Theory of spin-density waves in quasi-one-dimensional ferromagnetic metals

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A new model of the transition from a one-dimensional ferromagnetic metal to the insulating phase with a spin-density wave (SDW) is proposed. Below the transition point the spins develop a spiral structure, in which analogs of domain walls exist in the form of self-localized states of the order parameter (solitons and polarons). In systems with higher than twofold commensurability the SDW is an electrically active collective mode capable of giving rise to a nonlinear soliton conductivity. The SDW parameters depend on the external magnetic field. The role of magnetic anisotropy is investigated. The commensurability energy of the SDW is a nonmonotonic function of the occupation state of the electron energy band.

INTRODUCTION

It is well known that at sufficiently low temperatures quasi-one-dimensional metals undergo a transition to the insulating state. The main cause of such a transition is either a structural (Peierls) transition, Mott-Hubbard correlation interaction, or the effect of disorder. In a Peierls transition a charge-density wave (CDW) state characterized by a new lattice constant and by electron-density modulation specified by the quantity $2k_F$ (k_F is the Fermi momentum of the electrons in the direction of variation) arises.[1] The Coulomb correlation interaction of the electrons also gives rise to the formation of a superstructure with scale $2k_F$, but, in the subsystem of the electron spins, to a so-called spin-density wave (SDW).[2]

From a conceptual point of view the CDW and SDW are due to Umklapp processes between the "right" and "left" branches of the spectrum of the one-dimensional electrons and holes. The order parameter of the CDW is proportional to the anomalous average $\langle a^+_{k_F\tau} b_{-k_F\tau}\rangle$, where $a^+_{k_F\tau}$ is the creation operator for a "right" electron, $b_{-k_F\tau}$ is the annihilation operator for a "left" hole, τ is the spin-projection index, while the order parameter of the SDW is proportional to $\langle a^+_{k_F\tau} b_{-k_F-\tau}\rangle$, i.e., the SDW is formed by Umklapp processes with a spin flip.

The commom origin of CDW and SDW gives rise to the largely similar behavior of the collective degrees of freedom of the order parameter in these phases—in particular, the collective conductivity, as was noted in Ref. 3. In a Peierls-Fröhlich system (CDW with higher than twofold commensurability, or an incommensurable CDW), the collective conductivity is due to the dynamics of the phase φ (of the order parameter $\Delta e^{i\varphi}$) describing the large-scale variations of the Fermi energy.[4] In turn, the evolution of the phase φ is intimately related to Umklapp scatterings through $2k_F$, as is formally manifested in the appearance of the so-called CDW chiral-anomaly current,[4,5] i.e., the collective CDW current is an Umklapp current. The modulus Δ of the Peierls order parameter determines the gap in the spectrum of the electrons that have become nonconducting, and its nonuniform states are deep polarons of large radius $\sim\xi_0 = \hbar v_F/\Delta$ (v_F is the Fermi velocity). The theory of such self-localized states was expounded in detail in Ref. 6.

It is rather natural to expect analogous behavior from the SDW order parameter. However, in the framework of the existing approach to the description of SDW (Ref. 2) these analogies with Peierls systems have not been clearly traced, because of specific features of calculations in the Hubbard model. Therefore, it is of undoubted interest to create a theory of SDW on the basis of a scheme that is related to the Peierls scheme and in which the nontrivial role of Umklapp processes is considered explicitly. Such a description is proposed in the present paper. We use the *s-f* model of a ferromagnetic metal[7] and study the transition to the SDW phase in the system of *s* conduction electrons. The advantage of this description lies in the possibility of the explicity introduction of electron-magnon interaction and, consequently, Umklapp processes with emission or absorption of magnons with momenta $\pm 2k_F$. It is natural to call such an ordered phase a magnetic Peierls insulator (MPI), in order in distinguish it from the SDW state which arises in the Hubbard model. The term SDW in our approach will be reserved for the description of the dynamical phase degree of freedom, in analogy with the use of the term CDW in the Peierls-insulator (PI) state. In the framework of the *s-f* description we are able to formulate a scheme analogous in many respects to the Peierls scheme, i.e., to formulate a field model of an MPI and to investigate nonuniform and nonstationary states of the order parameter. It is found that the transition to an MPI in a one-dimensional ferromagnetic metal leads to the formation of a spiral structure in the magnetic subsystem, with a simultaneous transition of the conducting subsystem to the insulating state. The period of the spiral is specified by the quantity $2k_F$, as we should expect in accordance with the theory of Dzyaloshinskiĭ spiral structures.[7] Just as the Peierls transition is a consequence of the enhancement of the Kohn singularity in the one-dimensional case, so the transition to the MPI state is an enhancement of the interaction that is responsible in three-dimensional metals for the formation of Dzyaloshinskiĭ spirals. The self-localized states of the modulus of the order parameter of an MPI are to a certain degree analogous to domain structures in magnets.

In contrast to a Peierls-Frohlich system, in SDW there is no microscopic modulation of the electron-charge density, whence it follows that the collective current in transitions caused by mixing of branches of the electron spectrum is (contrary to what is sometimes stated) in no way connected with the average variations of the density. In addition, im-

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portant differences between the models are manifested in the commensurability effect and the role of the magnetic field. It is found that the parameters of the MPI are easily controlled by a magnetic field, while the commensurability effect differs for systems with even and odd commensurability indices, which does not happen in the Peierls phase. The latter follows from the fact that in a PI the spin does not change in Umklapp scatterings, and so the symmetry of an MPI, in contrast to a PI, is determined not only by the relationship of the Umklapp vector $2k_F$ to the Brillouin vector, but also by the number of spin flips in the Umklapp processes.

The proposed approach can be applicable to the description of the properties of quasi-one-dimensional conducting magnets (metals or degenerate semiconductors), in the form of rare-earth compounds of the type Eu_3O_4, $Eu_2Lu_2O_4$, and $EuGd_2O_4$ (Ref. 8).

1. DERIVATION OF A CONTINUUM MODEL OF A MAGNETIC PEIERLS INSULATOR

The nonuniform states of an MPI and the response to external fields can be studied conveniently in an continuum model. Below the metal-insulator transition point, the characteristic length scales over which the components of the order parameter vary become much greater than the interatomic distance $a \sim k_F^{-1}$. Therefore, it is natural to obtain a description of the system in terms of large-scale fields, by averaging the microscopic Hamiltonian over interatomic distances.

The starting point of this program is the one-dimensional *s-f* Hamiltonian

$$H=\sum_n J^\alpha S_n^\alpha S_{n+1}^\alpha + A\sum_{n,\tau,\tau'} a_{n\tau}^+ (\mathbf{S}\tau)_{\tau\tau'} a_{n\tau'} + t\sum_n (a_{n\tau}^+ a_{n+1\,\tau} + \text{h.c.}). \quad (1)$$

Here J^α are Heisenberg-exchange integrals between f atoms, A is the s-f-interaction constant, $\mathbf{S}_n$ is the spin operator of the f atom at the site n, $a_{n\tau}^+$ is the creation operator for an s electron whose z-component of spin is equal to τ at the site n, τ is the set Pauli matrices, and t is the Bloch transfer integral of the s electrons.

We shall consider a metal below the ferromagnetic-ordering temperature T_c. Let the spontaneous magnetic moment of the lattice be directed along the z axis and let the atomic spin $S \gg 1$. We represent the spin operators in the linearized Holstein-Primakoff form

$$\begin{aligned} S_n^z &\approx S - c_n^+ c_n, \\ S_n^- = S_n^x - iS_n^y &\approx (2S)^{1/2} c_n, \\ S_n^+ = S_n^x + iS_n^y &\approx (2S)^{1/2} c_n^+. \end{aligned} \quad (2)$$

In the one-dimensional problem the effective electron–scattering amplitude and the polarization operator of the magnons have a logarithmic singularity for momenta $\pm 2k_F$ (Ref. 7), analogous to the Kohn singularity in the electron-phonon system. Therefore, intending to study the rearrangement of the magnon and electron spectra, we shall retain in the Hamiltonian (1) only magnons with momenta $\pm 2k_F$, i.e.,

$$\begin{aligned} (2/S)^{1/2} S_n^x &= \exp(2ik_F na)\,(c_{2k_F} + c_{-2k_F}^+) \\ &\quad + \exp(-2ik_F na)\,(c_{2k_F}^+ + c_{-2k_F}), \\ (2/S)^{1/2} S_n^y &= i\exp(2ik_F na)\,(c_{2k_F} - c_{-2k_F}^+) \\ &\quad + i\exp(-2ik_F na)\,(c_{-2k_F} - c_{2k_F}^+). \end{aligned} \quad (3)$$

Following Ref. 4, we represent the electron operators in the form

$$a_{n\uparrow,\downarrow} = (2a)^{-1/2}(W_{\uparrow,\downarrow}\exp(ik_F na) + V_{\uparrow,\downarrow}\exp(-ik_F na)), \quad (4)$$

where W and V are smooth functions of the coordinate n. A quasiclassical continuum Hamiltonian is obtained after substitution of (3) and (4) into (1) and after averaging over rapid oscillations with period $1/k_F a$. Here an important role is played by the condition (always fulfilled in metals) $k_F a = \pi/N$ $(N>1)$, which can be represented in the form

$$2k_F/p_B = 1/N, \quad (5)$$

where $p_B = 2\pi/a$ is the size of the Brillouin zone. If N is a rational number, i.e., $N = M/L$, where M and L are integers $(M>L)$, then (5) is the condition for commensurability of the Peierls phase.[4] The physical meaning of the commensurability condition is transparent; it implies that an electron must participate N times in Umklapp processes with a change of momentum by $2k_F$ in order to return to its starting point in the Brillouin zone. An irrational value of N corresponds to an incommensurable Peierls. Below we shall consider only the transition to a commensurable phase. Here, as in the Peierls problem, in our case we can distinguish two possiblities: $M>2$ and $M=2$. In the former case the density of the continuum Hamiltonian of the s-f model is

$$a^{-1}h_{s-f} = A\left(\frac{S}{2}\right)^{1/2} \times \left\{\Psi^+\begin{pmatrix} 0 & c_{2k_F} \\ c_{2k_F}^+ & 0 \end{pmatrix}\Psi + \chi^+\begin{pmatrix} 0 & c_{-2k_F}^+ \\ c_{-2k_F} & 0 \end{pmatrix}\chi\right\}, \quad (6)$$

$$\Psi = \begin{pmatrix} W_\uparrow \\ V_\downarrow \end{pmatrix}, \quad \chi = \begin{pmatrix} W_\downarrow \\ V_\uparrow \end{pmatrix}.$$

The density h_{s-f} takes the same form in the incommensurable case.

For twofold commensurability we have

$$a^{-1}h_{s-f} = A\left(\frac{S}{2}\right)^{1/2}\left\{\Psi^+\begin{pmatrix} 0 & c_{2k_F} + c_{-2k_F} \\ c_{2k_F}^+ + c_{-2k_F}^+ & 0 \end{pmatrix}\Psi + \chi^+\begin{pmatrix} 0 & c_{2k_F}^+ + c_{-2k_F}^+ \\ c_{2k_F} + c_{-2k_F} & 0 \end{pmatrix}\chi\right\}. \quad (7)$$

In accordance with the principles of self-consistent field theory we represent the magnon operators in the form

$$c_{2k_F} = \Delta e^{i\varphi}, \quad c_{-2k_F} = \rho e^{-i\theta}. \quad (8)$$

Correspondingly, the spin components take the form

$$\begin{aligned} S_n^x &= (2/A)\,[\bar\Delta\cos(2k_F na+\varphi) + \bar\rho\cos(2k_F na+\theta)], \\ S_n^y &= (2/A)\,[\bar\rho\sin(2k_F na+\theta) - \bar\Delta\sin(2k_F na+\varphi)], \end{aligned} \quad (9)$$

where $\bar\Delta = \Delta A(S/2)^{1/2}$ and $\bar\rho = \rho A(S/2)^{1/2}$. From (9) we can see immediately what distinguishes the case of twofold

commensurability ($2k_Fa=\pi$); of the four independent quantities Δ, ρ, φ, and θ there remain two:

$$\Phi=\bar{\Delta}\cos\varphi+\bar{\rho}\cos\theta, \quad \eta=\bar{\rho}\sin\theta-\bar{\Delta}\sin\varphi. \tag{10}$$

It is Φ and η that appear in the Hamiltonian (7).

After substitution of (8) the Hamiltonians (6) and (7) acquire the form

$$a^{-1}h_{s-f}=\bar{\Delta}\bar{\Psi}\exp(-i\tau_z\varphi)\Psi+\bar{\rho}\bar{\chi}\exp(-i\tau_z\theta)\chi, \tag{11}$$

$$a^{-1}h_{s-f}=\bar{\Psi}(\Phi+i\eta\tau_z)\Psi+\bar{\chi}(\Phi-i\eta\tau_z)\chi, \tag{12}$$

where $\bar{\Psi}=\Psi^+\tau_x$ and $\bar{\chi}=\chi^+\tau_x$.

We now go over to the continuum limit ($\Sigma_n\ldots\to a^{-1}\int dx\ldots$) and in place of the Hamiltonians write the corresponding Lagrangians, since this is convenient for the subsequent calculations ($\hbar=1$). For $M>2$

$$\mathcal{L}=i\bar{\Psi}\tau_x\partial_t\Psi+i\bar{\chi}\tau_x\partial_t\chi-\bar{\Delta}\bar{\Psi}\exp(-i\tau_z\varphi)\Psi-\bar{\rho}\bar{\chi}\exp(-i\tau_z\theta)\chi$$
$$-v_F\bar{\Psi}\tau_y\left(\partial_x-i\frac{AS}{2}\right)\Psi-v_F\bar{\chi}\tau_y\left(\partial_x+i\frac{AS}{2}\right)\chi-\frac{\bar{\Delta}^2+\bar{\rho}^2}{g_1}$$
$$-\frac{\bar{\Delta}\bar{\rho}\cos(\varphi-\theta)}{g_2}\operatorname{sign}(J^x-J^y), \tag{13}$$

where

$$\frac{1}{g_1}=2\frac{(J^x+J^y)\cos(2k_Fa)-2J^z}{aA^2},$$
$$\frac{1}{g_2}=4\frac{|J^x-J^y|\cos(2k_Fa)}{aA^2}, \quad v_F=ta; \tag{14}$$

for $M=2$

$$\mathcal{L}=i\bar{\Psi}\tau_x\partial_t\Psi+i\bar{\chi}\tau_x\partial_t\chi-v_F\bar{\Psi}\tau_y\left(\partial_x-i\frac{AS}{2}\right)\Psi$$
$$-v_F\bar{\chi}\tau_y\left(\partial_x+i\frac{AS}{2}\right)\chi-\bar{\Psi}(\Phi+i\eta\tau_z)\Psi$$
$$-\bar{\chi}(\Phi-i\eta\tau_z)\chi-\frac{\Phi^2}{g_3}-\frac{\eta^2}{g_4}, \tag{15}$$

where

$$\frac{1}{g_3}=-4\frac{J^x+J^z}{aA^2}, \quad \frac{1}{g_4}=-4\frac{J^y+J^z}{aA^2}. \tag{16}$$

The Lagrangians (13) and (15) corresponds to the chirally invariant Gross-Neveu (GN) model of quantum field theory with different coupling constants. We first investigate the ground state of the models (13) and d(15) for isotropic ferromagnetic exchange $J^x=J^y=J^z=J<0$, and then indicate the result of including anisotropy.

2. ISOTROPIC MODEL OF A MAGNETIC PEIERLS INSULATOR

In the isotropic case, $g_2^{-1}=0$, while $g_1^{-1}=-8J\sin^2(k_Fa)/aA^2$ and $g_3^{-1}=g_4^{-1}=-8J/aA^2$, and so the ground state of the GN model is stable for $J<0$, which, naturally, coincides with the condition for ferromagnetism of the initial metallic phase. The model obtained has been investigated in detail (see, e.g., Ref. 4), and so we immediately give the expression for the order parameter in the uniform ground state of the system (13) at $T=0$:

$$\bar{\Delta}=\bar{\rho}=\Delta_0=2(\varepsilon_F^2-A^2S^2/4)^{1/2}\exp(-1/\lambda), \tag{17}$$

where $\lambda^{-1}=2g_1^{-1}N^{-1}(0)$, in which $N(0)$ is the density of states at the Fermi level. The expression (17) and the usual formula for the gap in the Peierls problem differ in the form of the pre-exponential factor (the Peierls pre-exponential factor is simply $2\varepsilon_F$). This difference is connected with the fact that in a ferromagnetic metal the number of electrons with spin along the spontaneous moment of the lattice differs from the number of electrons with the opposite spin, and the order parameter, as already stated, is due to pairing between these branches:

$$\bar{\Delta}\sim\langle\bar{\Psi}\Psi\rangle, \quad \bar{\rho}\sim\langle\bar{\chi}\chi\rangle. \tag{18}$$

It is clear that an MPI exists for so long as there are electrons with both spin components, i.e., for $\varepsilon_F>|A|S/2$ (in metals the stronger inequality $\varepsilon_F\gg|A|S/2$ is usually fulfilled[7]). In the case of period doubling the equations of the self-consistent model (15) have the form

$$\Phi=-\frac{g_3}{2}\langle\bar{\Psi}\Psi+\bar{\chi}\chi\rangle, \quad \eta=-\frac{g_3}{2}\langle\bar{\Psi}\tau_z\Psi-\bar{\chi}\tau_z\chi\rangle. \tag{19}$$

But it follows from the equations of motion that $\chi=\Psi^*$; therefore $\eta=0$ and the order parameter is only the field Φ:

$$\Phi=2(\varepsilon_F^2-A^2S^2/4)^{1/2}\exp(-2/N(0)g_3). \tag{20}$$

In this case $S_n^y=0$, and $S_n^x\neq0$ (see (9)).

We note that because of (18) and (19) the average value of the electron-density operator

$$\hat{n}=a_\uparrow^+a_\uparrow+a_\downarrow^+a_\downarrow \tag{21}$$

is constant ($\langle\hat{n}\rangle=n_0$) and, in contrast to the case of a PI, does not contain corrections oscillating with frequency $2k_Fx$. This is a very important difference between the model studied here and the Peierls model.

In the case of isotropic Heisenberg exchange it is not difficult to supplement the Lagrangians (13) and (15) with terms describing the kinetic energy of the magnetic subsystem. In fact, in the one-magnon approximation the Heisenberg Hamiltonian is

$$h_H=4\bar{J}(c_{2k_F}^+c_{2k_F}+c_{-2k_F}^+c_{-2k_F}), \quad \bar{J}=-JS\sin^2k_Fa. \tag{22}$$

Introducing generalized coordinates (see, e.g., Ref. 9) by the formulas

$$Q_1=2^{-1/2}(c_{2k_F}^++c_{-2k_F}), \quad Q_2=2^{-1/2}i(c_{-2k_F}-c_{2k_F}^+), \tag{23}$$

we write h_H in the form of the potential energy of a harmonic oscillator:

$$h_H=4\bar{J}(|Q_1|^2+|Q_2|^2). \tag{24}$$

Correspondingly, the kinetic energy of the oscillator is given by a quadratic form of the generalized momenta

$$P_1=2^{-1/2}i(c_{-2k_F}^+-c_{2k_F}), \quad P_2=2^{-1/2}(c_{-2k_F}^++c_{2k_F}), \tag{25}$$

while the Lagrangian of the magnon subsystem for $M>2$ is

$$\mathcal{L}_H=\frac{1}{2}\sum_i\left\{\frac{1}{2\bar{J}}|\dot{Q}_i|^2-8\bar{J}|Q_i|^2\right\}$$
$$=\frac{\dot{\bar{\Delta}}^2+\bar{\Delta}^2\dot{\varphi}^2}{g_1\omega_0^2}+\frac{\dot{\bar{\rho}}^2+\bar{\rho}^2\dot{\theta}^2}{g_1\omega_0^2}-\frac{\bar{\Delta}^2+\bar{\rho}^2}{g_1}. \tag{26}$$

where $\omega_0=-4JS\sin^2(k_fa)$. Correspondingly, for period

doubling we have

$$\mathscr{L}_H=\dot{\Phi}^2/g_3\bar{\omega}^2-\Phi^2/g_3, \quad \bar{\omega}=-4JS. \tag{27}$$

From (26) and (27) it can be seen that the kinetic terms in the Lagrangians have the same form as in the Peierls problem.

In contrast to the Peierls continuum model, an MPI is formally described by two fermion fields Ψ and χ, and the isotropic variant of the model (15) is a sum of two Lagrangians of the GN model with multiplicity $N_m = 1$ (see e.g., Ref. 10). However, because of (19) this sum reduces to a GN model with $N_m = 2(\Psi = \chi^*)$, and, when (27) is taken into account, coincides exactly with the Peierls model for systems with a doubled crystal-lattice period.[4,6] Consequently, in an MPI with doubled period there should exist an identical set of self-localized state—topological solitons and polarons of the order parameter.

The isotropic model (13) is the sum of two Lagrangians of the chiral GN model with $N_m = 1$ (Ref. 10). The coincidence of the vacua (17) also makes it possible to write the model in the form of a two-component chiral GN model ($N_m = 2$). Consequently, in the model, as in an incommensurable PI, there exist dynamical solitons of the order parameter. It follows from this reasoning that in an incommensurable isotropic MPI we have $\theta = -\varphi$ as a consequence of the equality $\Psi^* = \chi$, but since the vacuum energy does not depend on φ, the values of φ, and hence the values of the components $S_n^{x,y}$, are not fixed. This degeneracy is lifted when the commensurability energy of the SDW is taken into account.

3. COMMENSURABILITY ENERGY OF THE SDW

As is well known,[4] in field models with a linearized electron spectrum the commensurability effect (i.e., the dependence of the energy of the system on φ and θ) is due to the anharmonic terms in the Hamiltonian that describe Umklapp processes with the participation of several phonons (in the Peierls problem) or magnons (in our problem). Multimagnon terms arise from higher terms of the expansion in the Holstein-Primakoff representation:

$$\begin{aligned}\left(\frac{2}{S}\right)^{1/2} S_n^x &= (c_n^+ + c_n)\left(1-\frac{c_n^+ c_n}{2S}\right)^{1/2},\\ \left(\frac{2}{S}\right)^{1/2} S_n^y &= i(c_n - c_n^+)\left(1-\frac{c_n^+ c_n}{2S}\right)^{1/2}.\end{aligned} \tag{28}$$

It follows from (8) that

$$c_n^+ c_n = \Delta^2+\rho^2+2\Delta\rho\cos(4k_F na+\varphi+\theta). \tag{29}$$

In contrast to the Peierls problem, in which the anharmonicities are given by powers of linear phonon operators[4] and, consequently, by sums of Umklapp scatterings through $2k_F$, in our case we sum Umklapp scatterings through $4k_F$. This is not an accidental difference; it is connected with the conservation of the electron spin in the case of an even number of Umklapp scatterings between branches. It is also not accidental that the combination $\varphi + \theta$ appears in the anharmonic terms. In fact, in the ground state ($\Delta = \rho$) the formulas (9) take the form

$$\begin{aligned}S_n^x &= (2S)^{1/2}\Delta\cos\left(\frac{\varphi-\theta}{2}\right)\cos\left(2k_F na+\frac{\varphi+\theta}{2}\right),\\ S_n^y &= -(2S)^{1/2}\Delta\sin\left(\frac{\varphi-\theta}{2}\right)\cos\left(2k_F na+\frac{\varphi+\theta}{2}\right).\end{aligned} \tag{30}$$

For M-fold commensurability the unit cell of the MPI consists of M sites and contains two electrons—the condition for insulating behavior. Consequently, the expressions (30) should be periodic under the shift $n \to n + M$, i.e., the commensurability energy, which determines the minimum of the energy as a function of the position of the SDW relative to the original lattice, should be periodic in the argument $\varphi + \theta$; this is incorporated in (29) and is confirmed by the subsequent calculation.

We shall calculate the commensurability terms in the s-f-exchange Hamiltonian. Since the commensurability terms are sought using perturbation theory, in the expansion in the parameter $\Delta^2/S \ll 1$ in formulas (28) and (29) it is necessary to keep the largest terms in which rapid oscillations vanish when $k_F a = \pi/M$. It is not difficult to verify that in the general mth term of the perturbation-theory series it is necessary to keep only

$$\begin{aligned}S_n^{x(m)} &\approx \left(\frac{S}{2}\right)^{1/2}(c_n+c_n^+)\left(\frac{2\Delta^2}{S}\right)^m C_{1/2}^m\cos^m(4k_F na+\varphi+\theta),\\ S_n^{y(m)} &\approx i\left(\frac{S}{2}\right)^{1/2}(c_n-c_n^+)\left(\frac{2\Delta^2}{S}\right)^m C_{1/2}^m\cos^m(4k_F na+\varphi+\theta).\end{aligned} \tag{31}$$

The remaining terms of the binomial series for the minimum value m_0 (which, in the final analysis, is determined by the commensurability index M) are found to be of higher order of smallness in $\Delta^2/S \ll 1$. We must now substitute (31) into the combination representing the anharmonic correction to the Hamiltonian (6):

$$\begin{aligned}&S_n^{x(m)}\left[(W_\uparrow^+ V_\downarrow + W_\downarrow^+ V_\uparrow)\exp(-2ik_F na)\right.\\ &\quad\left.+(V_\uparrow^+ W_\downarrow + V_\downarrow^+ W_\uparrow)\exp(2ik_F na)\right]\\ &+iS_n^{y(m)}\left[(V_\downarrow^+ W_\uparrow - V_\uparrow^+ W_\downarrow)\exp(2ik_F na)\right.\\ &\quad\left.+(W_\downarrow^+ V_\uparrow - W_\uparrow^+ V_\downarrow)\exp(-2ik_F na)\right],\end{aligned} \tag{32}$$

and separate out the smooth contribution. The result has the form

$$\begin{aligned}h_{comm} = \bar{\Delta}2^{1/2}C_{1/2}^{m_0}\left(\frac{\bar{\Delta}^2}{A^2S^2}\right)^{m_0}&\{\bar{\Psi}\exp(i\tau_z(m_0(\varphi+\theta)+\theta))\Psi\\ &+\bar{\chi}\exp(i\tau_z(m_0(\varphi+\theta)+\varphi))\chi\} = -\mathscr{L}_{comm},\end{aligned} \tag{33}$$

where for even M we have $m_0 = M/2 - 1$, and for odd M we have $m_0 = M - 1$. The anharmonic terms of Heisenberg origin are small in the parameter $J/|A| \ll 1$ and are discarded. In real conducting magnetic systems this condition is, as a rule, fulfilled.[8]

Having added (33) to the isotropic Lagrangian (13), we can easily separate out from the total Lagrangian of the system the Lagrangian of the phase degree of freedom. Henceforth, we shall call the latter Lagrangian the SDW Lagrangian, by analogy with the terminology adopted for CDW. For this it is necessary to subject the total Lagrangian to the chiral transformation

$$\Psi \to \exp\left(i\frac{\tau_z}{2}\varphi\right)\Psi, \quad \bar{\Psi}\to\bar{\Psi}\exp\left(i\frac{\tau_z}{2}\varphi\right),$$
$$\chi \to \exp\left(i\frac{\tau_z}{2}\theta\right)\chi, \quad \bar{\chi}\to\chi\exp\left(i\frac{\tau_z}{2}\theta\right) \tag{34}$$

and integrate over the fermion fields Ψ and χ (see the details of the analogous calculation in Refs. 5 and 11). We obtain

$$\mathscr{L}_{SDW} = \frac{\bar{\Delta}^2}{\lambda\omega_0^2}N(0)\left\{\frac{1}{2}\dot{\varphi}^2+\frac{1}{2}\dot{\theta}^2-\frac{c_0^2}{2}(\varphi')^2-\frac{c_0^2}{2}(\theta')^2 + \zeta\frac{\bar{\Delta}^2}{\lambda}\cos((m_0+1)(\varphi+\theta))\right\}, \tag{35}$$

where $c_0 = v_F\lambda^{1/2}\omega_0/\bar{\Delta}$ is the SDW velocity, and

$$\zeta\sim(\bar{\Delta}^2/A^2S^2)^{m_0-1/2}\lambda\omega_0^2/\bar{\Delta}^2\ll 1.$$

As expected, the SDW potential is $\cos(\frac{M}{2}(\varphi+\theta))$, where the upper line pertains to even M and the lower line to odd M.

In the next section we shall discuss the role of anisotropy, and also of electric and magnetic fields, in the dynamics of SDW.

4. EFFECT OF ANISOTROPY AND EXTERNAL FIELDS ON THE CHARACTERISTICS OF MAGNETIC PEIERLS INSULATORS

As has been shown, in the presence of anisotropic Heisenberg exchange the continuum Lagrangian acquires a term that depends on the phase difference $\varphi-\theta$ (the last term in (13)). Consequently, the ground-state energy depends on the phases even in the absence of commensurability terms. It is clear that the phase fixing of the MPD vacuum is determined by the relative contribution of the anisotropy and the commensurability energy $\varepsilon_{\text{comm}}$. We shall call the anisotropy strong if

$$\bar{\Delta}^2/g_2 \gg \varepsilon_{comm}\sim\zeta N(0)\bar{\Delta}^2(\bar{\Delta}^2/\lambda\omega_0^2),$$

and weak if $\varepsilon_{\text{comm}} \gg \bar{\Delta}^2/g_2$.

In the case of strong anisotropy the difference $\varphi-\theta$ is fixed, while the vacuum value of φ (or θ) is determined from the equations of motion of the SDW (35). For weak anisotropy the last term in the Lagrangian (13) may be disregarded and the problem is isotropic.

It is not difficult to convince oneself that for strong anisotropy the difference $\varphi-\theta$ is determined by the sign of J^x-J^y. The minimum vacuum energy always corresponds to the conditions

$$\bar{\Delta}=\bar{\rho}, \quad \operatorname{sign}(J^x-J^y)\cos(\varphi-\theta)<0. \tag{36}$$

Correspondingly, the sign of J^x-J^y determines the polarization of the SDW: For $J^x>J^y$ we have $S_n^x=0$ and $S_n^y\neq 0$, while for $J^x<J^y$ we have $S_n^y=0$ and $S_n^x\neq 0$ (in the case of weak anisotropy, both components of the spiral are nonzero).

In the case of strong anisotropy the form of the dimensionless coupling constant λ in the self-consistency equation (17) also changes:

$$\lambda^{-1}=2N^{-1}(0)\,(g_1^{-1}-{}^1\!/_2 g_2^{-1}) \tag{37}$$

and the condition $g_1^{-1}>\frac{1}{2}g_2^{-1}$ for the existence of an MPI becomes a threshold condition. When (14) is taken into account this condition takes the form

$$(J^x+J^y)\cos 2k_Fa-2J^z-|J^x-J^y|\cos 2k_Fa>0,$$

i.e., it coincides with the criteria for stability of the ground state of an anisotropic Heisenberg system.

The above remarks pertain only to exchange anisotropy. It is not difficult to see that the addition of single-ion anisotropy $H_a = -K\Sigma_n (S_n^z)^2$ to the isotropic Heisenberg Hamiltonian only renormalizes the coupling constant g_1:

$$g_1^{-1}\to -\frac{8J\sin^2 k_Fa}{aA^2}+\frac{K}{aA^2}, \quad K>0, \quad J<0.$$

It is clear from the above discussion that in the region of existence of an MPI neither exchange anisotropy nor single-ion anisotropy has any influence on the spatial dependence of the self-localized states of the order parameter.

The behavior of the magnetic order parameter in an external electric field E directed along the axis of one-dimensional motion of the electrons is identical to the behavior of the Peierls order parameter—namely, the modulus of the order parameter is suppressed by the field,[4] and MPI with a doubled magnetic period is not electrically active, and the field E affects only the motion of the SDW (35). In the Lagrangian (35) the field appears in the combination $eE(\varphi+\theta)$ (e is the electron charge), in analogy with the case of CDW (Refs. 4, 11). In analogy with CDW, in SDW three-dimensional interactions and Coulomb effects associated with the quasi-one-dimensionality of real materials should be manifested. At low temperatures, nonlinear conductivity, due to phase solitons,[4,12] should be observed in SDW.

However, in contrast to CDW, in SDW there appears the interesting possibility of initiating collective nonlinear conductivity by a magnetic field. We suppose that the spontaneous ferromagnetic moment S is nonuniform and that there are domains in the system. In a magnetic field the domains move, i.e., we have $\dot{S}\neq 0$. In the Lagrangian (13) the chiral anomaly singles out the terms $A(\varphi-\theta)\dot{S}$, i.e., the SDW are acted upon by an effective electric field $E^*\sim A\dot{S}/e$ due to the imbalance of the branches of the electron spectrum. An experimental verification of this phenomenon would be of considerable interest.

We now discuss the effect of a magnetic field on SDW. A magnetic field **H** acts on both subsystems forming the SDW condensate, and should be included both in the Heisenberg Hamiltonian and in the electron Hamiltonian. Since the z axis is specified by the direction of the spontaneous moment of the lattice, we can consider different orientations of **H** with respect to the z axis. We begin with the electron subsystem. In a one-dimensional system of electrons a magentic field acts only on their spins and there is no orbital effect; therefore, the Hamiltonian of the interaction with the field has the same matrix structure as the s-f-exchange Hamiltonian[4]:

$$H_H^{el}=-\mu\sum_{n,\tau,\tau'} a_{n\tau}^+(\mathbf{H}\tau)_{\tau\tau'}a_{n\tau'}, \tag{38}$$

where μ is the Bohr magneton. The component H_z is added to the internal "magnetizing" field $AS/2\mu$ and, thus, controls the pre-exponential factor in (17). The physics of this

effect is trivial: The numbers of electrons with spins parallel and antiparallel to the spontaneous moment change.

The transverse components $H_{x,y}$ flip the electron spin, but, unlike the operator components $S^{x,y}$, are not connected with Umklapp scatterings through $2k_F$. Therefore, they do not mix the right and left branches of the spectrum, but merely link the components of the spinors $(W\uparrow, W\downarrow)$ and $(V\uparrow, V\downarrow)$ (in the field Lagrangian the components $H_{x,y}$ appear as interaction constants of the fields Ψ and χ):

$$\mu H_x(\bar{\Psi}\tau_x\chi e^{iASx}+\bar{\chi}\tau_x\Psi e^{-iASx}),\ i\mu H_y(\bar{\Psi}\tau_z\chi e^{iASx}-\chi\tau_z\Psi e^{-iASx}). \tag{39}$$

It is clear that the transverse components of **H** "distinguish" the fields Ψ and χ only for $\mu H_\perp \propto \bar{\Delta}$ (for $\bar{\Delta}\sim 1$ K, $H_\perp \sim 10^4$ Oe). In weaker fields, in (39) we can set $\Psi = \chi^*$, i.e., obtain a term $\mu H_x \Psi^+\Psi$ playing the role of a chemical potential. It leads to a nonuniform ground state of the soliton-lattice type.[6]

The magnetic field appears in the Heisenberg Hamiltonian in the standard way: $\mu\Sigma_n S_n H$. The component H_z renormalizes the coupling constant g_1 analogously to single-ion anisotropy. The transverse components of **H** formally drop out of the Lagrangian in the passage to the continuum model, because of the summation over the fast oscillations. In fact, however, by virtue of the self-consistency condition, the interaction of $H_\perp$ with the average lattice spin remains as before, but is contained only in the electron Hamiltonian.

CONCLUSION

In this paper we have constructed a new model of the metal-insulator transition in one-dimensional ferromagnetic metals. The metallic ferromagnetic state is found to be unstable at a certain temperature $T_{\text{MPI}} < T_c$ "dielectrization" of the electron spectrum and formation of a magnetic superstructure—a spiral with period π/k_F. Unlike the traditional description of SDW, based on the Hubbard interaction, in our model the spiral structure arises as a consequence of *s-f* exchange of conduction electrons with magnetic ions of the lattice, and is due to Umklapp processes between branches of the electron spectrum with absorption or emission of magnons with momenta $\pm 2k_F$. Formally, the model is found to be analogous in many respects to models of one-dimensional Peierls insulators and reduces to the exactly solvable GN models of one-dimensional quantum field theory.

Here it is necessary to emphasize the fundamental differences between an MPI and a PI. In a magnetic metal the crystal lattice does not change, and a new electron Brillouin zone of size $P_B^* = 2k_F = p_B L/M$ is formed by rearrangement of the atomic spins in the chain, i.e., the new symmetry of the electron wavefunction arises from the symmetry of the spin factor of the electron. As a result, the spatial translational symmetry with period a remains a true symmetry only for MPI with even commensurability indices, since it corresponds to Umklapp scatterings through the vector $L(2\pi/a)$ with conservation of the electron-spin projection. For an odd commensurability index the minimum number of Umklapp scatterings is doubled and the amplitude of the commensurability energy decreases sharply in comparison with the case of even M. This nonmonotonic behavior of the commensurability energy with the label M is specific for MPI only, and does not have analogs in the electron-phonon system.

Thus, the new state of an MPI is characterized by the following indicators: There exist two transition temperatures T_c and T_{MPI} ($T_{\text{MPI}} < T_c$); for $T_{\text{MPI}} < T < T_c$ the system is a ferromagnetic conductor, while below T_{MPI} it is a magnetic insulator with a spiral structure; inside the spiral there can exist macroscopic transitional regions, the analogs of domains (they are solitons and polarons of the order parameter of the MPI); these regions necessarily contain an electron (hole) level within the forbidden band, and this level can be determined from the optical or infrared spectra; in systems with higher than twofold commensurability there exists a collective electrically active SDW mode, nonlinear excitations of which (phase solitons) give rise to a nonohmic contribution to the conduction; all the parameters of the MPI and the SDW can be controlled by a magnetic field, as can be observed, e.g., in the dependence of the absorption frequencies on H.

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