; moreover, films could be separated from the substrate when their structure was investigated by electron microscopy. The evaporation took place through special heat-resistant masks with a selected geometry, which made it possible to use the four-probe method in the study of electrical properties of the evaporated samples. Typical dimensions of the samples were as follows: the length between the potential contacts was 5–6 mm and the width was 1–1.5 mm. In all the investigated superlattices the thickness of vanadium was $d \approx 220$ Å, but the thickness of silicon films was varied and it amounted to $s = 10$, 30, 40, 60, and 240 Å for different samples. The thicknesses were determined, using two independent quartz sensors, on the basis of the mass of the deposited substance. The error in the determination of the vanadium layer thickness was ± 1 Å, whereas in the case of silicon layers it was ± 2 Å. All the superlattices consisted of ten V/Si packets. Some of the Samples were evaporated on silicon substrates. The top layer of a superlattice always consisted of silicon. Superlattices with different values of s were obtained in the course of different evaporation samples.

 0038-5646/87/040821-06$04.00

To be able to compare the critical parameters and to carry out electron-microscopic examinations, in a given cycle we evaporated not only a superlattice but also single films of vanadium as well as V/Si sandwiches with just one metal layer. The thickness of the V layer was always the same for a superlattice and control samples prepared in the same evaporation cycle. Transmission electron microscopy showed that the Si films were condensed in the amorphous form, whereas the vanadium films were fine-grained (grain size $\sim 10^{-6}$ cm). Measurements of electrical properties were made in a cryostat fitted with a superconducting solenoid. During measurements a sample was at the center of the solenoid and a special rod with a rotatable device made it possible to alter the orientation of the sample relative to the applied magnetic field. Parallel orientation was deduced from the minimum of the resistance of a sample to within an angle of 0.5°. The dependences $R(T)$ and $R(H)$ were determined in the region of the transition to the superconducting state inside a liquid helium bath the temperature of which was varied in the range 4.2–1.6 K by pumping out the helium vapor. The voltage across a sample was measured with a digital voltmeter accurate to within 10^{-6} V. Temperatures were kept constant at a given value to within ~ 0.003 K.

The magnetic field dependences $R(H)$ were recorded using an X-Y potentiometric plotter. All the measurements were carried out under dc conditions and the current was $100\,\mu$A. A thermal hysteresis was not observed. The critical temperature T_c and the critical fields $H_{c\parallel}$ and $H_{c\perp}$ were determined from the dependences $R(T)$ and $R(H)$ at the points $R = 0.5R_n$ (R_n is the residual resistance).

EXPERIMENTAL RESULTS

Figure 1 shows the temperature dependences of the parallel critical field $H_{c\parallel}$ for the superlattices with the silicon layer thicknesses $s = 10$, 30, and 240 Å. Clearly, the behavior of $H_{c\parallel}$ depended strongly on the thickness s. For $s = 10$ Å the value of $H_{c\parallel}$ was proportional to T_c-T throughout the investigated range of temperatures, as in the case of the usual three-dimensional superconducting systems. For $s = 240$ Å, we found that throughout the investigated temperature range the dependence $H_{c\parallel} \propto (T_c - T)^{1/2}$ was obeyed and this dependence was typical of thin films [$d \ll \xi(T)$, where d is the thickness of the metal film, and $\xi(T)$ is the coherence length]. In the case of superlattic with $s = 30$, 40, and 60 Å, it was found that cooling changed the behavior of the critical field as reported earlier for Nb/Ge and Nb/Cu superlattices.[1-3] Near the superconducting transition temperature we found that $H_{c\parallel} \propto T_c - T$, whereas at lower temperatures the corresponding dependence was $H_{c\parallel} \propto (\tilde{T}_c - T)^{1/2}$, i.e., a transition from the 3D behavior of $H_{c\parallel}$ near T_c to the 2D behavior was observed when temperature was lowered. The critical temperature $\tilde{T}_c$ found by extrapolation of the dependence $H_{c\parallel}(T)$ in the range of the 2D behavior to the value $H = 0$ was much less than the superconducting transition temperature T_c of the superlattice. The difference between the temperatures $\tilde{T}_c$ and T_c was readily demonstrated (Fig. 2) by plotting the dependences of the critical magnetic field $H_{c\parallel}$ for two superlattices with $s = 30$ and 60 Å. We also included in Fig. 2 the dependences obtained for a single V film and for a V/Si sandwich fabricated during the same evaporation cycle as the superlattice with $s = 30$ Å. The temperature dependences of the critical field were plotted using the coordinates $H_{c\parallel}^2$ and T (Fig. 3). A comparison of the results for the superlattices with $s = 30$, 40, and 60 Å, which exhibited the crossover effect, demonstrated that a reduction in the thickness s expanded the range of the linear dependence $H_{c\parallel}(T)$ near T_c.

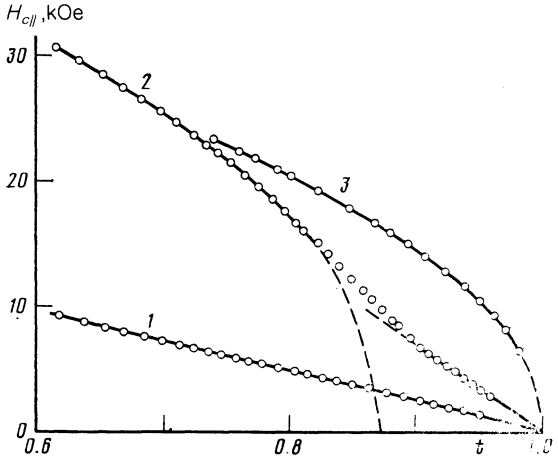

FIG. 1. Dependences of the critical magnetic field $H_{c\parallel}$ on the reduced temeprature $t = T/T_c$, plotted for three superlattices with the silicon layer thicknesses 10 Å (1), 30 Å (2), and 240 Å (3).

A comparison of the critical parameters of a superlattice (SL), a single film of vanadium, and a V/Si sandwich (Fig. 2) prepared in the same evaporation cycle led us to the conclusion that in all the investigated batches irrespective of s the critical temperatures obeyed the inequalities $T_c^v < T_c^{SL} \leqslant T_c^{V/Si}$. The inequality $T_c^v < T_c^{SL}$ was due to the fact that the V layers unprotected by silicon oxidized to a certain depth and this reduced T_c. (In the case of thin V films the critical temperature fell on reduction in the thickness.[7] Oxidation in fact reduced the thickness of the unprotected V films.) For the majority of samples we found that $T_c^{SL} = T_c^{V/Si}$, since in sandwiches the V layers were practically unoxidized. The inequality $T_c^{CP} < T_c^{V/Si}$ was clearly obeyed by those superlattice samples in which the V layers were not quite identical. The critical magnetic fields $H_{c\perp}$ of all the investigated samples varied linearly with temperature

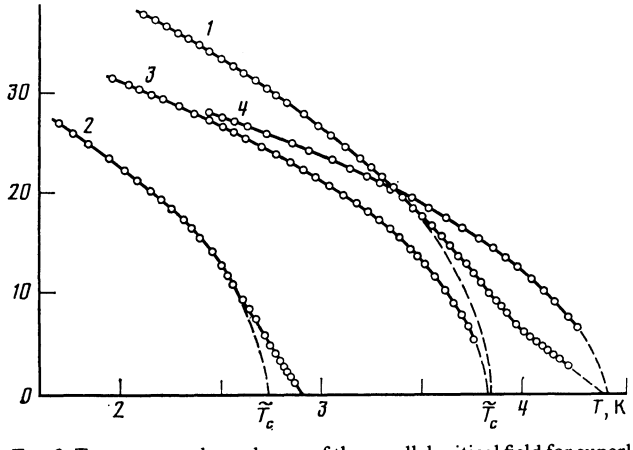

FIG. 2. Temperature dependences of the parallel critical field for superlattices with $s = 30$ Å (1) and $s = 60$ Å (2), for a single film of vanadium (3), and for a V/Si sandwich (4). Samples 1, 3, and 4 were prepared in the same deposition cycle.

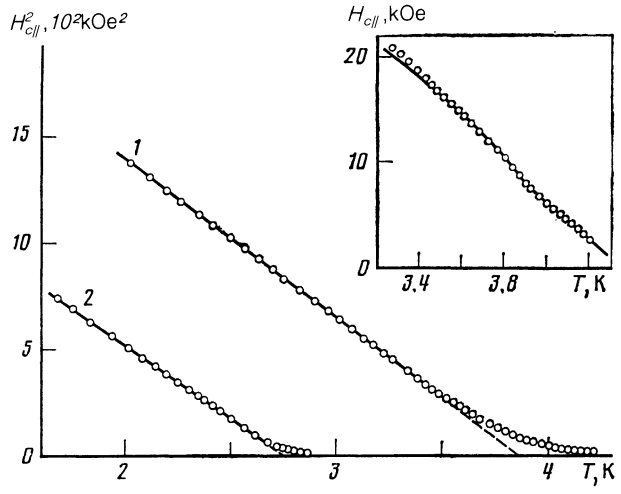

FIG. 3. Temperature dependences of $H_{c\parallel}^2$ for superlattices with $s = 30$ Å (1) and $s = 60$ Å (2). The inset shows the transition region of the dependence $H_{c\parallel}(T)$ for a superlattice with $s = 30$ Å. The continuous curve represents a theoretical calculation based on Eq. (18).

and, irrespective of s, the following empirical relationship was obeyed (Fig. 4):

$$\frac{dH_{c\perp}^{V}}{dT} \leqslant \frac{dH_{c\perp}^{V/Si}}{dT} < \frac{dH_{c\perp}^{SL}}{dT}.$$

THEORETICAL MODEL AND COMPARISON WITH EXPERIMENTAL RESULTS

Behavior of critical fields of superlattices with finite thicknesses of the metal layers was investigated theoretically by Deutscher and Entin-Wohlman[6] by solving the linearized Ginzburg-Landau (GL) equation for a periodic system of layers coupled by a weak Josephson interaction. This problem was solved in Ref. 6 assuming that the modulus of the order parameter $|\Psi|$ was the same for all the metal layers and that the change of $|\psi|$ across the layer thickness was negligible. It was found that in the case of a magnetic field parallel to the layers at temperatures close to T_c (weak fields) the $3D$ behavior should be observed, whereas at low temperatures (strong fields) the behavior should be of the $2D$ type. Moreover, a condition for the $8D$-$2D$ crossover was found. However, the simplification of the problem made by assuming that $|\psi| = \text{const}$ in a layer resulted (as shown below) in some error in the determination of $H_{c\parallel}$ in the $2D$ range because of inaccurate allowance for the energy of weak coupling. The hypothesis that $|\psi|$ is constant in different layers prevented us from estimating the influence of inhomogeneities associated with the inequivalence of the individual superlattice layers, which was quite likely in real samples for technological reasons.

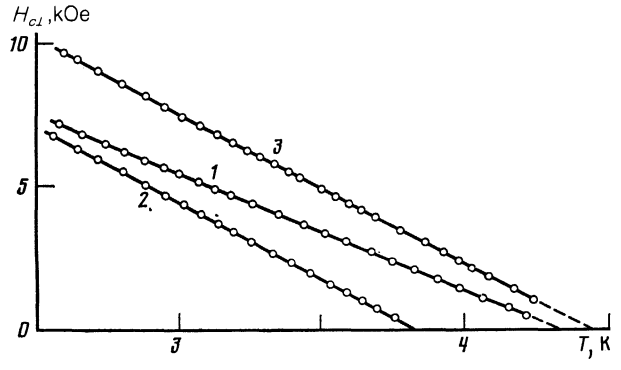

FIG. 4. Temperature dependences of the critical field $H_{c\perp}$ for a superlattice (1), a single film of vanadium (2), and a V/Si sandwich (3) for the series with $S = 30$ Å.

We shall determine the temperature dependence of $H_{c\parallel}$ for a superlattice by solving the GL equation using the same model as in Ref. 6, but we shall not make the assumption mentioned above. Moreover, we shall estimate the influence of a weak inhomogeneity of the layer thickness and coupling strength in a superlattice on the behavior of $H_{c\parallel}(T)$.

The linearized GL equation is similar to the Schrödinger equation for an electron. The spectrum of electrons in a semiconductor superlattice subjected to a magnetic field parallel to the layers was investigated in Ref. 8. The spectra were classified in Ref. 8 using the eigenvalues of the operator of magnetic translation by a period L_H^2/D (L_H is the magnetic length and D is the superlattice period). This made it possible to separate the variables in the Schrodinger equation and to reduce it to an ordinary differential equation. Using the same classification of the spectra, we obtained the following representation for the GL equation of each layer (i.e., for $|x - nD| \leqslant d/2$, where x is the coordinate along the normal to the superlattice layers, $n = 0, \pm 1, \pm 2, \ldots$):

$$-d^2\psi/dx^2 + [(x - \varkappa L_H^2)^2/L_H^4 - \tau/\xi^2]\psi = 0. \quad (1)$$

Here, $\tau = (T_c - T)/T_c$; T_c is the temperature of the superconducting transition in $H = 0$; $\xi = \xi(T = 0)$ is the coherence length; $L_H^2 = c\hbar/eH$; $\mathbf{H}$ is a magnetic field directed along the z axis; $\varkappa$ is a quasiwave vector which parametrizes[8] the solution of the GL equation; $|x| \leqslant D/2L_H^2$. The boundary condition matching the solutions of the system of equations (1) are

$$\psi'(Dn + d/2) = \psi'(D(n+1) - d/2),$$
$$\psi'(Dn + d/2) = l^{-1}[\psi(D(n+1) - d/2) - \psi(Dn + d/2)]. \quad (2)$$

Here, the extrapolation length l represents the force of the coupling between the layers. In the case of a weak coupling the values of l are larger: the length l is inversely proportional to the transparencey of the insulating layer to electrons[9]; in the "dirty" limit, we can use the formulas from Ref. 9 and readily obtain the relationship between the extrapolation length and the conductivity σ_n of a single layer and the tunnel resistance R_N per unit area of the dielectric layer $l = 2\sigma_n R_N$.

We shall assume that the metal layers are thin compared with the extrapolation length and with the coherence length $\xi(T)$:

$$d \ll l, \quad (3)$$

$$d \ll \xi\tau^{-1/2}. \quad (4)$$

Under these conditions the order parameter varies little in each of the layers. The inequality (4) means that vortices cannot penetrate into the individual superconducting layers.

The critical field can be found by determining first the minimum eigenvalue τ in Eq. (1) subject to the boundary conditions (2) when H is fixed. Obviously, the minimum value of τ corresponds to $\varkappa = 0$. In the absence of a magnetic

field ($H=0$) the smallest eigenvalue ($\tau=0$) corresponds to constancy of the order parameter throughout the superlattice: $\psi(Dn)=\text{const}$. In a magnetic field the order parameter is inhomogeneous in each of the layers and, moreover, it changes from one layer to the next. It follows from the condition (3) that in allowing for all the terms of the first order of smallness in respect of the parameter d/l in order to find $\psi(x)$ inside the metal layers it is sufficient to expand $\psi(x)$ to the term quadratic[1)] in respect of $x-nD$:

$$\psi(x)=\psi(Dn)+(x-nD)\psi'(Dn)+{}^1/_2(x-nD)^2\psi''(Dn). \quad (5)$$

The boundary conditions of Eq. (4) allow us to find the coefficients of the expansions of ψ' and ψ'' in Eq. (5):

$$\psi'(Dn)\approx(2l)^{-1}[\psi(D(n+1))-\psi(D(n-1))],$$

$$\psi''(Dn)\approx(ld)^{-1}[\psi(D(n+1))+\psi(D(n-1))-2\psi(Dn)]. \quad (6)$$

Next, using the variational principle we shall determine the influence of the field H on τ. This can be done by finding a minimum of the functional

$$(\psi, \mathcal{H}\psi)=-(\psi, d^2\psi/dx^2)+L_H^{-1}(\psi, x^2\psi) \quad (7)$$

corresponding to a fixed value $|\psi|$. If the first function in the variational method is Eq. (5) with the coefficients (6) in each segment ($nD-d/2, nD+d/2$) and the variable parameters are assumed to be the coefficients $\psi(Dn)\equiv\psi(n)$, then a magnetic field can be allowed for to within H^2. This allowance for the field is justified on the basis of the inequality (4). The variational procedure gives the following system of finite difference equations:

$$\left\{-\frac{1}{l}[\psi(n+1)+\psi(n-1)-2\psi(n)]+d\left[\left(\frac{D}{L_H^2}n\right)^2+\frac{1}{12}\left(\frac{d}{L_H^2}\right)^2-\frac{\tau}{\xi^2}\right]\right\}+\frac{1}{12l}[\psi(n+1)+\psi(n-1)-2\psi(n)]$$
$$\times\left[\frac{3}{20}\left(\frac{d}{L_H}\right)^4-\left(\frac{d}{\xi}\right)^2\tau+\left(\frac{dDn}{L_H^2}\right)^2\right]$$
$$+\frac{1}{6l}\frac{d^3D}{L_H^4}n[\psi(n+1)-\psi(n-1)]=0. \quad (8)$$

In Eq. (8) we have all the terms of the order of $1/l$ and H^2 including the cross terms. With the same precision we can obtain also each solution. Equation (8) was derived without imposing any restrictions on the rate of change of $\psi(n)$. The braces in Eq. (8) are used to collect the terms which are included in Ref. 6. If we retain only these terms, then Eq. (8) can be reduced by the Fourier transformation to Eq. (7) of Ref. 6. An analysis of all the other terms in Eq. (8) readily shows that they give rise to corrections to the values of τ which are of the order of $(d/l)(d\xi H/\Phi_0)^2$, where Φ_0 is a magnetic flux quantum.

In the range of weak fields (i.e., at temperatures close to T_c) these corrections are unimportant. The order parameter ψ depends weakly on n. Therefore, the finite differences can be replaced by the corresponding derivatives ψ with respect to the coordinate $X=nD$. In the main approximation with respect to H, we obtain from Eq. (8)

$$-\frac{D^2}{l}\frac{d^2\psi}{dX^2}+\frac{d}{L_H^4}X^2\psi-\frac{\tau d}{\xi^2}\psi=0. \quad (9)$$

As in Ref. 6, Eq. (9) yields the dependence

$$H_{c\parallel}(T)=\Phi_0(dl)^{1/2}/2\pi\xi^2(T)D, \quad (10)$$

which is typical of an anisotropic three-dimensional superconductor. The characteristic spatial scale of the variation of $\psi(x)$, defined by Eq. (9), is much greater than the superlattice period so that a superconducting nucleus should penetrate a large number N of the layers:

$$N(H)\sim L_H/(D^2ld)^{1/4}\gg 1.$$

When the field H is increased, the size of a nucleus (i.e., the value of N) decreases and when the field is sufficiently strong the nucleus is concentrated mainly in one layer [the amplitude ψ in the neighboring layers is proportional to $\sim(d/l)^2$]. We can then assume that $\psi(n)=0$ for all values of $n\neq 0$ in Eq. (8), which gives the following value of τ:

$$\tau\approx-2\xi^2/ld+{}^1/_{12}(d\xi/L_H^2)^2[1-{}^2/_{15}d/l]. \quad (11)$$

It is clear from Eq. (11) that in strong fields we have

$$H_{c\parallel}^2(T)=\frac{3}{\pi^2}\frac{\Phi_0^2}{d^2\xi^2}\left(1+\frac{2}{15}\frac{d}{l}\right)\frac{\tilde{T}_c-T}{T_c}. \quad (12)$$

Equation (12) contains a characteristic temperature

$$\tilde{T}_c=T_c(1-2\xi^2/ld), \quad (13)$$

which can be found by linear extrapolation of the dependence (12) to $H=0$. The functional dependence $H_{c\parallel}(T)$ is identical with that obtained for a single thin film, i.e., in strong fields the $2D$ behavior is observed. The values of the critical field given by Eq. (12) are higher than the values of $H_{c\parallel}$ reported in Ref. 6 for the same parameters of the system and this is due to inclusion of corrections to τ in our study. Two features of Eq. (12) should be remembered. Firstly, linear extrapolation of the formula (12) typical of the $2D$ range to the field $H=0$ yields a "critical" temperature $\tilde{T}_c$ which is lower than T_c. Secondly, the effective thickness of the layers can be deduced in the usual way from the derivative $dH_{c\parallel}^2/dT$, and this thickness is less than d:

$$d_{\text{eff}}\approx d(1-{}^1/_{15}d/l). \quad (14)$$

These two features are a consequence of a characteristic proximity effect: the layer which contains a nucleus comes into contact via insulating layers with those in which the superconductivity is suppressed by the field.

The transition from the $3D$ to the $2D$ behavior of $H_{c\parallel}(T)$ occurs in a fairly narrow range of fields near H_{cr} defined as follows[6]:

$$L_{H_{cr}}=2^{-1/4}(D^2ld)^{1/4}. \quad (15)$$

Equation (8) is easily generalized to that case if the parameters l, d, and D vary from layer to layer. This makes it possible to estimate the corrections to $H_{c\parallel}(T)$ due to small structural fluctuations of the superlattice parameters. Using the solution of Eq. (9) which is not perturbed by fluctuations, we find that in the first order with respect to the deviations δl, δd, and δD from the average values l, d, and D, we obtain the relative change in $H_{c\parallel}$:

$$\frac{\delta H_{fl}}{H_{c\parallel}}\sim\frac{1}{\mathrm{N(H)}}\frac{\delta(ldD)}{ldD}\propto H_{c\parallel}^{1/2}(T). \quad (16)$$

It is clear from Eq. (16) that the deviation of the dependence $H_{c\parallel}(T)$ from linearity in the $3D$ range is small not only be-

cause of the smallness of fluctuations of the superlattice parameters, but because there is an additional small factor $1/N$. Obviously, small deviations of δH_{reg} from the linear dependence of Ref. 10 are typical also of a perfectly regular superlattice and they are due to the discrete nature of its structure. Using Eq. (16) and also Eq. (10) from Ref. 6, we obtain

$$\delta H_{fl}/\delta H_{reg} \propto \delta dN(H)/d \qquad (17)$$

(for the sake of simplicity we shall assume that only the quantity d can fluctuate and, moreover, we shall assume that $D/d \sim 1$). It is also clear from Eq. (17) that the role of the correction to Eq. (10) due to the superlattice inhomogeneity decreases on increase in the field and on approach to the crossover region ($H \leqslant H_{cr}$), where $N(H) \sim 1$ the deviations of $H_{c\parallel}(T)$ from linearity are related to the properties of a homogeneous superlattice.

In the $2D$ range the homogeneities localize a nucleus in that superlattice layer for which the value of $H_{c\parallel}^2$ of Eq. (12) is maximal. It is clear from Eqs. (13) and (14) that the inhomogeneities of the layer thickness d have the greatest effect on the relationship between the bond length d_{eff} and the nominal value of d. If $\delta d/d \gtrsim d/15l$, the superlattice inhomogeneities affect the value of d_{eff} more strongly than the Josephson interaction in a regular superlattice.

Equations (10), (13), and (15) representing the $2D$ and $3D$ asymptotes of the temperature dependence $H_{c\parallel}(T)$ and the crossover point make it possible to consider several independent methods for the determination of the extrapolation length l, which represents the interlayer coupling forces of the superlattice. We shall now give these formulas in the form most convenient for the comparison with other experimental results:

$$l = \frac{D^2}{d}\left(\frac{dH_{c\parallel}/dT}{dH_{c\perp}/dT}\right)^2, \qquad (10a)$$

$$l = \frac{2\xi^2}{d}\frac{T_c}{T_c - \tilde{T}_c}, \qquad (13a)$$

$$l = \left(\frac{\Phi_0}{\pi d^2 H_{cr}}\right)^2 \frac{d^3}{D^2}. \qquad (15a)$$

In our experiments it was found that three series of samples with the superconductor layer thicknesses $s = D - d$ amounting to 30, 40, and 60 Å show clearly the existence of the crossover in the dependence $H_{c\parallel}(T)$ (see Figs. 1 and 2). Near T_c the $3D$ behavior of the $H_{c\parallel}(T)$ field is observed, whereas in strong fields $H_{c\parallel}^2$ is a linear function of T, i.e., the $2D$ behavior is exhibited (Fig. 3). The extrapolation temperature $\tilde{T}_c$ is found to be less than T_c (Figs. 2 and 3), which is in agreement with Eq. (10). In qualitative agreement with Eqs. (10), (13), and (15) it is found that the region of the transition from the $3D$ and $2D$ dependences correspond to a temperature close to $\tilde{T}_c$. A reduction in the thickness of the Si films increases the strength of the bonds and the $3D$ behavior of $H_{c\parallel}(T)$, which expands as shown in Figs. 2 and 3. When the thickness is $s = 10$ Å, then the crossover effect is not observed in the investigated range of temperatures (Fig. 1).

The values of l deduced from Eqs. (10a), (13a), and (15a) for two batches of superlattices are listed in Table I. The quantity d is attributed the values obtained by the standard method using the formula

TABLE I.

Superlattices with d/s (Å)	l_1, Å (15a)	l_2, Å (13a)	l_3, Å (10a)
220/30	860	970	800
220/60	1060	1150	1250

$$d^2 = \frac{6\Phi_0}{\pi}\left(\frac{dH_{c\perp}/dT}{dH_{c\parallel}^2/dT}\right)$$

and the data for one V/Si packet. The coherence length for superlattices was found from the corresponding derivative $dH_{c\perp}/dT$. It is worth noting the good agreement between the values of l obtained by different methods for samples in each batch. The region of the transition from the strong $3D$ behavior to the characteristic $2D$ dependence of the field $H_{c\parallel}(T)$ amounts to 0.2–0.5 K, depending on the sample. We calculated l from Eq. (15a) taking H_{cr} to be the field corresponding to the temperature $\tilde{T}_c$, because this temperature lies always approximately in the middle of the transition interval. The scatter of the values of l listed in Table I does not exceed 15%. [For a superlattice with $s = 40$ Å the values of l obtained from Eqs. (13a) and (15a) are the same and l estimated from Eq. (10a) differs considerably from the other two.] It should be pointed out that a better agreement could not be expected since the thickness of the semiconductor layers was determined only in the course of deposition and could not be monitored in any way in the finished superlattice Fluctuations of the thickness of the metal layers can be deduced from the experimental data and it was found that these were fairly small. This was confirmed by the good agreement (to within 1 Å) between the values of d_{eff} deduced respectively from the slopes $dH_{c\parallel}^2/dT$ in the case of superlattices with $s = 30$ Å in the $2D$ range and from Eq. (14) using the value of $d = 226$ Å obtained for a single packet of V/Si in the same batch of samples.

A typical value of d/l, assumed to be small in the theory, was close to 0.2 for all the investigated samples. It should be noted that the value of d/l deduced from the experiments of Beasley[1,2] is considerably less ($\sim 5\times 10^{-3}$). Nevertheless, both in our case and in Refs. 1 and 2 the model of weak coupling describes well not only the behavior of $H_{c\parallel}(T)$ in the $3D$ and $2D$ ranges, but in the region of the crossover itself. This is demonstrated by the inset in Fig. 3 for the sample with $s = 30$ Å when the transition region is most prominent. In plotting the graph in Fig. 3 we used the parametric equation[6] for the $H_{c\parallel}(T)$ curve:

$$T = T_c\left\{1 - \frac{2\xi^2}{ld}\left[\frac{a(q)}{2q} + 1\right] - \frac{\pi^2}{3}\frac{d^2\xi^2}{\Phi_0^2}H_{c\parallel}^2\right\},$$

$$H_{c\parallel} = \Phi_0/\pi D(dlq)^{1/2}. \qquad (18)$$

Here, $a(q)$ are the eigenvalues in the Mathieu equation written down in its canonical form.[10] We assumed that d is 226 Å, as deduced from measurements on a single V/Si packet, whereas the value of D was selected in such a way as to fit the experimental points to the calculated curve in the $3D$ region. We can see from Fig. 3 that the approximating function (18) describes well the transition region. The discrepancy between the experimental values and the function (18) observed at lower temperatures is associated with the differ-

ence between d_{eff} and d, which is ignored in Eq. (18).

It is worth noting the relatively small change in the coupling parameter l on increase in the thickness of the semiconductor layers in a superlattice. In the case of ideal layers free of short circuits the coupling would be reduced very rapidly: $l \propto \exp(s/a_0)$ (the characteristic length a_0 is of the order of the interatomic distance). The weak dependence $l(s)$ for the investigated samples is probably due to the presence of metallic short circuits in the Si layers.

The results for the samples with $s = 240$ Å show that in this case there is practically no coupling between the layers and the $2D$ behavior begins immediately from T_c. The critical field $H_{c\parallel}(T)$ is governed by the parameters in a single superconducting film.

The measurements of the critical fields $H_{c\parallel}(T)$ and their comparison with the theory reported above show that, in spite of the imperfection of the semiconductor layers, the crossover is observed clearly for some of the superlattice samples. Hence, it follows that this feature of the behavior of $H_{c\parallel}(T)$ is not affected by imperfections of the superstructure.

We shall conclude by noting that interpretation of our experimental results is based, as in Ref. 1, on the assumption that in the $3D$ behavior range the field is H_{c2} and not the surface superconductivity field H_{c3}. Good agreement is then obtained here and in Ref. 1 with the calculated curve $H_{c\parallel}(T)$ throughout the investigated range of temperatures, i.e., both in the $3D$ and $2D$ regions. However, it is still unclear why a superlattice does not exhibit surface superconductivity. This requires further study. It is possible that the answer may be provided by experiments involving determination of the angular dependences of H_c for a superlattice.

The authors are grateful to A. N. Stetsenko for the help in the preparation of the samples.

[1]It should be noted that the approximation used in Ref. 6 corresponds to negligibly small terms ψ' and ψ'' in Eq. (5).

[1]S. T. Ruggiero, T. W. Barbee, and M. R. Beasley, Phys. Rev. Lett. **45**, 1299 (1980).
[2]M. R. Beasley, in: *Percolation, Localization, and Superconductivity* (Proc. NATO Advanced Study Institute, Les Arcs, France, 1983, ed. by A. M. Goldman and S. A. Wolf), Plenum Press, New York (1984), p.115 (Vol. 109, NATO Advanced Study Institute Series B, Physics].
[3]I. Banerjee and I. K. Schuller, J. Low Temp. Phys. **54**, 501 (1984).
[4]D. E. Prober, M. R. Beasley, and R. E. Schwall, Phys. Rev. B **15**, 5245 (1977).
[5]R. V. Coleman, G. K. Eiserman, S. J. Hillenius, A. T. Mitchell, and J. L. Vincent, Phys. Rev. B **27**, 125 (1983).
[6]G. Deutscher and O. Entin-Wohlman, Phys. Rev. B **17**, 1249 (1978).
[7]A. N. Stetsenko, A. E. Kolin'ko, A. I. Fedorenko, and N. Ya. Fogel', Metallofizika **3**, 59 (1981).
[8]A. M. Berezhkovskii and R. A. Suris, Zh. Eksp. Teor. Fiz. **86**, 193 (1984) Sov. Phys. JETP **59**, 109 (1984)].
[9]A. V. Svidzinskii, Spatially Inhomogeneous Problems in the Theory of Superconductivity [in Russian], Nauka, Moscow (1982), p. 312.
[10]L. S. Bark, N. I. Dmitrieva, L. N. Zakhar'ev, and A. A. Lemanskiĭ, Tables of Eigenvalues for the Mathieu Equation [in Russian], Computing Center, Academy of Sciences of the USSR, Moscow (1970), p.150.

Translated by A. Tybulewicz