Effect of a microwave field on the quantum correction to the conductivity

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We study the effect of a high-frequency field on the anomalous magnetoresistivity of a thin film. We consider the effect of a microwave field on the quantum correction to the conductivity of a thin wire *e-e* scattering taken into account.

INTRODUCTION

The anomalous magnetoresistivity effect which is connected with weak localization effects is at the present time studied actively in experiments. One of the possible means to study this effect may be the study of the effect of high-frequency radiation on the conductivity of a sample.[1] In the present paper we give a theoretical calculation of the effect of a microwave field on the quantum correction to the conductivity.

The influence of a high-frequency electromagnetic field on the magnetoresistivity of a thin film is considered in section 1. It is known that the quantum correction to the conductivity is strongly sensitive to the *e-e* interaction, especially in one-dimensional systems. We consider in section 2 the change in the conductivity of a thin wire under the action of a microwave field in the case when in the relaxation processes *e-e* collisions with small energy transfers dominate.

1. EFFECT OF A MICROWAVE FIELD ON THE ANOMALOUS MAGNETORESISTIVITY OF A THIN FILM

The quantum correction to the conductivity arises due to the coherence of electrons traversing the same trajectories in opposite directions. Any violation of the symmetry with respect to time reversal leads to their dephasing. On the one hand, a constant magnetic field leads to dephasing and also a variable electromagnetic field. On the other hand, all inelastic processes lead to phase errors. We first of all consider the behavior of the quantum correction to the conductivity of a film in a classically weak transverse magnetic field, $\Omega\tau \ll (\tau\varepsilon_F)^{-1} \ll 1$ (τ is the free flight time, ε_F the Fermi energy, $\Omega = eH/mc$) under the action of microwave radiation, assuming that the phase relaxation when there are no fields can be described in the phase relaxation time approximation ($\tau_{\rm ph}$ approximation).

The quantum correction to the conductivity is connected with the electron correlation function in the Cooper channel by the relation

$$\delta\sigma = 4\sigma\tau(\pi\nu)^{-1}\int_0^\infty d\eta\, C^{t-\eta/2}_{\eta,-\eta}(\mathbf{r},\mathbf{r}). \tag{1.1}$$

It was shown in Ref. 2 that the cooperon $C^t_{\eta,\eta'}(\mathbf{r},\mathbf{r}')$ satisfies the equation

$$\left\{\frac{\partial}{\partial\eta} - D\left[i\nabla + \frac{e}{c}\mathbf{A}\left(t+\frac{\eta}{2}\right) + \frac{e}{c}\mathbf{A}\left(t-\frac{\eta}{2}\right)\right]^2 + \tau_{\rm ph}^{-1}\right\}C = \tau^{-1}\delta(\mathbf{r}-\mathbf{r}')\delta(\eta-\eta'). \tag{1.2}$$

We can write the solution of (1.2) in the form of a path integral

$$C=\tau^{-1}\int_{\mathbf{r}(\eta')=\mathbf{r}'}^{\mathbf{r}(\eta)=\mathbf{r}} \mathcal{D}r(t_1)\exp\left\{-\int_{\eta'}^{\eta} dt_1[\dot{\mathbf{r}}^2(4D)^{-1} + i\dot{\mathbf{r}}ec^{-1}\mathbf{A}_t(t_1) + \tau_{\rm ph}^{-1}]\right\}, \tag{1.3}$$

where we have introduced the notation $\mathbf{A}_t(t_1) = \mathbf{A}(t+t_1/2) + \mathbf{A}(t-t_1/2)$. It is convenient to split in (1.3) the integration over even and odd (in t_1) variables:

$$\mathbf{R} = \frac{\mathbf{r}(t_1)+\mathbf{r}(-t_1)}{\sqrt{2}}, \quad \boldsymbol{\rho} = \frac{\mathbf{r}(t_1)-\mathbf{r}(-t_1)}{\sqrt{2}}.$$

For the cooperon in coinciding points $\mathbf{r} = \mathbf{r}'$ and symmetric times $\eta' = -\eta$ one can easily integrate over even functions of $\mathbf{R}$ using the relation

$$\int d\mathbf{R}\int_{\mathbf{R}(0)=\mathbf{R}}^{\mathbf{R}(\eta)=0} \mathcal{D}\mathbf{R}(t_1)\exp\left\{-\int_0^\eta dt_1\,\dot{\mathbf{R}}^2(4D)^{-1}\right\} = 1.$$

As a result we get for the cooperon in a transverse magnetic field and a microwave field

$$C=\tau^{-1}\int_{\boldsymbol{\rho}(\eta)=\boldsymbol{\rho}(0)=0}\mathcal{D}\boldsymbol{\rho}(t_1)\exp\left\{-\int_0^\eta dt_1\left[\dot{\boldsymbol{\rho}}^2(4D)^{-1} + 2\tau_{\rm ph}^{-1} + e^2DH^2c^{-2}\boldsymbol{\rho}^2 + i2^{1/2}e(4D)^{-1}\sin\frac{\omega t_1}{2}\cos\omega t\mathbf{E}\boldsymbol{\rho}\right]\right\}. \tag{1.4}$$

One can simplify (1.4) by a change in variables $\boldsymbol{\rho}\to\boldsymbol{\rho}' = \boldsymbol{\rho} - \boldsymbol{\xi}$ such that in the exponent of the transformed path integral there are no terms linear in $\boldsymbol{\rho}'$. This leads to an equation for $\boldsymbol{\xi}$:

$$\ddot{\boldsymbol{\xi}} - \Omega^2\boldsymbol{\xi} - 2^{1/2}i\,De\,\mathbf{E}_t = 0, \quad \Omega = 2eDHc^{-1} \tag{1.5}$$

with initial conditions

$$\boldsymbol{\xi}(0) = \boldsymbol{\xi}(\eta) = 0. \tag{1.5'}$$

Substituting the solution of Eqs. (1.5), (1.5')

$$\boldsymbol{\xi} = -i\int d\omega\,\mathbf{E}(\omega)e^{i\omega t}\left[\Omega^2 + \left(\frac{\omega}{2}\right)^2\right]^{-1}\times\left[\sin\frac{\omega t_1}{2} - \sin\frac{\omega\eta}{2}\frac{\operatorname{sh}\Omega t_1}{\operatorname{sh}\Omega\eta}\right]$$

into (1.4) and using the fact that the remaining integral over ρ' describes a cooperon in a constant magnetic field which is evaluated in Ref. 3 we get

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$$C=2\Omega \exp\{-\eta \tau_{ph}^{-1}\} \operatorname{sh}^{-1}(2\Omega\eta) \times \exp\left\{-\int_0^\eta dt_1 \int d\omega\, d\omega'\, \mathbf{E}(\omega)\mathbf{E}(\omega')\right.$$

$$\times \exp\{it(\omega+\omega')\}\left[\Omega^2+\left(\frac{\omega}{2}\right)^2\right]^{-1}$$

$$\left.\times \sin\frac{\omega' t_1}{2}\left[\sin\frac{\omega t_1}{2}-\sin\frac{\omega\eta}{2}\frac{\operatorname{sh}\Omega t_1}{\operatorname{sh}\Omega\eta}\right]\right\}. \qquad (1.6)$$

After averaging

$$(2T)^{-1}\int_{-T}^{T} dt \qquad (1.6')$$

and introduction of the notation

$$x_\Phi=0{,}5\omega\tau_\Phi, \quad \xi=4\mathrm{De}\, H(c\omega)^{-1},$$

$$\alpha=E^2e^2D/2\omega^3(1+\xi^2), \quad x=\omega t_1$$

(1.6) takes the form

$$\delta\sigma=4\sigma\tau(\pi\nu)^{-1}\int_{\omega\tau}^{\infty} dx\,\xi \exp\{-x x_{ph}^{-1}\}\operatorname{sh}^{-1}(\xi x)$$

$$\times \exp\{-\alpha B(x)\} I_0[\alpha(P^2-A^2)^{1/2} B(x)], \qquad (1.7)$$

where

$$B(x)=x\left[1+((1+\xi^2)x)^{-1}(1-\xi^2)\sin x - 4\operatorname{cth}\left(\frac{\xi x}{2}\right)\frac{\xi \sin^2(x/2)}{(1+\xi^2)x}\right].$$

We have denoted the degree of polarization and the degree of circular polarization of the UHF field by P and A.

Under the action of strong radiation the loss of coherence occurs over times $x \ll x(\alpha)$,

$$x(\alpha)=\begin{cases}\alpha^{-1/5}, & \alpha\gg 1\\ \alpha^{-1}, & \alpha\ll 1\end{cases}.$$

When $\alpha \gg 1$, $\alpha^{-1/5} \ll \min\{\xi^{-1}, x_{ph}\} = x_1$

$$\delta\sigma=-4\sigma\tau(\pi\nu\omega)^{-1}\ln(\omega\tau\alpha^{1/5}), \qquad (1.8)$$

when $\alpha \ll 1$, $\alpha^{-1} \ll x_1$

$$\delta\sigma=-4\sigma\tau(\pi\nu\omega)^{-1}\ln(\omega\tau\alpha). \qquad (1.8')$$

When the intensity of the variable field is low $1 \gg \alpha B(x_1)$ the change in the conductivity is described by the formula

$$\Delta\sigma(\alpha)=\delta\sigma(\alpha)-\delta\sigma(0) = 4\sigma\tau(\pi\nu)^{-1}\alpha\int_{\omega\tau}^{\infty} dx\,\xi e^{-x/x_{ph}}\operatorname{sh}^{-1}(\xi x) B(x).$$

If $x_1 \ll 1$ we have

$$\Delta\sigma=(450\pi\nu)^{-1}\sigma\tau x_1^5\alpha, \quad \alpha x_1^5 \ll 1, \qquad (1.9)$$

when $x_1 \gg 1$

$$\Delta\sigma=4\sigma\tau(\pi\nu)^{-1}\alpha x_1, \quad \alpha x_1 \ll 1. \qquad (1.9')$$

We show in Fig. 1 the way the magnetoresistivity

$$\Delta\sigma=4\sigma\tau(\pi\nu)^{-1} f(\alpha, \xi, x_{ph}) \qquad (1.10)$$

changes under the action of microwave radiation of linear and circular polarizations for different choices of the parameters.

Taking e-e scattering into account leads in the two-dimensional case[2] to replacing the time for phase breaking τ_{ph} (due to e-ph processes) by $\tau_{eff} = (\tau_{ph}^{-1} + \tau_e^{-1})^{-1}$:

$$\tau_e^{-1}=\begin{cases} T(2\pi D\nu\hbar^2)^{-1}\ln(\pi D\nu\hbar), & a \ll L_T,\\ T(4\pi D\nu\hbar^2)^{-1}\ln(\pi D^2\nu\hbar^2 T^{-1}a^{-2}), & a \gg L_T,\end{cases}$$

where a is the film thickness and $L_T = (D\hbar T^{-1})^{1/2}$.

2. EFFECT OF A MICROWAVE FIELD ON THE QUANTUM CORRECTION TO THE CONDUCTIVITY OF A THIN WIRE

In section 1 we took in Eq. (1.2) the phase relaxation time due to inelastic processes into account through a quantity which was independent of the frequency and the momentum of the cooperon. Such an approximation would be fully justified if in the interaction processes of the electrons one could restrict oneself to e-e scattering with large energy transfers. However, in systems with a low dimensionality ($d = 1,2$) this approximation is inapplicable and τ_{ph}^{-1} is an operator. The e-e processes with a small energy transfer which are responsible for the operator form of τ_{ph}^{-1} can be taken into account as an interaction of the electron with the fluctuations of the classical electromagnetic field $\tilde{A}(r,t)$:

$$\tilde{A}_t(r, t_1)=\tilde{A}(r, t+t_1/2)+\tilde{A}(r, t-t_1/2), \qquad (2.1)$$

which is the same for all electrons. In the path intergral (1.2) we must therefore take into account the field (2.1) at the same time as the microwave field.

In a one-dimensional conductor the correlator

$$\langle \tilde{A}_\alpha \tilde{A}_\beta\rangle_{k\omega}=2Tc^2(\sigma\omega^2)^{-1}k_\alpha k_\beta k^{-2}.$$

Therefore, after averaging over the fluctuations of the field (2.1) (Nyquist noise) and integrating over the even variable R we can transform the integral (1.3) to the form

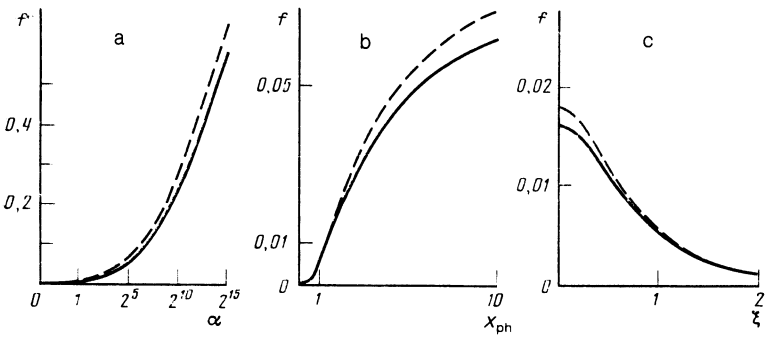

FIG. 1. Change of the magnetoresistivity (1.10) of a thin film under the action of a UHF field with linear (full drawn curve) or circular (dashed curve) polarizations. The calculation was performed for the following parameters: a: $\xi = 1$, $x_{ph} = 1$; b: $\xi = 1$, $\alpha = 1$; c: $\alpha = 1$, $x_{ph} = 1$. δ, B, x_φ to b, c, x_{ph}.

$$C=\int\limits_{\rho(0)=\rho(\eta)=0}\mathcal{D}\rho(t_1)\exp\left\{-\int\limits_0^\eta\left[\frac{\dot\rho^2}{4D}+2\tau_{\rm ph}^{-1}+u_0|\rho|\right.\right.$$
$$\left.\left.+i\sqrt{2}(4D)^{-1}eE\rho\sin\frac{\omega t_1}{2}\cos\omega t\right]\right\}, \qquad (2.2)$$

where $u_0=\sqrt{2c^2T\sigma^{-1}}$.

The cooperon (2.2) and with it the quantum correction to the conductivity (1.1) oscillate in a time t with the frequency ω of the external field. When averaging in the integral (1.6′) we can distinguish two characteristic ranges: a) such times t for which the phase relaxation occurs after a time $\tau_E\ll\tau_e=(Du_0^2)^{-1/3}$, i.e., due to the action of the UHF field; b) times t for which $\cos\omega t\ll 1$ and the dephasing is mainly determined by the e-e interaction. In strong fields the main contribution to the quantum correction comes from a), the region b) corresponds to small additions.

We can in the region b) use the results from Ref. 2 for $\delta\sigma(t)$:

$$\delta\sigma=\sqrt{2}(\pi\nu)^{-1}e^2(D\tau_e)^{1/2}\{[\ln \mathrm{Ai}(2\tau_e\tau_{\rm ph}^{-1})]'\}^{-1}$$
$$=2\sqrt{2}(3^{5/6}\Gamma^2(2/3)\hbar)^{-1}(D\sigma/\sqrt{2}e^2T)^{1/3},\quad \tau_e\ll\tau_{\rm ph}. \qquad (2.3)$$

It is convenient to change in the region a) where we can neglect the e-e processes the variables $\rho\to\rho'=\rho+\xi$ in such a way that the terms linear in ρ' drop out and the whole expression reduces to

$$\int\limits_{\rho'(0)=\rho'(\eta)=0}\mathcal{D}\rho'(t_1)\exp\left\{-\int\limits_0^\eta\frac{\dot\rho'^2}{4D}dt_1\right\},$$

describing pure diffusion. To do this ξ must satisfy Eqs. (1.5) and (1.5′) with $\Omega=0$ and for a monochromatic field

$$\xi=-4iEe\omega^{-2}\left[\sin\frac{\omega t_1}{2}-\sin\frac{\omega\eta}{2}\frac{t_1}{\eta}\right].$$

Therefore in region b)

$$\delta\sigma=4\sigma\tau(\pi\nu)^{-1}\int\limits_0^\infty\left(\frac{\pi}{x\omega D}\right)^{1/2}\exp\{-\alpha B_1(x)\}dx, \qquad (2.4)$$

where

$$B_1(x)=x\left(1+x^{-1}\sin x-8x^{-2}\sin^2\frac{x}{2}\right)\sim\begin{cases}x, & x\gg1\\ x^5, & x\ll1\end{cases}$$

and $\alpha=DE^2e^2(2\omega^3)^{-1}$.

In strong fields ($\alpha\gg1$)

$$\delta\sigma(t)\propto(\alpha\cos^2\omega t)^{-1/10},$$

so that when averaging over t the main contribution to $\int dt$ comes from the region $\cos\omega t\sim 1$ and

$$\delta\sigma=4\sigma\tau(\pi\nu)^{-1}c(\omega D)^{-1/2}\alpha^{-1/10}[1$$
$$+O((\alpha^{1/5}\omega\tau_e)^{-2})], \qquad (2.5)$$

where

$$c=2\Gamma(1/10)360^{1/10}(5\sqrt{\pi})^{-1}$$
$$\times\int\limits_0^{\pi/2}dx\cos^{-1/5}x=4{,}49.$$

in moderate fields ($\alpha\ll1$) Eq. (2.4) leads to

$$\delta\sigma(t)\propto(\alpha\cos^2\omega t)^{-1/2},$$

As a result of this, when one integrates over the region a) it becomes important to cut off the logarithmically diverging integral

$$\int\limits_0 dt\cos^{-1}t$$

at the boundary between the regions a) and b): $\omega t_c=\pi/2-(\omega\tau_e\alpha)^{-1/2}$. Thus

$$\delta\sigma=(4\pi)^{-1/2}\sigma\tau\nu^{-1}(\mathcal{D}\omega\alpha)^{1/2}[1/2\ln(\tau_eE^2e^2D(2\omega^2)^{-1})+O(1)]. \qquad (2.5')$$

Therefore we take for the parameters which separate the characteristic regions for the behavior of the quantum correction

$$\alpha=E^2e^2D(2\omega^3)^{-1},\qquad x_e\alpha=\tau_e\omega\alpha,\qquad x_e^5\alpha=\alpha(\omega\tau_e)^5.$$

We show in Fig. 2 these characteristic regions (in the coordinates which are the intensity E^2 and the frequency ω of the UHF field). In the regions 2 ($\alpha\gg1$, $x_e^5\alpha\gg1$) and 3 ($\alpha\ll1$, $x_e\alpha\gg1$) we must use, respectively, Eqs. (2.5) and (2.5′). In the region 4 ($\alpha x_e^5\ll1$, $\alpha\gg1$) of a weak field the change in the quantum correction can be calculated using perturbation theory and

$$\Delta\sigma(\alpha)=e^2(2\hbar)^{-1}(D\tau_e)^{1/2}\alpha x_e^5. \qquad (2.6)$$

The region 1 ($\alpha x_e\ll1$, $x_e\gg1$) corresponds to a high frequency of the external field. In that case we can in the integral (2.2) integrate over the "fast" variables which leads to replacing $\tau_{\rm ph}^{-1}$ by $\tau_{\rm eff}^{-1}=\tau_{\rm ph}^{-1}+E^2e^2D(4\omega^2)^{-1}\cos^2\omega t$ and we can by using (2.3) obtain

$$\Delta\sigma(\alpha)=(2\pi\sqrt{2}\hbar)^{-1}e^2(D\tau_e)^{1/2}\alpha x_e. \qquad (2.6')$$

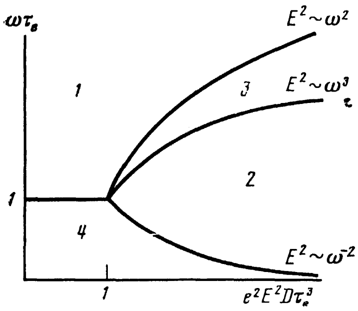

FIG. 2. Characteristic regions of parameters of the UHF field in which the quantum correction to the conductivity is described by Eqs. (2.5) to (2.6′).

CONCLUSION

We have thus in the present paper considered the effect of a UHF field on the quantum correction to the conductivity of a thin film (in a transverse magnetic field) and of a thin wire, taking e-e scattering into account. We have distinguished the characteristic regions for the quantum correction as function of the frequency and intensity of the UHF field.

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