Drift of Bloch lines in an oscillating field

S. V. Iordanskiĭ and V. I. Marchenko

Institute of Solid-State Physics, Academy of Sciences of the USSR, Chernogolovka, Moscow Province

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A theory is developed of the drift of Bloch lines in a ferromagnet subjected to an external magnetic field which oscillates in time, but is uniform in space. The dependences of the drift velocity on the frequency, amplitude, and direction of the field are determined.

Dedukh, Gornakov, and Nikitenko[1,2] discovered a directional drift of Bloch lines in a cubic ferromagnet subjected to a magnetic field which oscillates in time and is homogeneous in space. A similar effect was predicted earlier by Schlömann for Bloch walls. An important feature of the Schlömann theory[3] is that the Landau-Lifshitz equations have an exact solution for Bloch walls. An exact solution for a Bloch line has not yet been found. Nevertheless, we shall show below that only very general representations of the structure of a Bloch line are sufficient to account for and describe qualitatively the drift effect mentioned above.

1. The Landau-Lifshitz equations in spherical coordinates are

$$-\dot\theta\sin\theta-\varkappa\dot\varphi\sin^2\theta=\delta\tilde E/\delta\varphi,\qquad \dot\varphi\sin\theta-\varkappa\dot\theta=\delta\tilde E/\delta\theta,\quad (1)$$

where $\varkappa \ll 1$ is the damping constant and the energy $\tilde E$ is equal to the sum of the energy of a ferromagnet in an external magnetic field $\int \mathbf{M}\cdot\mathbf{H}dV$ and the intrinsic energy E, which is given by

$$\int dV\left\{\frac{\beta}{2}\sin^2\theta-K\left[\frac{\sin^4\theta}{4}+\frac{\cos^4\theta}{3}-\frac{2^{1/2}}{3}\cos\theta\sin^3\theta\cos^3\varphi\right.\right.$$
$$\left.+2^{1/2}\cos\theta\sin^3\theta\cos\varphi\sin^2\varphi\right]+{}^1/_2[\sin^2\theta(\nabla\varphi)^2+(\nabla\theta)^2]$$
$$\left.+\int dV'|\mathbf{r}-\mathbf{r}'|^{-1}\operatorname{div}\mathbf{M}(\mathbf{r})\operatorname{div}\mathbf{M}(\mathbf{r}')\right\};\qquad (2)$$

here, β is the effective uniaxial anisotropy[4] due to the magnetostriction. The state $\theta = 0$ corresponds to the magnetization directed along the [111] easy axis (x axis); the last term is the demagnetization energy. The y axis in the Cartesian coordinate system (x,y,z) is directed along $[1\bar10]$. In Refs. 1 and 2 it is assumed that domain walls lie in the (z,x) plane and the Bloch lines are oriented along the z axis. We shall assume that the equilibrium structure of a line is described by the functions

$$\{\theta_0(\mathbf{r}),\varphi_0(\mathbf{r})\};$$
$$\mathbf{M}_0=(M_0^x, M_0^y, M_0^z)=(\cos\theta_0, \sin\theta_0\sin\varphi_0, \sin\theta_0\cos\varphi_0).$$

For the line orientation assumed in Refs. 1 and 2, the functions $\{\theta_0, \varphi_0\}$ depend only on (x,y) (we shall ignore the edge effects). We can easily see that functions of the type

I: $\{\theta_0,\varphi_0\}$; II: $\{\pi+\theta_0(x,-y), -\varphi_0(x,-y)\}$;

III: $\{\pi+\theta_0(-x,-y), \varphi_0(-x,-y)\}$;

IV: $\{\theta_0(-x,y), -\varphi_0(-x,y)\}$;

V: $\{\pi+\theta_0(-x,y), -\varphi_0(-x,y)\}$;

VI: $\{\theta_0(-x,-y), \varphi_0(-x,-y)\}$;

VII: $\{\theta_0(x,-y), -\varphi_0(x,-y)\}$;

VIII: $\{\pi+\theta_0(x,y), \varphi_0(x,y)\}$

correspond to the same energies (2) and, therefore, represent different solutions of the equilibrium equations $\delta E/\delta\varphi = 0, \delta E/\delta\theta = 0$. The magnetization fields corresponding to these solutions are shown schematically in Fig. 1. The black and white parts of these Bloch walls correspond to the positive and negative values of the z component of the magnetization.

2. We shall assume now that $\theta = \theta_0 + \theta_1, \varphi = \varphi_0 + \varphi_1$, where θ_1 and φ_1 are arbitrary small functions of (x,y); then, in the approximation which is linear in respect of θ_1 and φ_1, we have

$$\delta E/\delta\varphi=L_{\varphi\varphi}\varphi_1+L_{\varphi\theta}\theta_1;\qquad \delta E/\delta\theta=L_{\theta\theta}\theta_1+L_{\theta\varphi}\varphi_1,$$

where the matrix integrodifferential operator $\hat L$ defined by the components of the second variational derivative of the energy E with respect to θ and φ is clearly self-adjoint. Since the solution $\{\theta_0, \varphi_0\}$ corresponds to a minimum of the energy E, the spectrum of the operator $\hat L$ has no negative eigenvalues. We shall assume that the position of a line on a wall is not pinned by any defects or external conditions so that in addition to the solution $\{\theta_0, \varphi_0\}$, we have a continuous series of degenerate solutions $\{\theta_0(x + X,y), \varphi_0(x + X,y)\}$, where X is an arbitrary constant. Hence, it obviously follows that the operator L has an eigenvector

$$(\partial\theta_0/\partial x,\ \partial\varphi_0/\partial x)\qquad (3)$$

with zero eigenvalue. It follows from the experimental data of Ref. 5 that the position of a domain wall is pinned either by growth defects or by the effects associated with the demagnetization so that there is an eigenfrequency of the wall vibrations amounting to about 1.8 MHz (Ref. 6). For the sake of simplicity, we shall add a term $A^2\int M_z^2 y^2 dv$, to the energy and this term represents the "attraction" of a domain wall to

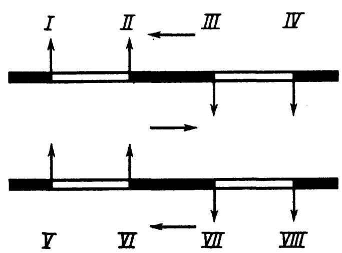

FIG. 1.

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the $y = 0$ plane. Then, the continuous spectrum of the operator $\hat{L}$ begins with a gap proportional to A. A discrete spectrum of local modes is not excluded; however, the important point in the subsequent analysis is that there are no other reasons for the existence of small eigenvalues.

Before investigating the dynamic problem, we shall first derive an identity which will be useful later. We shall expand the equilibrium equations for functions of the type $\{\theta_0(x+X+\zeta,y), \varphi_0(x+X+\zeta,y)\}$ for a small constant ζ accurate to within quadratic terms. Bearing in mind that

$$\theta_0(x+X+\zeta,y)\approx\theta_0(x+X,y)+\zeta\frac{\partial\theta_0}{\partial x}+\frac{1}{2}\zeta^2\frac{\partial^2\theta_0}{\partial x^2},$$

$$\varphi_0(x+X+\zeta,y)\approx\varphi_0(x+X,y)+\zeta\frac{\partial\varphi_0}{\partial x}+\frac{1}{2}\zeta^2\frac{\partial^2\varphi_0}{\partial x^2}$$

we obtain

$$\begin{aligned}\frac{\delta E}{\delta\theta}&\approx\frac{\zeta^2}{2}\left(L_{\theta\theta}\frac{\partial^2\theta_0}{\partial x^2}+L_{\theta\varphi}\frac{\partial^2\varphi_0}{\partial x^2}\right)+\left\{\frac{\delta E}{\delta\theta}\right\}_2=0,\\ \frac{\delta E}{\delta\varphi}&\approx\frac{\zeta^2}{2}\left(L_{\varphi\varphi}\frac{\partial^2\varphi_0}{\partial x^2}+L_{\varphi\theta}\frac{\partial^2\theta_0}{\partial x^2}\right)+\left\{\frac{\delta E}{\delta\varphi}\right\}_2=0.\end{aligned}\tag{4}$$

Here, we have separated the terms which are linear in the quadratic corrections to the function $\{\theta_0, \varphi_0\}$ and terms of the type $\{\ \}_2$ which are quadratic in the linear correction to the functions $\{\theta_0, \varphi_0\}$. We multiply the first equation in the system (4) by $\delta\theta_0/\delta x$ and the second by $\delta\varphi_0/\delta x$, add the resultant equations, and integrate over the volume. In view of the self-adjoint nature of the operator L and also the fact that the vector of Eq. (3) is an eigenvector with zero eigenvalue, we obtain the required identity:

$$\iint dx\,dy\left(\frac{\partial\theta_0}{\partial x}\left\{\frac{\delta E}{\delta\theta}\right\}_2+\frac{\partial\varphi_0}{\partial x}\left\{\frac{\delta E}{\delta\varphi}\right\}_2\right)=0.\tag{5}$$

3. An investigation of the motion of a Bloch line in an external field $\mathbf{H} = \mathbf{h}\sin\omega t$ will be carried out by expanding the solution of the system (1) in the small amplitude of the field and the low (compared with the eigenfrequency of the wall vibrations) frequency ω.

We first determine the low-frequency spectrum of a Bloch line. We seek a solution of equations linear in respect of the amplitude of motion and we assume that this solution can be described by a series in terms of the frequency and damping constant:

$$\begin{aligned}\theta(x,y,t)&\approx\theta_0(x,y)+c(t)(\partial\theta_0/\partial x)+\theta_1+\theta_2+\ldots,\\ \varphi(x,y,t)&\approx\varphi_0(x,y)+c(t)(\partial\varphi_0/\partial x)+\varphi_1+\varphi_2+\ldots\end{aligned}\tag{6}$$

In the first approximation, we obtain

$$\dot{c}\begin{pmatrix}+\dfrac{\partial\varphi_0}{\partial x}\sin\theta_0\\ -\dfrac{\partial\theta_0}{\partial x}\sin\theta_0\end{pmatrix}=L\begin{pmatrix}\theta_1\\ \varphi_1\end{pmatrix}.\tag{7}$$

Hence, introducing time-independent functions η^θ and η^φ such that $\theta_1 = \dot{c}\eta^\theta$, $\varphi_1 = \dot{c}\eta^\varphi$, we find that

$$\begin{pmatrix}\eta^\theta\\ \eta^\varphi\end{pmatrix}=L^{-1}\begin{pmatrix}\dfrac{\partial\varphi_0}{\partial x}\sin\theta_0\\ -\dfrac{\partial\theta_0}{\partial x}\sin\theta_0\end{pmatrix}.\tag{8}$$

The second approximation yields

$$\begin{pmatrix}\ddot{c}\eta^\varphi\sin\theta_0-\varkappa\dot{c}\dfrac{\partial\theta_0}{\partial x}\\ -\ddot{c}\eta^\theta\sin\theta_0-\varkappa\dot{c}\dfrac{\partial\varphi_0}{\partial x}\sin^2\theta_0\end{pmatrix}=L\begin{pmatrix}\theta_2\\ \varphi_2\end{pmatrix}.\tag{9}$$

Hence, following the procedure used to go from the system (4) to the identity (5), we obtain the usual equation of motion for a free Bloch line

$$m\ddot{c}+\alpha\dot{c}=0.\tag{10}$$

Therefore, the low-frequency spectrum reduces to two modes $\Omega_1 = 0$ and $\Omega_2 = i\tau^{-1}$, where $\tau = \alpha/m$ and

$$m=\int\sin\theta_0\left(\eta^\theta\frac{\partial\theta_0}{\partial x}-\eta^\varphi\frac{\partial\varphi_0}{\partial x}\right)dx\,dy;$$

$$\alpha=\varkappa\int\left[\left(\frac{\partial\varphi_0}{\partial x}\right)^2\sin^2\theta_0+\left(\frac{\partial\theta_0}{\partial x}\right)^2\right]dx\,dy.\tag{11}$$

The other eigenfrequencies of the system begin with a gap proportional to A.

Generally speaking, an external alternating field excites all the modes. However, if the field frequency is low compared with the gap, the amplitude of motion of the magnetization is mainly due to two modes, Ω_1 and Ω_2. The contribution of the remaining modes is obtained in the adiabatic approximation. We separate in the system (1) the terms with an external field

$$-\dot{\theta}\sin\theta-\varkappa\dot{\varphi}\sin^2\theta+\sin\theta\cos\varphi H_y-\sin\theta\sin\varphi H_z=\delta E/\delta\varphi,$$

$$\dot{\varphi}\sin\theta-\varkappa\dot{\theta}-\sin\theta H_x+\cos\theta\sin\varphi H_y+\cos\theta\cos\varphi H_z=\delta E/\delta\theta.\tag{12}$$

In the linear approximation, we have

$$\begin{pmatrix}\dot{\varphi}_\omega\sin\theta_0-\varkappa\dot{\theta}_\omega-\sin\theta_0H_x+\cos\theta_0\sin\varphi_0H_y+\cos\theta_0\cos\varphi_0H_z\\ -\dot{\theta}_\omega\sin\theta_0-\varkappa\dot{\varphi}_\omega\sin^2\theta_0+\sin\theta_0\cos\varphi_0H_y-\sin\theta_0\sin\varphi_0H_z\end{pmatrix}=L\begin{pmatrix}\theta_\omega\\ \varphi_\omega\end{pmatrix}.\tag{13}$$

Separating in the terms with the magnetic field the part orthogonal to the vector (3), we find that instead of Eq. (7) we now have

$$\begin{pmatrix}\dot{c}\dfrac{\partial\varphi_0}{\partial x}\sin\theta_0-\sin\theta_0H_x+\cos\theta_0\sin\varphi_0H_y+\cos\theta_0\cos\varphi_0H_z-I^{-1}\mu\mathbf{H}\dfrac{\partial\varphi_0}{\partial x}\\ -\dot{c}\dfrac{\partial\theta_0}{\partial x}\sin\theta_0+\sin\theta_0\cos\varphi_0H_y-\sin\theta_0\sin\varphi_0H_z-I^{-1}\mu\mathbf{H}\dfrac{\partial\theta_0}{\partial x}\end{pmatrix}=L\begin{pmatrix}\theta_1\\ \varphi_1\end{pmatrix}.\tag{14}$$

Here, the vector μ is a discontinuity of the magnetic moment of a domain wall on transition across a Bloch line:

$$\mu_x=\iint \sin\theta_0 \frac{\partial\theta_0}{\partial x}\,dx\,dy=\iint \frac{\partial M_0^x}{\partial x}\,dx\,dy$$
$$=\int (M_0^x|_{x=+\infty}-M_0^x|_{x=-\infty})\,dy,$$
$$\mu_z=\iint\left(-\sin\theta_0\sin\varphi_0\frac{\partial\varphi_0}{\partial x}-\cos\theta_0\cos\varphi_0\frac{\partial\theta_0}{\partial x}\right)dx\,dy$$
$$=\int (M_0^z|_{x=+\infty}-M_0^z|_{x=-\infty})\,dy. \tag{15}$$

The component μ_y vanishes because in domain walls far from a line we have

$$I=\iint[(\partial\theta_0/\partial x)^2+(\partial\varphi_0/\partial x)^2]dx\,dy.$$

We write the solution (θ_1,φ_1) of Eq. (14) in the following form:

$$\begin{pmatrix}\theta_1\\ \varphi_1\end{pmatrix}=\dot c\begin{pmatrix}\eta^\theta\\ \eta^\varphi\end{pmatrix}+\mathbf{H}\begin{pmatrix}\boldsymbol{\eta}^\theta\\ \boldsymbol{\eta}^\varphi\end{pmatrix}, \tag{16}$$

where the functions η and $\boldsymbol{\eta}$ are independent of time and frequency. The last term in Eq. (16) clearly corresponds to the contribution of high-frequency modes. Then, instead of the system (9), we have

$$\ddot c\sin\theta_0\begin{pmatrix}\eta^\varphi\\ -\eta^\theta\end{pmatrix}+\sin\theta_0\dot{H}\begin{pmatrix}\boldsymbol{\eta}^\varphi\\ -\boldsymbol{\eta}^\theta\end{pmatrix}-\varkappa\dot c\begin{pmatrix}\dfrac{\partial\theta_0}{\partial x}\\ \dfrac{\partial\varphi_0}{\partial x}\sin^2\theta_0\end{pmatrix}$$
$$+I^{-1}\mu\mathbf{H}\begin{pmatrix}\dfrac{\partial\theta_0}{\partial x}\\ \dfrac{\partial\varphi_0}{\partial x}\end{pmatrix}=L\begin{pmatrix}\theta_2\\ \varphi_2\end{pmatrix}. \tag{17}$$

Hence [compare the transition from Eq. (9) to Eq. (10)], we obtain the equation of motion of a Bloch line in an external field:

$$m\ddot c+\alpha\dot c+\boldsymbol{\mu}\mathbf{H}+\mathbf{v}\dot{\mathbf{H}}=0; \tag{18}$$

we have introduced here a vector

$$\mathbf{v}=\iint\sin\theta_0\left(\boldsymbol{\eta}^\varphi\frac{\partial\theta_0}{\partial x}-\boldsymbol{\eta}^\theta\frac{\partial\varphi_0}{\partial x}\right)dx\,dy.$$

Equation (18) is the condition of solubility of the system (17). Starting from Eq. (17) [in agreement with Eq. (18)] and eliminating a term proportional to $\boldsymbol{\mu}\cdot\mathbf{H}$, we obtain

$$\ddot c\begin{pmatrix}\sin\theta_0\eta^\theta-I^{-1}m\dfrac{\partial\theta_0}{\partial x}\\ -\sin\theta_0\eta^\varphi-I^{-1}m\dfrac{\partial\varphi_0}{\partial x}\end{pmatrix}$$
$$+\dot c\begin{pmatrix}\varkappa\dfrac{\partial\theta_0}{\partial x}-I^{-1}\alpha\dfrac{\partial\theta_0}{\partial x}\\ \varkappa\sin^2\theta_0\dfrac{\partial\varphi_0}{\partial x}-I^{-1}\alpha\dfrac{\partial\varphi_0}{\partial x}\end{pmatrix}$$
$$+\dot{\mathbf{H}}\begin{pmatrix}\sin\theta_0\boldsymbol{\eta}^\theta-I^{-1}\mathbf{v}\dfrac{\partial\varphi_0}{\partial x}\\ -\sin\theta_0\boldsymbol{\eta}^\varphi-I^{-1}\mathbf{v}\dfrac{\partial\theta_0}{\partial x}\end{pmatrix}=L\begin{pmatrix}\theta_2\\ \varphi_2\end{pmatrix}, \tag{19}$$

which shows that the correction (θ_2,φ_2) is of the form [compare with Eq. (16)]

$$\begin{pmatrix}\theta_2\\ \varphi_2\end{pmatrix}=\ddot c\begin{pmatrix}\eta_1^\theta\\ \eta_1^\varphi\end{pmatrix}+\varkappa\dot c\begin{pmatrix}f_1^\theta\\ f_1^\varphi\end{pmatrix}+\dot{\mathbf{H}}\begin{pmatrix}\boldsymbol{\eta}_1^\theta\\ \boldsymbol{\eta}_1^\varphi\end{pmatrix}, \tag{20}$$

where the functions η_1, f_1, and $\boldsymbol{\eta}_1$ are defined only by the characteristics of the equilibrium problem. From Eq. (18), we find $\mathbf{H}=\mathbf{h}\sin\omega t$ that

$$c=\left(\omega^2+\frac{1}{\tau^2}\right)^{-1}m^{-1}\left(\boldsymbol{\mu}-\frac{\mathbf{v}}{\tau}\right)\mathbf{h}\left(\sin\omega t-\frac{\cos\omega t}{\omega\tau}\right)$$
$$+\frac{\mathbf{vh}}{\omega m}\cos\omega t. \tag{21}$$

4. We seek a solution in the approximation which is quadratic in the field amplitude:

$$\theta=\theta_0(x-Vt,y)+\theta_\omega(t,x-Vt,y)+\theta'(t,x-Vt,y),$$
$$\varphi=\varphi_0(x-Vt,y)+\varphi_\omega(t,x-Vt,y)+\varphi'(t,x-Vt,y), \tag{22}$$

where V is the expected drift velocity (proportional to h^2); θ_ω and φ_ω are the corrections which are linear in the amplitude, as found above; θ' and φ' are the corrections which are quadratic in the field. We shall now average the solution (22) in a time interval which is long compared with the period of the field oscillations, but short compared with the characteristic drift time of a Bloch line which travels a distance of the order of its "thickness." For example, $\langle\theta\rangle$ is described by

$$\langle\theta\rangle=\theta_0(x-Vt,y)+\langle\theta'(t,x-Vt,y)\rangle.$$

Clearly, after such averaging the time dependence of the solution reduces simply to a drift at a velocity V, so that to within terms proportional to h^4, we have

$$\langle\dot\theta\rangle=-V\partial\theta_0/\partial x. \tag{23}$$

We now apply this time-averaging to the system (1). Retaining terms proportional to h, we obtain with the aid of Eq. (23)

$$V\sin\theta_0\begin{pmatrix}-\partial\varphi_0/\partial x\\ \partial\theta_0/\partial x\end{pmatrix}+\varkappa V\begin{pmatrix}\partial\theta_0/\partial x\\ \sin^2\theta_0(\partial\varphi_0/\partial x)\end{pmatrix}$$
$$-\left\langle\begin{pmatrix}(\partial E/\partial\theta)-\dot\varphi\sin\theta\\ (\partial E/\partial\varphi)+\dot\theta\sin\theta\end{pmatrix}\right\rangle_2=L\begin{pmatrix}\langle\theta'\rangle\\ \langle\varphi'\rangle\end{pmatrix}. \tag{24}$$

Here, in the angular brackets with the index 2, i.e., in $(\langle\ \rangle_2)$ we have arbitrarily separated the terms which are quadratic in the correction $(\theta_\omega,\varphi_\omega)$ and the terms with the magnetic field. Repeating the procedure used to go over from Eq. (4) to Eq. (5), we obtain the following expression for the velocity

$$V=\alpha^{-1}F, \tag{25}$$

where the effective force F consists of three parts:

$$F_1=\iint\left(\left\langle\frac{\delta E}{\delta\varphi}\right\rangle_2\frac{\partial\varphi_0}{\partial x}+\left\langle\frac{\delta E}{\delta\theta}\right\rangle_2\frac{\partial\theta_0}{\partial x}\right)dx\,dy,$$

$$F_2=-\iint\langle\dot\varphi_\omega\theta_\omega\rangle\cos\theta_0\frac{\partial\theta_0}{\partial x}dx\,dy,\tag{26}$$

$$\begin{aligned}F_3=\iint dx\,dy\Big\{&\langle H_x\theta_\omega\rangle\cos\theta_0\frac{\partial\theta_0}{\partial x}\\&+\Big\langle H_y,\sin\theta_0\sin\varphi_0\frac{\partial\varphi_0}{\partial x}\varphi_\omega\\&-\cos\theta_0\cos\varphi_0\frac{\partial\varphi_0}{\partial x}\theta_\omega+\sin\theta_0\sin\varphi_0\frac{\partial\theta_0}{\partial x}\theta_\omega\\&-\cos\theta_0\cos\varphi_0\frac{\partial\theta_0}{\partial x}\varphi_\omega\Big\rangle\\&+\Big\langle H_z,\sin\theta_0\cos\varphi_0\left(\frac{\partial\theta_0}{\partial x}\theta_\omega+\frac{\partial\varphi_0}{\partial x}\varphi_\omega\right)\\&+\cos\theta_0\sin\varphi_0\left(\frac{\partial\theta_0}{\partial x}\varphi_\omega+\frac{\partial\varphi_0}{\partial x}\theta_\omega\right)\Big\rangle\Big\}.\end{aligned}$$

If in Eq. (26) for F_1 in the expressions for $(\theta_\omega,\varphi_\omega)$ we retain the leading (in the amplitude) terms $(c\partial\varphi_0/\partial x, c\partial\theta_0/\partial x)$, it follows from the identity (5) that $F_1=0$, so that the finite contribution to F_1 is due to inclusion of the next corrections given by Eqs. (16) and (20). Substitution of the solutions $(\theta_\omega,\varphi_\omega)$ in Eq. (26) gives very cumbersome expressions. However, we may readily show that the result can be represented in the following simple form:

$$F=(\omega^2+1/\tau^2)^{-1}(\boldsymbol{\mu}\mathbf{h})(\mathbf{N}\mathbf{h}),\tag{27}$$

where the vector $\mathbf{N}$ is governed only by the characteristics of the equilibrium problem and there are no reasons to reduce to zero any of its components N_x, N_y, and N_z. The expression (27) determines completely the dependence of the effective force on the frequency, damping constant, direction, and field amplitude. The order of magnitude (on the assumption that $\beta\sim K\sim4\pi$) of the velocity expressed in the usual units is

$$V\sim\frac{\gamma M\delta}{\varkappa}\frac{(\gamma h)^2}{\omega^2+\tau^{-2}},$$

where γ is the gyromagnetic ratio and δ is the domain wall thickness.

We shall now determine the law describing the transformation of the expression for the force F on transition from the solution $\{\theta_0,\varphi_0\}$ [I in Fig. 1] to the other solutions given above (II–VIII). In the first term of the integrand in the equation for F_3, we have

$$\begin{aligned}\langle H_x\theta_\omega\rangle\cos\theta_0\frac{\partial\theta_0}{\partial x}&\approx\langle H_xc\rangle\cos\theta_0\left(\frac{\partial\theta_0}{\partial x}\right)^2\\&\approx\frac{1}{2}\left(\omega^2+\frac{1}{\tau^2}\right)^{-1}\frac{(\boldsymbol{\mu}\mathbf{h})}{m}\\&\quad\times h_x\cos\theta_0\left(\frac{\partial\theta_0}{\partial x}\right)^2.\end{aligned}\tag{28}$$

Here, the quantities $\boldsymbol{\mu}$ and $\cos\theta_0$ may differ in the sign for different Bloch lines. For example, if we go over from the solution I $\{\theta_0,\varphi_0\}$ to the solution III $\{\pi+\theta_0(-x,-y),\varphi_0(-x,-y)\}$, Eq. (28) changes its sign. We can analyze all the terms in F_3 in an equally simple manner.

In the integrand of Eq. (26) for F_2 we have to include also the following corrections in $(\theta_\omega,\varphi_\omega)$ of Eq. (16):

$$\begin{aligned}&\left\langle\dot c\frac{\partial\varphi_0}{\partial x}+\ddot c\eta^\varphi,c\frac{\partial\theta_0}{\partial x}+\dot c\eta^\theta\right\rangle\cos\theta_0\frac{\partial\theta_0}{\partial x}\\&=\langle\dot c\rangle^2\frac{\partial\varphi_0}{\partial x}\frac{\partial\theta_0}{\partial x}\cos\theta_0\eta^\theta+\langle c\ddot c\rangle\left(\frac{\partial\theta_0}{\partial x}\right)^2\cos\theta_0\eta^\varphi.\end{aligned}\tag{29}$$

The functions η^θ and η^φ of Eq. (8) satisfy equations of the type

$$\begin{aligned}-\frac{\partial\theta_0}{\partial x}\sin\theta_0&=\frac{\delta^2E}{\delta\varphi^2}\eta^\varphi+\frac{\delta^2E}{\delta\varphi\,\delta\theta}\eta^\theta,\\\frac{\partial\varphi_0}{\partial x}\sin\theta_0&=\frac{\delta^2E}{\delta\theta^2}\eta^\theta+\frac{\delta^2E}{\delta\theta\,\delta\varphi}\eta^\varphi,\end{aligned}\tag{30}$$

and hence it is obvious that transformations that do not alter the energy modify η^φ in the same way as the function $\sin\theta_0(\partial\theta_0/\partial x)$, whereas η^θ is modified as $\sin\theta_0(\partial\varphi_0/\partial x)$. In this way Eq. (29) transforms as the function $2\theta_0(\partial\theta_0/\partial x)$.

The first term in the integration of Eq. (26) describing F_1 is

$$\left\{\frac{1}{2}\frac{\delta^3E}{\delta\varphi^3}\langle\varphi_\omega^2\rangle+\frac{\delta^3E}{\delta\varphi^2\,\delta\theta}\langle\varphi_\omega\theta_\omega\rangle+\frac{1}{2}\frac{\delta^3E}{\delta\varphi\,\delta\theta^2}\langle\theta_\omega^2\rangle\right\}\frac{\partial\varphi_0}{\partial x}.$$

Let us now consider, for example, the first term in the above equation. Expanding up to $\ddot c$ in φ_ω, we obtain

$$\begin{aligned}&\frac{\delta^3E}{\delta\varphi^3}\left\langle\left(c\frac{\partial\varphi_0}{\partial x}+\dot c\eta^\varphi+\mathbf{H}\eta^\varphi+\ddot c\eta_1^\varphi\right)^2\right\rangle\frac{\partial\varphi_0}{\partial x}\\&\approx\frac{\delta^3E}{\delta\varphi^3}\frac{\partial\varphi_0}{\partial x}\left[\langle c^2\rangle\left(\frac{\partial\varphi_0}{\partial x}\right)^2+\langle\dot c^2\rangle(\eta^\varphi)^2\right.\\&\qquad\left.+2\langle c\mathbf{H}\rangle\eta^\varphi+\langle c\ddot c\rangle\frac{\partial\varphi_0}{\partial x}\eta_1^\varphi\right].\end{aligned}$$

The term proportional to $\langle c^2\rangle$ drops out from the final answer because of the identity (5). The next term proportional to $\langle\dot c^2\rangle$ transforms as $(\delta^3E/\delta\varphi^3)\partial\varphi_0/\partial x$, i.e., its sign changes as a result of those transformations which include the substitution $x\to-x$. We shall not consider further what is essentially a simple analysis of the other terms, but formulate the final result as follows. The components N_x and N_z of the vector N of Eq. (27) are the same for the solutions I, IV, VI, and VII, and their sign changes in the case of the other solutions. The component N_y is the same for the solutions I, II, V, VI and it sign changes in the case of III, IV, VII, and VIII, i.e., it is proportional to the magnetic moment of a line. A discontinuity of the magnetic moment μ at a wall changes sign as a result of the (I, III, V, VII) $\to$ (II, IV, VI, VIII) transition (see Fig. 1).

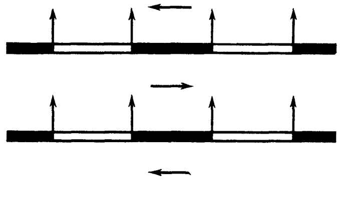

FIG. 2.

According to Refs. 1 and 2, all the lines in one wall move in one direction, whereas in the next wall they move in the opposite direction; the most effective is the x component of the external field. We can explain this rule if we perform a certain matching, namely the magnetic moments of all the Bloch lines should be directed in one way (Fig. 2). In fact, if an external field has only the x component, then the direction of the force of Eq. (27) is governed by the factor $\mu_x N_x h_x^2$, which is easily shown to transform in the necessary way. On appearance of the y component of the field, there is a correction proportional to $\mu_x h_x N_y h_y$, which for a structure of this type (Fig. 2) has different signs for the neighboring lines and depends on the sign of h_y. Therefore, when the field deviates slightly from the x axis, Bloch lines drift retaining their initial direction but with different velocities. The field component H_z has in prnciple the same effect as H_x, but in Refs. 1 and 2 it cannot be strong because of the demagnetization effects [the sample is in the form of a plate lying in the (x,y) plane]. This picture is in qualitative agreement with the experimental data of Refs. 1 and 2.

The drift of Bloch lines increases the density at one edge of a Bloch wall and reduces it at the other.[8] The large distance between the lines corresponds to a metastable state, because new lines are then formed.[2] The orientation of their magnetic moments corresponds to the drift from the edge of the plate, because otherwise the lines are expelled from the plate.

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