Two-photon production of $e^\pm$ pairs in a strong magnetic field

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Two-photon production of $e^\pm$ pairs in a strong magnetic field is discussed for the case in which the energy of each of the photons is insufficient for production of a pair as a consequence of single-photon production. Resonance behavior is observed in the cross section, a reduction of its energy threshold in comparison with the production of pairs in the absence of a field is established, and the dependence of the cross section on the polarization of the photons is investigated.

1. INTRODUCTION

At the present time conditions are known in which the magnetic field strength reaches a value close to critical: $B_c = m^2c^3/e\hbar = 4.414\cdot 10^{13}$ G.[1)] It has been established that near the surface of neutron stars the magnetic field can have a strength $B \lesssim B_c$. If the electric field has the critical strength it becomes possible to produce $e^\pm$ pairs spontaneously from vacuum (see for example Ref. 1). Polarization of the vacuum by a magnetic field is not accompanied by production of $e^\pm$ pairs, but in superstrong magnetic fields the processes of quantum electrodynamics differ substantially from the case $B = 0$. Certain processes (single-photon production of $e^\pm$ pairs, and photon splitting; see Ref. 2) which do not occur at all at $B = 0$ have been investigated comparatively recently. In addition to the fact that study of the processes of quantum electrodynamics in the presence of a strong magnetic field is of independent interest, without taking them into account it is impossible to explain the generation by neutron stars of hard x rays and γ rays and to investigate physical phenomena under extreme conditions of superstrong fields.

Three two-photon processes—scattering of a photon by $e^\pm$, annihilation of $e^\pm$, and production of $e^\pm$—play a major role in the production of hard radiation under the conditions indicated. None of these processes has been studied completely at $B \lesssim B_c$, although they have been investigated in many special cases. The two-photon production of $e^\pm$ pairs in a magnetic field has been discussed[3–5] only in the simplest special case of head-on collision of photons along the magnetic field (see below). In the present work this process is studied for arbitrary directions of propagation of the photons for the condition that the energy of each of the photons is insufficient for production of a pair in the single-photon process.

2. AMPLITUDE OF TWO-PHOTON PRODUCTION OF $e^\pm$ PAIRS IN A MAGNETIC FIELD

The two-photon production of $e^\pm$ pairs is described by two diagrams of second order (Fig. 1). In a magnetic field the formulation of the problem differs from the case $B = 0$ (page 281 of Ref. 6) in the following way.

1) The electron wave functions and electron propagator calculated with inclusion of a uniform magnetic field are used. The choice of wave functions corresponds to Refs. 7. Use of the Feynman representation of the electron propagator[7] permits preservation of the explicit space-time picture of the process.

2) It is assumed that the photons can have only linear polarization either in the plane formed by the wave vector **k** and the magnetic field **B** or perpendicular to it. These polarizations correspond to the polarizations of two normal waves (extraordinary and ordinary) propagated in a polarized vacuum.[2]

The two diagrams of the process considered (Fig. 1) are described by the amplitude

$$S_{fi}=ie^2\int d^4x'\int d^4x[\bar{\psi}_f(x')\gamma_\mu A_2^\mu(x')G_F(x'-x)\gamma_\nu A_1^\nu(x)\psi_i(x)+\bar{\psi}_f(x')\gamma_\mu A_1^\mu(x')G_F(x'-x)\gamma_\nu A_2^\nu(x)\psi_i(x)]. \quad (1)$$

The photon wave functions[6]

$$A^\mu_{1,2}(x)=\frac{\varepsilon_\mu^{(1,2)}\exp(ik_{1,2}x)}{(2\omega_{1,2}L^3)^{1/2}} \quad (2)$$

correspond to a plane wave with a wave vector **k** and frequency ω and have a normalization chosen from the condition of one photon in a volume L^3. The photon polarization vectors $\boldsymbol{\varepsilon}^{(1)}$ and $\boldsymbol{\varepsilon}^{(2)}$ are defined relative to a rectangular system of coordinates with z axis along the direction of the magnetic field **B** (Fig. 2). The time component of the polarization vectors is assumed equal to zero, and therefore the index μ in Eq. (1) runs over values 1, 2, and 3.

The wave functions $\psi^{(\pm)}(x)$, corresponding to solutions of the Dirac equation in a magnetic field with positive and negative energies have the form[7]

$$\psi^{(+)}(x)=[(E_n+1)/2E_n]^{1/2}u_n^{(s)}(x)\exp(-iE_nt),$$
$$\psi^{(-)}(x)=[(E_n+1)/2E_n]^{1/2}v_n^{(s)}(x)\exp(+iE_nt), \quad (3)$$

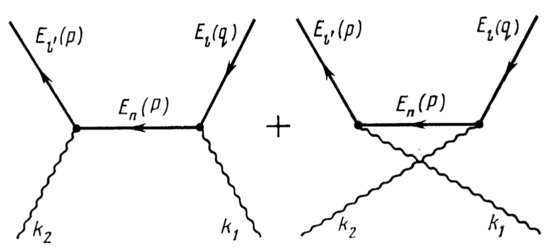

FIG. 1. Diagrams describing the two-photon production of $e^\pm$ pairs.

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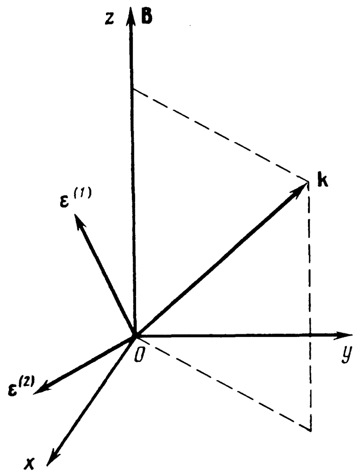

FIG. 2. The photon wave vector $\mathbf{k}$ and orthogonal linear polarizations $\boldsymbol{\varepsilon}^{(1)}$ and $\boldsymbol{\varepsilon}^{(2)}$ in the selected coordinate system: the vector $\boldsymbol{\varepsilon}^{(1)}$ lies in the plane $(\mathbf{k}, \mathbf{B})$, and the vector $\boldsymbol{\varepsilon}^{(2)}$ is perpendicular to this plane.

where x is the 4-coordinate, $E = (p^2 + 1 + 2nB)^{1/2}$, p is the longitudinal momentum, $n = 0, 1, 2, \ldots$, and $s = 1, 2$ are the principal quantum number and spin number of the electron ($s = 1$ corresponds to a spin orientation along $\mathbf{B}$ and $s = 2$—opposite $\mathbf{B}$), the spinors

$$u_n^{(1)} = \begin{pmatrix} \chi_{n-1} \\ 0 \\ \tilde{p}_n\chi_{n-1} \\ V_n\chi_n \end{pmatrix}, \quad u_n^{(2)} = \begin{pmatrix} 0 \\ \chi_n \\ V_n\chi_{n-1} \\ -\tilde{p}_n\chi_n \end{pmatrix},$$
$$v_n^{(1)} = \begin{pmatrix} -\tilde{p}_n\chi_{n-1} \\ -V_n\chi_n \\ \chi_{n-1} \\ 0 \end{pmatrix}, \quad v_n^{(2)} = \begin{pmatrix} -V_n\chi_{n-1} \\ \tilde{p}_n\chi_n \\ 0 \\ \chi_n \end{pmatrix} \qquad (4)$$

are expressed in terms of the scalar function

$$\chi_n(x) = \frac{i^n}{L(\lambda^2\pi)^{1/4}(2^n n!)^{1/2}} \times \exp\left[-\frac{(x-a)^2}{2\lambda^2} - \frac{iay}{\lambda^2} + ipz\right] H_n\left(\frac{x-a}{\lambda}\right) \qquad (5)$$

and the coefficients $\tilde{p}_n = p/(E_n + 1)$ and $V_n = (2nB)^{1/2}/(E_n + 1)$. In Eq. (5) $\lambda = \lambda\!\!\!^{-}_c B^{-1/2}$, $\lambda\!\!\!^{-}_c$ is the Compton wavelength a is the x-coordinate of the center of rotation of the electron, and H_n are Hermite polynomials. The wave functions of the $e^{\pm}$ satisfy the normalization condition

$$\int d^3x \psi^+(x)\psi(x) = 1, \qquad (6)$$

which corresponds to one elementary charge in a volume L^3.

The electron propagator in (1) has the form[7]

$$G_F(x'-x) = \left(\frac{L}{2\pi}\right)^2 \int \frac{dA}{\lambda^2} \int dP \times \sum_{n=0}^{\infty}\sum_{m=1}^{2} \{-i\theta(t'-t)u_n^{(m)}(x')\bar{u}_n^{(m)}(x) \times \exp[-iE_n(t'-t)]$$
$$+ i\theta(t-t')v_n^{(m)}(x')\bar{v}_n^{(m)}(x)\exp[iE_n(t'-t)]\}, \qquad (7)$$

where n is the principal quantum number, A is the x-coordinate of the center of rotation, and P is the longitudinal momentum of the virtual electron.

According to the Feynman rules, the first term of the amplitude (1) of interaction of two photons with energies ω_1 and ω_2 and polarizations $\boldsymbol{\varepsilon}^{(1)}$ and $\boldsymbol{\varepsilon}^{(2)}$, in which there is produced an e^- with energy $E_{l'}$, momentum p, x-coordinate of the center of the Larmor circle a, and spin number s' and an e^+ with energy E_l, momentum q, x-coordinate of the center of rotation b, and spin number s, can be represented in the form

$$S^{(1)(\mathbf{k}_1,\mathbf{k}_2,\boldsymbol{\varepsilon}^{(1)},\boldsymbol{\varepsilon}^{(2)})}_{(l',p,l,q)} = \frac{ie^2\delta(\omega_1+\omega_2-E_{l'}(p)-E_l(q))}{4\pi L(\omega_1\omega_2)^{1/2}} \qquad (8)$$
$$\times \int\frac{dA}{\lambda^2}\int dP \sum_{n=0}^{\infty}\sum_{m=1}^{2}\Big\{\frac{1}{E_{l'}(p)-\omega_2-E_n(P)}$$
$$\times \int d^3x' e^{ik_2x'} u_{l'}^{+(s')}(x') M_2 u_n^{(m)}(x')$$
$$\times \int d^3x e^{ik_1x} u_n^{+(m)}(x) M_1 v_l^{(s)}(x) + \frac{1}{\omega_1 - E_l(q) + E_n(P)}$$
$$\times \int d^3x' e^{ik_2x'} u_{l'}^{+(s')}(x') M_2 v_n^{(m)}(x') \int d^3x e^{ik_1x} v_n^{(m)}(x) M_1 v_l^{(s)}(x)\Big\}.$$

The polarization matrix M_i corresponding to a photon with polarization $\boldsymbol{\varepsilon}^{(i)}$ is

$$M_i = \boldsymbol{\varepsilon}^{(i)}\alpha = \begin{bmatrix} 0 & 0 & \varepsilon_z & \varepsilon_- \\ 0 & 0 & \varepsilon_+ & -\varepsilon_z \\ \varepsilon_z & \varepsilon_- & 0 & 0 \\ \varepsilon_+ & -\varepsilon_z & 0 & 0 \end{bmatrix}, \qquad (9)$$

where $\varepsilon_{\pm} = \varepsilon_x \pm i\varepsilon_y$, and $i = 1, 2$.

The spatial integrals entering into Eq. (8) for each of the vertices reduce to expressions of the form

$$J_{ln} = \int d^3x e^{ikx}\chi_l(x,p,a)\chi_n^*(x,P,A)$$
$$= \left(\frac{2\pi}{L}\right)^2 \left(\frac{2^S S!}{2^R R!}\right)^{1/2} \delta(k_z + p - P)$$
$$\times \delta\left(k_y - \frac{a-A}{\lambda^2}\right)\exp\left(-\frac{\lambda^2 k_\perp^2}{4} - \frac{i\lambda^2}{2}k_x k_y + ik_x a\right)$$
$$\times L_S^{R-S}\left(\frac{\lambda^2 k_\perp^2}{2}\right)\begin{cases} (\lambda k_-)^{n-l} & l<n, \\ (-\lambda k_+)^{l-n}, & l>n, \\ 1, & l=n, \end{cases} \qquad (10)$$

where $R = \max(n,l)$, $S = \min(n,l)$, $k_{\pm} = k_x \pm ik_y$, and L_k^i is the associated Laguerre polynomial. The second term $S_{fi}^{(2)}$ is obtained from (8) by permutation of all photon indices ($1 \leftrightarrow 2$).

The summation in (8) over the principal quantum numbers n of the virtual electron formally is not limited in any way. However, for $n > l$ the integrals J_{ln} fall off rapidly with increase of n. For $n = l + k$

$$|J_{ln}| \sim \left[\frac{l!}{2^k(l+k)!}\right]^{1/2}\exp\left(-\frac{\lambda^2 k_\perp^2}{4}\right)L_l^k\left(\frac{\lambda^2 k_\perp^2}{4}\right)$$
$$\leqslant \frac{1}{k!}\left[\frac{(l+k)!}{l!}\right]^{1/2} \sim \frac{1}{(k!)^{1/2}}, \qquad (11)$$

and therefore the infinite summation in (8) can be replaced by a finite sum with any specified accuracy.

It is necessary to distinguish four versions of the final states of the $e^\pm$ corresponding to different projections of their spins. Let the values $i = 1, 2, 3$, and 4 correspond to combinations $(++)$, $(+-)$, $(-+)$, and $(--)$, where the signs $+$ and $-$ correspond to spin projections respectively along and opposite to the magnetic-field direction. Each of the states enumerated corresponds to a squared amplitude

$$\left|S_{i(l',p,l,q)}^{(\mathbf{k}_1,\mathbf{k}_2,\boldsymbol{\varepsilon}^{(1)},\boldsymbol{\varepsilon}^{(2)})}\right|^2 = \frac{8\pi^5\alpha^2T}{L^8\omega_1\omega_2E_lE_{l'}}\,\delta[\omega_1+\omega_2-E_l(q)-E_{l'}(p)] \times\delta(k_{1z}+k_{2z}-p+q)\,\delta\left(k_{1y}+k_{2y}+\frac{a-b}{\lambda^2}\right) \times\exp\left[-\frac{\lambda^2}{2}(k_{1\perp}^2+k_{2\perp}^2)\right] \times|N_i^{(1)}\exp(-i\lambda^2k_{1y}k_{2x})+N_i^{(2)}\exp(-i\lambda^2k_{2y}k_{1x})|^2. \tag{12}$$

The expressions for $N_i^{(1)}$ and $N_i^{(2)}$ are given in the Appendix. Equations (12) in the special case $l = l' = 0$ coincides with the expression obtained in Ref. 7 for the amplitude of two-photon annihilation in a magnetic field from the ground state, with accuracy to obvious replacements.

3. CROSS SECTION FOR THE PROCESS

The cross section for production of $e^\pm$ pairs with quantum numbers (l',p) and (l,q) by photons with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ whose polarizations are described by vectors $\boldsymbol{\varepsilon}^{(1)}$ and $\boldsymbol{\varepsilon}^{(2)}$ is

$$\sigma_{(l',p,l,q)}^{(\mathbf{k}_1,\mathbf{k}_2,\boldsymbol{\varepsilon}^{(1)},\boldsymbol{\varepsilon}^{(2)})} = \frac{L^3}{cT(1-\cos\theta)}\cdot\int\frac{Ldp}{2\pi\hbar}\int\frac{Ldq}{2\pi\hbar}\int\frac{Lda}{2\pi\lambda^2}\int\frac{Ldb}{2\pi\lambda^2} \times\sum_{i=1}^{4}\left|S_{i(l',p,l,q)}^{(\mathbf{k}_1,\mathbf{k}_2,\boldsymbol{\varepsilon}^{(1)},\boldsymbol{\varepsilon}^{(2)})}\right|^2, \tag{13}$$

where θ is the angle between the detections of $\mathbf{k}_1$ and $\mathbf{k}_2$. After integration we obtain from (13) the following expression for the cross section:

$$\sigma_{(l',p,l,q)}^{(\mathbf{k}_1,\mathbf{k}_2,\boldsymbol{\varepsilon}^{(1)},\boldsymbol{\varepsilon}^{(2)})} = \frac{3}{16}\sigma_TB\frac{1}{|pE_l+qE_{l'}|(1-\cos\theta)\omega_1\omega_2} \times\exp\left[-\frac{\lambda^2}{2}(k_{1\perp}^2+k_{2\perp}^2)\right]\sum_{i=1}^{4}|N_i^{(1)}\exp(-i\lambda^2k_{1y}k + N_i^{(2)}\exp(-i\lambda^2k_{2y}k_{1x})|^2 \tag{14}$$

and the laws of conservation of energy

$$E_{l'}(p)+E_l(q)=\omega_1+\omega_2 \tag{15}$$

and of longitudinal momentum

$$p-q=k_{1z}+k_{2z}. \tag{16}$$

Equations (15) and (16) correspond to two independent pairs of solutions for the momenta of the $e^\pm$:

$$p_{1,2}(l,\ l')=\{\xi_1(\xi_2+\xi_3) \pm[(\xi_2+\xi_3)^2-(1+2l'B)(1-\xi_1^2)]^{1/2}\}(1-\xi_1^2)^{-1}, \tag{17}$$

$$q_{1,2}(l,\ l')=2\xi_1\xi_3(1+\xi_1^2)^{-1}-p_{2,1}(l,\ l'), \tag{18}$$

where

$$\xi_1=(k_{1z}+k_{2z})/(\omega_1+\omega_2),\quad \xi_2={}^1/_2[\omega_1+\omega_2 -(k_{1z}+k_{2z})^2/(\omega_1+\omega_2)],\quad \xi_3=B(l'-l)(\omega_1+\omega_2)^{-1}.$$

From this we obtain the relations

$$\begin{aligned}(p_1(l,\ l');\ q_1(l,\ l'))&=(-q_1(l',\ l);\ -p_1(l',\ l)),\\(p_2(l,\ l');\ q_2(l,\ l'))&=(-q_2(l',\ l);\ -p_2(l',\ l)),\end{aligned} \tag{19}$$

from which the symmetry of the partial cross sections for production of $e^\pm$ pairs in states (l,l') and (l',l) follows. In all subsequent formulas it is assumed that the quantities p and q satisfy Eqs. (17) and (18).

For given $\omega_{1,2}$ and $k_{z1,2}$ the total cross section for the process

$$\sigma(\mathbf{k}_1,\mathbf{k}_2,\boldsymbol{\varepsilon}^{(1)},\boldsymbol{\varepsilon}^{(2)})=\sum_{\{l,\ l'\}}\sigma_{(l',p,l,q)}^{(\mathbf{k}_1,\mathbf{k}_2,\boldsymbol{\varepsilon}^{(1)},\boldsymbol{\varepsilon}^{(2)})} \tag{20}$$

is determined by the sum of the partial cross sections (14) corresponding to the set of allowed channels $\{l,l'\}$. This set is determined by the condition of reality of (17) and (18). This condition, rewritten in the form

$$\begin{aligned}(\omega_1+\omega_2)^2-(k_{1z}+k_{2z})^2&\geqslant\mathscr{E}^2(l,\ l')\\&=[(1+2lB)^{1/2}+(1+2l'B)^{1/2}]^2,\end{aligned} \tag{21}$$

corresponds to the threshold of production of $e^\pm$ in states (l,l'). On the other hand, for fixed (l,l') one obtains from (21) values of $\omega_{1,2}$ and $k_{z1,2}$ for which this channel is permissible.

With decrease of the photon energy, the number of allowed reaction channels decreases. The minimum permissible threshold (the absolute threshold of the process) corresponds to the channel $l = l' = 0$; for it $\mathscr{E}(0,0) = 2$ and Eq. (21) has the form

$$(\omega_1+\omega_2)^2-(k_{1z}+k_{2z})^2\geqslant4. \tag{22}$$

For any $k_{z1,2}$ the region of permissible values of ω_1 and ω_2 for which the threshold condition (22) is satisfied is broader than the region of values corresponding to the threshold condition with $B = 0$:

$$\omega_1\omega_2(1-\cos\theta)\geqslant2. \tag{23}$$

The difference between the thresholds (22) and (23) is explained by the fact that in the presence of a field the conservation of 3-momentum is replaced by the condition of conservation of its longitudinal component (16). The minimum threshold, as follows from (22), corresponds to the condition $k_{1z} + k_{2z} = 0$; in this case for production of a pair it is sufficient to satisfy the energy condition: $\omega_1 + \omega_2 \geqslant 2$.

Simple asymptotic formulas for $B \gg B_c$ and $B \ll B_c$ for the cross section for the process under discussion exist only for the case of head-on collision of photons along the field. In the case $B \gg B_c$ we have $(\omega_1 = \omega_2 = \omega)$

$$\frac{\sigma}{\sigma_T} = \frac{3B\omega[(B+\omega^2)^2+\omega^2(\omega^2-1)]}{16(\omega^2-1)^{1/2}[B^2+\omega^2(2B+1)]^2}.$$

For the case $B \ll B_c$ the correction to the cross section is given, for example, in Ref. 4.

4. FEATURES OF THE TWO-PHOTON PRODUCTION OF $e^\pm$ PAIRS IN A MAGNETIC FIELD

The denominator $|pE_l(q) + qE_{l'}(p)|$ in the expression for the cross section (14) arises as the result of integration in (12) of the delta function $\delta[E_{l'}(p) + E_l(q) - \omega_1 - \omega_2]$ over p. Substituting into it expressions (17) and (18) for p and q, we can show that it vanishes if the equality is satisfied in (21), i.e., a new channel for production of $e^\pm$ pairs is opened. The behavior the cross section near threshold is described by the formula

$$\sigma \sim \{(\omega_1+\omega_2)^2-(k_{1z}+k_{2z})^2 - [(1+2lB)^{1/2}+(1+2l'B)^{1/2}]^2\}^{-1/2}. \quad (24)$$

The coefficient of absorption as a consequence of single-photon production of $e^\pm$ in a strong magnetic field has similar singularities.[8,9]

It is obvious that the cross section for the process must have a finite value; the existence of the threshold singularity (24) is due to the failure to take into account in the amplitude the contribution of diagrams of higher order. This singularity is integrable, and therefore in the case in which the spectral distribution of the interacting photons is sufficiently smooth in comparison with the width of the resonances of (24), its contribution to the cross section averaged over the spectrum can be neglected. The error due to taking into account diagrams of only second order (Fig. 1) makes a small contribution to such integrated quantities as, for example, the photon mean free path in a known radiation field with a smooth spectrum.

Each of the vertices of the diagrams describing the process considered corresponds to a resonance denominator of the amplitude $E_{l'} - \omega_{2,1} - E_n$ or $E_l - \omega_{1,2} - E_n$ (see Eq. (A1)). This singularity arises on coincidence of the frequency of one of the photons with the energy of the transition between the bound states of the virtual electron; it is similar to the cyclotron resonance in the cross section for scattering of photons by $e^\pm$ in a magnetic field. If the energy of one of the photons satisfies the condition of the indicated resonance $\omega_1 = E_{l'}(p) - E_n(P)$, then from energy conservation (15) one obtains the relation $\omega_2 = E_l(q) + E_n(P)$ for the second photon. Since at each of the vertices longitudinal momentum is conserved, the resulting relation means that the second photon can produce a real $e^\pm$ pair. It follows from this that at the photon energies considered when single-photon production of an $e^\pm$ pair by each of the photons is forbidden, the indicated denominators do not vanish.

The third singularity in the cross section (14) is the result of the denominator $1 - \cos\theta$, which vanishes when the photon wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ are parallel. For $B = 0$ the threshold condition (23) excludes this singularity: $1 - \cos\theta \geqslant 2/\omega_1\omega_2$. In the presence of a magnetic field the reaction amplitude in the general case for $\theta = 0$ does not vanish; as $\theta \to 0$ the cross section increases in proportion to θ^{-2}. However, in calculation of the mean free path of a photon ω_1 in a medium with a given spectral density of radiation $N(\omega_2, \mathbf{n}_2)$,

$$l^{-1} = \int d\mathbf{n}_2 \sigma(\omega_1, \omega_2, \mathbf{n}_2)(1 - \cos\theta)N(\omega_2, \mathbf{n}_2), \quad (25)$$

this singularity does not appear in the cross section $(\mathbf{n}_2 = \mathbf{k}_2/k_2)$.

A general investigation of the cross section for two-photon production of $e^\pm$ pairs was carried out numerically. First, we made a limiting transition of the cross section (20) to the known value $\sigma_0(\omega_1, \omega_2)$ for $B = 0$.[6] In the energy region where the process allowed for $B \sim B_c$ and forbidden for $B = 0$, a decrease of the magnetic field leads to a rapid decrease of the cross section for $B \lesssim 0\,0.3B_c$. For those energies at which the process is allowed for $B = 0$, a decrease of the field leads to a crowding together of the threshold resonances, which overlap and form a smooth dependence corresponding to $\sigma_0(\omega_1, \omega_2)$.

Second, we made a detailed study of the dependence of the cross section for the process on the directions of propagation and the polarizations of the colliding photons. The purpose of this investigation was to answer the question of what channels (l, l') contribute to the cross section under given conditions. The relations between the energies of the interacting photons ω_1 and ω_2 satisfying the threshold conditions (21) for arbitrary channels (l, l') are shown in Fig. 3 for various directions of propagation. We have shown also the pair-production thresholds for $B = 0$. At energies lying above the threshold curve for a given channel (l, l'), the cross section corresponding to the channel is greater than zero; below the threshold curve the cross section is $\sigma(l, l') = 0$. For some channel (l, l') the cross section can be equal to zero also at energies above threshold in the case in which this channel is forbidden for the chosen photon-polarization directions. For example, in a headon collision of photons transverse to the magnetic field the minimum reaction threshold, which corresponds to the threshold of the (0, 0) channel, is determined by the condition $\omega_1 + \omega_2 = 2$. For $\omega_1 = \omega_2$ in the case in which the photons are polarized parallel to each other, many channels turn out to be forbidden (for example, channels (1, 0), (3, 0), (2, 1), and so forth). This is due to the fact that for $\omega_1 = \omega_2$ the component of the combined photon momentum perpendicular to the magnetic field is equal to zero. For $\omega_1 \neq \omega_2$ all channels contribute to the cross section regardless of the polarization. An exception is the channel (0, 0). Near the threshold for parallel polarizations of the photons this channel is always forbidden. If the directions of the polarizations of the first and second photons are perpendicular to the magnetic field (the state $(\perp, \perp)$), the cross section for the channel (0, 0) is equal to zero at all energies above threshold. If the directions of the polarizations of both photons are parallel to the magnetic field (the state $(\|, \|)$), then at energies above threshold the cross section for this channel is nonzero. The dependence of

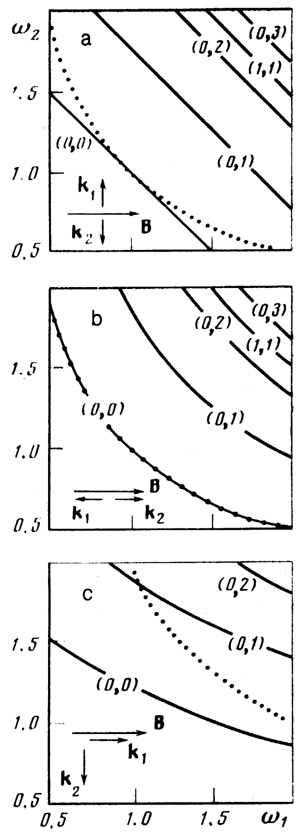

FIG. 3. Relations between the energies ω_1 and ω_2 for which the threshold conditions (21) are satisfied for various reaction channels for $B = B_c$: (a)—corresponds to a headon collision of photons transverse to the magnetic field, (b)—corresponds to a headon collision of photons along the magnetic field, and (c)—corresponds to collision of photons in the case in which the vector $\mathbf{k}_1$ is directed along the magnetic field and the vector $\mathbf{k}_2$ is transverse to the magnetic field; the dots denote the reaction threshold for $B = 0$.

the cross section on the frequency $\omega = \omega_1 = \omega_2$ for a collision of the photons transverse to the magnetic field is shown in Fig. 4.

In a headon collision of photons along the magnetic field, the threshold of the channel (0, 0) $\omega_1\omega_2 = 1$ coincides with the threshold corresponding to the case $B = 0$. As we have already mentioned, this case was discussed in the previous studies.[3–5] Its relative simplicity is due to the fact that with this collision geometry $k_\pm = 0$, and therefore the integrals $J_{ln} \sim \delta_{ln}$ (see Eq. (10)); the infinite sum over the states of the virtual electron is replaced by a sum of several terms. As a result additional selection rules arise for the states of the final particles: $l = l, l' \pm 2$. For parallel polarizations of the photons the channel (0, 0) is strongly suppressed (see Fig. 5). For energies above the threshold of the channel (0, 1) the difference of the cross sections for photons with parallel and perpendicular polarizations is small. For a collision along the magnetic field the restriction adopted by us on the photon energy, which corresponds to the possibility of single-photon production of a pair, does not occur.

The third spectral case is the collision of two photons, one of which is moving along the magnetic field and the other transverse to it. For the photon ω_1 any energy is permissible—it is moving along the field, and therefore it cannot produce a pair; the energy of the second photon ω_2 must be limited by the value of the threshold for single-photon production $\omega_2 < 2$. In this case the dependence of the cross section on the polarization of the photons is substantial only for the channel of $e^\pm$ production in the ground state. This channel has a negligible cross section for photons with parallel polarizations (Fig. 6); we can consider that in this case the reaction threshold is the threshold of the channel (0, 1): $\omega_{\min}^{(0.1)} = 1.58$. In the absence of a magnetic field $\omega_{\min} = 1.41$; this threshold is higher than the threshold $\omega_{\min}^{(0.0)} = 1.15$ for orthogonally polarized photons and is below the threshold $\omega_{\min}^{(0.1)} = 1.58$ for protons with parallel polarizations.

5. CONCLUSIONS

The two-photon production of $e^\pm$ pairs in a magnetic field depends both on the interacting-photon wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, and on their directions relative to the lines of force of

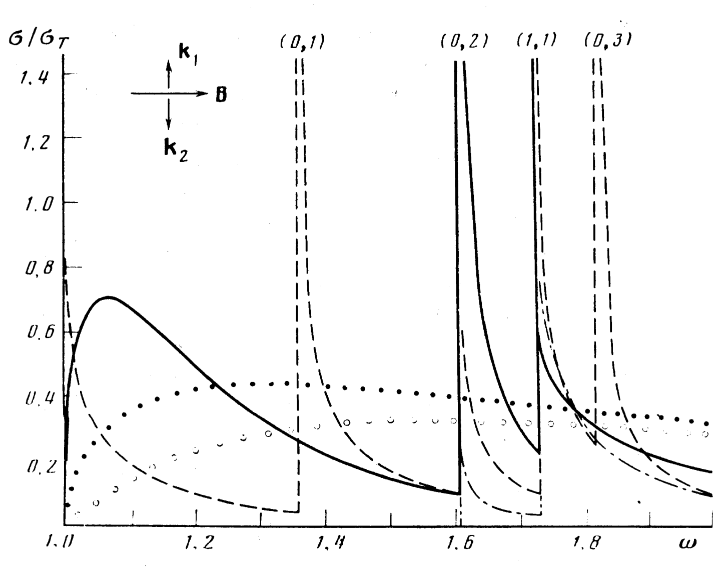

FIG. 4. Cross section for two-photon production of $e^\pm$ pairs in a magnetic field $B = B_c$ as a function of the energies, which are equal to each other $\omega = \omega_1 = \omega_2$, of photons colliding transverse to the field B, for various directions of photon polarizations: solid curve—($\|, \|$), dashed curve—($\|, \perp$), dot-dash curve—($\perp, \perp$). For comparison we have shown the dependence of the cross section on ω for mutually parallel (solid points) and mutually orthogonal (hollow points) polarizations for $B = 0$.

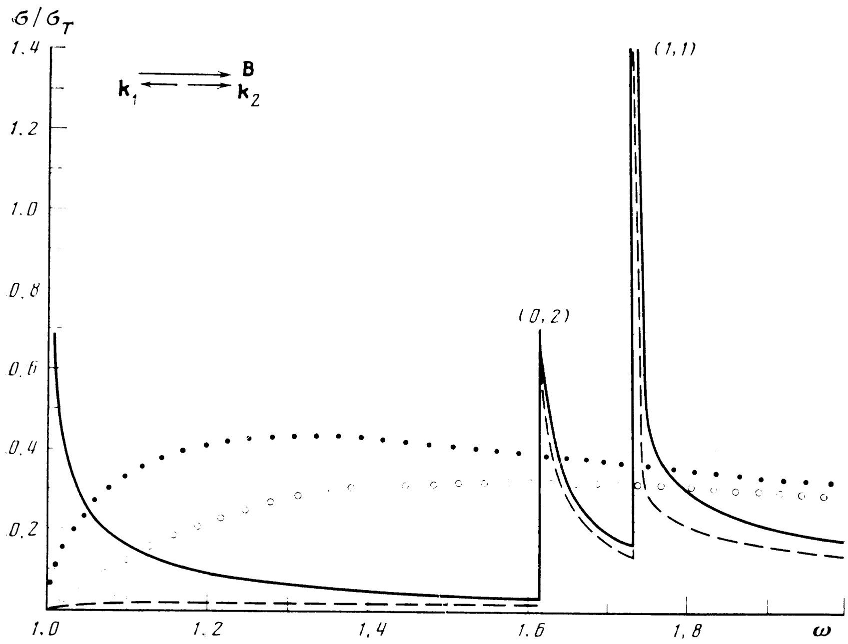

FIG. 5. The same as in Fig. 4 but for headon collision of photons of equal energies along the magnetic field: the solid curve—(⊥), and the dashed curve—(‖).

the magnetic field. However, as a result of the fact that a Lorentz transformation of the coordinate systems along **B** preserves the qualitative features of the considered processes (allowed channels, dependence on polarization), the set of all possible configurations can be reduced to several basic types. For example, for propagation of one of the photons along the magnetic field the process can be reduced to the case $\mathbf{k}_2 \| \mathbf{B}$, $\mathbf{k}_1 \perp \mathbf{B}$ shown in Fig. 3c.

Comparison of the cross sections for the typical configurations discussed above with similar cases for $B = 0$ leads to the following conclusions.

1) The cross section in a magnetic field has strong resonances corresponding to the threshold energies of production of $e^{\pm}$ in Landau levels (l, l').

2) For unpolarized photons the reaction threshold in a magnetic field is, as a rule, lower. If the combined longitudinal momentum of the photons is equal to zero, this threshold is determined only by conservation of energy $\omega_1 + \omega_2 = 2$.

3) The cross section in a magnetic field depends substantially on the polarization of the radiation. If the photons have parallel polarizations perpendicular to the magnetic-field direction, then the channel (0, 0) corresponding to the minimum threshold is forbidden.

In addition, an important feature of the process $\omega_1 + \omega_2 \to e^{\pm}$ in a magnetic field is the fact that the threshold does not depend on the azimuthal angles of the wave

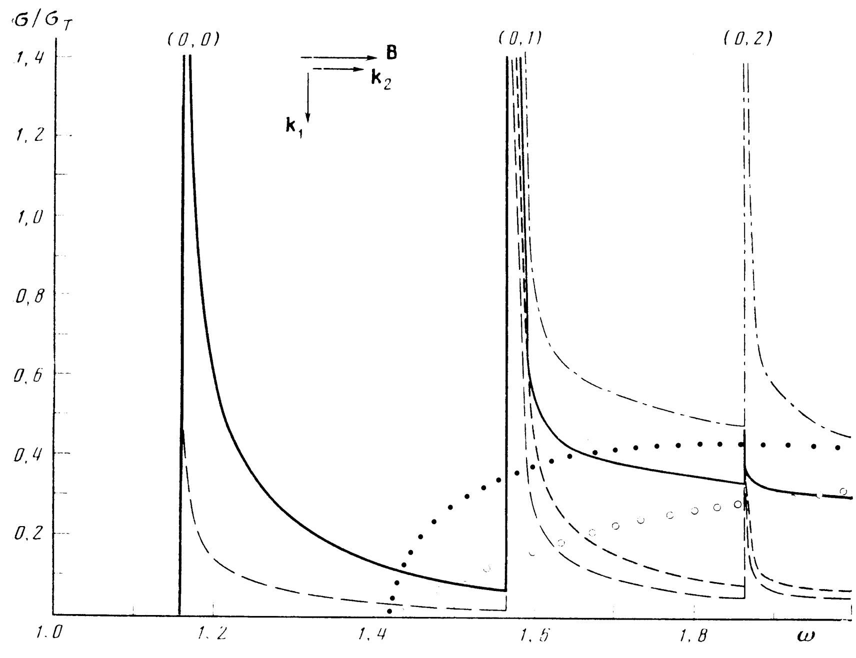

FIG. 6. The same as in Fig. 4 but for collision of photons of equal energies in which $\mathbf{k}_2 \| \mathbf{B}$ and $\mathbf{k}_1 \perp \mathbf{B}$: solid curve—(‖,⊥), long dashes—(⊥, ‖), short dashes—(⊥, ⊥), dot-dash—(‖, ‖) (the first sign in the parenthese refers to the first photon, and the second sign refers to the second photon).

vectors $\mathbf{k}_1$ and $\mathbf{k}_2$. Therefore the results of a calculation of the cross section for a headon collision transverse to the field $\sigma(\mathbf{k}_1\downarrow\uparrow\mathbf{k}_2;\ \mathbf{k}_1,\mathbf{k}_2\perp\mathbf{B})$ (Fig. 4) will permit a crude estimate of the cross section for photons propagated at small angles to a direction perpendicular to the magnetic field: $\sigma\sim\sigma(\mathbf{k}_1\downarrow\uparrow\mathbf{k}_2;\ \mathbf{k}_1,\mathbf{k}_2\perp\mathbf{B})(1-\cos\theta)^{-1}$. The greatest difference of pair production in a magnetic field from that observed for $B=0$ arises just in this case if $\theta\ll 1$. Since for $k_z\ll k$ the threshold condition practically coincides with energy conservation $\omega_1+\omega_2=2$, absorption of the photons will occur for $E_\gamma\gtrsim 1$. On the other hand, for $B=0$ the production of $e^\pm$ pairs occurs only in interaction of photons of much higher energies satisfying the threshold condition $\omega_1\omega_2\geqslant 4(1-\cos\theta)^{-1}$. Therefore the self-absorption of radiation propagating in a solid angle $\Delta\Omega<2\pi$ transverse to the magnetic field for $B\sim B_c$ is stronger than for $B=0$.

For unpolarized radiation propagating in a solid angle $\Delta\Omega<2\pi$ along the magnetic field, the thresholds of the process $\omega_1+\omega_2\to e^\pm$ for $B=0$ and $B\sim B_c$ practically coincide. However, in the case in which the photons are polarized parallel to each other and perpendicular to the magnetic field, the threshold for production of $e^\pm$ pairs corresponds to the channel (2,0), and therefore the self-absorption of polarized radiation propagating along the magnetic field for $B\sim B_0$ can be smaller than for $B=0$.

Contemporary models of radio pulsars are based on the assumption that the hard gamma radiation, generated by relativistic electrons near the poles of a neutron star which are accelerated along the magnetic field, is absorbed as a consequence of single-photon production of $e^\pm$ pairs. In the presence of x radiation in the near-polar region, two-photon pair production can provide competition to the single-photon process. The process considered is also important for construction of models of sources of cosmic gamma bursts. The condition of transparency of the region of generation of the gamma bursts with respect to the reaction $\omega_1+\omega_2\to e^\pm$ permits estimation of the distance to the sources.[10] It is obvious that taking into account the magnetic field can greatly influence this estimate.

It is well known that the single-photon production of $e^\pm$ pairs in a critical magnetic field is similar to the oscillations of magnetic absorption observed in semiconductors in magnetic fields $\sim 10^5$G.[11] In semiconductors quasiparticles with effective masses significantly smaller than m_e act as $e^\pm$ pairs. Here the resonance frequencies corresponding to transitions between the Landau levels are in the optical region. Having in mind this analogy, we can suggest that two crossed photon beams passing through a semiconductor in a magnetic field also should undergo resonance absorption as a consequence of the photon-photon excitation of diamagnetic excitons.

The authors are grateful to A. S. Pozanenko for helpful discussion of a number of questions entering into this work.

APPENDIX

The coefficients $N_i^{(1)}$, $N_i^{(2)}$, $i=1,2,3,4$.

Since the formulas have a cumbersome form, we shall write them out with a number of additional notations:

$$N_i^{(1)}=[(E_l+1)(E_{l'}+1)]^{1/2}\sum_{n=0}^{\infty}\left\{\frac{A_i^{(1)}}{(E_{l'}-\omega_2-E_n)(E_l-\omega_1-E_n)}+\frac{B_i^{(1)}}{(E_{l'}-\omega_2-E_{n+1})(E_l-\omega_1-E_{n+1})}\right\},\quad\text{(A1)}$$

where

$$A_1^{(1)}=-\varepsilon_1^{2}\Pi_{10}^{01}\tilde S_1+\varepsilon_3^{2}\Pi_{00}^{01}C^2+\varepsilon_1^{3}\Pi_{10}^{00}C_2+\varepsilon_3^{3}\Pi_{00}^{00}D,$$

$$B_1^{(1)}=-\varepsilon_3^{1}\Pi_{10}^{00}C_1-\varepsilon_2^{3}\Pi_{00}^{01}C^1-\varepsilon_3^{3}\Pi_{10}^{01}S_1+(\varepsilon_3^{3}\Pi_{01}^{01}-\varepsilon_2^{2}\Pi_{00}^{11})\tilde p_{l'}V_lg+(\varepsilon_3^{3}\Pi_{10}^{10}-\varepsilon_1^{1}\Pi_{11}^{00})\tilde q_lV_{l'}g+(\varepsilon_3^{1}\Pi_{01}^{00}+\varepsilon_2^{3}\Pi_{00}^{10})V_lV_{l'}g-\varepsilon_2^{1}\Pi_{00}^{00}D+(\varepsilon_3^{2}\Pi_{10}^{11}+\varepsilon_1^{3}\Pi_{11}^{01})(1-\tilde p_{l'}\tilde q_l)g,$$

$$A_2^{(1)}=\varepsilon_3^{2}\Pi_{00}^{01}C_1+\varepsilon_1^{3}\Pi_{10}^{00}C^1+\varepsilon_3^{3}\Pi_{00}^{00}S_1+\varepsilon_1^{2}\Pi_{10}^{01}D,$$

$$B_2^{(1)}=\varepsilon_2^{1}\Pi_{00}^{00}\tilde S_1-\varepsilon_3^{1}\Pi_{10}^{00}C^2-\varepsilon_2^{3}\Pi_{00}^{01}C_2+(\varepsilon_1^{1}\Pi_{11}^{00}-\varepsilon_3^{3}\Pi_{10}^{10})\tilde p_{l'}V_lg+(\varepsilon_2^{2}\Pi_{00}^{11}-\varepsilon_3^{3}\Pi_{01}^{01})V_{l'}\tilde q_lg-(\varepsilon_1^{3}\Pi_{11}^{01}+\varepsilon_3^{2}\Pi_{10}^{11})V_lV_{l'}g-\varepsilon_3^{3}\Pi_{10}^{01}D-(\varepsilon_2^{3}\Pi_{00}^{10}+\varepsilon_3^{1}\Pi_{01}^{00})(1-\tilde p_{l'}\tilde q_l)g,$$

$$A_3^{(1)}=-\varepsilon_3^{3}\Pi_{00}^{00}C_1-\varepsilon_1^{2}\Pi_{10}^{01}C^2+\varepsilon_3^{2}\Pi_{00}^{01}\tilde S_2-\varepsilon_1^{3}\Pi_{10}^{00}D,$$

$$B_3^{(1)}=-\varepsilon_2^{1}\Pi_{00}^{00}C_2-\varepsilon_2^{3}\Pi_{00}^{01}S_2-\varepsilon_3^{3}\Pi_{10}^{01}C^1-(\varepsilon_3^{2}\Pi_{10}^{11}+\varepsilon_1^{3}\Pi_{11}^{01})\tilde p_{l'}V_lg-(\varepsilon_3^{1}\Pi_{01}^{00}+\varepsilon_2^{3}\Pi_{00}^{10})V_{l'}\tilde q_lg+(\varepsilon_3^{3}\Pi_{10}^{10}-\varepsilon_1^{1}\Pi_{11}^{00})V_{l'}V_lg-\varepsilon_3^{1}\Pi_{10}^{00}D+(\varepsilon_2^{2}\Pi_{00}^{11}-\varepsilon_3^{3}\Pi_{01}^{01})(1-\tilde p_{l'}\tilde q_l)g,$$

$$A_4^{(1)}=-\varepsilon_1^{2}\Pi_{10}^{01}C_2-\varepsilon_1^{3}\Pi_{10}^{00}S_2-\varepsilon_3^{3}\Pi_{00}^{00}C^1-\varepsilon_3^{2}\Pi_{00}^{01}D,$$

$$B_4^{(1)}=-\varepsilon_2^{1}\Pi_{00}^{00}C^2+\varepsilon_3^{1}\Pi_{10}^{00}\tilde S_2-\varepsilon_3^{3}\Pi_{10}^{01}C_1-(\varepsilon_3^{1}\Pi_{01}^{00}+\varepsilon_2^{3}\Pi_{00}^{10})\tilde p_{l'}V_lg-(\varepsilon_1^{3}\Pi_{11}^{01}+\varepsilon_3^{2}\Pi_{10}^{11})\times V_{l'}\tilde q_lg+(\varepsilon_3^{3}\Pi_{01}^{01}-\varepsilon_2^{2}\Pi_{00}^{11})V_{l'}V_lg-\varepsilon_2^{3}\Pi_{00}^{01}D+(\varepsilon_1^{1}\Pi_{11}^{00}-\varepsilon_3^{3}\Pi_{10}^{10})(1-\tilde p_{l'}\tilde q_l)g$$

in the notation

$$\varepsilon_j^i=\varepsilon_{2i}\varepsilon_{1j},\quad \varepsilon_{n1}=\varepsilon_{nx}+i\varepsilon_{ny},\quad \varepsilon_{n2}=\varepsilon_{nx}-i\varepsilon_{ny},$$

$$\varepsilon_{n3}=\varepsilon_{nz},\quad \Pi_{\alpha\beta}^{\gamma\delta}=J_{n+\alpha,\,l'-\beta}(k_2)J_{l-\gamma,\,n+\delta}(k_1),$$

$$D=V_{l'}V_lP,\quad V_k=(2kB)^{1/2}(E_k+1)^{-1},$$

$$C_{1,2}=V_{l'}(E_l-\omega_1-1\pm P\tilde q_l),\quad C^{1,2}=V_l(E_{l'}-\omega_2-1\pm P\tilde p_{l'}),$$

$$S_1=[\omega_2/(E_{l'}+1)-1]p+[\omega_1/(E_l+1)-1]q-P(1-\tilde p_{l'}\tilde q_l),$$

$$\tilde S_1=S_1+2P(1-\tilde p_{l'}\tilde q_l),\quad \tilde S_2=S_2+2P(1+\tilde p_{l'}\tilde q_l),$$

$$S_2=[\omega_2/(E_{l'}+1)-1]p-[\omega_1/(E_l+1)-1]q-P(1+\tilde p_{l'}\tilde q_l),$$

$$\tilde p_{l'}=p/(E_{l'}+1),\quad \tilde q_l=q/(E_l+1),\quad g=[2(n+1)B]^{1/2},$$

and $N_i^{(2)}$ differs from $N_i^{(1)}$ in the permutation of the photon indices $(1\leftrightarrow 2)$.

[1)] Below we use units $\hbar=1$, $mc^2=1$, and $B=B/B_c$.

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