Resonance effects in spectral lines of a hydrogen atom acted upon by both a static and an hf harmonic electric field

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The wave functions of the hydrogen atom and the spectral-line shapes L_α and L_β in an electric field $\mathbf{E}_D(t) = \vec{\mathscr{E}} + \mathbf{E}\cos(\Omega t + \varphi)$ are calculated for $\Omega \gg CE$ ($C = 3/2ean/\hbar$ is the Stark-effect constant, n the principal quantum number, and a the Bohr radius). At $|C\mathscr{E} - \Omega| \gg CE\,|\sin\vartheta\,|$ (ϑ is the angle between $\mathbf{E}$ and $\vec{\mathscr{E}}$) the lines contain principal Stark components due to splitting of the upper energy level by the static field $\vec{\mathscr{E}}$, and also weaker satellites whose frequencies differ from those of the principal components by integer multiples of Ω. As $C\mathscr{E} \to \Omega$ the principal components come closer to the satellite groups. If $|C\vec{\mathscr{E}} - \Omega| \lesssim CE\,|\sin\vartheta\,|$, however, and $\vartheta \neq 0$, the approaching components repel one another and do not cross. The satellite intensities are then resonantly enhanced and become of the same order as those of the principal components. The results are of interest in atomic physics, plasma physics, and atomic spectroscopy.

1. INTRODUCTION

The electric fields of a plasma, of laser emission, and others are easily determined[1] by measuring the Stark broadening of hydrogen spectral lines. We calculate here the hydrogen line shape in the presence of an electric field

$$\mathbf{E}_D(t) = \vec{\mathscr{E}} + \mathbf{E}\cos(\Omega t + \varphi), \tag{1}$$

which is the sum of a static field and an alternating harmonic one, with an angle ϑ between their directions (Fig. 1). It is assumed that the field $\mathbf{E}_D(t)$ produces no transitions between levels with different principal quantum numbers n, and that the frequency of the harmonic component of (1) is high,

$$\Omega \gg CE \equiv \frac{3}{2}\frac{ean}{\hbar}E, \tag{2}$$

where C is the Stark-effect constant and a is the Bohr radius.

The harmonic component of (1) may be produced, for example, by Langmuir oscillations of a plasma[1] or by laser emission (see, e.g., Ref. 2). The static component $\vec{\mathscr{E}}$ can be, in particular, the Holtsmark field of the ions or the electric field of low-frequency plasma oscillations that act on the atom quasistatically.[1]

The profile of the hydrogen ($n \to n'$) emission line is given by the familiar expression[1]

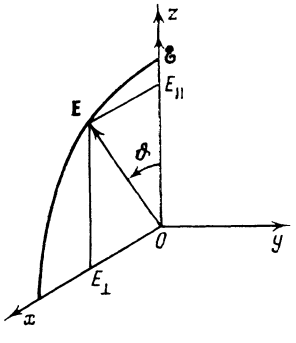

FIG. 1. Geometry of electric field (1).

$$I(\omega) = 3\,\mathrm{Re}\sum_{\alpha\alpha'\beta\beta'}(\mathbf{r}_{\beta'\alpha'}\mathbf{e}^*)(\mathbf{r}_{\alpha\beta}\mathbf{e})\int_0^\infty dt\, e^{i\omega t}\langle U_{\alpha'\alpha}(t)\,U^*_{\beta'\beta}(t)\rangle \times\left\{\pi\sum_{\alpha\beta}|\mathbf{r}_{\alpha\beta}|^2\right\}^{-1}, \tag{3}$$

where ω is the radiation frequency measured from the line center, $\mathbf{e}$ is the polarization unit vector, $\mathbf{r}_{\alpha\beta}$ are the matrix elements of the coordinate operator, $U_{\alpha'\alpha}(t)$ are the matrix elements of the atom evolution operator in the interaction representation, and α,α' and β,β' are the states of the upper and lower levels. The quantities $U_{\alpha'\alpha}(t)$ are determined by a Schrödinger equation with initial conditions

$$i\hbar\dot{U}_{\alpha'\alpha}(t) = e\sum_{\alpha''}\mathbf{E}_D(t)\,\mathbf{r}_{\alpha'\alpha''}U_{\alpha''\alpha}(t), \qquad U_{\alpha'\alpha}(t) = \delta_{\alpha'\alpha}. \tag{4}$$

Equations (4) are standard and describe the evolution of pure quantum states of one emitter in the field (1) with fixed phase φ (there is no need here to derive equations for $U_{\alpha'\alpha}(t)$ in the presence of effects that randomize strongly the states of one emitter). The angle brackets $\langle\ldots\rangle$ in (3) denote averaging over an ensemble (assumed statistically stationary) of fields $\mathbf{E}_D(t)$ (or of emitters). The averaging for the field (1) is over the random phase (and also with the distribution function of the vectors $\vec{\mathscr{E}}$, if $\vec{\mathscr{E}}$ is a quasistatic electric-field components). Note that the line profile (3) is normalized to

$$\int_{-\infty}^{+\infty} I(\omega)\,d\omega = 1.$$

The problem posed was considered in a number of papers,[2–6] but none is complete (see §5 for details). In §2 of the present paper we reduce Eqs. (4) to a more manageable form. These equations are solved in §3 by perturbation theory in terms of the interaction of an atom with a harmonic field, an approach valid if $|C\mathscr{E} - \Omega| \gg CE\,|\sin\vartheta\,|$. Equations (4) are solved in §4 in the resonance approximation[7] for $|C\mathscr{E} - \Omega| \ll \Omega$. The solutions are used in §5 to calculate the shapes of the lines L_α and L_β; the results are compared with those of Refs. 2–6.

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§2. FUNDAMENTAL EQUATIONS

In Eqs. (4) the states α on a level with principal quantum number n are defined by the parabolic quantum numbers q and m ($q \equiv n_1 - n_2, n = 1 + n_1 + n_2 + |m|$) in a coordinate frame whose z axis is parallel to $\vec{\mathscr{E}}$ and the x axis is in the $\vec{\mathscr{E}} \cdot \mathbf{E}$ plane (Fig. 1). The number q is an integer in the interval $|q| \leqslant n-1$. At fixed q, the range of the magnetic quantum number is $m = 1 + |q| - n, 3 + |q| - n, \ldots, n - 1 - |q|$. The matrix elements $\mathbf{r}_{\alpha\alpha'}$ are readily calculated with the aid of the generating functions for Laguerre polynomials:

$$z_{\alpha\alpha'} = {}^3/_2 anq\delta_{\alpha\alpha'}, \quad x_{\alpha\alpha'} = -{}^3/_2 an\xi_{qm}^{q'm'}, \qquad (5)$$
$$\xi_{qm}^{q'm'} = {}^1/_4[n^2 - (q^2 + m^2 - q'^2 - m'^2)^2/4]^{1/2}.$$

The expression for $\xi_{qm}^{q'm'}$ is valid only if $|q - q'| = |m - m'| = 1$; in all other cases $x_{\alpha\alpha'} = 0$. Note that expressions for $x_{\alpha\alpha'}$ and for their quadratic combinations were first given in Refs. 8 and 9.

With (5) taken into account, Eqs. (4) for the matrix elements

$$U_{\alpha'\alpha}(t) \equiv U_{q'm'}^{qm}(t)$$

can be rewritten in the form

$$i\dot{U}_{q'm'}^{qm}(t) = Cq'[\mathscr{E} + E_\parallel \cos(\Omega t + \varphi)] U_{q'm'}^{qm}(t)$$
$$- CE_\perp \cos(\Omega t + \varphi) \sum_{q''m''} \xi_{q'm'}^{q''m''} U_{q''m''}^{qm}(t), \qquad (6)$$

where $E_\parallel$ and $E_\perp$ are the components of $\mathbf{E}$ along z and x (Fig. 1).

To solve (6) it is convenient to use the substitution

$$U_{q'm'}^{qm}(t) = A_{q'm'}^{qm}(t) \exp(-iCq'\mathscr{E}t), \qquad (7)$$

which reduces Eq. (6) and the initial conditions [see (4)] to the form

$$i\dot{A}_{q'm'}^{qm}(t) = Cq'E_\parallel \cos(\Omega t + \varphi) A_{q'm'}^{qm}(t)$$
$$- CE_\perp \cos(\Omega t + \varphi) \sum_{q''m''} \exp[iC\mathscr{E}(q' - q'')t] \xi_{q'm'}^{q''m''} A_{q''m''}^{qm}(t), \qquad (8)$$
$$A_{q'm'}^{qm}(0) = \delta_{qq'}\delta_{mm'}.$$

If $\mathbf{E} = 0$, Eqs. (8) have the obvious solution

$$A_{q'm'}^{qm}(t) = \delta_{qq'}\delta_{mm'}, \quad U_{q'm'}^{qm}(t) = \delta_{qq'}\delta_{mm'} \exp(-iCq'\mathscr{E}t), \qquad (9)$$

which describes the linear Stark effect in a static field. According to (9), the level splits into $2n - 1$ sublevels with frequency spacing $Cq\mathscr{E}$.

Equations (8) describe the action of the harmonic field on the atom, in a representation in which the action of the static field $\vec{\mathscr{E}}$ is taken into account exactly. Firstly, the harmonic field modulates the states (9) by a periodic perturbation of frequency Ω and, secondly, mixes these states (if $\vartheta \neq 0$). Strictly speaking, there are no stationary energy levels of the atom in the field (1), but one can introduce the concept of quasilevels[10] resulting from virtual absorption and emission of harmonic-field quanta by the atom.

§3. DETERMINATION OF THE EVOLUTION OPERATOR BY PERTURBATION THEORY

Equations (8) can easily be solved if the interaction of the atom with the harmonic field can be treated by perturbation theory. The zeroth approximation should be the solution (9) with $\mathbf{E} = 0$. According to (8), the components $U_{q'm'}^{qm}(t)$ can be represented as sums of terms that vary harmonically with time at different frequencies. In first-order perturbation theory, the frequency spectrum contains the frequencies $C\mathscr{E}q$ that correspond to the principal Stark sublevels [see (9)], and also the frequencies $C\mathscr{E}q \pm \Omega$ corresponding to quasilevels separated from the principal sublevels by a frequency $\pm\Omega$. In second-order perturbation theory, the components $U_{q'm'}^{qm}(t)$ acquire additional quasilevels with frequencies $C\mathscr{E}q \pm 2\Omega$. More and more new quasilevel appear in the succeeding orders of perturbation theory. The number of quasilevels is formally infinite, and their spectrum is given by

$$\omega_{q,j} = C\mathscr{E}q + \Omega j, \quad j = 0, \pm 1, \pm 2, \ldots \qquad (10)$$

when perturbation theory is valid (see below), however, the amplitudes of the corresponding terms in $U_{q'm'}^{qm}(t)$ decrease as $|j|$ increases. Note that the quasilevels (q, j) are in fact of the same type as the well known Blokhintsev satellites[11] obtained from Eqs. (4) in the absence of a static field. The spectrum of the Blokhintsev satellites agrees with (10) if $\mathscr{E} = 0$. We present the components $A_{q'm'}^{qm}(t)$ with account taken of lower-order terms that correspond to $j = 0$ and ± 1 and do not contain the factors $\exp(\pm 2i\varphi)$. Under condition (2), these terms suffice for investigating the most significant features of the line shape (3). In this approximation, the nonzero values of $A_{q'm'}^{qm}(t)$ are:

$$A_{q\pm2,m'}^{qm}(t) = \frac{2e^{\pm iC\mathscr{E}t}}{\theta_+\theta_-}(\cos C\mathscr{E}t - \cos\Omega t)$$
$$\times \sum_{m''} \xi_{q\pm1,m''}^{qm} \xi_{q\pm2,m'}^{q\pm1,m''} \quad (|m - m'| = 0, 2), \qquad (11a)$$

$$A_{q,m\pm2}^{qm}(t) = \sum_{\sigma=\pm} \frac{\cos\varepsilon_\sigma t - 1}{\theta_\sigma^2} \sum_{q'm'} \xi_{q'm'}^{qm} \xi_{q,m\pm2}^{q'm'}, \qquad (11b)$$

$$A_{qm}^{qm}(t) = e^{-i\Delta\omega_q t} - iq\zeta[\sin(\Omega t + \varphi) - \sin\varphi] + \frac{q^2\zeta^2}{2}(\cos\Omega t - 1)$$
$$+ \sum_{\gamma=\pm1} \left[\frac{e^{-it\gamma\varepsilon_+} - 1}{\theta_+^2} + \frac{e^{-it\gamma\varepsilon_-} - 1}{\theta_-^2}\right] \sum_{m'} (\xi_{qm}^{q+\gamma,m'})^2, \qquad (11c)$$

$$A_{q\pm1,m'}^{qm}(t) = \xi_{q\pm1,m'}^{qm}\left\{\pm\left[e^{\pm i\varphi}\frac{e^{\pm i\varepsilon_+ t} - 1}{\theta_+} + e^{\mp i\varphi}\frac{e^{\pm i\varepsilon_- t} - 1}{\theta_-}\right]\right.$$
$$- \frac{\zeta q}{2}\left[\frac{e^{\pm i\varepsilon_+ t} - 1}{\theta_+} - \frac{e^{\pm i\varepsilon_- t} - 1}{\theta_-}\right]$$
$$\left. - \frac{\zeta(q\pm1)}{2} e^{\pm iC\mathscr{E}t}\left[\frac{e^{\mp i\varepsilon_+ t} - 1}{\theta_+} - \frac{e^{\mp i\varepsilon_- t} - 1}{\theta_-}\right]\right\}, \qquad (11d)$$

$$\Delta\omega_q = C\mathscr{E}/\theta_+\theta_-, \quad \theta_\pm = 2\varepsilon_\pm/CE_\perp,$$
$$\varepsilon_\pm = C\mathscr{E} \pm \Omega, \quad \zeta = CE_\parallel/\Omega. \qquad (12)$$

Equations (11b) and (11d) were derived by using Eq. (5) for $\xi_{qm}^{q'm'}$. Included among the quantities of second order in $A_{qm}^{qm}(t)$ is the term $-i\Delta\omega_q t$. It can be treated[5] as the linear term in the expansion of an exponential expression and yields $1 - i\Delta\omega_q t = \exp(-i\Delta\omega_q t)$. when combined with the zeroth-order term in (11c). The quantity $\Delta\omega_q$ de-

scribes the changes induced by the harmonic field in the position of the principal Stark level [$j = 0$ in (10)]. In the region where perturbation theory is valid, the sublevel shifts $\Delta\omega_q$ are much smaller than the distances $C\mathscr{E}$ between neighboring sublevels.

According to (11), the criterion for the validity of perturbation theory

$$|A_{q'm'}^{qm}(t)-\delta_{qq'}\delta_{mm'}|\ll 1,$$

is satisfied if

$$CE_{\parallel}\ll\Omega,\quad CE_{\perp}\ll|C\mathscr{E}\pm\Omega|. \tag{13}$$

At $\vartheta = 0$, when the harmonic field does not mix the Stark states (9) in the field $\vec{\mathscr{E}}$, the condition (13) is equivalent to (2). If, however, $\vartheta \neq 0$, we have formally $|A_{q'm'}^{qm}|\to\infty$ as $\mathscr{E}\to\Omega/C$. From the physical point of view the cause of this singularity of the solution is that, according to (10), at $\mathscr{E} = \Omega/C$ the principal Stark sublevels numbered $(q,j = 0)$ cross (quasicross) the quasilevels (q',j') corresponding to all possible $q'\neq q$ and $j' = q - q'$. The frequency of the external field then becomes equal to (resonant with) the frequency of the transition between neighboring Stark sublevels in the field $\mathscr{E}$. Equations (4) near resonance are solved in §4.

§4. EVOLUTION OPERATOR NEAR RESONANCE

Near resonance (at $|C\mathscr{E}-\Omega|\ll\Omega$) we use the resonance approximation (see, e.g., Ref. 7, problem at the end of §40) which is widely used for two-level systems. Using this approximation, we represent $\cos(\Omega t+\varphi)$ as a sum of exponentials and retain the terms that vary slowly with time. We get

$$i\dot{A}_{q'm'}^{qm}(t)=-\frac{CE_{\perp}}{2}\sum_{m''q''}\xi_{q'm'}^{q''m''}A_{q''m''}^{qm}(t)$$

$$\times\exp[i(q'-q'')(\varepsilon t-\varphi)], \tag{14}$$

where $\varepsilon\equiv\varepsilon_{-} = C\mathscr{E}-\Omega$, with $|\varepsilon|\ll\Omega$. The substitution

$$A_{q'm'}^{qm}(t)=G_{q'm'}^{qm}(t)\exp[i\varepsilon q't-i\varphi(q'-q)] \tag{15}$$

reduces Eqs. (14) and the initial conditions [see (8)] to the form

$$i\dot{G}_{q'm'}^{qm}(t)=\varepsilon q'G_{q'm'}^{qm}(t)-\frac{CE_{\perp}}{2}\sum_{q''m''}\xi_{q'm'}^{q''m''}G_{q''m''}^{qm}(t),$$

$$G_{q'm'}^{qm}(0)=\delta_{qq'}\delta_{mm'}. \tag{16}$$

We can seek the solution of the system (16) in the form

$$G_{q'm'}^{qm}(t)=\sum_{\varkappa\mu}B_{\varkappa\mu}^{qm}H_{q'm'}^{(\varkappa\mu)}e^{-i\lambda_{\varkappa}t}, \tag{17}$$

where $\lambda_{\varkappa}$ are the eigenvalues and $H_{qm}^{(\varkappa\mu)}$ the eigenvectors of the system of homogeneous linear equations

$$(\lambda-\varepsilon q)H_{qm}^{(\varkappa\mu)}+\frac{CE_{\perp}}{2}\sum_{q'm'}\xi_{qm}^{'m'}H_{q'm'}^{(\varkappa\mu)}=0. \tag{18}$$

The indices $\varkappa$ and μ number respectively the values $\lambda_{\varkappa}$ and the eigenvectors corresponding to the same $\lambda_{\varkappa}$ in the presence of degeneracy. The coefficients $B_{\varkappa\mu}^{qm}$ of the expansion (17) should be obtained from the initial conditions [see (16)].

Equating the determinant of the system (18) to zero, by direct calculation we find that at $n = 2$, 3, and 4 the $\lambda_{\varkappa}$ are given by

$$\lambda_{\varkappa}=\varkappa\left[\varepsilon^2+\left(\frac{CE_{\perp}}{2}\right)^2\right]^{1/2}=\varkappa\frac{CE_{\perp}}{2}\eta,$$

$$\eta=(1+\theta^2)^{1/2},\quad \theta=\frac{2\varepsilon}{CE_{\perp}}\equiv\theta_{-} \tag{19}$$

and are $(n-1-|\varkappa|)$-fold degenerate. In this case $\varkappa$ (as well as q) is an integer in the interval $|\varkappa|\leqslant n-1$. It is natural to expect the simple expression (19) to be valid for all n. It is convenient to assume that, given the index $\varkappa$, the index μ runs through the same values as the quantum number m at $q=\varkappa$(§2).

Since the $\lambda_{\varkappa}$ are degenerate, the eigenvectors $H_{qm}^{(\varkappa\mu)}$ cannot be uniquely determined. The Appendix contains a method of finding $H_{qm}^{(\varkappa\mu)}$ for any value of n, if the following symmetry and orthonormalization conditions are imposed:

$$H_{qm}^{(\varkappa\mu)}=H_{\varkappa\mu}^{(qm)},\quad \sum_{qm}H_{qm}^{(\varkappa\mu)}H_{qm}^{(\varkappa'\mu')}=\delta_{\varkappa\varkappa'}\delta_{\mu\mu'}. \tag{20}$$

In this case $B_{\varkappa\mu}^{qm}=H_{qm}^{(\varkappa\mu)}$ according to (16). We have then by virtue of (15) and (7)

$$U_{q'm'}^{qm}(t)=\exp\{i\varphi(q-q')\}\sum_{\varkappa\mu}H_{q'm'}^{(\varkappa\mu)}H_{qm}^{(\varkappa\mu)}\exp\{-i\omega_{q'\varkappa}t\}, \tag{21}$$

where the frequencies

$$\omega_{q\varkappa}=q\Omega+\lambda_{\varkappa}=q\Omega+\varkappa[(C\mathscr{E}-\Omega)^2+(CE_{\perp}/2)^2]^{1/2} \tag{22}$$

determine the energy spectrum of the quasilevels near resonance.

Expressions (10), (11) and (21), (22) obtained respectively for

$$|C\mathscr{E}-\Omega|\gg CE_{\perp},\quad |C\mathscr{E}-\Omega|\ll\Omega,$$

permit an evolution operator $\hat{U}(t)$ to be specified for any value of $\mathscr{E}$. In the common range of the parameters

$$CE_{\perp}\ll|C\mathscr{E}-\Omega|\ll\Omega$$

these expressions are in agreement. As an example, Figs. 2 and 3 show the spectra of the quasilevels (with number $s = a,b,c,\ldots$) as functions of $\mathscr{E}$ at $CE/\Omega = \frac{1}{4}$ for the cases $n = 2$ and 3. The indices (q,j) and $(q,\varkappa)$ in (10) and (22), corresponding to the quasilevels on Fig. 3 ahead of the resonance $(\Omega - C\mathscr{E}\gg CE_{\perp})$, in the resonance region $(|C\mathscr{E}-\Omega|\ll\Omega)$, and past the resonance $(C\mathscr{E}-\Omega\gg CE_{\perp})$, are listed in Table I. According to Figs. 2 and 3, the principal Stark sublevels approach the groups of quasilevels as $\mathscr{E}\to\Omega/C$. If $\vartheta\neq 0$, however, the approaching quasilevels do not cross, but are mutually repelled. The closest approach distance of neighboring quasilevels is reached at $\mathscr{E} = \Omega/C$ and equals $CE_{\perp}/2$.

At $|C\mathscr{E}-\Omega|\lesssim CE_{\perp}$ all the terms in (21) which describe the approaching quasilevles have, generally speaking, amplitudes of the same order, whereas at $|C\mathscr{E}-\Omega|\gg CE_{\perp}$ the amplitudes of the terms in (11) and (21), corresponding to the quasilevels (q,j) in (10), decrease drastically with $|j|$.

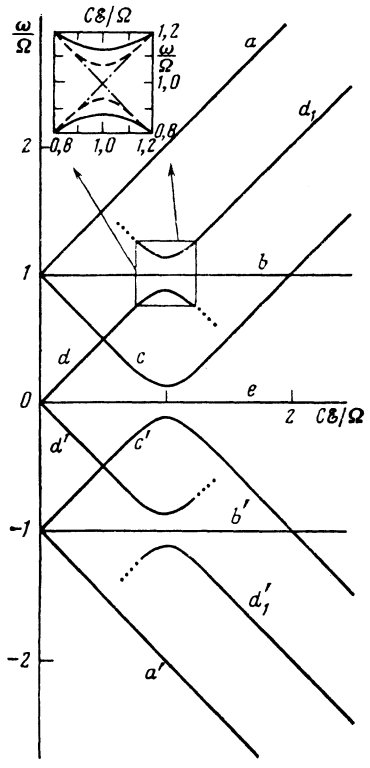

FIG. 2. Quasilevel spectrum vs the strength of the stationary field $\mathscr{E}$ at $CE/\Omega = \frac{1}{4}, n = 2$ and $\vartheta = 90°$ (see Fig. 1). The dots denotes the positions of the quasilevels in those regions where they are extremely weak (§2). When ϑ is varied the positions of the quasilevles change only near the quasicrossings. In the upper left corner this change is shown for three values of ϑ: 90° (solid lines), 30° (dashed) and 0° (dash-dot).

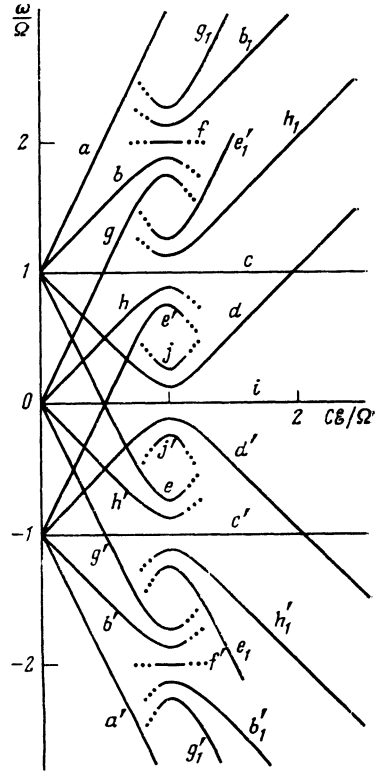

FIG. 3. The same as Fig. 2, for $n = 3$ and $\vartheta = 90°$.

§5. PROFILES OF THE LINES L_α AND L_β

Using (3) and the results of §§3 and 4 we calculate the profiles of the lines L_α and L_β of the hydrogen atom in the electric field (1). The ground level of the atom is not perturbed by the field (1) in this approximation. Hence $U_{\beta'\beta}(t) \equiv 1$ in (3). To be definite, we average (3) over the directions and polarizations of the radiation. Then

$$I(\omega) = \left(\pi \sum_\alpha |\mathbf{r}_\alpha|^2\right)^{-1} \mathrm{Re} \sum_{\alpha\alpha'} \mathbf{r}_\alpha \mathbf{r}_{\alpha'}^* \int_0^\infty dt\, e^{i\omega t} \langle U_{\alpha'\alpha}(t) \rangle, \quad (23)$$

where $\mathbf{r}_\alpha \equiv \mathbf{r}_{\alpha\beta}$.

According to (7), (11) and (21), the values of $U_{\alpha'\alpha}(t)$ averaged over the phase φ of the harmonic field are of the form

$$\langle U_{q'm'}^{qm}(t) \rangle = \sum_s u_{q'm'}^{qm}(s) e^{-i\omega(s)t}, \quad (24)$$

where $s = a, b, c, \ldots$ number the quasilevels of the upper level (Figs. 2 and 3); $u_{q'm'}^{qm}(s)$ are real coefficients. Substituting (24) in (23), we find that the spectral line splits into a series of components s in accordance with the energy spectrum of the upper level:

$$I(\omega) = \sum_s I_s \delta[\omega - \omega(s)]. \quad (25)$$

Here

$$I_s = \left(\pi \sum_{qm} |\mathbf{r}_{qm}|^2\right)^{-1} \mathrm{Re} \sum_{qmq'm} u_{q'm'}^{qm}(s) \mathbf{r}_{qm} \mathbf{r}_{q'm'}^* \quad (26)$$

is the intensity of the component s, and in accordance with the assumed normalization of $I(\omega)$ (see §1) we have $\sum_s I_s = 1$. Tables II and III list for the lines L_α and L_β the explicit forms of I_s calculated from Eqs. (7), (11), and (26) off resonance and, from (21) and (26), near resonance. Plots of I_s vs $\mathscr{E}$ at $CE_\perp/\Omega = \frac{1}{4}$ are shown in Figs. 4 to 7.

On Figs. 2–7 we can trace in detail the behavior of the components of lines L_α and L_β as functions of $\mathscr{E}$. Not too close to resonance, when $|C\mathscr{E} - \Omega| \gg CE_\perp$, the most intense are the principal Stark components (e.g., $d(d_1)$, $d'(d'_1)$ and e of the L_α line) that correspond to $j = 0$ in (10). Components separated from them in frequency by $\pm\Omega$ (for example a, a', c, c' of L_α) are much less intense, and those separated by $\pm 2\Omega$ (e.g., f and f' of L_β) are negligibly weak. Therefore only the principal Stark components and their less intense first satellites actually appear in the line.

If $\mathscr{E} \to \Omega/C$, the intensities of the group of weak components that approach the principal Stark components increase resonantly and at $|C\mathscr{E} - \Omega| \lesssim CE_\perp$ become of the same order as the intensities of the principal Stark components. It is important that in this case there appear components (e.g., f of the L_β line) that are extremely weak far from resonance. The component intensities near resonance can have a complicated dependence on $\mathscr{E}$ (just like, e.g., the components h and h_1 of the L_β line) as a result of quantum interference effects that appear when several quasilevels approach one another simultaneously. This distinguishes our results from the familiar results (see, e.g., Ref. 7) for two-level systems.

The dependence of the component intensities on the angle between the harmonic and constant fields is of interest.

TABLE I. Quasilevels with principal quantum number $n = 3$, shown in Fig. 3. (The signs of q, j, and $\varkappa$ should be reversed for the quasilevels marked by primed letters).

s	$\mathscr{E}$: $\Omega - C\mathscr{E} \gg CE_\perp$		$\lvert\Omega - C\mathscr{E}\rvert \ll \Omega$		$C\mathscr{E} - \Omega \gg CE_\perp$		s	$\mathscr{E}$: $\Omega - C\mathscr{E} \gg CE_\perp$		$\lvert\Omega - C\mathscr{E}\rvert \ll \Omega$		$C\mathscr{E} - \Omega \gg CE_\perp$	
	q	j	q	$\varkappa$	q	j		q	j	q	$\varkappa$	q	j
a	2	1	–	–	2	1	f	0	2	2	0	0	2
b	1	1	2	−1	−1	3	g	2	0	2	−2	−2	4
b_1	−1	3	2	1	1	1	g_1	−2	4	2	2	2	0
c	0	1	1	0	0	1	h	1	0	1	−1	−1	2
d	−1	1	0	1	1	−1	h_1	−1	2	1	1	1	0
e	−2	1	−1	2	2	−3	i	0	0	0	0	0	0
e_1	2	−3	−1	−2	−2	1	j	−2	2	0	2	2	−2

TABLE II. Intensities of the Stark components of line L_α.

s	$\mathscr{E}$: $\Omega - C\mathscr{E} \gg CE_\perp$	$\lvert\Omega - C\mathscr{E}\rvert \ll \Omega$	$C\mathscr{E} - \Omega \gg CE_\perp$
a, a'	$\frac{1}{6}\left(\frac{\zeta^2}{4} + \frac{1}{\theta_+^2}\right)$	–	$\frac{1}{6}\left(\frac{\zeta^2}{4} + \frac{1}{\theta_+^2}\right)$
b, b'	$\frac{1}{12}\left(\frac{1}{\theta_-} + \frac{1}{\theta_+}\right)^2$	$\frac{1}{12\eta^2}$	$\frac{1}{12}\left(\frac{1}{\theta_-} + \frac{1}{\theta_+}\right)^2$
c, c'	$\frac{1}{6}\left(\frac{\zeta^2}{4} + \frac{1}{\theta_-^2}\right)$	$\frac{1}{6\eta^2}$	$\frac{1}{6}\left(\frac{\zeta^2}{4} + \frac{1}{\theta_-^2}\right)$
d, d'	$\frac{1}{6}\left[1 - \frac{\zeta^2}{2} - \frac{1}{2}\left(\frac{1}{\theta_-} + \frac{1}{\theta_+}\right)^2\right]$	$\frac{1}{24}\frac{(\eta-\theta)^2}{\eta^2}$	–
d_1, d_1'	–	$\frac{1}{24}\frac{(\eta+\theta)^2}{\eta^2}$	$\frac{1}{6}\left[1 - \frac{\zeta^2}{2} - \frac{1}{2}\left(\frac{1}{\theta_-} + \frac{1}{\theta_+}\right)^2\right]$
e	$\frac{2}{3}\left(1 - \frac{1}{2\theta_-^2} - \frac{1}{2\theta_+^2}\right)$	$\frac{2\theta^2+1}{3\eta^2}$	$\frac{2}{3}\left(1 - \frac{1}{2\theta_-^2} - \frac{1}{2\theta_+^2}\right)$

TABLE III. Intensities of the Stark components of line L_β.

s	$\mathscr{E}$: $\Omega - C\mathscr{E} \gg CE_\perp$	$\lvert\Omega - C\mathscr{E}\rvert \ll \Omega$	$C\mathscr{E} - \Omega \gg CE_\perp$
a, a'	$\frac{1}{6}\left(\zeta^2 + \frac{1}{\theta_+^2}\right)$	–	$\frac{1}{6}\left(\zeta^2 + \frac{1}{\theta_+^2}\right)$
b, b'	$\frac{1}{6}\left(\frac{\zeta^2}{2} + \frac{1}{\theta_-^2}\right)$	$\frac{1}{24}\frac{(\eta-\theta)^2}{\eta^4}$	–
b_1, b_1'	–	$\frac{1}{24}\frac{(\eta+\theta)^2}{\eta^4}$	$\frac{1}{6}\left(\frac{\zeta^2}{2} + \frac{1}{\theta_-^2}\right)$
c, c'	$\frac{1}{3}\left(\frac{1}{\theta_-} - \frac{1}{\theta_+}\right)^2$	$\frac{4\theta^2+1}{12\eta^4}$	$\frac{1}{3}\left(\frac{1}{\theta_-} - \frac{1}{\theta_+}\right)^2$
d, d'	$\frac{1}{6}\left(\frac{\zeta^2}{2} + \frac{1}{\theta_+^2}\right)$	–	$\frac{1}{6}\left(\frac{\zeta^2}{2} + \frac{1}{\theta_+^2}\right)$
e, e'	$\frac{1}{6}\left(\zeta^2 + \frac{1}{\theta_-^2}\right)$	$\frac{1}{24}\frac{(\eta-\theta)^2}{\eta^4}$	–
e_1, e_1'	–	$\frac{1}{24}\frac{(\eta+\theta)^2}{\eta^4}$	$\frac{1}{6}\left(\zeta^2 + \frac{1}{\theta_-^2}\right)$
f, f'	–	$\frac{1}{16\eta^4}$	–
g, g'	$\frac{1}{6}\left(1 - 2\zeta^2 - \frac{1}{\theta_-^2} - \frac{1}{\theta_+^2}\right)$	$\frac{1}{96}\frac{(\eta-\theta)^4}{\eta^4}$	–
g_1, g_1'	–	$\frac{1}{96}\frac{(\eta+\theta)^4}{\eta^4}$	$\frac{1}{6}\left(1 - 2\zeta^2 - \frac{1}{\theta_-^2} - \frac{1}{\theta_+^2}\right)$
h, h'	$\frac{1}{6}\left[2 - \zeta^2 - \frac{1}{\theta_-^2} - \frac{1}{\theta_+^2} - 2\left(\frac{1}{\theta_-} + \frac{1}{\theta_+}\right)^2\right]$	$\frac{\theta^2(\eta-\theta)^2+1}{12\eta^4}$	–
h_1, h_1'	–	$\frac{\theta^2(\eta+\theta)^2+1}{12\eta^4}$	$\frac{1}{6}\left[2 - \zeta^2 - \frac{1}{\theta_-^2} - \frac{1}{\theta_+^2} - 2\left(\frac{1}{\theta_-} + \frac{1}{\theta_+}\right)^2\right]$

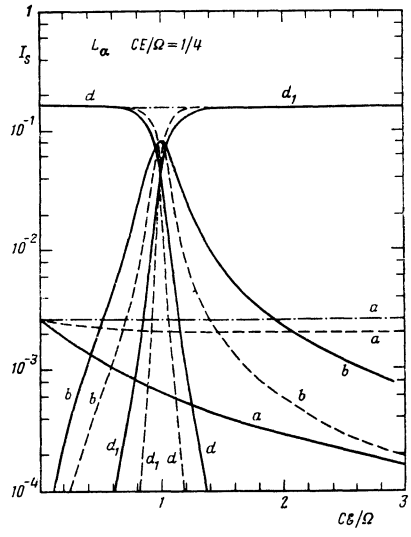

FIG. 4. Intensities I_s of a number of components of the line L_α (see Fig. 2) vs $\mathscr{E}$ at $CE/\Omega = \frac{1}{4}$ for three values of the angle between the harmonic and static fields: $\vartheta = 90°$ (solid lines), 30° (dashed), and 0° (dash-dot).

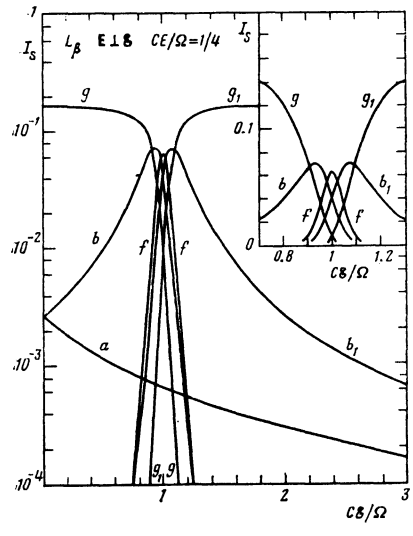

FIG. 6. Intensities I_s of a number of components as functions of the line L_β (see Fig. 3) on $\mathscr{E}$ at $CE/\Omega = 1/4$ and $\vartheta = 90°$. The upper corner shows a linear plot of the same dependence near the resonance.

When θ decreases the resonant changes of the intensities take place in an ever narrower interval of $\mathscr{E}$ ($|C\mathscr{E} - \Omega| \lesssim CE|\sin\vartheta|$, Figs. 2,4,5), and vanish at $\vartheta = 0$. In addition, the first satellites of the unshifted components n and n' of L_α and c and c' of L_β, which are absent at $\vartheta = 0$, appear in the lines at $\vartheta \neq 0$. According to Figs. 2 and 3, the positions of these satellites do not depend on the value of $\mathscr{E}$. It is possible therefore that these satellites will appear even in those cases when $\mathscr{E}$ is a quasistatic component of an electric field (§1). In these cases the details of the remaining components may become strongly washed out by additional averaging of the contours (25) with the distribution function of the vectors $\vec{\mathscr{E}}$.

The contours of hydrogen lines in the electric field (1) were investigated earlier both numerically[3,4] and analytically.[2,6] In Ref. 3 the positions of the L_α-line components were calculated as functions of $\mathscr{E}$ at $\vartheta = 90°$ and $CE/\Omega = 0.5$, 1, 1.5, 2, 2.5, 3.5, and 4.5. For $CE/\Omega = 0.5$ the results of Ref. 3 fit well the analytic formulas (10) and (22). A comparison of the results for the remaining CE/Ω is improper by virtue of (2). The positions and intensities of the components of the H_α line were obtained numerically in Ref. 4 for several values of $\mathscr{E}$, E, Ω, and ϑ. Nonetheless, the results of Ref. 3 and 4 do not permit a detailed investigation of the behavior of the line components when the parameters of the field (1) are varied.

An analytic solution of Eqs. (4) was obtained in Ref. 6 by a method similar to that used in §4, under exact resonance conditions ($\mathscr{E} = \Omega/C$ at $\vartheta = 90°$). The separation obtained in Ref. 6 between neighboring components (Eq. (8) of Ref. 6) differs from the value $CE_\perp/2$ obtained from (22). The discrepancy arises because the authors of Ref. 6 solved a system of equations similar to (14) by improperly subdividing the system into individual blocks (see (5)–(7) in Ref. 6), reducing them to the known equations for two-level sys-

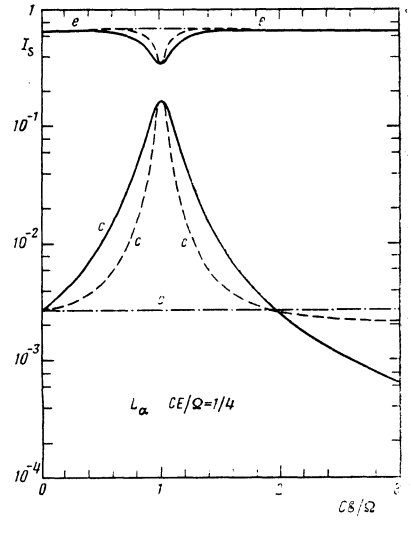

FIG. 5. The same as Fig. 4, for other components of the line L_α.

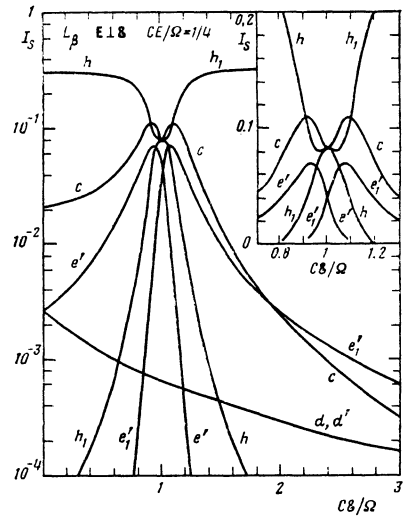

FIG. 7. The same as Fig. 6, for other components of the line L_β.

tems. Actually, however, the problem is substantially different from the two-level one (see above).

A somewhat different approach was used in a subsequent paper,[2] where the phase of the harmonic field in (1) was assumed equal to zero. Equations (4) were solved for the level $n = 2$ either far from resonance by perturbation theory, or at exact resonance by transforming to a rotating reference frame with z axis parallel to $\mathbf{E}_D(t)$. The assumption $\vartheta = 90°$ was made and the constraint (2) was not imposed. The authors of Ref. 2 calculated the profiles of the L_α line far from resonance and at exact resonance, and also indicated the resonant positions of the L_β components. Under condition (2), the resonant positions of the components of L_α and L_β agreed with those calculated from Eq. (22) of the present paper, but differed (in the case of L_β) from those obtained in the earlier paper.[6] This difference, due to the inaccuracy of the calculations of Ref. 6 (see above), were improperly attributed in Ref. 2 to the different action on the atom by the field (1) with arbitrary phase $\varphi \neq 0$ (in Ref. 6) and with fixed phase $\varphi = 0$ (in Ref. 2). It is clear, however, that one and the same problem was solved in Refs. 2 and 6 and the results should be identical.

In addition, the authors of Refs. 2 and 6, having observed closely-spaced line components under conditions of exact resonance. treated them as a special splitting of the principal Stark components by the resonant field. According to our results, however, the principal components are split by a harmonic field at any $\mathscr{E}$. The split components have in fact the same properties as the Blokhintsev satellites[11] at $\mathscr{E} = 0$. As $\mathscr{E} \to \Omega/C$ the components only come closer together and manifest themselves most distinctly because of the resonant interaction.

In addition, resonance effects can appear not only at $\mathscr{E} = \Omega/C$, but also at other $\mathscr{E}$ for which a tendency to level crossing takes place. Using (10), it can be easily shown (see also Ref. 6) that in case (2) all possible "resonant" values of $\mathscr{E}$ are $\mathscr{E} = \Omega k/Cs$, where k and s are integers, $k \neq 0$, $s \neq 0$, and $|s| \leqslant 2n - 2$. The resonance considered corresponds to $k = s = 1$ and is much more strongly pronounced than the remaining ones. In the opposite limit $CE \gtrsim \Omega$ several resonances can predominate simultaneously, but the resonant values of $\mathscr{E}$ are apparently not describable by such simple equations and depend nonlinearly on $\mathscr{E}$ and Ω (Refs. 2 and 3).

It would be of interest to study the effects calculated above under laboratory conditions, by recording the line shapes of hydrogen atoms placed in the field (1). By varying the parameters of this field and creating resonance conditions, it is possible to observe lines with a large number of components that are indicative of the complex structure of the quasilevels produced in the field (1).

APPENDIX

Eigenvectors of the system (18)

Using Eqs. (5), it can be easily shown that the eigenvectors $H_{qm}^{(\varkappa\mu)}$ that satisfy Eq. (18) and conditions (20) are subject to the following symmetry relations:

$$H_{-q,-m}^{(\varkappa\mu)}(\theta,\eta) = (-1)^{n-1-\varkappa} H_{qm}^{(\varkappa\mu)}(-\theta,\eta), \tag{A.1}$$

$$H_{qm}^{(-\varkappa,-\mu)}(\theta,\eta) = (-1)^{n-1-\varkappa} H_{qm}^{(\varkappa\mu)}(\theta,-\eta), \tag{A.2}$$

$$H_{q,-m}^{(\varkappa,-\mu)} = H_{qm}^{(\varkappa\mu)}, \tag{A.3}$$

$$H_{qm}^{(qm)} = (-1)^{n-1} H_{mq}^{(mq)}, \tag{A.4}$$

$$H_{q}^{(\varkappa\mu)} = H_{q+\tau,\mu+\tau}^{(\varkappa-\tau,\mu+\tau)} \equiv H^{(\varkappa'\mu')}. \tag{A.5}$$

In the last relation, $m_q = n - |q| - 1$ is the maximum value of m at a given q, while $\varkappa'$ and μ' are the minimum and maximum values of the corresponding indices, given the sum $\varkappa' + \mu' = \varkappa + \mu$:

$$\varkappa' = (\varkappa+\mu-n+1)/2, \quad \mu' = (\varkappa+\mu+n-1)/2, \quad \gamma = (\varkappa-\mu+n-1)/2. \tag{A.6}$$

A simple procedure can be proposed to determine the components $H_{qm}^{(\varkappa\mu)}$ for any n. We find first $H_{qm}^{(\varkappa\mu)}$ for the values $\varkappa = 1 - n$ and $\mu = 0$. These values are

$$H_{qm}^{(1-n,0)} = (2\eta)^{1-n} h_{qm} (\eta-\theta)^q \equiv (2\eta)^{1-n} h_{qm} (\eta+\theta)^{-q}. \tag{A.7}$$

Here θ and η are given by Eqs. (12) and (19); $h_{qm} = h_{|q|,|m|}$ are constant coefficients, with

$$h_{n-1,0} = 1, \tag{1.8}$$

while the remaining coefficients h_{qm} are easily obtained from the recurrence relation

$$(n-1-q)\, h_{qm} = 2 \sum_{m'} \xi_{qm}^{q+1,m'} h_{q+1,m'}, \tag{A.9}$$

obtained by substituting (A.7) in (1.8) and equating coefficients of like powers of $\eta - \theta$. In particular,

$$h_{n-2,1} = (n-1)^{1/2},$$

$$h_{n-3,0} = n-1, \quad h_{n-3,2} = \left[\frac{(n-1)(n-2)}{2}\right]^{1/2},$$

$$h_{n-4,1} = (n-1)\left(\frac{n-2}{2}\right)^{1/2},$$

$$h_{n-4,3} = \left[\frac{(n-1)(n-2)(n-3)}{6}\right]^{1/2}; \tag{A.10}$$

$$\cdots\cdots\cdots$$

$$h_{0,m} = C_{n-1}^{m},$$

where C_{n-1}^{m} is the binomial coefficient.

Knowing $H_{qm}^{(1-n,0)}$ we can determine $H_{qm_q}^{(2-n,-1)}, H_{qm_q}^{(3-n,-2)}, \ldots, H_{qm_q}^{(0,1-n)}$ with the aid of (A.5). All the remaining components $H_{qm}^{(2-n,-1)}, H_{qm}^{(3-n,-2)}, \ldots, H_{qm}^{(0,1-n)}$ are obtained without difficulty by successive application of Eqs. (18) (as recurrence relations) and (A.1). The values of $H_{qm}^{(2-n,1)}, H_{qm}^{(3-n,2)}, \ldots, H_{qm}^{(0,n-1)}$ are then directly obtained with the aid of (A.3). Now, knowing $H_{qm}^{(2-n,1)}$, we can determine $H_{qm_q}^{(3-n,0)}, H_{qm_q}^{(4-n,-1)}, \ldots, H_{qm_q}^{(0,3-n)}$, etc., with the aid of (A.5). Repeating this procedure, we can calculate all the elements $H_{(qm)}^{\varkappa\mu}$ with $\varkappa \leqslant 0$, and then use (A.2) to find the remaining elements with $\varkappa > 0$.

We have determined the eigenvectors of the system (18) for the cases $n = 2$, 3, 4, and 5. It is convenient to express the vector components in the form

$$H_{qm}^{(\varkappa\mu)} = \frac{1}{2^{n-1}(1+\theta^2)^{(n-1)/2}} \mathscr{H}_{qm}^{(\varkappa\mu)}. \tag{A.11}$$

TABLE IV. Certain components of $\mathscr{H}_{qm}^{(\varkappa\mu)}$ for the level with $n = 3$.

qm	$\varkappa\mu$				
	−2,0	−1, −1	−1, 1	0, −2	0,0
−2,0	$(\eta+\theta)^2$	$2^{1/2}(\eta+\theta)$	$2^{1/2}(\eta+\theta)$	1	2
−1, −1	$2^{1/2}(\eta+\theta)$	$-2\theta(\eta+\theta)$	2	$-2^{1/2}(\eta+\theta)$	$-2^{3/2}\theta$
−1, 1	$2^{1/2}(\eta+\theta)$	2	$-2\theta(\eta+\theta)$	$2^{1/2}(\eta-\theta)$	$-2^{3/2}\theta$
0, −2	1	$-2^{1/2}(\eta+\theta)$	$2^{1/2}(\eta-\theta)$	$(\eta+\theta)^2$	−2
0,0	2	$-2^{3/2}2\theta$	$-2^{3/2}\theta$	−2	$4\theta^2$

By way of example, Table IV lists certain $\mathscr{H}_{qm}^{(\varkappa\mu)}$ for $n = 3$. The remaining quantities can be easily obtained with the aid of (A.1) and (A.2).

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