Some features of stimulated Brillouin scattering in light-absorbing media

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It is shown experimentally and theoretically that the influence of light absorption on the stimulated Brillouin scattering (SBS) process does not reduce merely to extinction of the light beams, but constitutes primarily phase-synchronism loss due to the change of the refractive index of the medium that has become heated by the radiation. The presence of even an insignificant absorption of light lowers noticeably the effectiveness of the scattering and changes the instantaneous frequency of the SBS radiation in the course of the scattering.

1. INTRODUCTION

Radiation conversion with the aid stimulated Brillouin scattering (SBS) is now widely used in connection with the feasibility of wavefront reversal. In practice, SBS is of value in the saturation regime, when the conversion coefficient η, ususally taken to mean the ratio of the peak SBS radiation intensity to the pump intensity, is close to unity. According to predictions of the theory[1] the conversion coefficient increases with increasing pump intensity and tends in the limit to unity. The role of light absorption is usually reduced in SBS theory merely to extinction of the light beams,[2,3] and it might seem that even at comparatively large ($\alpha L \sim 1$) values of the absorption coefficient its influence should be small, since as a rule the SBS gain in experiment is $GIL = 25$–30. In practice, however, it is found that in many substance the conversion coefficient remains less than unity for all intensities. Moreover, an insignificant increase of the absorption can suppress the SBS entirely.[4,5] This is not accompanied by pump exhaustion due to the stimulated temperature scattering (STS) of light, a scattering that is either not excited at all[4] or has little effect.[5]

SBS can also be used to determine the hypersound velocity v from the backward shift of the SBS frequency ω_S relative to the pump frequency ω_L, since $\omega_L - \omega_S = 2n\omega_L v/c$. The hypersound velocity obtained in such experiments,[2,6] however, was found to be less than determined from measurements of the spectra of thermal scattering of light, meaning a smaller shift of the SBS line. The cause of this behavior has remained unexplained. The assumption that the difference is due to the change of hypersound velocity as the medium is heated by the laser radiation did not always lead to quantitative agreement with experiment; furthermore, it could not explain the decrease of the shift in gases and in water, where $\partial v/\partial T < 0$. Still, when SBS is excited by focusing a laser beam with a spherical lens, the SBS and STS radiation propagate in the same backward direction and do not interact directly. A direct influence of light absorption on hypersound excitation, and hence on SBS, is noticeable only at rather large absorption coefficients, $\alpha \sim 1\ \text{cm}^{-1}$ and, on the contrary, increases the shift of the SBS line.[8] In addition, an investigation of the spectral distribution of the radiation of the SBS first Stokes component has shown that, notwithstanding the predictions of the theory, the spectrum has a fine structure,[9] and the total width of the spectrum in the saturation region increases with pump intensity.[10]

We show in the present paper that the change, heretofore unaccounted for by the theory, of the refractive index of the medium by light absorption upsets the phase sychronism in the SBS process, and even relatively small absorption, $\alpha \sim 10^{-2}$–$10^{-3}\ \text{cm}^{-1}$ influences strongly the effectiveness and the spectral properties of the scattering.

2. THEORY

To describe SBS in a medium with linear light absorption we use the following equations for the slowly varying complex amplitudes E_L and E_S of the pump and scattered waves, respectively:

$$\frac{\partial E_L}{\partial z} = -i\frac{\omega}{2nc}(\delta\varepsilon_1^* E_S + \delta\varepsilon_0 E_L),$$
$$\frac{\partial E_S}{\partial z} = i\frac{\omega}{2nc}(\delta\varepsilon_1 E_L + \delta\varepsilon_0 E_S), \tag{1}$$

where in addition to the term $\delta\varepsilon_1$ corresponding to the interaction of the light waves on the traveling dielectric-constant lattice, we take into account the change $\delta\varepsilon_0$ of the dielectric constant by the heating. From the linearized hydrodynamics equation for the density perturbation we obtain an expression for $\delta\varepsilon_1$:

$$\left(\frac{\partial}{\partial t} + \Gamma\right)\delta\varepsilon_1 = i\frac{\rho(\partial\varepsilon/\partial\rho)^2 n\omega}{16\pi cv} E_L^* E_S, \tag{2}$$

where v and $1/2\ \Gamma$ are the velocity and the damping time of the free hypersonic wave. The uniform heating of the medium by the absorbed energy alters the dielectric constant $\delta\varepsilon_0$:

$$\frac{\partial}{\partial t}\delta\varepsilon_0 = \left(\frac{\partial\varepsilon}{\partial T}\right)\frac{cn\alpha}{8\pi\rho c_P}(|E_L|^2 + |E_S|^2). \tag{3}$$

Here α is the light-absorption coefficient, while ρ and c_P are the density and the specific heat of the medium.

We have left out of the system (1) terms that describe the pump-wave and scattered-wave damping by light absorption, since it will be shown below that in this case the dominant role is played by phase effects. The system (1) has therefore an obvious integral that postulates the conservation law for the number of photons:

$$\frac{cn}{8\pi}(|E_L|^2 - |E_S|^2) = I_L(L)(1-\eta) = \text{const}, \tag{4}$$

where

$$I_L = cn|E_L|^2/8\pi,\ I_S = cn|E_S|^2/8\pi,$$
$$\eta = I_S(L)/I_L(L).$$

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We seek for the system (1) a solution that is stationary in the intensity of the interacting waves, in the form

$$E_L(z,t)=\mathscr{E}_L(z,t)\exp\left[i\int_z^L\frac{\omega}{2cn}\delta\varepsilon_0\,dz'\right],$$
$$E_S(z,t)=\mathscr{E}_S(z,t)\exp\left[-i\int_z^L\frac{\omega}{2cn}\delta\varepsilon_0\,dz'\right], \tag{5}$$

here $z=0$ and $z=L$ are the boundaries of the nonlinear-interaction region, the stationary and monochromatic pump enters through the boundary $z=L$ and propagates in the negative z direction. It can be shown (see the Appendix) that in this case the complex amplitude $\mathscr{E}_L(z,t)$ is also stationary: $\partial\mathscr{E}_L/\partial t=0$.

Using (5) and the integral (4), we obtain an equation for the scattered-wave intensity:

$$\frac{dI_S}{dz}=GI_S(z)\left[I_L(L)(1-\eta)+I_S(z)\right]$$
$$\times\left\{1+\left[\left(\delta\Omega+2h\int_z^L\left[I_L(L)(1-\eta)+2I_S(z)\right]dz'\right)\Big/\Gamma\right]^2\right\}^{-1}, \tag{6}$$

where $G=\rho(\partial\varepsilon/\partial\rho)^2\omega^2/2c^3vn\Gamma$ is the gain,

$$h=\alpha\omega(\partial\varepsilon/\partial T)/2cn\rho c_P,$$

$\delta\Omega$ is the detuning of the SBS frequency from the center of the Brillouin thermal-scattering line. Integration of (6) yields

$$D+\ln\frac{\eta}{(1-\eta)^2}=\frac{G\Gamma}{h}\operatorname{arctg}\left\{h\left[I_L(L)(1-\eta)L+2\int_0^L I_S(z)\,dz\right]\right\}, \tag{7}$$

where $D=ln[I_L(0)/I_S(0)]$. According to Ref. 11 $I_L(0)/I_S(0)=10^{-11}-10^{-13}$, therefore $D=25$–30. Calculation shows that a $\eta\lesssim 0.8$ we have for the solution (1), which does not take the influence of absorption into account,

$$2\int_0^L I_S(z)\,dz\ll I_L(L)(1-\eta)L.$$

Neglecting the integral

$$\int_0^L I_S(z)\,dz$$

compared with $I_L(L)(1-\eta)L$ in the right-hand side of (7) and the term $2\ln(1-\eta)$ compared with D in the left-hand side, we obtain an expression for the conversion coefficient η, maximized with respect to $\delta\Omega$, as a function of the intensity $I_L(L)$ of the pump wave incident on the medium:

$$h(D+\ln\eta)=\Gamma G\operatorname{arctg}\frac{I_L(L)(1-\eta)Lh}{\Gamma}, \tag{8}$$

which goes over into the known solution (1) in the limit as $h\to 0$. For the optimal frequency detuning corresponding to maximum scattering efficiency we have

$$\delta\Omega=-I_L(L)(1-\eta)hL. \tag{9}$$

Equation (9) jointly with (8) describes the displacement of the SBS line from the center of the thermal scattering line as a function of the pump intensity. In the region of practical importance, $0.1<\eta<0.8$, we can also neglect the term $\ln\eta$ compared with D in Eq. (8), after which we obtain for the conversion coefficient η and for the frequency detuning $\delta\Omega$ the simple expressions

$$\eta=1-\frac{\Gamma}{I_L(L)Lh}\operatorname{tg}(Dh/\Gamma G),\qquad \delta\Omega=-\Gamma\operatorname{tg}(Dh/\Gamma G).$$

3. EXPERIMENT

The effect of light absorption on the SBS process was investigated in experiments on acetone with the scattering excited by a single-frequency single-mode neodymium laser at $\lambda=1.06\ \mu$m, pulse duration $t_p=46$ nsec, beam divergence ≈ 0.5 mrad, and spectral width less than $3\cdot10^{-3}$ cm^{-1}. In acetone at $\lambda=1.06\ \mu$m we have $\alpha=10^{-2}$ cm^{-1}, $G=2.2\cdot10^{-2}$ cm/MW, $\Gamma/2\pi=47$ MHz, and $v=1.2\cdot10^5$ cm/s. The laser radiation was focused with a lens of focal length $f=10$ cm into a cell 10 cm long filled with acetone. Between the cell and the laser was placed a polarizing shutter, consisting of a Glan prism and of a Fresnel rhomb that prevented stray radiation from entering the laser cavity. The experimental setup could record SBS radiation going both into the pump solid angle ≈ 0.5 mrad, and into the entire recording-system aperture 20 mrad. In addition, the energy was recorded, and the pump pulse shape was monitored simultaneously.

Figure 1 shows the coefficient η of pump conversion into SBS radiation propagating exactly backward into a 20 mrad solid angle (upper curve) and into a 0.5 mrad angle (lower) curve, as a function of the pump power W_L. It can be seen from the figure that in the region where η grows rapidly practically all the scattered radiation goes into the pump solid angle. In the saturation region, on the other hand, the fraction of the reversed SBS component is decreased to 20%. Figure 2 shows oscillograms of the reversed (a) and nonreversed (b) SBS components, obtained simultaneously with the aid of two FK-19 coaxial photocells and an SB-14 two-beam oscilloscope. Figure 2c shows for comparison an oscillogram of the pump pulse. It can be seen that at the initial instant all the SBS radiation goes into the invert-

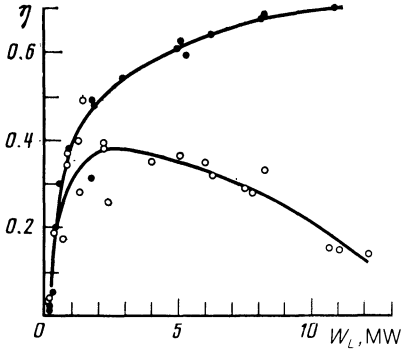

FIG. 1. Coefficient η of conversion of pump radiation into SBS radiation vs. the pump power W_L: the dark circles correspond to scattered radiation going into the entire aperture of the recording system (20 mrad); the light circles correspond to scattered radiation going into a solid angle with aperture 0.5 mrad.

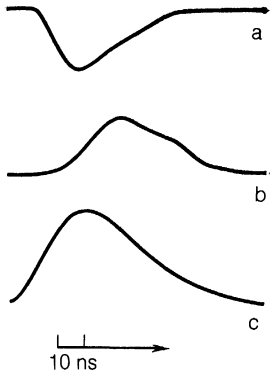

FIG. 2. Time scan of the inverted component (a) and of the noninverted component (b) of the SBS radiation, and also of the pump pulse (c).

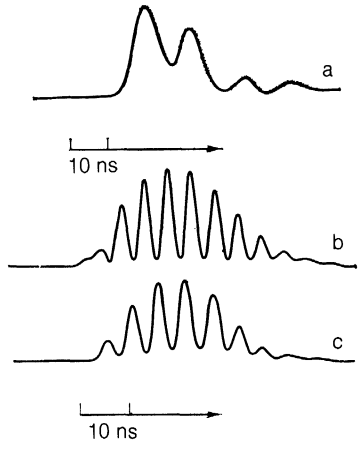

FIG. 4. Oscillograms of beats produced in heterodyning of SBS radiation (see the text).

ed component, followed as it were by a switchover of the scattering regime—the power of the noninverted component increases and that of the inverted decreases. An increase of the pump power shifted the instant of this reversal towards the start of the pulse. It can thus be concluded that a decrease of the fraction of the inverted component in Fig. 1 with increase of power is due to the ever earlier switching of the scattering regime from the inverted to the noninverted component.

The phase-frequency characteristics of the SBS were investigated by heterodyning by a Michelson-interferometer technique (Fig. 3a). The laser radiation was split by a semitransparent mirror into two beams of approximately equal intensity, and each beam was focused to a corresponding liquid-filled cell. The SBS radiation of these beams were mixed past the semitransparent mirror in an FK-26 square-law detector and recorded with S8-14 and S7-10b oscilloscopes. The reference beam for the heterodyning was SBS

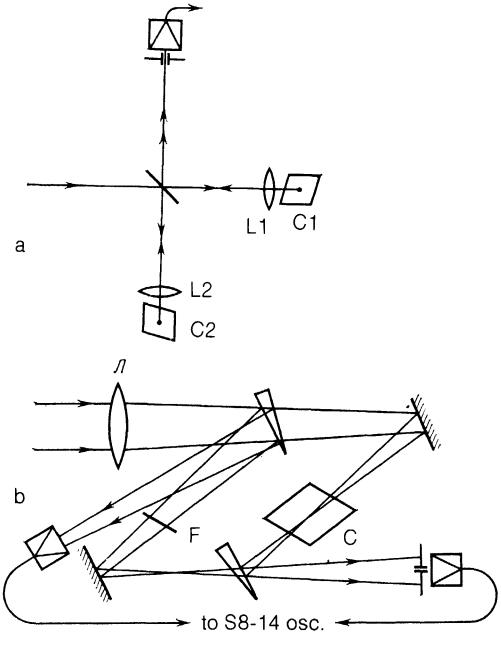

FIG. 3. Experimental setup for the investigation of the phase-frequency characteristics of SBS radiation (a) and of laser radiation passing through an absorbing substance (b).

radiation excited by a collimated pump beam (the focal length of lens L1 in Fig. 3a was 75 cm) either in acetone or in hexane, for in this case a spectrally bounded SBS pulse was emitted and there were no phase fluctuations due either to the statistical nature of the SBS nor to other nonlinear processes. The absence of phase fluctuations from the reference beam was also verified by a heterodyning method. To this end, cells with identical substance were placed in both arms of the interferometer, and the focal lengths of lenses L1 and L2 were 75 cm. No beats were observed under these conditions in the interferometer output.

When the signal beam was excited by focusing the pump radiation into the acetone-filled cell C2 by a short-focus lens with $f = 5$ cm, and the reference beam was also excited in acetone, beats of frequency 70–90 MHz were produced (Fig. 4a). To determine the magnitude and direction of the instantaneous SBS frequency in acetone under conditions of strong focusing of the pump radiation, the reference beam used for the heterodyning was collimated-pump SBS in hexane. The SBS frequency deviation from the pump frequency is 200 MHz smaller in hexane than the corresponding deviation in acetone.[2] Therefore, when the SBS signal pulse in the acetone was excited also by a collimated pump, a regular sinusoidal modulation of the signal was recorded, with a period that remained constant during the SBS pulse and corresponded to the difference between the SBS frequencies in acetone and in hexane (Fig. 4b). When the pump was focused into the acetone-filled cell C2 by a short-focus lens, however, the modulation frequency varied during the SBS pulse (Fig. 4c). Figure 5 shows the results of reduction of six oscillograms—three each at pump energy 30–40 mJ and 100–120 mJ. The ordinate is the instantaneous frequency shift in the acetone, calculated from the difference between the signal modulation periods under conditions of strong focusing of the pump beam and of the collimated beam. At low values of the energy at the start and at the end of the pulse, the instantaneous SBS frequency had the same value as in the case of scattering by a collimated beam. At the maximum pump intensity, however, a shift of the instantaneous SBS frequency towards the blue was observed, corresponding to the increase of the modulation period in Fig. 4c. At high laser-pulse energy, the shift of the instantaneous frequency towards the end of the pulse remained practically

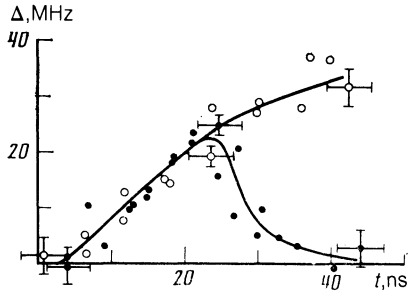

FIG. 5. Dynamics of the instantaneous frequency of SBS radiation: the dark and light circles correspond respectively to pump-pulse energies 30–40 and 100–120 mJ.

the same as at the maximum of the pump intensity.

To study the influence of light absorption on the phase-frequency characateristics of laser radiation passing through a layer of light-absorbing medium, a corroborative experiment was performed, in which no backscattering was excited. The laser pulse used for this purpose did not exceed 0.3 MW in power. The coefficient of pulse-radiation conversion into SBS radiation was then less than 10^{-4}, and for the thermal effect to be of approximately the same magnitude as in the experiments described above, the absorption coefficient of the acetone was increased to $\alpha = 0.16$ cm^{-1} by adding copper chloride.

In this experiment, a heterodyning method based on the Mach-Zehnder interferometry (Fig. 3b) the dynamics was recorded of the phase of a light beam passing through a cell with a solution. The laser radiation was focused by a lens L of focal length 30 cm and split by a glass plate, directly behind the lens, into two beams—reference and signal. A solution-filled cell 1 cm thick was placed in the interferometer arm through which the more intense (signal) light beam passed. The reference beam passing through the second arm of the interferometer was attenuated by a filter F to make the intensities of the beams equal as they interfered on the photoreceiver. Figure 6 shows an oscillogram of the signal produced by interference between the laser beam passing through the cell and the reference beam. For comparison, the figure shows the laser-beam time scan simultaneously obtained with a two-beam oscilloscope. The intensity difference oscillations observed in Fig. 6a attest to the onset of a frequency difference between the laser beam and the beam passing through a layer of the ligh-absorbing medium. When the power of the laser pulse incident on the cell was increased, the beat frequency of the light beam increased and amounted to about 80 MHz at $W_L = 0.28$ MW.

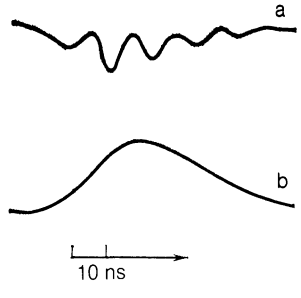

FIG. 6. Oscillogram of beats produced by heterodyning laser radiation passing through an absorbing medium (a). Time scan of pump pulse (b).

4. DISCUSSION

Before we compare theory with experiment, we present the numerically calculated conversion coefficient η and frequency detuning $\delta\Omega$ for different pump wave intensitites. Our expressions (7) and (8) for η and $\delta\Omega$ contain a parameter h that depends in turn on the quantity $(\partial\varepsilon/\partial T)$. If the pump-pulse duration is such that over the characteristic scale d of the light-field inhomogeneity the pressure can become equalized, $d \ll vt_p$, it is necessary to substitute in h the isobaric value, which amounts to $(\partial\varepsilon/\partial T)_P = 1.3\cdot10^{-3}$ deg^{-1} for acetone. In the opposite case we must use $(\partial\varepsilon/\partial T)_V$, which as a rule is smaller by two orders than $(\partial\varepsilon/\partial T)_P$.

In a numerical calculation of the dependences of η and $\delta\Omega$ on the pump intensity, we used for the main parameters contained in Eqs. (7) and (8) the values for acetone, and chose for $(\partial\varepsilon/\partial T)$ the isobaric value. The results are qualitatively correct also for other SBS-active substances, since the parameters used in the calculations differ insignificantly for different liquids.

Figure 7a shows the dependence, obtained from Eq. (8), of the absorption coefficient η on the excess above the "threshold" $x = GI_L(L)L/D$ for different values of the absorption coefficient. It can be seen that the presence of even insignificant absorption can lower the effective scattering noticeably. This effect explains apparently the suppression of the SBS in the experiments of Refs. 4 and 5.

Using the $\eta(x)$ dependence obtained from Eq. (8), we

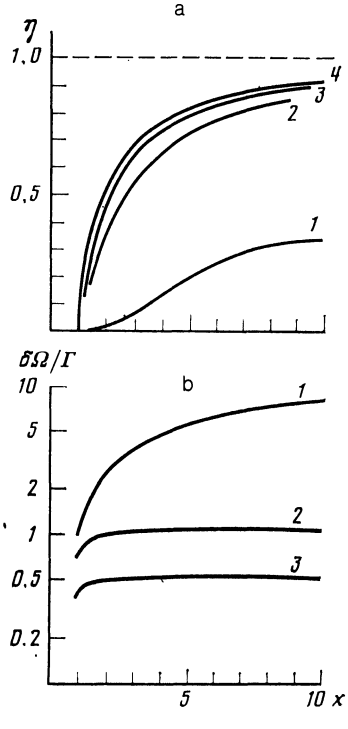

FIG. 7. Dependences of the coefficient of conversion of pump radiation into SBS radiation, and of the deviation $\delta\Omega$ of the SBS frequency from the center of the thermal-scattering line, on the excess above the "threshold" $x = GI_L L/D$, $D = 25$ for different values of the absorption coefficient $\alpha = 1.9\cdot10^{-2}$ cm^{-1} (1), 10^{-2} cm^{-1} (2), $6\cdot10^{-3}$ cm^{-1} (3), 0 cm^{-1} (4).

can calculated with the aid of (9) the detuning $\delta\Omega$ of the SBS from the thermal-scattering line center. Figure 7b shows the dependence, obtained in this manner, of $\delta\Omega/\Gamma$ on the "excess" x above threshold. The rapid growth of the displacement of the line in the region $x = 1$–2 gives way to saturation. the reason is that at a near-unity conversion coefficient the product $I_L(L)(1-\eta)$ becomes almost independent of the pump intensity.

The experimental results are in qualitatively good agreement with the constructed theory. A quantitative comparison, however, is made difficult by the fact that in the experiment the pump-pulse duration is of the order of the time of travel of the sound over the diameter of the caustic. Therefore $(\partial\varepsilon/\partial T)$ must be substituted by some value intermediate between $(\partial\varepsilon/\partial T)_p$ and $(\partial\varepsilon/\partial T)_v$. We compare the theory with experiment by using the fact that $(\partial\varepsilon/\partial T)$ is contained in Eqs. (8) and (9) only as a factor in $\alpha(\partial\varepsilon/\partial T)$. Therefore, even though $\alpha = 10^{-2}$ cm^{-1} in the experiment, the plots in Fig. 7 which agree best with experiment are those for $\alpha = 6\cdot 10^{-3}$ cm^{-1}. The change of the instantaneous SBS radiation frequency, corresponding to a pump energy 30–40 mJ, agrees with that predicted by Eq. (8)—as the wave incident on the medium increases in intensity, the shift of the SBS lines increases; next, when the pump intensity decreases, the line shifts also decrease. Note that increasing the pump energy to 100–120 mJ does not increase the SBS line shift corresponding to the maximum pump intensity ($t\approx 25$ ns). This is precisely the behavior predicted by Eq. (9). At pump energy as low as 30–40 mJ the conversion coefficient, according to Fig. 1 ($W_L \approx 0.8$ MW) reaches 0.3–0.4 and, as follows from (9), the frequency detuning ceases to depend on the pump intensity. The discrepancy between the two curves of Fig. 5 is due to the onset, at the end of the pulse, of small-scale inhomogeneities both in the SBS beam and apparently in the pump beam. This is manifested in the dynamics of the noninverted SBS component. Whereas at a pump energy 100–120 mJ the coefficient of conversion of the exciting radiation into the noninverted SBS component was approximately 30% (Fig. 1), by the end of the pulse, as follows from Fig. 2, the noninverted component should predominate. The appearance of small-scale inhomogeneities of the field distribution causes phase enhancement on account of the "turning-on" of the isobaric derivative $(\partial\varepsilon/\partial T)_p$. A similar effect was apparently observed also in the study of the phase-frequency characteristics of a laser beam after passage through a light-absorbing medium—the time of appearance of the first minimum on Fig. 6a (the phase at the beam center was changed by π) was noticeably longer than the period of the ensuing oscillations (each of which corresponds to a phase shift or 2π). The scanning of the SBS frequency by the phase modulation should, according to Ref. 12, produce line structures in the SBS spectra integrated over time; these were observed in Ref. 9.

Thus, allowance for phase modulation of the pump radiation and of the scattered radiation, due to the heating of the medium, explains many observed SBS effects heretofore not explained by the conventional SBS theory in which no account was taken of the influence of light absorption on the phase-frequency characteristics of the interacting waves. These include the anomalous position of the SBS line, the broadening of the scattered radiation spectrum, the appearance in the latter of a fine structure, the earlier saturation of the absorption coefficient, and the suppression of the SBS at relatively low light-absorption coefficients.

APPENDIX

We shall show that, within the framework of the employed set of equations (1), it follows from the assumed stationary character of the intensity of the interacting wave and from the boundary condition $\partial E_L/\partial t|_{z=L} = 0$ that the complex amplitude $\mathscr{E}_L(z,t)$ of the pump wave is stationary:

$$\partial\mathscr{E}_L(z,t)/\partial t = 0.$$

Carrying out the substitution (5) in the first equation of the system (1) and using expressions (2) and (3) for the change of the dielectric constant, we obtain an equation for the complex amplitudes:

$$\frac{\partial^2\mathscr{E}_L}{\partial z\,\partial t} - \frac{1}{\mathscr{E}_S}\frac{\partial\mathscr{E}_S}{\partial t}\frac{\partial\mathscr{E}_L}{\partial z} + 2iF\frac{\partial\mathscr{E}_L}{\partial z} + \Gamma\frac{\partial\mathscr{E}_L}{\partial z} + \gamma\mathscr{E}_L|\mathscr{E}_S|^2 = 0, \tag{A1}$$

where

$$F = \int_z^L \frac{\omega\alpha}{16\pi\rho c_p}\left(\frac{\partial\varepsilon}{\partial T}\right)(|E_L|^2+|E_S|^2)\,dz',$$

$$\gamma = \frac{\rho(\partial\varepsilon/\partial\rho)^2\omega^2}{32\pi c^2 v}.$$

We introduce the substitutions

$$\begin{aligned}\mathscr{E}_L(z,t) &= \tilde{E}_L(z)\exp i\varphi_L(z,t),\\ \mathscr{E}_S(z,t) &= \tilde{E}_S(z)\exp i\varphi_S(z,t).\end{aligned} \tag{A2}$$

Here $\tilde{E}_L$, $\tilde{E}_S$ and φ_L, φ_S are real since we have assumed a stationary intensity of the interacting wave. Separating the real and imaginary parts of Eq. (A1) we obtain

$$\frac{\partial\tilde{E}_L}{\partial z}\left[\frac{\partial\varphi_L}{\partial t} - \frac{\partial\varphi_S}{\partial t} + 2F\right] + \tilde{E}_L\left[\frac{\partial^2\varphi_L}{\partial z\,\partial t} + \Gamma\frac{\partial\varphi_L}{\partial z}\right] = 0, \tag{A3}$$

$$\tilde{E}_L\frac{\partial\varphi_L}{\partial z}\left[\frac{\partial\varphi_L}{\partial t} - \frac{\partial\varphi_S}{\partial t} + 2F\right] - \Gamma\frac{\partial\tilde{E}_L}{\partial z} - \gamma\tilde{E}_L|\tilde{E}_S|^2 = 0. \tag{A4}$$

Note that since E_L and E_S depend only on z, it follows from (A4) that the expression

$$f = (\partial\varphi_L/\partial z)\,[\partial\varphi_L/dt - \partial\varphi_S/dt + 2F]$$

is also a function of only the spatial variable. Substituting (A3) in (A4), we obtain an equation for the phase of the pump wave:

$$\frac{\partial}{\partial t}\left(\frac{\partial\varphi_L}{\partial z}\right)^2 + 2\Gamma\left(\frac{\partial\varphi_L}{\partial z}\right)^2 + \frac{2f(z)}{\Gamma}\,[f(z) - \gamma|E_S|^2] = 0. \tag{A5}$$

It can be seen from (A5) that at $t \gg 1/2\Gamma$ the quantity $(\partial\varphi_L/\partial z)^2$ depends only on z. The pump-wave phase can therefore be represented in the form $\varphi_L = A(z) + B(t)$. From the boundary condition $\partial E_L/\partial t|_{z=L} = 0$ we obtain then $\partial\mathscr{E}_L(z,t)/dt = 0$.

It was assumed in Ref. 1 that the complex amplitudes of the interacting waves are stationary. It follows from our proof that in the particular case when there is not absorption, $h = 0$, a sufficient condition for stationary complex amplitude is a stationary intensity. In conclusion, the authors

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