Mesoscopic fluctuations of current-voltage characteristics

A. I. Larkin and D. E. Khmel'nitskiĭ

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR, Moscow, and Institute of Solid-State Physics, Academy of Sciences of the USSR, Moscow

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Fluctuations of the current-voltage characteristic (CVC) of a mesoscopic specimen are considered. Such a specimen can be made in the form of a contact involving two normal metals. The dependence of the current I on the voltage V is a random function, and the CVC has the form of "grass" on the usual ohmic pedestal. The scale of the "grass" with respect to the voltage is of the order of $V_c \sim \hbar/\tau_f e$, where τ_f is the time of flight across the contact and e is the electron charge. The current scale δI depends on the voltage V and the temperature T and is found to be of the order of $(e^2/\hbar)(VV_c)^{1/2}$, if $V \gg V_c$, T/e. As a result, at sufficiently large voltages regions of negative differential resistance appear.

The current-voltage characteristic of a contact between two normal metals, calculated by using the kinetic equation, is found to be ohmic. The deviations of the CVC from linearity that arise from the nonequilibrium character of the distribution function in the contact and from the dependence of the rate of inelastic processes on the electron energy have a large scale with respect to the voltage and will not be considered here. Another source of nonlinearity is associated with quantum interference effects. In the present article we consider the case when the contact resistance $R_0 \ll \hbar/e^2$ and the interference effects emerge as small corrections to Ohm's law.

These corrections are greatest when the interference occurs over a long time exceeding the time of flight τ_f of an electron across the region of the contact. Such interference is possible only for electrons with approximately equal energies such that $|\varepsilon_1 - \varepsilon_2| \lesssim eV_c = \hbar/\tau_f$. If the mean free path l is smaller than the length L of the contact, then $\tau_f \sim L^2/D$, where D is the electron diffusion coefficient. The interference effects depend on the specific arrangement of the impurities, and this leads to non-self-averaging of the CVC. If the voltage V across teh contact is much smaller than V_c, the CVC is linear and the conductance $g = 1/R$ contains a non-self-averaging correction δg. The theory for this case[1–5] predicts an irregular dependence of $\delta g(\mu, H)$ on the magnetic field H and on the Fermi energy μ of the electrons.

The current across the contact is formed by electrons from the region of energies $|\varepsilon_1 - \varepsilon_2| \sim$ eV near the Fermi level. If $V \gg V_c$, this region breaks down into V/V_c intervals of values of ε_1 and ε_2 for which interference is important. Each such interval gives an independent random contribution of order $e^2 V_c/\hbar$ to the total current. Therefore, the current I is a random function of the voltage V with characteristic scale $\Delta V \sim V_c$ and amplitude $\Delta I \sim (e^2/\hbar)(VV_c)^{1/2}$. For $V > (\hbar/e^2 R_0) V_c$ there are parts of the CVC that have a negative differential resistance.

The random function $I(V)$ is characterized by the correlator

$$K(V_1, V_2) = \overline{I(V_1)I(V_2)} - \overline{I(V_1)}\,\overline{I(V_2)}. \quad (1)$$

In this paper we find the dependence of K on $V = (V_1 + V_2)/2$, $\Delta V = V_1 - V_2$, and the temperature T. It is assumed that the contact has the form of a bridge between massive "banks", with cross section S and length $L \gg S^{1/2}$. The mean free path l is assumed to be smaller than L.

In the study of a nonlinear CVC it is convenient to use the diagram technique of Keldysh.[6,7] In this technique the Green's function has the matrix form

$$\hat{G} = \begin{pmatrix} G^R & G^K \\ 0 & G^A \end{pmatrix}.$$

Here

$$G^{R(A)}(1,2) = \mp i\theta(\pm t_1 \mp t_2)\langle \psi(1)\psi^+(2) + \psi^+(2)\psi(1)\rangle,$$
$$G^K(1,2) = -i\langle \psi(1)\psi^+(2) - \psi^+(2)\psi(1)\rangle,$$

where ψ^+ and ψ are creation and annihilation operators in the Heisenberg picture. In conditions of thermodynamic equilibrium,

$$G_\varepsilon^K = (G_\varepsilon^R - G_\varepsilon^A)[1 - 2n(\varepsilon)], \quad (2)$$

where $n(\varepsilon)$ is the Fermi distribution function.

In a microcontact, inelastic-relaxation processes are unimportant, and therefore $\hat{G}$ satisfies the equation

$$[\varepsilon + (\hbar^2/2m)\nabla^2 - U(\mathbf{r}) - e\varphi(\mathbf{r})]\hat{G}_\varepsilon(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}'), \quad (3)$$

where $\varphi(\mathbf{r})$ is the electrostatic potential and $U(\mathbf{r})$ is the random potential of the impurities; the distribution of U is assumed to be Gaussian, and

$$\overline{U(\mathbf{r})} = 0, \quad \overline{U(\mathbf{r})U(\mathbf{r}')} = 2\pi\nu\tau\delta(\mathbf{r} - \mathbf{r}').$$

Here ν is the density of one-electron states and τ is the mean free time. Calculating the average Green's functions $\overline{G}$ in the approximation $\mu\tau/\hbar \gg 1$, we obtain

$$\overline{G}_\varepsilon^{R(A)}(\mathbf{r}, \mathbf{r}') = \int (d\mathbf{p}) \exp[i\mathbf{p}(\mathbf{r} - \mathbf{r}')]\{\varepsilon - p^2/2m \pm i/2\tau\}^{-1}, \quad (4)$$

while $\overline{G}_\varepsilon^K(\mathbf{r}, \mathbf{r}')$ satisfies the diffusion equation

$$D\nabla^2 \overline{G}^K(\mathbf{r}, \mathbf{r}) = 0, \quad D = v_F^2\tau/3. \quad (5)$$

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Taking into account the boundary conditions at the banks of the contact, we find

$$\bar{G}_\varepsilon^K(\mathbf{r},\mathbf{r})|_{x=0,L}=2\pi i\nu[1-2n(\varepsilon\pm eV/2)], \tag{6}$$

where $V=\varphi(L)-\varphi(0)$ is the potential difference across the contact. The current I is expressed in terms of the Green's function be means of the formula

$$I=-i\frac{e\hbar}{2m}\int d\varepsilon\int dS(\nabla-\nabla')G_\varepsilon^K(\mathbf{r},\mathbf{r}')|_{\mathbf{r}=\mathbf{r}'}. \tag{7}$$

Averaging (7) over the distribution of the random potential and using (5) and (6), we obtain for the total current $\bar{I}$ across the contact the usual expression

$$\bar{I}=DS\nabla\int\bar{G}_\varepsilon^K(\mathbf{r},\mathbf{r})\,d\varepsilon$$

$$=(e\nu DS/L)\int_{-\infty}^{+\infty}\{n(\varepsilon-eV/2)-n(\varepsilon+eV/2)\}d\varepsilon. \tag{8}$$

The current correlation function $K(V_1,V_2)$ corresponds to the diagrams depicted in the figure. The spurs on these diagrams correspond to $\bar{G}^K(\mathbf{r},\mathbf{r})$ determined by formulas (5) and (6). The other ladders on the diagram correspond to the two-particle Green's functions

$$P_{\varepsilon_1-\varepsilon_2}(\mathbf{r},\mathbf{r}')=\overline{G_{\varepsilon_1}^R(\mathbf{r},\mathbf{r}')G_{\varepsilon_2}^A(\mathbf{r},\mathbf{r}')}. \tag{9}$$

Here the one-particle Green's functions in (9) describe the motion of an electron in different electrostatic potentials φ_1 and φ_2. In the diffusion approximation $P_\omega(\mathbf{r},\mathbf{r}')$ satisfies the equation

$$\{D\nabla^2+i\omega/\hbar+i(e/\hbar)[\varphi_1(\mathbf{r})-\varphi_2(\mathbf{r})]\}P_\omega(\mathbf{r},\mathbf{r}')=-2\pi\nu\delta(\mathbf{r}-\mathbf{r}'). \tag{10}$$

On a boundary with massive banks,

$$P_\omega(\mathbf{r},\mathbf{r}')|_{x=0,L}=0. \tag{11}$$

The sum of the diagrams a–d yields the expression

$$K(V_1,V_2)=\frac{2}{(2\pi)^4}\left(\frac{eSD}{\nu\hbar L^2}\right)^2\int_0^L\int dx_1\,dx_2\int_{-\infty}^{+\infty}\int d\varepsilon_1\,d\varepsilon_2$$

$$\times\left\{|P_{\varepsilon_1-\varepsilon_2}(x_1,x_2)|^2+\frac{1}{2}\operatorname{Re}P^2_{\varepsilon_1-\varepsilon_2}(x_1,x_2)\right\}$$

$$\times\left[n\left(\varepsilon_1-\frac{eV_1}{2}\right)-n\left(\varepsilon_1+\frac{eV_1}{2}\right)\right]$$

$$\times\left[n\left(\varepsilon_2-\frac{eV_2}{2}\right)-n\left(\varepsilon_2+\frac{eV_2}{2}\right)\right]. \tag{12}$$

For $V_{1,2}\ll V_c$, in Eq. (10) we can neglect the potential φ and the expression (12) goes over into the formula for the current correlation function in the ohmic region.[3-5,8] If $V_{1,2}\gg V_c$, ΔV, then the main contribution to the formula is given by the term $|P_{\varepsilon_1-\varepsilon_2}(x_1,x_2)|^2$ and the important region of energies is $|\varepsilon_1-\varepsilon_1|\ll eV$. Therefore, the dependence of K on V, T, and ΔV factors:

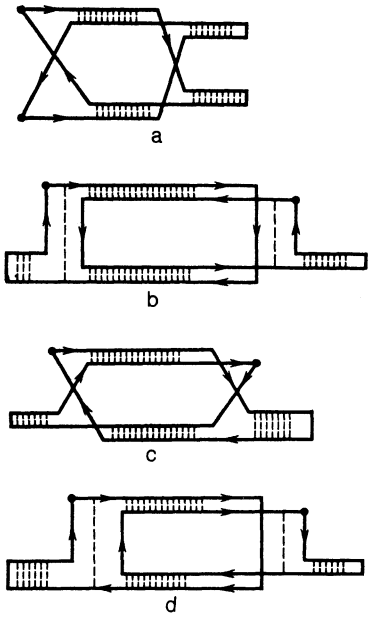

FIG. 1.

$$K(V,\Delta V,T)=\frac{4}{(2\pi)^4}\left(\frac{e^2}{\hbar}\right)^2VV_c$$

$$\times\left[\operatorname{cth}\left(\frac{eV}{T}\right)-\frac{T}{eV}\right]f\left(\frac{\Delta V}{V_c}\right). \tag{13}$$

Here the dimensionless function $f(\alpha^3)$ is expressed by the integral

$$f(\alpha^3)=\alpha^{-2}\int_{-\alpha/2}^{\alpha/2}\int dy_1\,dy_2\int_{-\infty}^{+\infty}dz\,|\Pi_z(y_1,y_2)|^2, \tag{14}$$

where $\Pi_z(y,y')$ satisfies the equation

$$\{\partial^2/\partial y^2-i(y-z)\}\Pi_z(y,y')=-2\pi\delta(y-y'). \tag{15}$$

The coefficient in Eq. (14) has been chosen so that $f(0)=1$. For $\alpha\gg1$ the important region in the integral (14) is $|y_1-z|\sim|y_2-z|\sim1\ll\alpha$. Therefore,

$$f(\Delta V/V_c)=C(V_c/\Delta V)^{1/2},\quad \Delta V\gg V_c; \tag{16}$$

$$C=\frac{V}{V}\int_{-\infty}^{+\infty}\int dy_1\,dy_2\,|\Pi_z(y_1,y_2)|^2, \tag{17}$$

where the integral (17) the function $\Pi_z(y_1,y_2)$ depends weakly on z in the interval $-\alpha/2<z<\alpha/2$. In the region of high temperatures $T\gg V_c$, V, factorization occurs irrespective of the relative magnitudes of V and ΔV. In this case we can also neglect the term with Re P^2 in (12) and rewrite this equation in the form

$$K(V_1,V_2,T)=\frac{1}{(2\pi)^4}\left(\frac{e^2}{\hbar}\right)^2V_1V_2\frac{eV_c}{T}f\left(\frac{\Delta V}{V_c}\right). \tag{18}$$

The correlator K_g of the differential conductances is connected with the function $K(V_1,V_2)$ by the relation

$$K_g(V_1, V_2) = \overline{g(V_1)g(V_2)} - \overline{g(V_1)}\,\overline{g(V_2)} = \frac{\partial^2}{\partial V_1 \partial V_2} K(V_1, V_2). \quad (19)$$

For $V \gg V_c$ the largest contribution to K_g arises upon differentiationof the function $f(\Delta V/V_c)$:

$$K_g = -\frac{4}{(2\pi)^4}\left(\frac{e^2}{\hbar}\right)^2 \frac{V}{V_c}\left[\operatorname{cth}\frac{eV}{T} - \frac{T}{eV}\right] f''\left(\frac{\Delta V}{V_c}\right). \quad (20)$$

With increase of V the correlator K_g grows, and for

$$V[\operatorname{cth}(eV/T) - T/eV] > V_c(\hbar/e^2R_0)^2 \quad (21)$$

the fluctuations of g exceed $\bar{g}$. This implies that if the condition (21) is fulfilled, then on the CVC there are parts, of width $\Delta V \sim V_c$, on which the differential resistance is negative. With increase of voltage V or temperature, inelastic processes, which have not been taken into account above, can become important. These processes are characterized by a drift time τ_{in}. The time τ_{in} is determined by the average electron energy, i.e., by the temperature or voltage. When the time τ_{in} becomes smaller than the time of flight, τ_f, τ_{in} must be taken into account in the equation for P, which takes the form

$$\{D\nabla^2 - 1/\tau_{in} + i\omega/\hbar - eEx/\hbar\} P_\omega(x, x') = -2\pi\nu\delta(x - x'). \quad (22)$$

When τ_{in} is taken into account the function $f(\Delta V/V_c)$ msut be replaced by a fucntion $\Phi(L/L_{\text{in}}, L_{\text{in}}/L_e)$, where $L_{\text{in}} = (D\tau_{\text{in}})^{1/2}$ is the diffusion length of the inelastic processes, and $L_E = (D\hbar/eE)^{1/2}$ is the field length. For large ΔV, the asymptotic form of $\Phi(\xi,\eta)$ coincides with the asymptotic form (16). However, this asymptotic form is reached only for $L_E \ll L_{\text{in}}, L$, when

$$\Delta V \gg V_{c\,max}\{1, (L/L_{in})^3\}. \quad (23)$$

In the opposite limit, then

$$\Phi(\xi, 0) = (3/\xi)(\operatorname{cth}\xi - 1/\xi). \quad (24)$$

For $\xi \ll 1$ and arbitrary $\eta = (L_{\text{in}}/L_E)^3$ the function $\Phi(\xi,\eta)$ takes the form

$$\Phi(\xi, \eta) = (3/\xi)F(\eta), \quad \eta = \Delta V(e^3V_c\tau_{in}{}^3/\hbar^3)^{1/2}, \quad (25)$$

$$F(\eta) = \begin{cases} 1, & \eta \ll 1 \\ C/3\eta^{1/3}, & \eta \gg 1 \end{cases}. \quad (26)$$

The coefficient C is defined by formula (17).

The temperature factor in (13) is changed somewhat when energy relaxation is taken into account, if $V \gg T$ and $L \gg L_{\text{in}}$. This change is connected with the change of the electron distribution function in the contact and depends on the relative magnitudes of the electron-electron collision time τ_{ee} and the electron-phonon collision time $\tau_{e,ph}$.

Thus, inelastic processes lead to weakening of the current fluctuations and to increase of the voltage scale of the CVC fluctuations. The correlator K_g of the differential conductances is then further decreased:

$$K_g = -\frac{12}{(2\pi)^4}\left(\frac{e^2}{\hbar}\right)^2 \frac{e^3VV_c{}^2\tau_{in}{}^3}{\hbar^3}\frac{L_{in}}{L}F''[\Delta V(e^3V_c\tau_{in}{}^3/\hbar^3)^{1/2}]. \quad (27)$$

Qualitatively, the form of the CVC does not depend on the explicit form of the function $K(V_1,V_2)$, but it would be interesting to find this correlation function experimentally and, in particular, to check the asymptotic formula (17). The calculation of $K(V_1,V_2)$ was carried out above by averaging over realizations of the random potential. The correlation function can also be found by measuring the CVC of the same contact in a wide range of voltages V and averaging over V for a fixed ΔV.

The shape of the contact is not important. For an arbitrary shape, in place of L one must use the characteristic size of the contact region that determines the resistance of the contact. For example, for a hole in a thin insulating layer between normal metals this size of of the order of the diameter of the hole. The assumption that the mean free path satisfies $l \ll L$ is also not a restriction. If $l \gg L$, an irregular CVC should also be observed, but the times of flight $\tau_f \sim L/V_f$ in this case are shorter and the characteristic scale $V_c \sim \hbar v_F/L$ of the fluctuations is greater than for a dirty contact.

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