The growth dynamics of a quantum crystal

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The dynamic properties of an atomically rough interface have been analyzed for three different quantum models, including a model of the interface between a quantum crystal and a superfluid Bose liquid. It is shown that the movement of the interface (growth or melting of the crystal) is inevitably accompanied by energy dissipation, and the temperature dependences of the relevant kinetic coefficient have been found. Far from the roughening transition, the interface mobility decreases with increasing temperature, but it remains finite on approaching the transition point. The absolute value of the mobility near the transition is exponentially large with respect to a parameter which is determined by the ratio of the amplitude of the zero-point oscillations of the surface to the interplanar distance, which accords with the possibility of crystallization waves propagating along faces which have Miller indices which are not small.

1. INTRODUCTION

It is well known (see, e.g., Refs. 1 and 2) that crystal growth proceeds in an appreciably different way for faces which are in atomically smooth and atomically rough states. In an atomically smooth state (characterized by a finite free energy per unit step length) growth takes place in layers through the formation of critical nuclei; the rate of growth is then exponentially small relative to the applied chemical potential difference. At the same time, an atomically rough state is characterized by the possibility of continuous growth, for which the kinetic coefficient

$$K=\lim_{\Delta\mu\to 0}(v/\Delta\mu) \tag{1}$$

(the growth coefficient) has a finite value. In Eq. (1), v is the normal velocity of the interface (relative to the crystal), while $\Delta\mu$ is the difference between the chemical potentials of the liquid and the solid phases (per atom). In this case, for small rates of interface motion the surface density of the rate of energy dissipation r is quadratic in v:

$$r=K^{-1}V_0^{-1}v^2, \tag{2}$$

where V_0 is the specific volume of the crystal (per atom). It is the value of K^{-1} which is additive, i.e., it is made up of terms corresponding to the various dissipative mechanisms.

In 1978 Andreev and Parshin proposed that in view of the large amplitude of quantum fluctuations, nearly all the faces of a ^{4}He crystal which is in equilibrium with the liquid (superfluid) phase should be in the atomically rough state, even at absolute zero.[3] Such a specific (quantum-rough) state of the surface is characterized by a strictly non-dissipative growth (melting) process of the crystal at absolute zero ($K=\infty$), and this enabled Andreev and Parshin[3] to predict the possibility of the propagation along the interface of the phases of ^{4}He of crystallization waves, analogous in their properties to capillary waves on the free surface of a liquid. Marchenko and Parshin[4] predicted another experimentally observed consequence of the non-dissipative melting, the anomalous temperature dependence of the Kapitza jump at the interface in ^{4}He.

It was proposed both by Andreev and Parshin[3] and in subsequent work on the dynamics of the surface of a quantum crystal,[5,6] that even at finite temperature a pure non-dissipative melting (growth) of a crystal is possible, where the gases of excitations on both sides of the interface move with the same velocity as the boundary itself. The dissipation at the boundary is then considered to be produced by the absence of complete equilibrium between the gases of excitations and the interface, which will unavoidably occur for processes characterized by a finite frequency and (or) by a finite wave vector. It turns out that for frequencies exceeding some characteristic inverse time, the dissipation at the boundary can also be described by Eqs. (1) and (2), with a frequency-independent coefficient K. As was noted by Andreev and Knizhnik,[5] such a consideration assumes the neglect of umklapp processes in the solid, the inclusion of which into the discussion leads to both the motion of the phonon gas in the crystal and the motion of the interface being dissipative. Their contribution is, however, exponentially small in T_D/T (T_D is the Debye temperature of the ^{4}He crystal), and can be omitted for the temperatures of interest to us.

So far, three transitions have been recorded on the surfaces of a ^{4}He crystal in the atomically smooth state, taking place at temperatures of 1.2, 0.9, and 0.35 K (Refs. 7–11). No new transitions are observed on lowering the temperature further to 0.07 K (Ref. 11). Experimental confirmation has been obtained for both the existence of crystallization waves[9] and for an anomalous Kapitza jump.[12–14] The crystallization wave spectrum coincides with that predicted,[3] while the temperature dependence of their attenuation agrees with results of Refs. 3, 5, and 6. The collection of experimental facts does not, thus, contradict the hypothesis of quantum roughness.

At the same time, from the theoretical viewpoint, both considerations of the general characteristics,[15] and investi-

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gations of a number of concrete microscopic models of a quantum interface,[16–18] indicate that at absolute zero any face should be in the atomically smooth state, while the roughening transition temperature decreases only insignificantly when quantum effects are taken into account. It should be noted that Bol'shov *et al.*[19] discussed the possibility of zero-point roughness of weakly inclined faces of a quantum crystal. However, as V. L. Pokrovskiĭ explained to the present author, the confirmation in that work of the possibility of zero-point roughness must be understood in a somewhat different way, namely: although every face with whole-number indices will be in the atomically smooth state at absolute zero, the total area of all the plane sections of the faceting can be less than the total area of the surface, and a finite area will have to be used on atomically rough states (corresponding to irrational indices).

As we observed earlier,[18] the essential transition of a face into the atomically smooth state on lowering the temperature does not contradict the possibility of crystallization waves propagating at temperatures exceeding the transition temperature T_R. A stricter basis for this assertion requires studies of microscopic models of an interface in the context of verifying the possibility of its dissipation-free (quasi-dissipation-free) motion, which is one of the aims of the present work.

We consider three different models of a quantum interface. Two of them [with discrete (Sec. 2) and continuous (Sec. 3) variables] consist of a description only of the interface itself, and are applicable for the description of quantum interfaces between media with different density, i.e., of domain walls in magnetic substances, planar defects in quantum crystals, etc. In Sec. 4 a model of an interface between a quantum crystal and a superfluid Bose liquid is considered, proposed by Iodanskiĭ and Korshunov.[18]

The quantity directly studied is the temperature-dependent kinetic coefficient K^{-1} at zero frequency, assumed elsewhere[3,5,6] to be equal to zero. The suggestion of the impossibility of a strictly non-dissipative motion of an interface in ^{4}He was proposed earlier by Castaing,[20] but in an entirely different context, namely in connection with some special properties of the interaction between rotons and moving interfaces. Nevertheless, the inverse growth coefficient is exponentially small for large amplitudes of quantum tunneling, which provides an explanation for the possibility of crystallization waves propagating along faces with not too small Miller indices, as well as other phenomena which are regarded below as a manifestation of non-dissipative melting. The possibilities of an experimental observation of the predicted temperature dependence $K^{-1}(T)$ are discussed in Sec. 5.

2. THE SIMPLEST MODEL OF A QUANTUM INTERFACE

A) Formulation of the model

The simplest of the models of a quantum interface which have been studied is described by the Hamiltonian[16,17]

$$\hat{H}=\hat{U}+\hat{W}=\frac{J}{2}\sum_{(\mathrm{jj}')}(n_{\mathrm{j}}-n_{\mathrm{j}'})^2-\frac{Y}{2}\sum_{\mathrm{j}}(\hat{a}_{\mathrm{j}}^{+}+\hat{a}_{\mathrm{j}}^{-}). \tag{3}$$

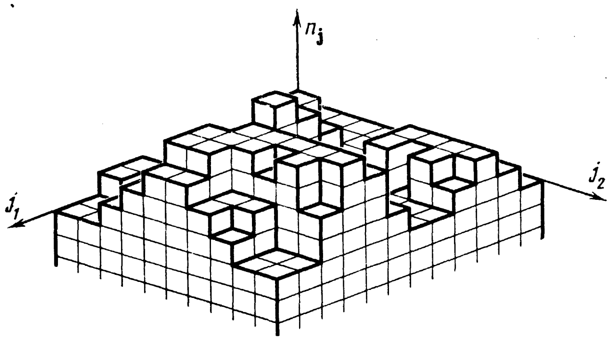

FIG. 1. Schematic representation of the basal plane of a crystal with a simple cubic lattice and the presence of developed surface fluctuations (atomically rough state). In the SOS approximation it is assumed that the configuration of the surface can be described with the help of a collection of integral variables n_j, which indicate the departure of the surface from some fixed position, i.e., the height of the jth "block of cubes."

Here the integer variables n_j, defined at points **j** of a regular plane lattice, indicate the distance between the surface (in atomic units) and some fixed position (Fig. 1), while the operators $\hat{a}_j^+$ and $\hat{a}_j^-$ describe tunneling processes which take place with a change in $n\mathbf{j}$ by unity and only have the matrix elements $\langle n_j \pm 1|\hat{a}_j^{\pm}|n_j\rangle = 1$ nonzero. The potential energy of the interface is represented in the first term of Eq. (3), with the summation called out over pairs of nearest neighbors, indicated by $(\mathbf{jj}')$.

In the model of Eq. (3), as in other models discussed below, the interface is atomically smooth at absolute zero for any ratio between the parameters Y and J. The transition temperature into the atomically rough state (the roughening temperature, also called the faceting transition), does fall with increasing Y but only insignificantly, and for $Y/J \to \infty$ tends to $(2/\pi)J$.[17] As in the case of a classical discrete Gaussian model ($Y \equiv 0$), the phase transition in model (3) belongs to the same universality class as the phase transition in a classical two-dimensional XY model.[21–23] The influence of quantum effects only shows up in that a large value of the ratio Y/J leads to an exponentially small equilibrium size of a plane faceting section corresponding to a given face, and to a temperature region of critical behavior of thermodynamic quantities. The latter fact means that one can expect everywhere, except in an exponentially small vicinity of the transition point, satisfaction of the dependences predicted by the self-consistent approximation[16,17] (but not by the phenomenological theory of the faceting transition[24]). In particular, one should expect that for $T > T_R$, the surface rigidity $\tilde{\alpha} = \text{const}$. Results of experimental investigations of the shape of ^{4}He crystals in the region of faceting transitions[25–27] also provide evidence of the absence of a temperature dependence of $\tilde{\alpha}$ for $T > T_R$.

B) The mechanism of dissipation

The dynamical properties of a one-dimensional analog of model (3) have been studied for $Y \gtrsim J$ by, in particular, Korshunov[28] using the instanton technique in real time, introduced by Chakravarty and Leggett[29] for studying tunneling processes in quantum crystals (see also Ref. 30). It was shown that macroscopic motion in such a system is dissipative, and the dependence of the frictional force on velocity and temperature was found. At finite temperature and at low velocity the frictional force is proportional to the velocity, meaning a finite growth coefficient. If the same condition, $Y \gtrsim J$, is satisfied, the analogous approach can be extended to the two-dimensional model (3), but only for temperatures corresponding to the atomically rough phase ($T > T_R$).

In order to explain the dissipation mechanism, the second term of Eq. (3) must be rewritten in the form

$$\hat{W} = -Y \sum_{j} \cos \varphi_j, \tag{4}$$

where φ_j is the phase change canonically conjugate to an integral variable n_j (i.e., $n_j \equiv -i\partial/\partial\varphi_j$).[16,17]

Although the classical equation of motion for φ_j is

$$\hbar^2 \partial^2 \varphi_j / \partial t^2 = JY \Delta_L (\sin \varphi_j)$$

(where Δ_L is the lattice Laplacian), apart from a solution corresponding to a stationary interface (all $\varphi_j = 0$), there are also stationary solutions $\varphi_j = \Phi = \text{const}$, corresponding to its motion with velocity $v = (Yd/\hbar)\sin \Phi$, and tunneling of the variables φ_j into neighboring minima of the periodic potential (4) can lead to a change in the mean velocity

$$\langle v \rangle = (Yd/N\hbar) \sum_{j} \langle \sin \varphi_j \rangle.$$

Here N is the total number of lattice sites in the plane of the face, while d is the interplanar distance.

Because of the necessity of satisfying the conservation law

$$\sum_{j} \frac{\partial}{\partial t} \varphi_j = 0,$$

the existence of which is due to the invariance of the Hamiltonian (3) to a simultaneous shift of all the variables n_j, a change in one of the quantities φ_j by $\pm 2\pi$ arising on tunneling into a neighboring minimum, must be compensated by a change in each of them by $\pm 2\pi/N$. A change in the kinetic (in n-representation) energy of motion of the interface as a whole by $\Delta W = \mp 2\pi Y \sin \Phi$ and a change in v by $\mp (2\pi Yd/N\hbar) \cos \Phi$ must then take place.[28] In general it is not the amplitude Y itself which must enter into these formulae, but its value renormalized by fluctuations $\tilde{Y}$. We neglect this difference (of little significance for $Y \gg J$, T), all the more so because for the relation $\Delta W = \mp 2\pi\hbar v/d$ of interest to us, these renormalizations are reduced.

The probability of tunneling (per lattice site) ν, is naturally different for processes taking place with an increase and with a decrease in W, and for $\nu_0 \equiv \nu(\Phi = 0) \neq 0$ for small v linear in ΔW:

$$\nu = \nu_0 (1 - \Delta W / 2T) \tag{5}$$

(here and everywhere in what follows, the temperature T is considered to be expressed in energy units). Equation (5) is a consequence of the principle of detailed balancing. The rate of dissipation of kinetic energy is then, as should be expected, quadratic in v:

$$\frac{1}{N} \frac{\partial W}{\partial t} = -\nu_0 \frac{(\Delta W)^2}{T} = -\left(\frac{2\pi\hbar}{d} \right)^2 \frac{\nu_0}{T} v^2.$$

Comparison with Eq. (2) shows that

$$K^{-1} = \left(\frac{2\pi\hbar}{da} \right)^2 V_0 \frac{\nu_0}{T} = \frac{(2\pi\hbar)^2 \nu_0}{dT}, \tag{6}$$

where a is the lattice constant (in the plane of the face), and ν_0, according to Wolf *et al.*,[28] can be expressed in terms of instantons in real time:

$$\nu_0 = \Omega_c^2 \sum_{\mathbf{R}} \int_{-i\infty}^{i\infty} d\tau \exp[-S_2^E(\mathbf{R}, \tau)]. \tag{7}$$

Here $S_2^E(\mathbf{R},\tau)$ is the action on the classical trajectory, cyclic in imaginary time, on which the variables φ_j and $\varphi_1(\mathbf{j} - \mathbf{l} = \mathbf{R})$ pass through maxima of the potential $-Y \cos \varphi$ at moments in time separated by the interval τ. In the atomically rough phase, such saddle points (instantons) form bound pairs on typical trajectories which provide the main contribution to the determining functional sum over states integral.

To each tunneling act (in real time) corresponds a pair of instantons. If the pairs are of small dimensions and are far apart (in space-time), then the separate tunneling acts take place practically independently of one another, and the tunneling process itself is classical and incoherent (a pure exponential relaxation).[29] It is only under these conditions [corresponding in the case of model (3) to the atomically rough phase] that it is possible to use Eqs. (6) and (7) where $S_2^E(\mathbf{R},\tau)$ can be considered to be equal to[17]:

$$S_2^E(\mathbf{R}, \tau) = (2\pi)^2 \int \frac{d\mathbf{k}^2}{(2\pi)^2} \beta^{-1} \sum_{\omega_s} \{1 - \exp[i(\mathbf{kR} - \omega_s \tau)]\}$$
$$\times G_0(\mathbf{k}, \omega_s)$$
$$= 2\pi^2 Y \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\operatorname{ch}(\beta\omega_{\mathbf{k}}/2) - \cos(\mathbf{kR}) \operatorname{ch}[(\beta/2 - \tau)\omega_{\mathbf{k}}]}{\hbar\omega_{\mathbf{k}} \operatorname{sh}(\beta\omega_{\mathbf{k}}/2)}. \tag{8}$$

Here ω_s takes the value $2\pi\beta^{-1}s$ (s is an integer), $\beta = \hbar/T$, the unperturbed Green's function

$$G_0(\mathbf{k}, \omega_s) = (Y/\hbar)(\omega_{\mathbf{k}}^2 + \omega_s^2)^{-1} \tag{9}$$

is the correlation function for the variables n_j, calculated in the harmonic approximation without taking account of the discreteness, and ω_k is the vibration spectrum in this same approximation. For a square lattice

$$\omega_{\mathbf{k}}^2 = (JY/\hbar^2)[4\sin^2(k_x/2) + 4\sin^2(k_y/2)] \approx (\Omega_k)^2;$$
$$\Omega^2 = JY/\hbar^2. \tag{10}$$

In Eqs. (8)–(10) and what follows, the quasimomenta $\mathbf{k} = (k_x, k_y)$ are expressed in inverse lattice distances.

Extending Eq. (8) analytically to imaginary values of the imaginary time (i.e., to real times) and bearing in mind that the main contribution to the integral comes from the region of small quasimomenta, we obtain that to first order

$$\operatorname{Re} S_2^E(\mathbf{R}, i\tau) \propto (2\pi T/J) \ln[(R^2+\Omega^2\tau^2)^{1/2}],$$

so that the condition for the mutual independence of different tunneling acts is the better satisfied the higher the temperature T compared with the roughening transition temperature $T_R \approx (2\pi)J$.

Insofar as the variables φ_j relative to the variables n_j play the role of quasimomenta, the tunneling processes responsible for dissipation are no other than umklapp processes.

C) Temperature dependence of the growth coefficient

The use of the concept of instantons and also of structures localized in time, corresponding to a strong departure of any of the quantities φ_j from the equilibrium (quasi-equilibrium) values and the saddle point through the maxima of the periodic potential of Eq. 4, is only possible if the conditions $J \ll Y$, $T \ll (YJ)^{1/2}$ are satisfied, which ensure a small value for the dimensions of an instanton compared with the inverse temperature β, and small fluctuations of φ compared with unity. As we have already verified, the calculation of K^{-1} in this case comes down to a calculation of ν_0 [see Eq. (6)]. We assume in what follows that $J \ll Y$.

For $T \ll J(Y/J)^{1/3}$, on substituting expression (8) into Eq. (7), the summation over $\mathbf{R}$ in Eq. (7) can be replaced by an integral. At the same time, for $T \gg J$, after shifting the contour of integration by τ parallel to itself, in order that it should pass through the point $\tau = \beta/2$ (where $S_2^E(\mathbf{R},\tau)$ has a maximum for real τ), all three integrations can be performed by the method of steepest descents. We thus find for $J \ll T \ll J(Y/J)^{1/3}$

$$K^{-1} = A(T)\exp[-B(T)], \tag{11}$$

where

$$A(T) \propto J^3 Y^{5/2} T^{-11/2}, \tag{12}$$

$$B(T) = 2S_1 - 4\pi(\ln 2)(T/J) + \dots . \tag{13}$$

Here the constant

$$S_1 = 2\pi^2 \int \frac{d^2\mathbf{k}}{(2\pi)^2} \int \frac{d\omega}{2\pi} G_0(\mathbf{k}, \omega) \propto \left(\frac{Y}{J}\right)^{1/2} \tag{14}$$

is the action corresponding to a single instanton at absolute zero, while only the first two terms in the expansion in terms of $T/(Y/J)^{1/2}$ are written out in the expression for B.

If the ratio Y/J is sufficiently great, then the temperature interval $J(Y/T)^{1/3} \ll T \ll J(Y/J)^{1/2}$ can also be considered, in which in Eq. (7) only one term in the sum over $\mathbf{R}$ (with $\mathbf{R} = 0$) can be preserved, since the rest are exponentially small. Integration over τ can, as in the previous case, be performed by the method of steepest descents; unlike the dependence of Eqs. (11)–(13), this leads to a change in the temperature dependence of the pre-exponential:

$$A(T) \infty J Y^{3/2} T^{-5/2}. \tag{15}$$

We have thus verified that for $T \gg J(Y/J)^{1/3}$, the tunneling probability ν_0 (which gives the growth coefficient K) is real, to exponential accuracy, over a cyclic closed (in imaginary time) classical trajectory, over which one of the variables (for example, φ_{j_0}) passes twice through a maximum of the periodic potential $-Y\cos\varphi_{j_0}$ (in different directions), see Fig. 2. The decisive role of a single classical trajectory and its immediate vicinity will be preserved over the whole range $J(Y/J)^{1/3} \ll T \ll Y$ where the fluctuations in φ are small. It is then only for $J(Y/J)^{1/3} \ll T \ll J(Y/J)^{1/2}$ does φ_{j_0} lie over a large part of this trajectory in the immediate vicinity of one or another minimum in $-Y\cos\varphi_{j_0}$, which enables Eq. (8) to be used to evaluate ν_0 for the action over a two-instanton trajectory.

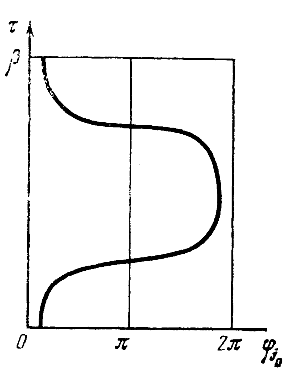

FIG. 2. An extremal (saddle) trajectory determining the probability of tunneling for $J \ll T \lesssim (JY)^{1/2}$.

As T approaches $J(Y/J)^{1/2}$, even the minimum value of φ_{j_0} over the trajectory of interest to us becomes no longer small (of the order of unity), so that such an approach is already inapplicable. For $T \ll Y$, however, the effective action for the variable φ_{j_0} can be used to study the character of the extremal trajectory, obtained from the total action of model (3) after substituting $-Y\cos\varphi_j \to -Y[1-\varphi_j^2/2]$ for all the variables besides φ_{j_0}, and performing Gaussian integrations over these variables in the sum over states. This procedure was carried out by Korshunov[28] in studying the one-dimensional analog of the Hamiltonian of Eq. (3). It leads to an effective action for $\varphi \equiv \varphi_{j_0}$:

$$S_{eff}^E = -\frac{Y}{\hbar} \int_0^\beta d\tau \cos\varphi(\tau) + \frac{1}{2\beta} \sum_{\omega_s} G_{eff}^{-1}(\omega_s) \varphi_{\omega_s} \varphi_{-\omega_s}, \tag{16}$$

with the propagator, not explicitly dependent on temperature,

$$G_{eff}(\omega_s) = \frac{\hbar}{Y} \left\{ \left[\int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\omega_\mathbf{k}^2}{\omega_\mathbf{k}^2 + \omega_s^2} \right]^{-1} - 1 \right\}. \tag{17}$$

For ω_k given by Eq. (10) we have

$$\begin{aligned} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\omega_\mathbf{k}^2}{\omega_\mathbf{k}^2+\omega_s^2} = 1 - \omega_s^2 \Big\{ & \frac{1}{2} [(\omega_s^2+4\Omega^2)^2 - 8\Omega^4 \\ & + (\omega_s^2+4\Omega^2)|\omega_s|(\omega_s^2+8\Omega^2)^{1/2}] \Big\}^{-1/2} \\ \times \frac{2}{\pi} K & \left[\frac{(\omega_s^2+4\Omega^2)^2 - 8\Omega^4 - (\omega_s^2+4\Omega^2)|\omega_s|(\omega_s^2+8\Omega^2)^{1/2}}{(\omega_s^2+4\Omega^2)^2 - 8\Omega^4 + (\omega_s^2+4\Omega^2)|\omega_s|(\omega_s^2+8\Omega^2)^{1/2}} \right], \end{aligned} \tag{18}$$

where

$$K(m) = \int_0^{\pi/2} d\theta (1 - m\sin^2\theta)^{-1/2}$$

is the complete elliptical integral of first order, and $\Omega = (JY)^{1/2}/\hbar$.

On rise of temperature, a transition must occur from the regime of quantum (although also incoherent) tunneling, corresponding to an extremal trajectory, shown in Fig. 2, to a thermal activation regime corresponding to the trajectory $\varphi = \pi$. Such a transition to a quantum system with non-local dissipation was first studied by Larkin and Ovchinnikov.[31]

In the activated tunneling regime, the action on the extremal trajectory is equal to $2Y/T$. The temperature T_0 of the transition between the two different regimes, when fluctuations are not taken into account, can be found as the temperature at which a second negative eigenvalue appears in the quadratic form, obtained by the expansion of the action of Eq. (16) near the stationary trajectory $\varphi = \pi$.[31] Since $G_{\rm eff}(\omega_s)$ depends monotonically on ω_s, this corresponds to the satisfaction of the equation

$$G_{eff}^{-1}(2\pi\beta^{-1}) = Y/\hbar. \tag{19}$$

We find from Eqs. (17)–(19) $T_0 \approx 0.2935\hbar\Omega$.

For $T_0 < T \ll Y$, the exponent $B(T)$ in the relation of Eq. (12) has the traditional activation form:

$$B(T) = 2Y/T.$$

Taking account of fluctuations leads to a smoothing-out of the singularities in $K^{-1}(T)$ at the point T_0 and to a smooth transition between the quantum and thermal tunneling regimes.

We have thus demonstrated that for $T_R \ll T \ll Y$, the tunneling probability ν_0, and with it the inverse growth coefficient K^{-1} grow with increasing temperature. Comparison of the pre-exponential and exponential factors in Eqs. (11)–(13) indicates that the function $K^{-1}(T)$ can have a minimum in the immediate vicinity of T_R. We shall now consider the question of the behavior of K for $T \to T_R + 0$.

As we note earlier, the summation over **R** in Eq. (7) for $T \ll J(Y/J)^{1/3}$ can be replaced by integration. The form of the asymptotic dependence of the second term in the expression

$$S_2^E\left(\mathbf{R}, \frac{\beta}{2} + i\tau\right) = \frac{2\pi^2 Y}{\hbar}\left[\int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\operatorname{th}(\beta\omega_\mathbf{k}/2)}{\omega_\mathbf{k}} + \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{1 - \cos\mathbf{kR}\cos\omega_\mathbf{k}\tau}{\omega_\mathbf{k}\operatorname{sh}(\beta\omega_\mathbf{k}/2)}\right] \tag{20}$$

on **R** and τ (which determines the convergence or divergence of the corresponding integrals) will be evaluated by substituting into Eq. (20) $\omega_k = \Omega k$ and replacing $\sinh(\beta\omega_k/2)$ by $(\beta\omega_k/2)\exp(\beta\omega_k/2)$, which preserves the exponential character of the cut-off of the integral by thermal pulses. By performing the integration over infinite limits, we obtain

$$S_2^E\left(\mathbf{R}, \frac{\beta}{2} + i\tau\right) \approx S_2^E\left(0, \frac{\beta}{2}\right) + \frac{\pi T}{J} \times \left\{\ln \frac{1 + i2\tau/\beta + [(1 + i2\tau/\beta)^2 + (2\mathbf{R}/\beta\Omega)^2]^{1/2}}{2} + \text{c.c.}\right\}. \tag{21}$$

For $S_2^E(\mathbf{R}, \beta/2 + i\tau)$, given by Eq. (21), the expression of Eq. (7) is convergent for $T > (3/2\pi)J$, while the roughness transition temperature to a self-consistent approximation is $(2/\pi)J$ (Ref. 17). The value of K^{-1} thus remains finite up to $T = T_R$ [for $T < T_R$, Eq. (7) is inapplicable and $K^{-1} = \infty$].

On taking renormalization into account, the value of T shifts relative to $(2/\pi)J$ (by less, the greater the ratio Y/J, see Ref. 17). The interaction of instantons then also changes, which manifests itself in Eq. (21) which is of interest to us to first order in the replacement of the coefficient J by its renormalized value $\tilde{J}(\mathbf{R},\tau)$. Since at the transition point the values of $\tilde{J} \equiv \tilde{J}(\infty)$ are related by the universal relation $(2/\pi)\tilde{J} = T_R$ (Refs. 15 and 17), the value of $K^{-1}(T_R + 0)$ remains constant as before. In our formulation K^{-1} is a function of $\tilde{J}$, having a square root singularity at the transition point; the quantity K^{-1} must, therefore, also have an analogous square root singularity:

$$K^{-1}(T) \approx K^{-1}(T_R + 0) - \gamma(T - T_R)^{1/2}.$$

It is interesting that such a law of critical behavior of the mobility of an interface is also obtained for a purely classical formulation of the dynamical problem within the framework of the Langevin equation for the sine-Gordon Hamiltonian.[32]

D) The correlation function

The value of the growth coefficient K can be found not only with the help of an actual evaluation of the tunneling processes responsible for dissipation, but also by using a formal calculation of the temperature (Matsubara) Green's function, which will be carried out here.

Neglecting the discreteness of n_j (i.e., the existence of instantons), the correlation function for the variables n_j in Euclidian space-time (coiled into a cylinder for minimum time) is given by Eq. (9) as

$$\tilde{G}_0(X_1 X_2) \equiv \langle n_{\mathbf{R}_1}(\tau_1)\, n_{\mathbf{R}_2}(\tau_2)\rangle = \int \frac{d^2\mathbf{k}}{(2\pi)^2}\, \beta^{-1} \sum_{\omega_s} G_0(\mathbf{k}, \omega_s) \times \exp\{i[\mathbf{k}(\mathbf{R}_1 - \mathbf{R}_2) - \omega_s(\tau_1 - \tau_2)]\} = \tilde{G}_0(X_1 - X_2), \tag{22}$$

where $X = (\mathbf{R}, \boldsymbol{\tau})$.

When studying the atomically rough phase ($T > T_R$), we must take account of corrections to Eq. (22) which arise when bound instanton pairs are taken into account. Since for a single instanton placed at the point X_3, the distribution of the n field far from X_3 is $\pm 2\pi i\tilde{G}_0(X_1 - X_3)$ (Ref. 17), for pairs of instantons of opposite sign positioned at the points X_3 and X_4, it will be equal to

$$\delta n(X_1) = \pm 2\pi i[\tilde{G}_0(X_1 - X_3) - \tilde{G}_0(X_1 - X_4)].$$

Averaging the product $\delta n(X_1)\delta n(X_2)$ by the positions of the instantons X_3 and X_4 which go into the pair with the weight

$$w(X_3; X_4) = \Omega_c^2 \exp[-S_2^E(\mathbf{R}_3 - \mathbf{R}_4;\ \tau_3 - \tau_4)] = \Omega_c^2 \exp\{-4\pi^2[\tilde{G}_0(0) - \tilde{G}_0(X_3 - X_4)]\} \tag{23}$$

and going over to the Fourier representation, we find that the correction to $G_0(\mathbf{k}, \omega_s)$ in the approximation linear in the concentration of instanton pairs is

$$\delta G(\mathbf{k},\omega_s)=-G_0^2(\mathbf{k},\omega_s)\Sigma(\mathbf{k},\omega_s),$$

where

$$\Sigma(\mathbf{k},\omega_s)=8\pi^2\Omega_c^2\sum_{\mathbf{R}}\int_0^\beta d\tau\{1-\exp[i(\mathbf{kR}-\omega_s\tau)]\}$$
$$\times\exp[-S_2^E(\mathbf{R},\tau)]. \qquad (24)$$

This result corresponds to the taking into account of the corrections from any number of instanton pairs on neglecting the interaction of the pairs among themselves. On taking account of the lowest non-vanishing order of interaction of instantons going into different pairs, we obtain after summation of all terms in the series (according to the number of pairs)

$$G^{-1}(\mathbf{k},\omega_s)=G_0^{-1}(\mathbf{k},\omega_s)+\Sigma(\mathbf{k},\omega_s), \qquad (25)$$

where $\Sigma(\mathbf{k},\omega_s)$ is given by the same expression, Eq. (24).

Eqs. (24) and (25) already contain the information necessary to us.

The non-dimensional inverse interface mobility, η, connected with K by the relation

$$K^{-1}=(\hbar/d)\eta, \qquad (26)$$

can be found as a coefficient for ω_s in the Taylor expansion of the corresponding analytic continuation of $G^{-1}(0,\omega_s)$ to continuous frequencies (cf. Ref. 33). It is evident that $G^{-1}(0,\omega_s)$ does not give a contribution to this coefficient.

Since the analytic continuation of the function

$$\tilde{w}(\tau)\equiv\sum_{\mathbf{R}}w(\mathbf{R},\tau;0,0)$$

(see Kosterlitz[23]) falls rapidly (in a stepwise fashion) at complex frequencies $\tilde{f}(t)\equiv\tilde{W}(\beta/2+it)$ for $t\to+\infty$, it can be represented in the form of a Fourier integral:

$$\tilde{f}(t)=\int\frac{d\omega}{2\pi}f(\omega)e^{-i\omega t},$$

where $f(\omega)$ is a real and even function, falling exponentially for large ω. We then have for the Fourier components of $\tilde{\omega}(\tau)$, calculated for the interval $(0,\beta)$

$$w(\omega_s)=\int_0^\beta d\tau\,\tilde{w}(\tau)\exp(i\omega_s\tau)$$
$$=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{2\omega}{\omega^2+\omega_s^2}\operatorname{sh}\left(\frac{\beta\omega}{2}\right)f(\omega). \qquad (27)$$

The integral (27) can be used for the continuation of the function $w(\omega_s)$ to complex frequencies. For $\operatorname{Re}\omega_s>0$, the function (27) is analytic in ω_s and its modulus is a maximum for $\omega_s\to 0$ (where it is nevertheless exponentially small in Y/J). As $|\omega_s|$ increases, the absolute value of $w(\omega_s)$ falls rapidly, from which it follows that the function determined by the present analytic continuation

$$G^{-1}(0,\omega_s)=G_0^{-1}(0,\omega_s)+8\pi^2[w(0)-w(\omega_s)] \qquad (28)$$

will not have zeros in the right-hand half-plane. The analytic properties of function (28) constructed in this way indicate that the delayed and advanced Green's functions are expressed through it as

$$G^{R(A)}(0,\omega_s)=G(0,\pm i\omega_s).$$

Since for $\omega_s>0$

$$w(0)-w(\omega_s)=\omega_s^2\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\operatorname{sh}(\beta\omega/2)}{(\omega/2)(\omega^2+\omega_s^2)}f(\omega)\approx\frac{\beta}{2}f(0)\omega_s,$$

we will then have for the dimensionless mobility η

$$\eta=(2\pi)^2\beta\int_{-i\infty}^{i\infty}d\tau\,\tilde{w}(\tau). \qquad (29)$$

Equations (26) and (27) give for K^{-1} the same expression as Eqs. (6) and (7) which we used before. On the whole, Eqs. (24), and (25) provide a greater possibility for obtaining information about the properties of an interface than the use of the earlier approach, since in principle it enables the dependence of the dissipation coefficient on frequency and wavelength to be determined, which can be important when going outside the limits of the hydrodynamic approximation.

3. QUANTUM SINE-GORDON MODEL

The thermodynamic properties of an interface between the phases of ^{4}He have been studied in a number of works[15,16,34] by using a two-dimensional sine-Gordon quantum model (SG model) with the Hamiltonian

$$\hat{H}=\frac{J}{2}\sum_{(jj')}(n_j-n_{j'})^2+\sum_j\left[\frac{Y}{2}\hat{p}_j^2-z\cos 2\pi n_j\right]. \qquad (30)$$

The model (30) has the same symmetry as model (3) considered earlier, namely invariance to a simultaneous shift in all the n_j by an integer. Unlike in Eq. (3), the variables n_j in Eq. (30) take on continuous values, in accordance with which the conjugate variables p_j ($\hat{p}_j\equiv-i\partial/\partial n_j$) have the meaning of ordinary momenta (and not quasi-momenta), i.e., take on arbitrary real values. The last term in Eq. (30) makes integer values of n_j preferable, thus imitating the influence of the periodic structure of the crystal. It follows from the equations of motion that for $z\neq 0$, the value of $\Sigma_j\,p_j$ is no longer conserved, i.e., the addition of the last term in Eq. (30) appreciably influences the law of the macroscopic motion of the interface.

The renormalization group equations for a classical two-dimensional SG model were obtained by several authors,[35] who showed that they are isomorphous with the renormalization group equations for a two-dimensional XY model.[23] This indicates that both models belong to one and the same class of universality, which also includes both a discrete Gaussian model and its quantum generalization of Eq. (3). A generalization of the $2D$ SG model to the quantum case does not lead to a change in its critical properties, but enables one to study the dynamical properties of the system, while staying within the framework of the Hamiltonian of Eq. (30) and not resorting to additional assumptions. Models (3) and (30) are very similar in their properties. If the last term in Eq. (30) is taken into account with the help of perturbation theory for the representation of functional integration, then one can go over to the instanton gas, analogous to that obtained when studying model (3), with the difference only that the chemical activity of instantons in the present case is proportional to z. It is more convenient to evaluate K^{-1} from the correlation function, as was done in Sec. 2d.

A diagrammatic technique for writing out any terms of the perturbation-theory series in z for a two particle correlation function of a one-dimensional quantum $(1+1)$ SG model at absolute zero, was developed by Amit *et al.*[36] The results can by trivial means be rewritten for the case of a two-dimensional quantum SG model at finite temperature. The particle summation of the perturbation theory series enables a self-consistent equation to be written for the correlation function with the required order of accuracy. The self-consistent first-approximation equation[16,17]

$$G^{-1}(\mathbf{k},\ \omega_s)=G_0^{-1}(\mathbf{k},\ \omega_s)+4\pi^2 z\exp\left[-2\pi^2\tilde{G}(0)\right] \qquad (31)$$

enables the existence of the roughening transition to be revealed, but does not allow the properties of the atomically rough phase to be studied since it leads at $T>T_R$, independently of T, to $G(\mathbf{k},\omega_s)=G_0(\mathbf{k},\omega_s)$.

The self-consistent second-approximation equation, which differs from Eq. (31) by the addition to the right-hand side of an additional term of the form

$$(4\pi^2 z)^2\exp[-4\pi^2\tilde{G}(0)]$$
$$\times\sum_{\mathbf{R}}\int_0^\beta d\tau\{[\operatorname{ch}(4\pi^2\tilde{G}(\mathbf{R},\tau))-8\pi^4\tilde{G}(\mathbf{R},\tau)-1]$$
$$-[\operatorname{sh}(4\pi^2\tilde{G}(\mathbf{R},\tau))-4\pi^2\tilde{G}(\mathbf{R},\tau)]\exp[i(-\mathbf{kR}+\omega_s\tau)]\},$$

leads, for $T>T_R$ ($\tilde{G}(0)=\infty$) to

$$G^{-1}(\mathbf{k},\omega_s)=G_0^{-1}(\mathbf{k},\omega_s)$$
$$+8\pi^4 z^2\sum_{R}\int_0^\beta d\tau\{1-\exp[i(-\mathbf{kR}+\omega_s\tau)]\}$$
$$\times\exp\{-4\pi^2[\tilde{G}(0)-\tilde{G}(\mathbf{R},\tau)]\}. \qquad (32)$$

Equation (32) is the main result of this section, differing from Eq. (25) in which (24) is substituted only by the factor in front of the exponential and by the fact that the complete and not the bare Green's function appears under the exponential. The latter fact, corresponding to taking partial account of the renormalization of the interaction of instantons, is clearly immaterial for $T\gg T_R$. This indicates that the results obtained in Sec. 2C for the temperature range $J\ll T\ll J(Y/J)^{1/2}$ will also be valid for model (3), with an accuracy up to a temperature-independent factor proportional to z^2. In fact, we will have for $J\ll T\ll J(Y/J)^{1/3}$

$$A(T)=\frac{8\pi^4}{[14\zeta(3)]^{3/2}}\frac{\hbar^2}{d}\frac{z^2J^{5/2}Y}{T^{11/2}},$$

and for $J(Y/J)^{1/3}\ll T\ll J(Y/J)^{1/2}$

$$A(T)=\frac{8\pi^4}{[14\zeta(3)]^{1/2}}\frac{\hbar^2}{d}\frac{z^2J^{1/2}}{T^{5/2}}$$

with the same exponent as Eq. (13). One can hope that the substitution of $\tilde{G}_0$ by $\tilde{G}$ does not lead to the divergence of K^{-1} for $T_R\to T+0$.

Equation (32) is also applicable for model (30) in the temperature interval $J(Y/J)^{1/2}\ll T\ll Y$ and leads to

$$A(T)=4\pi^{7/2}\frac{\hbar^2}{d}\frac{z^2}{Y^{1/2}T^{3/2}}, \qquad (33)$$

$$B(T)=\frac{\pi^2}{2}\frac{Y}{T}. \qquad (34)$$

At higher temperatures, as also for $T\to T_R+0$, the instanton gas can no longer be considered rarefied. It is interesting that relation (34) has a form corresponding to thermal (activated) tunneling, although there is no barrier of height $(\pi^2/2)Y$ in the system.

It follows from the analysis in this section that the temperature dependence of the mobility and the dependence of the frictional force on the velocity at $T=0$, found[28] for the one-dimensional version of the Hamiltonian of Eq. (3) will also be valid for a one-dimensional SG model (with an accuracy up to a factor proportional to z^2).

4. QUANTUM MODEL OF AN INTERFACE IN ^{4}He

A quantum model of the interface between a crystal and a superfluid Bose liquid, taking account of the motion and compressibility of both media (QSOS/L model) was formulated and studied by Iordanskiĭ and Korshunov.[18] It is a generalization of model (3). In it, it is assumed that the tunneling amplitude Y is a complex quantity: $Y=y\times\exp(\pm i\chi_j)$, the phase of which coincides with the phase of the condensate χ at the point $\mathbf{j}$, and the total action of the system apart from the purely surface contribution

$$S_S=-i\int dt\left\{\frac{J}{2}\sum_{(\mathbf{jj}')}(n_{\mathbf{j}}-n_{\mathbf{j}'})^2\right.$$
$$\left.+\sum_{\mathbf{j}}\left[-y\cos(\varphi_{\mathbf{j}}-\chi_{\mathbf{j}})+\hbar n_{\mathbf{j}}\frac{\partial\varphi_{\mathbf{j}}}{\partial t}\right]\right\}$$

includes the action of the liquid, S_L, taken in the hydrodynamic approximation, and the action of the crystal, S_C, taken in the elasticity theory approximation.

The equations obtained by varying $S_S+S_L+S_C$ relative to the surface variable with neglect of the discreteness of $n_{\mathbf{j}}$, coincide with the boundary conditions given by a phenomenological analysis of the interface in ^{4}He.[37,38] The retarded Green's function for the variables $n_{\mathbf{j}}$, calculated to the same approximation and being directly related to the interaction of instantons, has for frequencies small compared with the Debye frequency, the form

$$G_0(\mathrm{k},\ \omega_s)=\hbar\{Jk^2+[\hbar^2/ya^2+\rho_{eff}d^2(k^2+\omega_s^2/c_L^2)^{-1/2}]\omega_s^2\}^{-1}, \qquad (35)$$

where $\rho_{\mathrm{eff}}=(\rho_C-\rho_L)^2/\rho_L$, $\rho_{L(C)}$ is the liquid (crystal) density, c_L is the velocity of sound in the liquid, the quasimomentum $\mathbf{k}$ is expressed in dimensional units and the crystal is considered to be incompressible. Although the approximation of an instanton gas only works in the model considered for a compressible crystal (in the opposite case the φ-χ fluctuations diverge), there exists some intermediate region for the parameters in which the effect of the compressibility of crystal and liquid on the instanton interaction can be neglected.

The poles of function (35) determine the spectrum of crystallization waves on neglecting the discreteness of the n_j (and as a consequence, dissipation). It agrees with earlier predictions.[3,38]

The dissipation at the interface in the QSOS/L model, as in model (3), can be associated with tunneling processes of the variables φ_j into neighboring minima of the periodic potential. Although, as in model (3), a change in one of the variables φ_j by $\pm 2\pi$ leads to a change in each of them by $\mp 2\pi/N$, no change in the kinetic energy of the interface itself then takes place, since in the tunneling process the quantities χ_j change in such a way as to compensate the change in φ_j. If this did not take place, a change in the rate of motion of the interface would lead (according to conservation laws) to a change in velocity of both media, so that the total change in energy of the system would be infinite. The change in the kinetic energy of the liquid (in a reference system in which the interface is at rest) is expressed through a change in χ_j:

$$\Delta W=\delta\left[\int dV\frac{\rho_L}{2}\left(\frac{\hbar}{m}\nabla\chi\right)^2\right]$$

$$=-\frac{\rho_L\hbar}{m}v_L a^2\sum_j \delta\chi_j=\pm\frac{2\pi\hbar}{d}v,$$

where v_L is the velocity of the homogeneous one-dimensional liquid (relative to the interface), and v is the velocity of the normal motion of the interface (relative to the crystal). The total change in the kinetic energy of the system in the tunneling process is thus the same as in model (3), so that the equations of Sec. 2B can be used in the calculation of the growth coefficient K, into which Eq. (35) must be substituted as the Green's function determining the interaction of instantons.

The calculation is appreciably simplified if $y=\infty$ and $c_L=\infty$ are substituted in Eq. (33) (which is possible to do if y is large and J is small, i.e., in the case of an ultraquantum face). As an illustration, we will consider this limiting case, differing most from the other limiting case $\rho_{\text{eff}}=0$, of the corresponding model (3), and then discuss the situation in ^{4}He. For $y=\infty, c_L=\infty$, turning to the method of steepest descents, as in Sec. 2C, we find that the temperature dependences of the coefficients $A(T)$ and $B(T)$ entering Eq. (12) for $J\ll T\ll JS_1^{4/7}$ have the form

$$A(T)\propto T^{-29/6},\quad B(T)=2S_1-{}^8/_3\pi(\ln 2)T/J+\ldots,\tag{36}$$

where

$$S_1\propto\hbar/(J\rho_{eff}d^2a^3)^{1/2}\approx\hbar/d^2(\tilde{\alpha}\rho_{eff}a^3)^{1/2}\tag{37}$$

is the action corresponding to a single instanton (at absolute zero), and $\tilde{\alpha}=\alpha+\alpha''$ is the surface rigidity.

For $T\sim JS_1^{4/7}$ the temperature dependence of the pre-exponential $A(T)$ changes and in the interval $JS_1^{4/7}\ll T\ll JS_1$ it has the same form as in model (3) [see Eq. (15)]. It is interesting that the coefficient in the first temperature-dependent term in the expansion of $B(T)$ in powers of T, written out in Eq. (36), turns out to be dependent only on J. On raising the temperature further, as in models (3) and (30), a transition takes place to the regime of activated tunneling.

As is easy to notice, the temperature dependences found are only insignificantly different from those obtained in Sec. 2C. For finite magnitudes of the tunneling amplitude y and of the velocity of sound, the values of K^{-1} as a function of the ratio of the parameters will be close either to that found here or to that found in Sec. 2C. In particular, in the case of an interface in ^{4}He, because of the smallness of the quantity $\rho_{\text{eff}}\sim 10^{-2}\rho_L$, for nearly all wave vectors the surface inertia $\hbar^2/ya^2$ will predominate over the attached mass of liquid, so that the factor S_1 will be close to the value given by Eqs. (14) and not by Eq. (37).

5. CONCLUSIONS

We have thus considered the dynamic properties of an atomically rough interface on three different quantum models, have shown that its motion is necessarily accompanied by dissipation, and have found the temperature dependence of the growth coefficient K for the case of the ratio between the parameters corresponding to a large amplitude of quantum fluctuations. Outside the immediate vicinity of the transition temperature, K falls with increasing temperature. A small slowing down of the growth rate must also occur for $T\to T_R+0$; the value of K then remains finite and only has a square root singularity. The form of the temperature dependence of K is very close for the different models. The scheme discussed also allows the inclusion into the treatment of the strictional interaction of steps on the crystal surface, which does not lead to a qualitative change.

The magnitude of the inverse growth coefficient K^{-1} for $T\sim T_R$ (in the atomically rough phase) in the ultraquantum case is exponentially small in the parameter S_1, the same parameter that determines the exponential smallness of the temperature region for critical behavior and the equilibrium dimensions of a face. We note, however, that the equilibrium size of a face is proportional to $\exp(-S_1)$, while $K^{-1}\propto\exp(-2S_1)$. From the means of evaluation, $2S_1$ is (accurate to a factor $(2\pi)^2$) the square of the zero-point oscillations of the surface in units of the interplanar distances d (calculated with neglect of the transition to the atomically smooth state).

The models studied in Secs. 2–4 were introduced here as models of the basal plane of the crystal. The possibility of applying them for describing faces with large Miller indices has been discussed,[17] the basic state of which is represented by a regular lattice of jogs on steps on the basal face (Fig. 3). In such a situation the variables n_j describe a shift of the jth kink from the equilibrium position, while the first term in Eq. (3) is the interaction of neighboring jogs, in which only quasi-momentum is conserved in the shifts, i.e., the first nonvanishing term. The interaction of non-neighboring jogs can

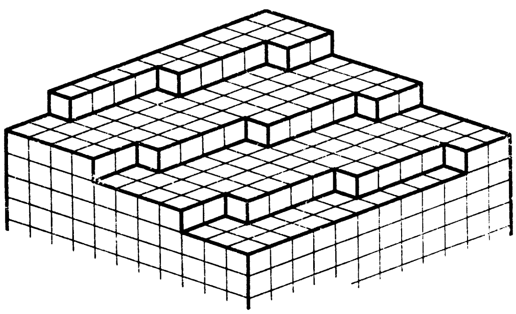

FIG. 3. In the absence of quantum fluctuations, the ground state of a face of the form ($1\ M\ ML$), and also of faces close in orientation, are a regular lattice of jogs on steps of the (001) face. For the face represented in the drawing $M = L = 4$.

also be included in the Hamiltonian but this does not lead to qualitative changes. Since the interaction of point surface defects is inversely proportional to the cube of the distance between them,[39] we have $J \propto M^{-5}$ for $L \sim M$. At the same time, the amplitude of tunneling of a jog into a neighboring position, Y, is a quantity characterizing a single jog and is not dependent on the Miller indices, so that $S_1 \propto M^{-5/2}$. Such an approach is, naturally, only possible if the basal face is in the atomically smooth state and fluctuations on it are small. Nevertheless, a qualitative conclusion can be drawn that the growth coefficient which we have studied at zero frequency should be a very strongly anisotropic (although, evidently, continuous) function of the orientation of the face.

The exponential growth of K with an increase in the Miller indices agrees with the possibility of propagating crystallization waves and with the existence for $T \lesssim 0.2$ K of an anomalous temperature dependence of the Kapitza jump, observed in experiments on different (apart from some excluded) faces of a ^{4}He crystal.[9,12-14]

It can be stated that although quantum roughness and crystallization waves were predicted in one work or another,[3] there is no strict reason for a connection between these phenomena. We may note that even in a system in which quantum roughness is possible [a one-dimensional version of model (3)], macroscopic motion is dissipative and is characterized at finite temperature by a finite growth coefficient.[28] The postulated connection between quantum roughness and non-dissipative growth[3] can, therefore, not be considered justified.

An experimental detection of the predicted temperature dependence of $K^{-1}(T)$ could be expected on studying the rate of growth of a crystal under quasi-equilibrium conditions (for example, with such an experimental arrangement as used by Balibar *et al.*,[26] Wolf *et al.*[27] or by Dyumin *et al.*[40]). It is not impossible that a repetition of the experiments to study the damping of crystallization waves[9] or on the conversion of sound at an interface[41,42] to lower frequencies and (or) lower temperatures might make it possible to separate the contribution to K^{-1} unconnected with collisions with excitations. In both cases the experiments should be carried out with strictly controlled orientation of the specimen, on faces with small Miller indices. In particular, it would be most promising to study the dynamic properties of a $(10\bar{1}1)$ face, on which the third faceting transition takes place at $T = T_{R3} \approx 0.35$ K.

The fact that new faceting transitions were not observed on lowering the temperature from 0.35 K to 0.07 K (Ref. 11) can be interpreted as evidence of a very rapid rise in S_1 with an increase in Miller indices. We note that the large value of S_1 makes it difficult to observe not only the equilibrium but also the kinetic faceting, since the magnitude of the critical supersaturation at which the transition from nucleated to continuous growth takes place, is proportional to $\exp(-2S_1)$. Since the equilibrium dimensions of a $(10\bar{1}1)$ face is not so small as to render it unobservable, it would be expected that the growth coefficient would not also be too small for it at zero frequency. As shown by experimental investigations,[26,27] the dynamics of an atomically rough interface in ^{4}He at $T \approx 1$ K is determined by collisions with rotons, but for $T \to T_{R3} + 0$ the roton contribution would be considerably weaker.

Departures were already found at low temperatures from the predicted temperature dependence of $K^{-1}(T)$ (Refs. 3, 5, 6) in experiments on the attenuation of melting waves. However, a systematic study of the additional contribution to K^{-1} from the orientation of the specimen was not carried out.

It is also not impossible that the activated $K^{-1}(T)$ dependence observed in experiments[27,40] under conditions of slow (quasi-equilibrium) growth, has as its cause not collisions with rotons, which in the hydrodynamic regime do not lead to dissipation,[3,5,6] but umklapp processes for the conjugate variable φ, discussed in the present paper. This would be evidence that the tunneling amplitude Y has a magnitude close to the size of the roton gap.

In conclusion we note that the dissipation mechanism considered only operates effectively for not too large frequencies. For $T \ll JS_1^{4/7}$ (or $T \ll JS_1^{1/3}$, depending on the model) limitation of the wave vector also arises.

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