Amorphous magnets with strong random anisotropy

M. V. Feigel'man and M. V. Tsodyks

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR, Moscow
(Submitted 30 April 1986)
Zh. Eksp. Teor. Fiz. **91**, 955–967 (September 1986)

A study is made of amorphous magnets having a strong random anisotropy. On the assumption of a long-range interaction it is shown that the correlations in such a system at high temperatures are ferromagnetic in nature, and the maximum correlation length is large compared to the range of the interaction. The phase transition to the disordered phase turns out to be equivalent to the phase transition in an Ising spin glass. The ordinary and nonlinear susceptibilities are calculated for different temperature regions above the transition point.

1. INTRODUCTION

Concentrated amorphous magnetic alloys based on rare earth metals are disordered systems for which a configurational disorder substantially alters the form of the magnetic state. In some cases this is due to an exchange interaction which can be of either sign, as in classical spin glasses. In another class of systems, which we shall consider here, the exchange interaction is ferromagnetic in nature and the configurational disorder enters through the creation of a single-ion anisotropy with randomly directed axes. The simplest Hamiltonian for such a system is that proposed by Harris, Plischke, and Zuckermann[1]:

$$H = -\frac{1}{2}\sum_{i,j} J(r_{ij})\mathbf{S}_i\mathbf{S}_j - D\sum_i (\mathbf{S}_i\boldsymbol{\xi}_i)^2 - \mathbf{h}\sum_i \mathbf{S}_i. \quad (1.1)$$

Here $\mathbf{S}_i$ is the Heisenberg spin at site i, $J_{ij} > 0$ are the exchange integrals, $\boldsymbol{\xi}_i$ is a random unit vector (director) indicating the direction of the anisotropy (of magnitude D) at site i, and $\mathbf{h} = g\mu_B\mathscr{H}$ is the reduced magnetic field.

As we know,[2] the ferromagnetic state of such a system is disrupted (for $h = 0$) even if $D \ll J$. In this case a frozen state should arise[3–5] at low temperatures, with a spin correlation length $L \sim (J/D)^2$ and a magnetic susceptibility $\chi \sim (J/D)^4$. Experiments[6,7] on the system $Dy_xGd_{1-x}Ni$, for which D varies with x, have confirmed these predictions. The critical behavior of such a system [for the case of planar (XY) spins] was investigated theoretically in Ref. 4, and it was predicted that the susceptibility should have a peak at $T \approx T_c$.

A number of experiments have not been done on the amorphous systems DyNi (Refs. 6 and 7), DyCu (Refs. 8 and 9), TbCoGa (Ref. 10), and DyFeB (Ref. 11), which are characterized by a large value of the random anisotropy: $D \gtrsim J$ (or even $D \gg J$). It has been shown[6,8,11] that in this case the susceptibility increases on decreasing temperature in the paramagnetic phase and then goes to saturation at $T \approx T_c$. At low temperatures the magnetic structure has an appreciable short-range order, with a correlation length $L \sim 15$ Å (Ref. 9), substantially larger than the average distance between spins. Measurements of the behavior of the magnetic moment in an external field as a function of temperature[7,10] have revealed the existence of a characteristic line on the h, T plane beyond which the irreversible magnetic response typical of spin glasses appears. The $T(h)$ curves obtained in Refs. 7 and 10 are very similar to the de Almeida–Thouless (AT) line $T(0) - T(h) = \text{const}\, h^{2/3}$ predicted[12] for an Ising spin glass.

Amorphous magnets with $D \gg J$ thus combine properties of the Heisenberg ferromagnet and Ising spin glass. In this paper we show that this behavior can be explained with the aid of the Hamiltonian (1) for $D \gg J$ if one assumes a rather slowly decaying exchange interaction $J(r)$. Assuming that the interaction range $\varkappa^{-1}$ is much larger than the distance a between neighboring spins, we show that the susceptibility grows appreciably in the paramagnetic region,

$$\chi(T_c) \sim (\varkappa a)^{-6}\chi(2T_c),$$

and that the maximum spin correlation length is substantially larger than the range of the interaction: $L \sim \varkappa^{-1}(a\varkappa)^{-3}$. We then find the slow variables which describe the thermodynamics of the system near the phase transition (for this we must use the $1/N$ expansion, where N is the number of spin components). It turns out that the effective Hamiltonian for these variables is the same as for an Ising spin glass with a long-range interaction.[13] We thereby explain the obervation[7] of the AT line noted above and predict the asymptotic form of the static and dynamic anomalies at $T \to T_c$.

We note that the idea that Ising spin glasses and magnets with random anisotropy belong to the same universality class (in the sense of phase transition theory) has also been stated previously: in Ref. 14, on the basis of a 6-ε expansion for the case of weak anisotropy, and in Ref. 15, on the basis of a numerical simulation of a two-dimensional system (where a phase transition was seen at $T = 0$). In the present paper this statement is first proved directly for a three-dimensional system with strong anisotropy.

2. FERROMAGNETIC CORRELATION REGION

Under the condition $D \gg J$ it can be assumed that each spin is directed along its own easy axis: $\mathbf{S}_i = \sigma_i\boldsymbol{\xi}_i$, where $\sigma_i = \pm 1$. Therefore, we obtain from (1.1) the Hamiltonian for the Ising variables σ_i in the form

$$H_\sigma = -\frac{1}{2}\sum_{ij} J(r_{ij})\boldsymbol{\xi}_i\boldsymbol{\xi}_j\sigma_i\sigma_j - \mathbf{h}\sum_i \boldsymbol{\xi}_i\sigma_i. \quad (2.1)$$

 0038-5646/86/090562-08$04.00

The random vectors ξ_i are assumed uncorrelated:

$$\overline{\xi_i^{\alpha}\xi_j^{\beta}}={}^1/_3\delta^{\alpha\beta}\delta_{ij} \tag{2.2}$$

(the superior bar denotes a configurational average). The interaction range $\varkappa^{-1}$ is defined through the relation

$$\varkappa^{-2}=\frac{1}{6J_0}\int r^2J(r)\,d^3r, \qquad J_0=\int J(r)\,d^3r. \tag{2.3}$$

In what follows we set $J_0=1$ and assume that $c\varkappa^{-3}\gg 1$ (c is the volume density of spins). As usual in problems with long-range interactions, it is convenient to change from the discrete variables σ_i to a continuous field $\mathbf{S}(\mathbf{r})$ [$\mathbf{S}(\mathbf{r}_i)$ is the molecular field acting at site i]:

$$\begin{aligned}Z&=\sum_{\{\sigma_i\}}\exp\left(-\frac{H_\sigma}{T}\right)\\&=\int D\mathbf{S}(r)\sum_{\{\sigma_i\}}\exp\left\{-\frac{1}{2T}\iint\mathbf{S}(\mathbf{r})\,(J^{-1})_{rr'}\mathbf{S}(\mathbf{r}')\,d^3r\,d^3r'\right.\\&\left.+\frac{1}{T}\sum_i[\mathbf{h}+\mathbf{S}(\mathbf{r}_i)]\xi_i\sigma_i\right\}\\&=\int D\mathbf{S}(\mathbf{r})\exp\left\{\frac{1}{2T}\int d^3r\left(\mathbf{S}^2+\frac{1}{\varkappa^2}(\nabla\mathbf{S})^2\right)\right.\\&\left.-\sum_i\ln\operatorname{ch}\frac{1}{T}\xi_i(\mathbf{S}(\mathbf{r}_i)+\mathbf{h})\right\}.\end{aligned} \tag{2.4}$$

In deriving the last relation in (2.4) we have used (2.3) and neglected the higher derivatives of the field $\mathbf{S}(\mathbf{r})$, as we may do for $c\varkappa^{-3}\gg 1$. We expand the argument of the exponential function in (2.4) in a power series in $\mathbf{S}$ up to the $\mathbf{S}^4$ term (we assume an N-component spin and $h=0$):

$$\begin{aligned}\frac{H[\mathbf{S}]}{T}&=\frac{1}{T}\int d^3r\left\{\frac{\tau}{2}\mathbf{S}^2+\frac{1}{2\varkappa^2}(\nabla\mathbf{S})^2+\frac{1}{2T}\left[\frac{c}{N}\delta_{\alpha\beta}-\sum_i\delta(\mathbf{r}\right.\right.\\&\left.\left.-\mathbf{r}_i)\xi_i^{\alpha}\xi_i^{\beta}\right]S_\alpha S_\beta+\frac{1}{12T^3}\sum_i\delta(\mathbf{r}-\mathbf{r}_i)\,(\xi_i\mathbf{S})^4\right\}.\end{aligned} \tag{2.5}$$

Here $\tau=1-c/NT$, and the expression multiplying $S_\alpha S_\beta$ in the third term averages to zero. For $T=c/N$ ($\tau=0$) the average value of the coefficient of $\mathbf{S}^2$ vanishes. We shall study the system in the vicinity of the presumed phase transition, $|\tau|\ll 1$. Here we will be interested in the large-scale fluctuations of the field $\mathbf{S}(\mathbf{r})$, and we can therefore neglect the fluctuations of the coefficient of $\mathbf{S}^4$ in (2.5) and treat the coefficient of $S_\alpha S_\beta$ as a Gaussian random field $V_{\alpha\beta}(r)$. Then the Hamiltonian becomes

$$\begin{aligned}\frac{H[\mathbf{S}]}{T}&=\int d^3r\left\{\frac{\tau}{2}S_\alpha^2+\frac{1}{2\varkappa^2}(\nabla S_\alpha)^2\right.\\&\left.+V_{\alpha\beta}(\mathbf{r})S_\alpha(\mathbf{r})S_\beta(\mathbf{r})+\frac{g}{4}(S_\alpha^2)^2\right\},\end{aligned} \tag{2.6}$$

where $g=N/c(N+2)$ and we have made the change of notation $\mathbf{S}\to T^{1/2}\mathbf{S}$. The correlator of the field $V_{\alpha\beta}(\mathbf{r})$ is

$$V_{\alpha\beta}(\mathbf{r})\,V_{\gamma\delta}(\mathbf{r}')=\frac{g}{4}\delta(r-r')\,(\delta_{\alpha\beta}\delta_{\gamma\delta}+\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma}). \tag{2.7}$$

The "bare" correlator, corresponding to (2.6), of the field $S_\alpha(\mathbf{r})$ has the usual form for an isotropic ferromagnet. In the Fourier representation we have

$$\langle S_\alpha(\mathbf{p})S_\beta(-\mathbf{p})\rangle=G^{(0)}_{\alpha\beta}(\mathbf{p})=\frac{\delta_{\alpha\beta}}{(p/\varkappa)^2+\tau}. \tag{2.8}$$

There are two kinds of corrections to (2.8): from the interaction term and the random potential. We can easily satisfy ourselves that these corrections are of the same order of magnitude and are small if

$$\tau\gg\tau_c=(\varkappa^3/c)^2(N/4\pi)^2(N-1)^2/(N+2)^2. \tag{2.9}$$

In the actual situation for amorphous magnets we have $N=3$ and $c\approx 0.5a^{-3}$ (a is the characteristic distance between atoms in the alloy), so that $\tau_c\approx(\varkappa a)^6/25$; the region of isotropic ferromagnetic correlations exists for $\varkappa^{-1}\gtrsim a$. We note that $\tau_c\approx 10^{-2}$ at an interaction range of only $\varkappa^{-1}=1.5a$. In the region $\tau\gg\tau_c$ the correlation length r_c and susceptibility χ are given by

$$r_c=\varkappa^{-1}\tau^{-1/2}, \qquad \chi\sim\tau^{-1} \qquad (\tau\gg\tau_c). \tag{2.10}$$

We show below that these quantities do not grow for $\tau\lesssim\tau_c$, so that the maximum value of the correlation length is

$$L=r_{c\,\max}\approx 5\varkappa^{-1}(\varkappa a)^{-3}\gg\varkappa^{-1}. \tag{2.11}$$

We shall show that an external magnetic field h will alter the pure Lorentzian shape of the correlator (2.8). We add to (2.6) a term $-hS_1(\mathbf{r})$ and write the field $S_\alpha(\mathbf{r})$ in the form $S_\alpha(\mathbf{r})=m\delta_{1\alpha}+\tilde{S}_\alpha(\mathbf{r})$, where $\langle\tilde{S}_\alpha(\mathbf{r})\rangle=0$. For $\tau\gtrsim\tau_c$ the third term in (2.6) can be treated as a small perturbation, so that m is given by the usual equation

$$\tau m+gm^3=h. \tag{2.12}$$

Assuming $\tilde{S}_\alpha(r)$ to be small, we obtain from (2.6)

$$\begin{aligned}\frac{H[S]}{T}&=\int d^3r\left\{\left[\frac{1}{2}(\tau+gm^2)\delta_{\alpha\beta}+gm^2\delta_{1\alpha}\delta_{1\beta}\right]\tilde{S}_\alpha\tilde{S}_\beta\right.\\&\left.+\frac{1}{2\varkappa^2}(\nabla\tilde{S}_\alpha)^2+V_{\alpha\beta}\tilde{S}_\alpha\tilde{S}_\beta+2V_{\alpha1}m\tilde{S}_\alpha\right\}.\end{aligned} \tag{2.13}$$

The last term in (2.13) means that a random external field has arisen, acting on the variables $\tilde{S}_\alpha$. As we know, this will give rise to a squared Lorentzian term in the correlator:

$$\langle\tilde{S}_\alpha(\mathbf{p})\tilde{S}_\beta(-\mathbf{p})\rangle=G_\perp(\mathbf{p})\,(\delta_{\alpha\beta}-\delta_{1\alpha}\delta_{1\beta})+G_\parallel(\mathbf{p})\delta_{1\alpha}\delta_{1\beta},$$

$$G_\perp(\mathbf{p})=\frac{1}{\tau_\perp+p^2/\varkappa^2}+\frac{gm^2}{(\tau_\perp+p^2/\varkappa^2)^2}, \tag{2.14}$$

$$G_\parallel(\mathbf{p})=\frac{1}{\tau_\parallel+p^2/\varkappa^2}+\frac{4}{3}\frac{gm^2}{(\tau_\parallel+p^2/\varkappa^2)^2}, \tag{2.15}$$

where

$$\tau_\perp=\tau+gm^2, \qquad \tau_\parallel=\tau+3gm^2.$$

The additional terms are not small in strong fields which satisfy $h\gtrsim h_c(T)=(\tau^3/g)^{1/2}$. Expressions (2.14) and (2.15) for the total correlators determine the form of the structure factors for scattering; the irreducible correlators

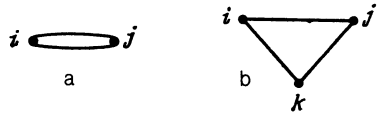

FIG. 1.

which determine the values of the differential susceptibilities are given by the first terms in (2.14) and (2.15). We note that the squared Lorentzian term is observed more easily in $G_\perp(\mathbf{p})$, since $\tau_\perp \approx \tau_\parallel/3$ in the high-field region.

From a theoretical standpoint it is interesting to consider condition (2.9) at large N and $\varkappa^{-1}$. We see tht $\tau_c \ll 1$ for $N \ll Z$, where $Z = 4\pi c\varkappa^{-3}/3$ is the effective number of neighbors. In the opposite limit $N \gg Z$ there is no ferromagnetic correlation region, and our model is completely equivalent to an Ising spin glass.

Indeed, if we consider the high-temperature expansion for the initial Hamiltonian (2.1), taking into account only those diagrams in which every bond $\tilde{J}_{ij} = J(r_{ij})\xi_i\xi_j$ appears twice (as for spin glasses; see Fig. 1a), we get a singularity of the high-temperature series at $\tilde{T} \approx J_0(Z/N)^{1/2}$. Now let us estimate the contribution of diagrams for which there are correlations between different $\tilde{J}_{ij}$ (Fig. 1b). The contribution of each triangle goes as $\sim T^{-3}\tilde{J}_{ij}\tilde{J}_{jk}\tilde{J}_{ki} \sim J_0^3 T^{-3} N^{-2}$ and the number of such triangles as Z^2, so that their total contribution goes as $\sim (J_0/T)^3 (Z/N)^2 \sim (Z/N)^{1/2}$ and is small for $Z \ll N$. Analogously, the contribution from any m-gons is of order $(Z/N)^{(m-2)/2} \ll 1$. Thus the leading contribution to the high-temperature series is from the same diagrams as for the Ising spin glass. We note that the decrease of the correlation length with increasing N has been detected previously[15] in a numerical simulation of a two-dimensional system.

3. SLOW VARIABLES IN THE CRITICAL REGION

1. In the last section we showed that appreciable corrections to the behavior of an isotropic ferromagnet should arise for $\tau \lesssim \tau_c \sim (\varkappa a)^6/25$. The problem of summing the diagrams in this region is equivalent to the problem considered in Ref. 16 for a model with a weak random anisotropy. In Ref. 16 it was shown (by means of a 4-ε expansion) that the renormalization group equations have no fixed points corresponding to a phase transition to a ferromagnetic phase. Here we consider the region $\tau \lesssim \tau_c$ in a three-dimensional system with the aid of a $1/N$ expansion and show that the critical behavior of the system for $N \gg 1$ is the same as in an Ising spin glass with a long-range interaction.[13] Here the role of the individual spins in the spin glass is played by ferromagnetically correlated regions having a dimension of the order of $L = r_{c\max}$ [see (2.11)]. We shall start from the Hamiltonian (2.6) but with the constant g and the coefficient in correlator (2.7) redefined in such a way as to have a well-defined limit for $N \to \infty$:

$$\overline{V_{\alpha\beta}(\mathbf{r})V_{\gamma\delta}(\mathbf{r}')} = \alpha N^{-1}\delta(\mathbf{r}-\mathbf{r}')(\delta_{\alpha\beta}\delta_{\gamma\delta}+\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma}),$$
$$g=\beta/N, \quad \alpha=9/20c\approx 1, \quad \beta=9/5c\approx 4. \tag{3.1}$$

These are the usual dependences on N chosen in the study of the spherical model with a weak random anisotropy,[17] where T_c is assumed independent of N, whereas for the initial Hamiltonian (2.1) [and, accordingly, for (2.6)] we would have $T_c \sim N^{-1}$. The numerical coefficients in (3.1) are chosen so that (3.1) goes over to (2.7) for $N = 3$. For $N = \infty$ the problem can be solved exactly[17]; the phase transition is due to a macroscopic occupation of the mode $\Psi_\alpha^{(0)}(\mathbf{r})$ corresponding to the maximum eigenvalue of the operator $\hat{M}_{\alpha\beta} = (\nabla^2/\varkappa^2)\delta_{\alpha\beta} - V_{\alpha\beta}(\mathbf{r})$. Before constructing the $1/N$ expansion with the eigenfunctions $\Psi_\alpha^\lambda(\mathbf{r})$ of the operator $\hat{M}_{\alpha\beta}$ ($\hat{M}_{\alpha\beta}\Psi_\beta^\lambda = E_\lambda\Psi_\alpha^\lambda$); as the basis, let us first examine the properties of these functions.

For $N = \infty$ all the eigenfunctions $\Psi_\alpha^\lambda(\mathbf{r})$ are delocalized, and the density of states $\rho(E)$ has a precise edge E_c, near which $\rho(E) \sim (E_c - E)^{1/2}$ (Ref. 17). For finite N the edge of the spectrum is smeared, and localized states appear. This effect is conveniently studied by the technique that was used in Ref. 13 to analyze the spectrum of the matrix J_{ij} for a long-range interaction. In this technique the average Green's function

$$R_E(\mathbf{r}) = \frac{1}{NV}\int d^3r' \sum_\lambda \frac{\Psi_\alpha^\lambda(\mathbf{r}')\Psi_\alpha^\lambda(\mathbf{r}+\mathbf{r}')}{E-E_\lambda+i\delta} \tag{3.2}$$

is written in the form of a functional integral

$$R_E(\mathbf{r}) = \lim_{m\to 0}\int DQ_{\alpha\beta}(\mathbf{r})\left[(E+i\delta)\delta_{\alpha\beta} - \frac{\nabla^2}{\varkappa^2}\delta_{\alpha\beta}+Q_{\alpha\beta}\right]^{-1}$$
$$\times\exp\left\{-N\int d^3r\left[\frac{1}{4\alpha}\mathrm{Sp}\,Q_{\alpha\beta}^2 + \frac{1}{2}\mathrm{Sp}\ln\left[\left(-\frac{\nabla^2}{\varkappa^2}+E+i\delta\right)\delta_{\alpha\beta}+Q_{\alpha\beta}\right]\right]\right\}. \tag{3.3}$$

Here (and below) lengths are measured in units of a, and $Q_{\alpha\beta}(\mathbf{r})$ is an $\mathbf{r}$-dependent tensor in replica space, with the number of replica indices m assumed equal to zero in the final calculations. Evaluating (3.3) by the method of steepest descent for $N \gg 1$, we obtain the density of (delocalized) states $\rho(E)$ in the form

$$\rho(E) = -\pi^{-1}\,\mathrm{Im}\,R_E(0) = \frac{\varkappa^3}{4\pi^2}(E_1-E)^{1/2}, \quad E_1 = -\alpha^2\varkappa^6/(8\pi)^2. \tag{3.4}$$

Estimating the fluctuations of $Q_{\alpha\beta}(\mathbf{r})$ near the saddle point, we find (in analogy with Ref. 13) that formula (3.4) is valid for

$$-\varepsilon = E_1 - E \gtrsim \varepsilon_0 = \left(\frac{3}{2}\alpha\right)^2 \frac{1}{16\pi^2}\frac{\varkappa^6}{N^{4/3}}. \tag{3.5}$$

In the region $\varepsilon \gg \varepsilon_0$ the states are localized, their density being determined by nontrivial saddle points in the integral (3.3). In analogy with Ref. 13 we find

$$\rho(\varepsilon) \sim \exp\left[-A(\varepsilon/\varepsilon_0)^{3/4}\right], \quad A \sim 1. \tag{3.6}$$

The localization length of these states is

$$l(\varepsilon) = \tilde{A}\varkappa^{-4}(\varepsilon/\varkappa^6)^{-1/4} = L(\varepsilon/\varkappa^6)^{-1/4}, \quad \tilde{A} \sim 1. \tag{3.7}$$

In the energy region $\varepsilon \sim \varepsilon_0$ there is a localization threshold ε_c which separates the delocalized states $\Psi_\alpha^\lambda(r)$ (for $\varepsilon < \varepsilon_c$)

from the localized states. For energies near the threshold $(\varepsilon-\varepsilon_c)\ll\varepsilon_0$ we use the scaling hypothesis of localization theory, according to which

$$l(\varepsilon)\approx l_0\left(\frac{\varepsilon_0}{\varepsilon-\varepsilon_c}\right)^{\nu}, \quad l_0\approx LN^{1/3}. \tag{3.8}$$

The value of l_0 is determined by joining this expression with (3.7) at $\varepsilon\sim\varepsilon_0$. The exponent ν is evidently close to unity; for simplicity we shall henceforth assume $\nu=1$ (see Ref. 13 for details). We will also have need of a correlation function of the form

$$K_E(r)=\overline{N^{-1}\Psi_\alpha{}^E(\mathbf{r})\Psi_\alpha{}^E(\mathbf{r}+\mathbf{r}')}=-(\pi\rho(E))^{-1}\operatorname{Im}R_E(r). \tag{3.9}$$

Evaluating $K_E(r)$ for $-\varepsilon\gg\varepsilon_0$ by the method of steepest descent [using (3.3) and (3.9)], we get

$$K_E(r)=\frac{\sin(\varkappa(-\varepsilon)^{1/2}r)}{4\pi\varkappa(-\varepsilon)^{1/2}r}e^{-r/L}, \quad L=\frac{8\pi}{\alpha}\varkappa^{-4}. \tag{3.10}$$

For $\varepsilon\ll\varkappa^6$ Eq. (3.10) reduces to

$$K_E(r)=(4\pi)^{-1}e^{-r/L}; \tag{3.11}$$

as this expression is independent of ε, we expect that (3.11) will remain valid for $|\varepsilon|\sim\varepsilon_0$.

2. We write the vector field $S_\alpha(r)$ as the spectral decomposition:

$$S_\alpha(r)=\sum_\lambda a_\lambda\Psi_\alpha{}^\lambda(r), \tag{3.12}$$

$$\hat{M}_{\alpha\beta}\Psi_\beta(r)=\frac{\nabla^2}{\varkappa^2}\Psi_\alpha(r)-V_{\alpha\beta}(r)\Psi_\beta(r)=E_\lambda\Psi_\alpha(r).$$

As we have mentioned, for $N=\infty$ the phase transition is due[17] to the onset of a macroscopic average of the amplitude $a_{\lambda 0}$ corresponding to the maximum eigenvalue $E_{\lambda 0}=E_1$. We shall show that for a large but finite N the critical behavior is governed by a macroscopic number of amplitudes (modes) a_λ with energies E_λ in the interval $|\varepsilon_\lambda|=|E_\lambda-E_1|\sim\varepsilon_0$. These amplitudes are the correct slow variables of our problem. Let us write (3.12) in the form

$$S_\alpha(\mathbf{r})=\sum_\lambda a_\lambda\Psi_\alpha{}^\lambda(\mathbf{r})+\sum_\mu b_\mu\Phi_\alpha{}^\mu(\mathbf{r})=S_\alpha^{(0)}(\mathbf{r})+\tilde{S}_\alpha(\mathbf{r}), \tag{3.13}$$

where the summation in the first term is over states with $\varepsilon_\lambda>-\varepsilon_d$ (corresponding to the slow variables), and that in the second term goes over the rest of the spectrum: $-\varepsilon_\mu>\varepsilon_d$; the parameter ε_d is chosen in the interval $\varepsilon_0\ll\varepsilon_d\ll\varkappa^6$.

We must now take the thermodynamic average over amplitudes b_μ and obtain an effective Hamiltonian for the slow variables a_λ. For this it is convenient to write the partition function of the field $S_\alpha(r)$ with the Hamiltonian (2.6), (3.1) in the form

$$Z=\int DS_\alpha(\mathbf{r})D\Phi(\mathbf{r})\exp\left\{-\int d^3r\left[\frac{1}{2}S_\alpha(\tau\delta_{\alpha\beta}-\hat{M}_{\alpha\beta})S_\beta+\frac{1}{2}S^2\Phi-\frac{N}{4\beta}\Phi^2(\mathbf{r})\right]\right\}. \tag{3.14}$$

We now substitute (3.13) into (3.12) and integrate over b_μ to get

$$Z=\int DS_\alpha{}^{(0)}D\Phi(\mathbf{r})\times\exp\left\{-\int d^3r\left[\frac{1}{2}S_\alpha^{(0)}(\delta_{\alpha\beta}(\tau+\Phi(\mathbf{r}))-\hat{M}_{\alpha\beta}(\mathbf{r}))S_\beta^{(0)}-\frac{N}{4\beta}\Phi^2(\mathbf{r})\right]-\frac{1}{2}\widetilde{\operatorname{Sp}}\ln(\delta_{\alpha\beta}(\tau+\Phi(\mathbf{r}))-\hat{M}_{\alpha\beta}(\mathbf{r}))\right\}. \tag{3.15}$$

The wavy line over the trace symbol Sp means that the summation over the spectrum is restricted to the interval $-\varepsilon_\mu>\varepsilon_d$. The condition $N\gg 1$ allows us to evaluate the integral over $\Phi(\mathbf{r})$ by the method of steepest descent; this same condition allowed us to neglect the cross terms $S^{(0)}\tilde{S}$ in deriving (3.15). For sufficiently large τ (a criterion is given below) we have for the saddle-point configuration $\Phi_0(\mathbf{r})\approx\Phi_0=\text{const}$. The equation for Φ_0 is of the form

$$\frac{N}{2\beta}\Phi_0=\frac{1}{2}\widetilde{\operatorname{Sp}}\hat{G}+\frac{1}{2V}\sum_\lambda a_\lambda{}^2=\frac{N}{2}\int_{-\infty}^{E_1-\varepsilon_d}\frac{\rho(E)\,dE}{\tau+\Phi_0-E}+\frac{1}{2V}\sum_\lambda a_\lambda{}^2. \tag{3.16}$$

Here V is the volume of the system, and

$$\hat{G}=\hat{L}^{-1}=[\delta_{\alpha\beta}(\tau+\Phi_0)-\hat{M}_{\alpha\beta}]^{-1}=NR(E=\tau+\Phi_0)$$

[see (3.2) and (3.4)]. In the region under consideration the last term in (3.16) is small, and the integral is determined by the spectral region in which $\rho(E)$ is given by (3.4). The integral over E formally diverges for $|E|\to\infty$, and it must be regularized by subtracting the same integral with $\tau+\Phi_0=0$, leading to a τ-independent shift Φ_0. Actually, this same shift was made back in the calculation of E_1 in (3.4). We are interested below in the quantity $\omega=\tau+\Phi_0-E_1$ (which has the meaning of a renormalized temperature), in which these shifts cancel. As a result, we obtain for ω the following equation [assuming that $\omega\gg\varepsilon_d$ and neglecting the last term in (3.15)]

$$\omega-\tilde{\tau}=-\frac{\beta}{4\pi}\varkappa^3\omega^{1/2}, \quad \tilde{\tau}=\tau-E_1 \tag{3.17}$$

which has the solution

$$\omega=\left\{\left[\tilde{\tau}+\frac{\beta^2}{(8\pi)^2}\varkappa^6\right]^{1/2}-\frac{\beta}{8\pi}\varkappa^3\right\}^2. \tag{3.18}$$

For $\tilde{\tau}\ll(\beta/8\pi)^2\varkappa^6$ we have $\omega=(4\pi/\beta)^2\tilde{\tau}^2/\varkappa^6$. The main contribution to the integral (3.15) is from the region $-\varepsilon=E_1-E\sim\omega$, and so the approximations we have made are valid for $\omega\gg\varepsilon_0$. As a result, the applicability condition of solution (3.18) is

$$\tilde{\tau}\gg\tau_0=\frac{3\alpha\beta}{32\pi^2}\frac{\varkappa^6}{N^{2/3}}. \tag{3.19}$$

Substituting $\Phi(\mathbf{r})=\Phi_0$ into (3.15), we obtain the quadratic part $H_0[S_\alpha^{(0)}]$ of the effective Hamiltonian $H[S_\alpha^{(0)}]$ in the form

$$H_0[S_\alpha^{(0)}]=\tfrac{1}{2}\sum_\lambda(\omega-\varepsilon_\lambda)a_\lambda^2. \tag{3.20}$$

In region (3.19) there are exponentially small [see (3.6)] eigenmodes a_λ with $\varepsilon_\lambda>\omega$. For these modes the states with $a_\lambda=0$ are unstable [see (3.20)]; to find the correct states we must evaluate the Gaussian integral over $\varphi(r)=\Phi(r)-\Phi_0$ with the term $\varphi(r)(S^{(0)}(\mathbf{r}))^2$ in the argument of the exponential function in (3.15) treated as a perturbation. We obtain the following contribution to $H[S_\alpha^{(0)}]$:

$$H_{int}[S_\alpha^{(0)}]=\frac{1}{8}\iint(S_\alpha^{(0)}(\mathbf{r}))^2(S_\beta^{(0)}(\mathbf{r}))^2D(\mathbf{r},\mathbf{r}')\,d^3r\,d^3r', \tag{3.21}$$

where D, the correlation function of the field $\varphi(r)$, is given by the equation [see (3.15) and (3.16)]

$$D^{-1}(\mathbf{r},\mathbf{r}')=\frac{N}{2\beta}\delta(\mathbf{r}-\mathbf{r}')+\frac{1}{2}\operatorname{Sp}G^2(\mathbf{r},\mathbf{r}'). \tag{3.22}$$

Let us examine the second term in (3.22) as a function of $\mathbf{r}-\mathbf{r}'$ and $\mathbf{r}$. In region (3.19) the $\mathbf{r}$ dependence is weak, and in (3.22) we can go over to the Fourier components $D(\mathbf{q})$. Using (3.10) and (3.11), we can show that the characteristic scale of the function $D(\mathbf{r})$ is smaller than $L\sim\varkappa^{-4}$, while $(\mathbf{S}^{(0)}(\mathbf{r}))^2$ varies over lengths of the order of $l(\omega)\gg L$ [see (3.7)]. Here we assume that $\varepsilon_0\ll\omega\ll\varkappa^6$. Therefore, in (3.21) we can make the replacement $D(\mathbf{r},\mathbf{r}')\to D(q=0)\delta(\mathbf{r}-\mathbf{r}')$, where

$$D^{-1}(q=0)\approx\frac{1}{2}\overline{\int\widetilde{\operatorname{Sp}}\,G^2(\mathbf{r},\mathbf{r}')\,d^3r'}\approx\frac{N}{2}\frac{\varkappa^3\omega^{-1/2}}{8\pi}. \tag{3.23}$$

Collecting (3.20), (3.22), and (3.23), we obtain the effective Hamiltonian of the field $\mathbf{S}^{(0)}(\mathbf{r})$ in the form

$$H[S_\alpha^{(0)}]=\frac{1}{2}\sum_\lambda(\omega-\varepsilon_\lambda)a_\lambda^2$$

$$+2\pi\frac{\omega^{1/2}}{N\varkappa^3}\sum_{\lambda_1\lambda_2\lambda_3\lambda_4}a_{\lambda_1}a_{\lambda_2}a_{\lambda_3}a_{\lambda_4}\int\Psi_{\lambda_1}^\alpha\Psi_{\lambda_2}^\alpha\Psi_{\lambda_3}^\beta\Psi_{\lambda_4}^\beta\,d^3r. \tag{3.24}$$

Let us now consider the consequences of the instability of the states with $a_\lambda=0$ for $\varepsilon_\lambda>\omega$. For $\omega\gg\varepsilon_0$ (i.e., $\tilde\tau\gg\tau_0$), such states are few in number, and we can therefore neglect the overlap of different Ψ_λ^α; then (3.24) reduces to

$$\sum_\lambda H_0^{(\lambda)}=\sum_\lambda\left\{\frac{1}{2}(\omega-\varepsilon_\lambda)a_\lambda^2+2\pi\frac{\omega^{1/2}}{N\varkappa^3}a_\lambda^4I_\lambda\right\}, \tag{3.25}$$

where

$$I_\lambda=\int d^3r[(\Psi_\lambda^\alpha(\mathbf{r})\Psi_\lambda^\alpha(\mathbf{r})]^2\sim l^{-3}(\varepsilon_\lambda). \tag{3.26}$$

Minimizing (3.25) with respect to a_λ, we get [see (3.7)]

$$a_\lambda^2=\frac{N\varkappa^3}{4\pi\omega^{1/2}}(\varepsilon_\lambda-\omega)I_\lambda^{-1}\approx\frac{N\varkappa^3}{4\pi\omega^{1/2}}(\varepsilon_\lambda-\omega)l^3(\varepsilon_\lambda). \tag{3.27}$$

We note that expression (3.27) holds for $\varepsilon-\omega\sim\omega\gg\varepsilon_0$, where the fluctuation contribution to $\langle a_\lambda^2\rangle$ can be neglected.

3. With decreasing temperature, $\tilde\tau$ and ω decrease, and the number of modes with $\langle a_\lambda\rangle\neq0$ ("condensed modes") increases. For $\tilde\tau\sim\tau_0$ the average number of such modes $\Psi_\lambda^\alpha(\mathbf{r})$ in a volume $l^3(\varepsilon_\lambda)$ is of order 1, and we must take into account the interaction of different modes, i.e., the terms with $\lambda_i\neq\lambda_j$ in (3.24). We note that on scales larger than L, $\Psi_\lambda^\alpha(\mathbf{r})$ is a random N-component vector with zero mean [see (3.10)], and the main contribution to the interaction is therefore from the terms in (3.24) with $\lambda_1=\lambda_2$ and $\lambda_3=\lambda_4$:

$$\frac{\omega^{1/2}}{N\varkappa^3}\sum_{\lambda,\mu}a_\lambda^2a_\mu^2I_{\lambda\mu}^{(0)},\qquad I_{\lambda\mu}^{(0)}=\int(\Psi_\lambda^\alpha)^2(\Psi_\mu^\beta)^2d^3r. \tag{3.28}$$

The remaining "matrix elements" in (3.24) are much smaller than $I_{\lambda\mu}^{(0)}$, by a factor of order N^{-1}. This means that in the leading approximation the interaction of the condensed modes can be taken into account in a Hartree-type approximation, which reduces to a renormalization of ω. We note that such a renormalization occurs automatically if the last term in (3.16) is taken into account in the calculation of ω. Instead of (3.18) we then get

$$\omega=\left\{\left[\tilde\tau+q+\frac{\beta^2}{8\pi^2}\varkappa^6\right]^{1/2}-\frac{\beta}{8\pi}\varkappa^3\right\}^2, \tag{3.29}$$

where

$$q=\frac{1}{NV}\sum_\lambda a_\lambda^2=\frac{1}{N}\langle(\mathbf{S}^{(0)}(\mathbf{r}))^2\rangle.$$

Now we must replace $(S_\alpha^{(0)}(\mathbf{r}))^2$ in H_{int} with $\delta(S_\alpha^{(0)}(\mathbf{r}))^2=(S_\alpha^{(0)}(\mathbf{r}))^2-q$ [cf. (3.21)]:

$$H[S_\alpha^{(0)}]=\frac{1}{2}\sum_\lambda(\omega-\varepsilon_\lambda)a_\lambda^2$$

$$+2\pi\frac{\omega^{1/2}}{N\varkappa^3}\int\left[\sum_{\lambda_1\lambda_2}a_{\lambda_1}a_{\lambda_2}\Psi_{\lambda_1}^\alpha\Psi_{\lambda_2}^\alpha-Nq\right]^2d^3r \tag{3.30}$$

and determine ω from (3.29), with q calculated using (3.27). Equation (3.27) retains its form. Strictly speaking, Eqs. (3.29) and (3.30) no longer apply when $\tau\sim\tau_0$, since condition (3.19) is violated, and the method of steepest descent cannot be used to calculate the functional integral over $\Phi(\mathbf{r})$ in (3.15). However, on qualitative considerations we can understand what happens in this temperature region and progress to the most interesting region, $-\tau\gg\tau_0$.

It is extremely important that the leading (in magnitude) mode-interaction terms which we have taken into account depend only on the absolute values $|a_\lambda|$ and do not determine the signs $\sigma_\lambda=a_\lambda/|a_\lambda|$. Therefore, these terms cannot lead to a real phase transition; those terms in H_{int} that do depend on the signs are small at $|\tilde\tau|\sim\tau_0$. Thus, for $|\tilde\tau|\sim\tau_0$ there is no phase transition but rather a smooth growth of the spin correlations.

We now recall[17] what happens for $N=\infty$, i.e., $\tau_0=\omega_0=0$. In this case Eq. (3.29) holds everywhere, and for $\tilde\tau<0$ we get

$$a_\lambda^2=qV\delta(\varepsilon_\lambda),\qquad q=|\tilde\tau|,\qquad \omega=0, \tag{3.31}$$

i.e., there occurs a macroscopic condensation into the mode with the maximum eigenvalue $E_\lambda=E_1$.

Returning to the case of finite N, let us seek a solution analogous to (3.31) for q in the region $-\tilde{\tau} \gg \tau_0$, i.e., let us assume that

$$q \approx |\tilde{\tau}| \gg \tau_0. \tag{3.32}$$

We note that relation (3.32) does not imply a phase transition, since $q \neq 0$ does not require $\langle a_\lambda \rangle \neq 0$. We shall show that (3.32) holds only for $\omega - \varepsilon_c \ll \omega_0$. We write the self-consistency equation with allowance for (3.26) and (3.28):

$$q = \frac{1}{NV} \sum_\lambda a_\lambda^2 = \int \rho(\varepsilon) a^2(\varepsilon) d\varepsilon$$

$$= \frac{1}{4\pi} \int_\omega^\infty \rho_0 \frac{N\varkappa^{-3}}{\omega^{1/2}} (\varepsilon - \omega) l_0^3 \left(\frac{\varepsilon_0}{\varepsilon - \varepsilon_c} \right)^{3\nu} d\varepsilon. \tag{3.33}$$

We have replaced $\rho(\varepsilon)$ by $\rho_0 \sim \varkappa^3 \varepsilon_0^{1/2}$ [see (3.4)], since the density of states has no singularities near the localization threshold. The integral (3.33) is of the order of τ_0 for $\omega \sim \varepsilon_0$ and increases as $\omega \to \varepsilon_c$:

$$q = C_1 \tau_0 \left(\frac{\varepsilon_0}{\omega - \varepsilon_c} \right)^{3\nu - 2} \sim \tau_0 \zeta^{2-3\nu}, \tag{3.34}$$

where $C_1 \sim 1$, $\zeta(\omega - \varepsilon_c)/\varepsilon_0$, and the exponent ν is known to be larger than 2/3; we shall henceforth set $\nu = 1$ (see Ref. 13 for details). Taking (3.32) into account, we get

$$\zeta \sim \tau_0 / |\tilde{\tau}|. \tag{3.35}$$

Thus, with decreasing temperature the condensation of localized modes occurs closer and closer (in "energy") to the localization threshold. The signs σ_λ of the amplitudes a_λ fluctuate strongly at the absolute values $|a_\lambda|$ determined by equation (3.27). Thus for $-\tilde{\tau} \gg \tau_0$ a superparamagnetic picture appears: there are many independent collective degrees of freedom $\sigma_\lambda = \pm 1$, and the change of each sign σ_λ leads to a rearrangement of the spin configuration in a large volume $\sim l_\lambda^3$. We emphasize that this picture differs from an ordinary superparamagnet in that in our case there are no rigid, spatially separate clusters (see Ref. 13 for details).

In deriving (3.34) and (3.35) we used expression (3.27), which we obtained by minimizing $H_0^{(\lambda)}$ from (3.25). Actually, the expression for the "interaction vertex" (i.e., for the coefficient of $a_\lambda^4 I_1$) given in (3.25) does not apply when $\omega \approx \varepsilon_c$, but it can be shown that the corrections reduce to a factor of order unity, which is unimportant for our analysis.

4. When the temperature and the value of ζ decrease [see (3.35)] the sign-dependent part of the mode interaction grows. The binary interaction of the signs $\sigma_\lambda = a_\lambda / |a_\lambda|$ is given by the second term in (3.30) and is of the form ($\omega \sim \varepsilon_0$)

$$H_\sigma = \frac{C_\sigma}{N^{1/2}} \sum_{\lambda \neq \mu} a_\lambda a_\mu \int d^3 r \Psi_\lambda^\alpha(\mathbf{r}) \Psi_\mu^\alpha(\mathbf{r}) \delta q(\mathbf{r}) = \sum_{\lambda \neq \mu} I_{\lambda\mu} \sigma_\lambda \sigma_\mu. \tag{3.36}$$

Here

$$C_\sigma \sim 1, \quad \delta q(\mathbf{r}) = N^{-1} \sum_\nu a_\nu^2 (\Psi_\nu^\beta(\mathbf{r}))^2 - q.$$

We recall that the absolute values $|a_\lambda|$ are determined by the sign-independent terms of the interaction. From the second statement in (3.36), which defines the quantities $I_{\lambda\mu}$, we see that $I_{\lambda\mu}$ takes both signs and has a zero mean. Moreover, the presence of the random function $\delta q(\mathbf{r})$ in the integral implies that $I_{\lambda\mu}$ is not correlated for different pairs (λ, μ). Interaction (3.36) is thus of the same form as for an Ising spin glass. The effective value of this interaction is determined by the parameter $I_1 = \tilde{Z}^{1/2} \overline{(I_{\lambda\mu}^2)}^{1/2}$, where $\tilde{Z}$ is the effective number of interacting "neighbors," i.e., the number of states with energies in the interval $(\varepsilon_\lambda - \varepsilon_c)/\varepsilon_0 \sim \zeta$ and in a volume of the order of $l^3(\zeta)$. It is easy to see that $\tilde{Z} \sim \zeta^{-2}$. In estimating $\overline{I_{\lambda\mu}^2}$ we should take into account that the wave functions $\Psi_\lambda^\alpha(\mathbf{r})$ and $\Psi_\lambda^\alpha(\mathbf{r})$ are correlated within a length $L \sim \varkappa^{-4}$ and are not correlated over larger scales [see (3.10)]; the scale of the fluctuations $[(\delta q(\mathbf{r}))^2]^{1/2}$ goes as $\sim \tau_0$ for all τ, since the states with $(\varepsilon_\lambda - \varepsilon_c)/\varepsilon_0 \sim \zeta$, which give the leading contribution to q, overlap one another strongly, and their contribution to $\delta q(\mathbf{r})$ is of order $q\tilde{Z}^{-1/2} \sim |\tilde{\tau}| \zeta \sim \tau_0$. Finally, for the "glass" interaction parameter I_1 we have

$$I_1 = \tilde{Z}^{1/2} \overline{(I_{\lambda\mu}^2)}^{1/2} \infty \frac{1}{N} \zeta^{-1/2} \sim |\tilde{\tau}| / \varkappa^6. \tag{3.37}$$

For $-\tilde{\tau} \sim \varkappa^6 \gg \tau_0$ the interaction of the "spins" σ_λ becomes strong, and one is dealing with the problem of a phase transition in a spin glass. Here the effective number of neighbors is $\tilde{Z} \sim \zeta^{-2} \sim N^{4/3} \gg 1$, so that we may apply the results of Ref. 13 for an Ising glass with a long interaction range. A freezing phase transition will occur at $-\tau = \tau_f \sim \tau_c$.

4. PHYSICAL QUANTITIES IN THE CRITICAL REGION

In Sec. 2 we calculated the magnetic susceptibility and structure factor in the ferromagnetic correlation region. Let us do the analogous calculations for the superparamagnetic region $-\tau_f < \tau \lesssim \tau_c$, using the representation of eigenmodes a_λ. For the average spin correlator we have

$$G(\mathbf{r}) = \langle \overline{S(\mathbf{r}) S(0)} \rangle = \frac{1}{V} \sum_{\lambda\mu} \langle a_\lambda a_\mu \rangle \overline{\Psi_\lambda^\alpha(\mathbf{r}) \Psi_\mu^\alpha(0)}$$

$$= \frac{1}{V} \sum_\lambda \langle a_\lambda^2 \rangle \overline{\Psi_\lambda^\alpha(\mathbf{r}) \Psi_\lambda^\alpha(0)}. \tag{4.1}$$

In the region $\tilde{\tau} \gg \tau_0$ the sum over the spectrum is predominantly from delocalized states with $-\varepsilon_\lambda \gg \varepsilon_0$, for which $\langle a_\lambda \rangle^2 = (\omega + |\varepsilon_\lambda|)^{-1}$. Using (3.4) for the density of states and (3.10) for the correlator of the wave functions, we obtain

$$G(\mathbf{r}) = N \frac{\varkappa^2}{4\pi r} \int_0^\infty \frac{\sin(r\varkappa\varepsilon^{1/2})}{\varepsilon + \omega} d\varepsilon e^{-r/L}$$

$$= N \frac{\varkappa^2}{4\pi r} \exp\{-r(L^{-1} + \varkappa\omega^{1/2})\}, \tag{4.2}$$

where ω is given in (3.18). For the homogeneous susceptibility χ and correlation length r_c we have

$$\chi \propto \int G(\mathbf{r})\, d^3r \propto \frac{1}{[(\varkappa L)^{-1}+(\beta^2\varkappa^6/64\pi^2+\tilde{\tau})^{1/2}-\beta\varkappa^3/8\pi]^2};$$

$$r_c \propto \chi^{1/2}. \tag{4.3}$$

For $\tilde{\tau} \gg \tau_c$ expression (4.3) goes over to the paramagnetic formula (2.10), and for $\tau \lesssim \tau_c$ it goes to a constant. Behavior of just this kind was observed in a recent experiment.[6]

In the temperature region $\tilde{\tau}<0$, $\tau_f \gtrsim |\tilde{\tau}| \gg \tau_0$ the sum over the spectrum in (4.1) also contains a contribution from the condensed states, for which the wave-function correlator is of the form (3.11). As a result, for the structure factor $S(\mathbf{p})$ we have

$$S(\mathbf{p}) = \int G(\mathbf{r})\, d^3r\, e^{i\mathbf{pr}} = \frac{1}{p^2/\varkappa^2+\chi^{-1}} + \frac{|\tilde{\tau}|}{(p^2/\varkappa^2+\chi^{-1})^2}. \tag{4.4}$$

It is also of interest to calculate the nonlinear susceptibility $\tilde{\chi} = -\partial^2\chi/\partial h^2$, which increases anomalously[13] at the phase transition in a spin glass. Above the freezing point it can be written in the form

$$\tilde{\chi} \sim \frac{1}{V}\sum_{\lambda} \langle a_\lambda^2\rangle^2 = N\int \rho(\varepsilon)\langle a^2(\varepsilon)\rangle^2\, d\varepsilon. \tag{4.5}$$

For $\tilde{\tau} \gg \tau_0$ the leading contribution to (4.5) is from the region of delocalized states with $-\varepsilon \sim \omega$. In analogy with (4.3) we have

$$\tilde{\chi} \sim \frac{N\varkappa^3}{(\tilde{\tau}+\beta^2\varkappa^6/64\pi^2)^{1/2}-\beta\varkappa^3/8\pi} = \begin{cases} N\varkappa^3/\tilde{\tau}^{1/2} & \text{for } \tilde{\tau} \gg \varkappa^6, \\ N\varkappa^6/\tilde{\tau} & \text{for } \tau_0 \ll \tilde{\tau} \ll \varkappa^6. \end{cases} \tag{4.6}$$

For $\tilde{\tau}<0$, $\tau_0 \ll |\tilde{\tau}| \lesssim \tau_f$ the main contribution to $\tilde{\chi}$ is from condensed states with $(\varepsilon-\varepsilon_c)/\varepsilon_0 \sim \zeta$. A calculation analogous to (3.33) gives

$$\tilde{\chi} \sim N^{11/3}(|\tau|/\varkappa^6)^3. \tag{4.7}$$

Formula (4.7) is valid in a region not too close to the phase transition, viz., for $(\tilde{\tau}-\tau_f) \gtrsim \varkappa^6$, where the interaction of the signs σ_λ can be neglected [see (3.37)]. Closer to T_f the nonlinear susceptibility behaves the same as in an Ising spin glass.[13]

5. CONCLUSION

We have considered a three-dimensional amorphous magnet with a strong random anisotropy on the assumption of a long interaction range $1/\varkappa \gg c^{-1/3}$. At a temperature close to $T_1 = (c/3)\int J(r)d^3r$ is the density of magnetic ions) there is a transition to a disordered phase. This phase is macroscopically similar to a spin glass but has a relatively large spin correlation length, so that it could naturally be called a correlated spin glass. We have shown that at temperatures $\tau = (T-T_1)/T_1 \gg 0.01(c^{-1}\varkappa^3)^2$ the random anisotropy is unimportant, and the correlations in the system are the same as in a pure ferromagnet. Here the maximum ferromagnetic correlation length is found to be $L \sim 10\, c/\varkappa^4$, i.e., much larger than the range of the interaction. At distances smaller than L the system thus behaves the same as a ferromagnet. For $\tau \lesssim 0.01\ (\varkappa^3/c)^2$ we used the $1/N$ expansion to separate out the slow degrees of freedom responsible for the critical behavior of the system. The effective Hamiltonian for these variables turned out to be the same as for an Ising spin glass.[13] It was thereby demonstrated that these two problems belong to the same universality class from the standpoint of the theory of phase transitions. We are hopeful that this conclusion wil remain valid at $N=3$. The equivalence of the two Hamiltonians can explain the experimentally observed[7,11] AT line, which is characteristic of Ising spin glasses. In addition, amorphous magnets with a strong anisotropy are described by the same hierarchical picture that was studied in Ref. 13. At $T = T_f = T_1(1-\tau_f)$, where $\tau_f \sim 0.01(c^{-1}\varkappa^3)^2$, a freezing phase transition occurs, characterized by the appearance of correlations of arbitrarily large spatial dimensions.

We have found the temperature dependence of the magnetic susceptibility χ above the freezing point T_f [see (4.3)]. In the ferromagnetic correlation region $\tau \gg 0.1(c^{-1}\varkappa^3)^2$ we found $\chi \sim \tau^{-1}$, while at lower temperatures χ approaches a constant. This behavior of the susceptibility has been observed experimentally.[6] The nonlinear susceptibility $\tilde{\chi} = -\partial^2\chi/\partial h^2$ begins to grow strongly in the region where χ approaches a constant [see (4.6) and (4.7)]. We also found the form of the structure factor $S(\mathbf{p})$ in different temperature regions above T_f. For $\tau \gg 0.01\ (c^{-1}\varkappa^3)^2$ and in zero external field, $S(\mathbf{p})$ has a Lorentzian shape with a correlation length $r_c \sim \tau^{-1/2}$. The application of an external field will lead to a decrease in r_c and to the appearance of an additional term having the form of a squared Lorentzian [see (2.14) and (2.15)]. For $\tau<0$, $|\tau| \lesssim \tau_f$ the additional term in $S(\mathbf{p})$ should be observed even without an external field [see (4.4)]. We emphasize that this additional term arises in the region above the freezing point (in our notation $T = T_f$ corresponds to $\tau = -\tau_f$) and is due to the purely exponential behavior of the correlator of the eigenfunctions $\Psi_\alpha^\lambda(\mathbf{r})$ with "energies" E_λ near the edge of the spectrum [see (3.10)]. We assume that as the temperature is decreased further in the region $T < T_f$ the relative contribution of the squared Lorentzian should increase smoothly.

We have thus shown that the assumption of a relatively long interaction range in amorphous rare earth magnets with strong random anisotropy permits explanation of both the ferromagnetic and "glass-like" properties of these systems. In fact, it is sufficient to have an interaction range $\varkappa^{-1} \sim 1.5$ lattice constants, which is quite realistic. It was recently pointed out[18] that one can obtain a qualitative explanation of these properties in a model with a nearest-neighbor interaction if it is assumed that the random anisotropy axes are correlated over a large length L equal to the low-temperature spin correlation length (recall that we assumed these axes to be completely uncorrelated). Setting aside the question of whether such correlations are possible, we note that in this case the temperature dependence of the susceptibility and correlation length in the paramagnetic region should be of the scaling type with the exponents of the "pure" Ising model, while the nonlinear susceptibility should not exhibit any critical anomalies.

We wish to thank L. B. Ioffe for helpful discussions.

[1]R. Harris, M. Plischke, and H. J. Zuckermann, Phys. Rev. Lett. **31**, 160 (1973).
[2]Y. Imry and S.-K. Ma, Phys. Rev. Lett. **35**, 1399 (1975).
[3]A. Aharony and E. Pytte, Phys. Rev. B **27**, 5872 (1983).
[4]Vik. S. Dotsenko and M. V. Feĭgel'man, J. Phys. C **16**, L803 (1983); Vik. S. Dotsenko and M. V. Feĭgel'man, Zh. Eksp. Teor. Fiz. **86**, 1544 (1984) [Sov. Phys. JETP **59**, 904 (1984)].
[5]E. M. Chudnovsky and R. A. Serota, J. Phys. C **16**, 4181 (1983).
[6]B. Barbara and B. Dieny, Physica B **130**, 245 (1985).
[7]B. Dieny and B. Barbara, J. Phys. (Paris) **46**, 293 (1985).
[8]J. M. D. Coey, T. R. McGuire, and B. Tissier, Phys. Rev. B **24**, 1261 (1981).
[9]S. J. Pickart, H. A. Alperin, R. J. Gambino, and T. R. McGuire, J. Appl. Phys. **55**, 1763 (1984).
[10]D. J. Sellmyer and S. Nafis, J. Appl. Phys. **57**, 3584 (1985).
[11]D. J. Sellmyer and S. Nafis, J. Magn. Magn. Mater. **54-57**, 113 (1986).
[12]J. R. L. de Almeida and D. J. Thouless, J. Phys. A **11**, 983 (1978).
[13]L. B. Ioffe and M. V. Feĭgel'man, Zh. Eksp. Teor. Fiz. **89**, 654 (1985) [Sov. Phys. JETP **62**, 376 (1985)].
[14]J. H. Chen and T. C. Lubensky, Phys. Rev. B **16**, 2106 (1977).
[15]A. J. Bray and M. A. Moore, J. Phys. C **18**, L139 (1985).
[16]A. Aharony, Phys. Rev. B **12**, 1038 (1975).
[17]S. L. Ginzburg, Zh. Eksp. Teor. Fiz. **80**, 244 (1981) [Sov. Phys. JETP **53**, 124 (1981)].
[18]E. M. Chudnovsky, Phys. Rev. B **33**, 2021 (1986).

Translated by Steve Torstveit