Magnetic symmetry of domain walls with Bloch lines in ferromagnets and ferrites

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A generalized concept of a Bloch line is presented, and a condition is found for Block lines to be present in ferromagnets and ferrites. The relationship between the specific magnetic structure in magnetically ordered crystals and the properties of domain walls with Bloch lines is investigated. Magnetic symmetry classes are constructed for planar 180° domain walls with Block lines in ferromagnets and ferrites, and the coordinate dependence of the magnetization density in domain walls belonging to each of the classes is described qualitatively. It is shown that there are 32 magnetic classes for domain walls with Bloch lines in ferromagnets and ferrites. Kinematic magnetic classes are also found which describe a domain wall with a Bloch line moving at constant velocity.

INTRODUCTION

Domain walls in magnetically ordered materials are currently under active study.[1] Two types of domain structures with different walls can be distinguished. In the first type the domain structures are thermodynamically stable and form near first-order phase transitions induced by an external magnetic field **H** (Ref. 2). The usual domain structure in ferromagnets and ferrites is stable even when $\mathbf{H} = 0$ and is of this type. These domains are stabilized by the magnetic dipole interaction field $\mathbf{H}_m$ in magnets of finite size, and also by the fact that at least two of the distinct magnetic phases are energy-degenerate; we take the magnetization vectors in these phases to be $\mathbf{M}_1$ and $\mathbf{M}_2$, respectively ($\mathbf{M}_2 = -\mathbf{M}_1$ for ferromagnets). The condition for a first-order phase transition is that the thermodynamic potentials Φ of the different phases be equal.

Another type of domain structure is also possible for which the magnetic states have the same magnetization **M** but different antiferromagnetic vectors **L**. Then $\mathbf{H}_m = 0$ and domains form randomly as the system goes from a disordered to an ordered state; such a domain structure is said to be "kinetic" (Ref. 3).

The crystal symmetry determines the energy degeneracy of magnetic phases with different **M** or **L**; similarly, the boundary conditions for a domain wall in a crystal determine the possible energy degeneracy for walls with different **M** or **L** distributions (this is discussed in more detail below).

In this case the kinetic mechanism may give rise to a complex domain wall which consists of regions that have the same surface energy but different magnetization distributions. The boundaries separating these regions are called Bloch lines. Bloch lines formation in a domain wall thus requires that the latter be energy-degenerate but is not thermodynamically inevitable. This process is analogous to kinetic formation of domain walls in antiferromagnets or between crystallite boundaries in polycrystals during phase transitions. Although it is possible that energy factors may also stabilize Bloch walls, their thermodynamic stability remains an open question.

Although studies of domain walls with Bloch lines are of great interest, no accurate description is available. It may therefore be helpful to develop a classification system (magnetic symmetry classes) for such walls to qualitatively describe the distribution of the magnetization.

A complete symmetry classification was given in Ref. 4 for plane 180° domain walls in magnetically ordered crystals. The purpose of the present paper is to develop an analogous classification for walls with Bloch lines in ferromagnets and ferrites. As in Ref. 4, we limit ourselves to domain walls with dimensions large compared to interatomic distances; we do not consider the space groups, but only the symmetry classes of domain walls with Bloch lines. We show that there are 32 such symmetry classes altogether. Finally, we analyze how the symmetry and spatial structure of a domain wall at rest are altered by the uniform motion of a Bloch line.

1. CONDITION FOR EXISTENCE OF BLOCH LINES

Bloch lines are quite common in domain walls in ferromagnets and ferrites.[1] However, it was shown in Ref. 5 that they cannot exist in magnetic materials belonging to certain crystallographic classes, because the different directions along which the magnetization vector rotates are not energetically equivalent.

We will find a general condition for Bloch lines to be present. First we observe that in terms of their symmetry properties, ferromagnets and ferrites (systems which have inequivalent magnetic atoms) are identical. We will therefore use the abbreviation FM to denote both ferromagnets and ferrites. Let $G_{\rm pm}$ be the symmetry class for the paramagnetic phase of an FM crystal, let $G_{\rm bc}$ be the symmetry class of the boundary conditions for a plane domain wall in an FM with no Bloch lines, and let $G_{\rm k}$ be the symmetry class for a planar domain wall without Bloch lines (see Ref. 4 for the definition of $G_{\rm bc}$ and $G_{\rm k}$). Then

$$G_{\rm k} \subset G_{\rm bc} \subset G_{\rm pm}, \quad G_{\rm bc} = \sum_{l=1}^{m} g_l G_k^4, \tag{1}$$

Eq. (1) shows that $G_{\rm bc}$ is a sum over $G_{\rm k}$ whose coefficients g_l are representatives of the coset classes (they are called lost operations in the theory of phase transitions), and m is the index of the subgroup $G_{\rm k}$ in $G_{\rm bc}$ (Ref. 6).

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By analogy with the theory of phase transitions,[6] one can show that when the boundary conditions in the domain (i.e., G_{bc}) are specified, there exist m energetically equivalent domain walls which have the same symmetry G_k but different magnetization distributions $\mathbf{M}_l(x)$, where

$$\mathbf{M}_l(x)=g_l\mathbf{M}_1(x);\ l=1,2,\ldots,m;\ g_1=1,$$

and the x axis is normal to the plane of the wall.

Bloch lines form when regions with different $\mathbf{M}_l(x)$ coexist in a domain wall. Since for each region l with distribution $\mathbf{M}_l(x)$ there are $m-1$ regions l' with distribution $\mathbf{M}_{l'}$ ($l'\neq 1$), we have $m(m-1)$ types of boundary conditions altogether for a Bloch line in a domain wall when the boundary conditions (G_{bc}) in the domains are specified. Each type generates a symmetry group G_{bc}^{BL} that characterizes the boundary conditions for the Bloch line in the wall; the elements of this group do not alter the magnetization distribution in domains far from the domain wall nor the magnetization distribution in the domain wall itself far from the Bloch line.

We denote the symmetry group of the Bloch line by G_k; it is a subgroup of G_{bc}^{BL},

$$G_{bc}^{BL}=\sum_{\mu=1}^{\nu} g_\mu U_k.$$

Its index ν in G_{bc}^{BL} gives the number of possible energetically equivalent types of Bloch line for a given G_{bc}^{BL}. Thus,

$$N=m(m-1)\nu \tag{2}$$

different Bloch lines are possible for a specified G_{bc}. It is important to note that all of these lines are energetically equivalent. Bloch lines can thus exist only when G_k is a proper subgroup of G_{bc} (i.e., its index ν is $\geqslant 2$).

We consider two examples illustrating the above condition. Let us suppose that the symmetry group G_{bc} for the boundary conditions coincides with the group G_1 in the classification in Ref. 4: $G_1=(1,\bar{2}_z,\bar{2}_y,\bar{2}_x\cdot(1,\bar{1}'))$. Here the z axis is parallel to $\mathbf{M}$ inside the domains, and the normal n to the wall points along $\mathbf{e}_x$, the unit vector along the x axis; the y axis is parallel to the wall ($y\perp x,z$).

As usual, the symbol 2_α denotes a rotation of order 2 about the α axis ($\alpha=x,y,z$); $\bar{2}_\alpha=2_\alpha\cdot\bar{1}$ is a reflection plane perpendicular to the α axis, and $\bar{1}$ is an inversion. A prime on a symmetry element indicates that a time-reversal operation is simultaneously performed.

Let the symmetry group G_k of the domain wall coincide with the symmetry group $G_7=(1,2_z',2_y,2_x')$ for a Bloch wall. Since G_7 has index 2 in G_1, there are 2 energetically equivalent domain walls. The anti-inversion $\bar{1}'$ is the lost symmetry operation here; it changes the component M_y of the magnetization distribution in the G_7 magnetic class which is symmetric with respect to x: $M_y\to -M_y$. As a result, two energetically equivalent domain walls with different magnetization distributions can coexist. This result reflects the familiar fact that the energies are equal for Bloch walls in which the $\mathbf{M}$ vector rotates in opposite directions. The number of Bloch lines in this magnetic class will be examined in more detail below.

As a second example, consider the following. As before, let the symmetry group G_{bc} coincide with G_1 but take the symmetry group of the domain wall to be $G_k=G_3=(1,\bar{2}_z,2_y,\bar{2}_x)$. As in the previous example, G_k has index 2 in G_{bc} and the lost symmetry operation can again be taken to be an anti-inversion. However, in this case only $M_z(x)$ differs from zero in the domain wall, and the antisymmetric function $M_z(x)$ is left unchanged by $\bar{1}'$. However, $\bar{1}'$ does change certain other characteristics such as the electric polarization $\mathbf{P}$ or the deformation, for example. It was shown in Refs. 7 and 8 that in such a domain wall, $\mathbf{P}$ has a component lying in the plane of the wall and is a symmetric function of x. Since the $\bar{1}'$ operation changes the sign of this component, such a domain wall can also posses Bloch lines which, however, differ essentially from Bloch lines of the normal type.

In the most general case, a Bloch line separates regions of a domain wall which are characterized by different physical parameters that transform under irreducible representations of the symmetry group G_{bc} for the boundary conditions. It is important to note that (as follows from the above example) Bloch lines may separate regions of the wall that have equivalent magnetization distributions but differ in other respects.

We will thus distinguish between two kinds of lines—magnetic Bloch lines, which separate regions with different magnetization distributions, and nonmagnetic Bloch lines, which separate regions which have identical magnetization distributions but in which certain other characteristics of the domain wall differ.

Expression (2) describes both magnetic and nonmagnetic Bloch lines. We will confine ourselves henceforth to magnetic Bloch lines. The quantity

$$\mathbf{q}_l(x)=\pi^{-1}M_0^{-2}[\mathbf{M}_l d\mathbf{M}_l/dx] \tag{3}$$

is convenient for characterizing the different energetically equivalent distributions $\mathbf{M}_l(x)$ in a domain wall; here M_0 is the magnitude of the magnetization inside the domains. We call $\mathbf{q}_l(x)$ the differential helicity of the domain wall; the corresponding total helicity is

$$\mathbf{Q}_l=\int_{-\infty}^{\infty}\mathbf{q}_l(x)\,dx. \tag{4}$$

Like the magnetization $\mathbf{M}_l(x)$, the differential helicity $\mathbf{q}_l(x)$ is left invariant by the group G_k. However, the lost operations g_l change both $\mathbf{M}_l(x)$ and $\mathbf{q}_l(x)$. Each of the energetically equivalent walls is thus characterized by its own differential helicity $\mathbf{q}_l(x)$, and the same is true for the total helicity $\mathbf{Q}_l$. The total helicity is related conceptually to the notion of the winding number of the vector $\mathbf{M}$ in closed domains.[9]

Let us calculate the differential and total helicity for Bloch and Néel walls for the case when $G_{bc}=G_1$. In a Cartesian coordinate system, we have

$$M_{1,2}^{(x)}=0,\quad M_{1,2}^{(y)}=\pm M_0\sin\theta,\quad M_{1,2}^{(z)}=M_0\cos\theta, \tag{5}$$

$$\cos\theta=-\mathrm{th}(x/\Delta), \tag{6}$$

for a planar Bloch domain wall; here θ is the rotation angle of $\mathbf{M}$ in the wall, which is of width Δ. From (5) we obtain

$$\mathbf{q}_{1,2}=q_{1,2}^{(B)}\mathbf{e}_x,\quad \mathbf{Q}_{1,2}=Q_{1,2}^{(B)}\mathbf{e}_x,$$
$$q_{1,2}^{(B)}=\frac{1}{\pi M_0^2}\left(M_{1,2}^{(y)}\frac{dM_{1,2}^{(z)}}{dx}-M_{1,2}^{(z)}\frac{dM_{1,2}^{(y)}}{dx}\right)=\pm\frac{1}{\pi}\frac{d\theta}{dx}. \tag{7}$$

For a Bloch domain wall we readily see that $Q_{1,2}^{(B)}=\pm 1$, depending on the direction of the rotation.

Similarly, for a Néel wall

$$M_{1,2}^{(x)}=\pm M_0\sin\theta,\quad M_{1,2}^{(y)}=0,\quad M_{1,2}^{(z)}=M_0\cos\theta, \tag{8}$$

$$\mathbf{q}_{1,2}=q_{1,2}^{(N)}\mathbf{e}_y,\quad \mathbf{Q}_{1,2}=Q_{1,2}^{(N)}\mathbf{e}_y,$$

$$q_{1,2}^{(N)}=\frac{1}{\pi M_0^2}\left(M_{1,2}^{(z)}\frac{dM_{1,2}^{(x)}}{dx}-M_{1,2}^{(x)}\frac{dM_{1,2}^{(z)}}{dx}\right)=\pm\frac{1}{\pi}\frac{d\theta}{dx}. \tag{9}$$

Here $\mathbf{e}_y$ is the unit vector along the y axis; $Q_{1,2}^{(N)}=\pm 1$.

2. MAGNETIC SYMMETRY CLASSES FOR DOMAIN WALLS WITH BLOCH LINES

Before developing a symmetry classification for domain walls with magnetic Bloch lines, one should note that the symmetry class U_k of a wall with a Bloch line is a subgroup of the magnetic symmetry class for the paramagnetic phase of the crystal. The same assertion also holds for the class G_k. However, in general U_k is not contained in G_k:

$$U_k \subset G_{bc} \subset G_{pm}, \quad U_k \not\subset G_k. \tag{10}$$

We will therefore not use the classes G_k directly to construct the magnetic symmetry classes U_k below but will proceed instead as follows. First we observe that since the problem here is two-dimensional (unlike the case in Ref. 4), an approach based on classifying the possibly boundary conditions for the vector $\mathbf{M}$ is awkward because one must analyze the various orientations of the Bloch line relative to the directions specified in the boundary conditions. For the same reason, it is inconvenient to use a coordinate system[4] in which one axis moves together with $\mathbf{M}(\pm\infty)$. We therefore choose the coordinate system shown in Fig. 1 for an FM with a plane domain wall containing a Bloch line: the wall lies in the yz plane, the Bloch line is along the z axis, $\mathbf{e}_x$ is parallel to the normal $\mathbf{n}$ to the wall, and the y coordinate is parallel to the wall and perpendicular to the Bloch line.

The elements of the magnetic symmetry classes must necessarily preserve the geometry shown in Fig. 1 and must thus be contained in the following set of 16 elements:

$$U:\ 1,\ 2_z,\ 2_x,\ 2_y,\ \bar{1},\ \bar{2}_z,\ \bar{2}_x,\ \bar{2}_y,$$
$$1',\ 2_z',\ 2_x',\ 2_y',\ \bar{1}',\ \bar{2}_z',\ \bar{2}_x',\ \bar{2}_y'. \tag{11}$$

To find the possible symmetry of the magnetization distributions corresponding to these elements, we first introduce some terminology. Distributions that are symmetric (antisymmetric) under the transformation $x\to -x$ are denoted by S_x (A_x). Similarly, functions symmetric (antisymmetric) under $y\to -y$ are denoted by S_y (A_y), and functions invariant under $(x,y)\to(-x,-y)$ are denoted by $S_0(A_0)$.

We can divide the 16 operations in (11) into four sets.

1. We denote by g_1 the elements that do not interchange the positions of the domains or the domain wall segments (by segments we mean the portions of the wall to the right and left of a Bloch line). We thus have the operations

$$g_1:\ 1,\ 1',\ \bar{2}_z,\ \bar{2}_z'. \tag{12}$$

They constrain the magnetization distribution as follows:

a) If $g_1M_\alpha = M_\alpha$ then g_1 imposes no constraints on the form of M_α; such functions will be denoted by the symbol $\oplus$;

b) If $g_1M_\alpha = -M_\alpha$, then $M_\alpha = 0$.

2. Elements that interchange domains but not segments are denoted by g_2. Thus,

$$g_2:\ 2_y,\ \bar{2}_x,\ 2_y',\ \bar{2}_x'. \tag{13}$$

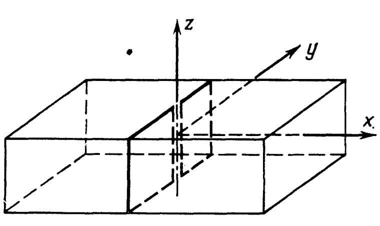

FIG. 1. Coordinate system for a crystal with a Bloch line.

TABLE I. Symmetry of the components of the magnetization corresponding to elements and subgroups of second order in the starting group U.

k	Symmetry elements	$M_z(x,y)$	$M_x(x,y)$	$M_y(x,y)$
1	$(1,)2_z$	S_0	A_0	A_0
2	$(1,)2_x$	A_y	S_y	A_y
3	$(1,)2_y$	A_x	A_x	S_x
4	$(1,)\bar{2}_z$	$\oplus$	0	0
5	$(1,)\bar{2}_x$	A_x	S_x	A_x
6	$(1,)\bar{2}_y$	A_y	A_y	S_y
7	$(1,)2_z'$	A_0	S_0	S_0
8	$(1,)2_x'$	S_y	A_y	S_y
9	$(1,)2_y'$	S_x	S_x	A_x
10	$(1,)\bar{1}'$	A_0	A_0	A_0
11	$(1,)\bar{2}_z'$	0	$\oplus$	$\oplus$
12	$(1,)\bar{2}_x'$	S_x	A_x	S_x
13	$(1,)\bar{2}_y'$	S_y	S_y	A_y
(14)	$(1,)\bar{1}$	S_0	S_0	S_0
(15)	$(1,)1'$	0	0	0

The constraints imposed by g_2 are as follows:

a) if $g_2M_\alpha = M_\alpha$ then $M_\alpha \in S_x$;

b) if $g_2M_\alpha = -M_\alpha$ then $M_\alpha \in A_x$.

3. Elements that interchange segments but not domains are denoted by g_3. Thus

$$g_3:\ 2_x,\ \bar{2}_y,\ 2_x',\ \bar{2}_y'. \tag{14}$$

For these operations we have the rules

a) if $g_3M_\alpha = M_\alpha$ then $M_\alpha \in S_y$;

b) if $g_3M_\alpha = -M_\alpha$ then $M_\alpha \in A_y$.

4. Elements that interchange both the domains and the domain wall segments are denoted by g_4:

$$g_4:\ \bar{1},\ \bar{1}',\ 2_z,\ 2_z'. \tag{15}$$

We have the rules

a) if $g_4M_\alpha = M_\alpha$ then $M_\alpha \in S_0$;

b) if $g_4M_\alpha = -M_\alpha$ then $M_\alpha \in A_0$.

Using these rules, one can determine the symmetry of the components of the magnetization vector corresponding to the operations in (11) (Table I). Naturally, we are interested only in the symmetry classes for which a magnetic Bloch line can exist in the domain wall. We will use the fact that the magnetization distribution in a domain wall with a Bloch line must satisfy the following two conditions.

I. For a domain wall to exist, it is necessary and sufficient that the magnetization distribution $\mathbf{M}$ have at least one component $M_\alpha \neq 0$ whose symmetry is consistent with the constraint A_xS_y. Consistency with the condition A_x ensures that $\mathbf{M}$ is aligned oppositely in the domains to the left and right of the wall, while the condition S_y ensures that the magnetization is uniformly distributed within the domains.

II. In order for a Bloch line to be present in a domain wall, it is necessary and sufficient that $\mathbf{M}$ have at least one component $M_\beta \neq 0$ whose symmetry is consistent with A_y (in particular, one may have $\alpha = \beta$). This condition is necessary to ensure that $\mathbf{M}$ varies spatially along the domain wall, as required for a magnetic Bloch line. This variation is of course described by a function that contains an antisymmetric part. It should be noted that the conditions mentioned above constrain the magnetization distribution in the Bloch line itself and at distances that are not too large compared to the width of the line. Far from the Bloch line (but within the domain wall), the magnetization distribution is described by the classes G_k. Far from the domain wall (i.e., inside other domains), $\mathbf{M}$ is of course described by one of the Shubnikov symmetry classes

TABLE II. Symmetry of the components of the magnetization specified by subgroups of U of order 4 and 8 and corresponding to domain walls with Bloch lines.

k	Symmetry elements	$M_z(x, y)$	$M_x(x, y)$	$M_y(x, y)$
14	$1,\ 2_z,\ 2_x,\ 2_y$	$S_0A_x(A_y)$	$A_0A_x(S_y)$	$A_0S_x(A_y)$
15	$1,\ 2_z,\ \bar{2}_x,\ \bar{2}_y$	$S_0A_x(A_y)$	$A_0S_x(A_y)$	$A_0A_x(S_y)$
16	$1,\ \bar{1}',\ 2_z,\ \bar{2}_z'$	0	A_0	A_0
17	$1,\ \bar{1}',\ 2_x,\ \bar{2}_x'$	$A_0S_x(A_y)$	$A_0A_x(S_y)$	$A_0S_x(A_y)$
18	$1,\ \bar{1}',\ 2_y,\ \bar{2}_y'$	$A_0A_x(S_y)$	$A_0A_x(S_y)$	$A_0S_x(A_y)$
19	$1,\ 2_z,\ \bar{2}_x',\ \bar{2}_y'$	$S_0S_x(S_y)$	$A_0A_x(S_y)$	$A_0S_x(A_y)$
20	$1,\ 2_x,\ \bar{2}_z',\ \bar{2}_y'$	0	S_y	A_y
21	$1,\ 2_y,\ \bar{2}_z',\ \bar{2}_x'$	0	A_x	S_x
22	$1,\ 2_z,\ 2_x',\ 2_y'$	$S_0S_x(S_y)$	$A_0S_x(A_y)$	$A_0A_x(S_y)$
23	$1,\ 2_y,\ 2_z',\ 2_x'$	$A_0A_x(S_y)$	$S_0A_x(A_y)$	$S_0S_x(S_y)$
24	$1,\ \bar{2}_x,\ 2_z',\ \bar{2}_y'$	$A_0A_x(S_y)$	$S_0S_x(S_y)$	$S_0A_x(A_y)$
25	$1,\ 2_x',\ \bar{2}_y,\ \bar{2}_z'$	0	A_y	S_y
26	$1,\ \bar{2}_x,\ 2_y',\ \bar{2}_z'$	0	S_x	A_x
27	$1,\ \bar{1}',\ \bar{2}_z,\ 2_z'$	A_0	0	0
28	$1,\ \bar{1}',\ \bar{2}_x,\ 2_x'$	$A_0A_x(S_y)$	$A_0S_x(A_y)$	$A_0A_x(S_y)$
29	$1,\ \bar{1}',\ \bar{2}_y,\ 2_y'$	$A_0S_x(A_y)$	$A_0S_x(A_y)$	$A_0A_x(S_y)$
30	$1,\ \bar{1}',\ 2_z,\ 2_x,\ 2_y,\ \bar{2}_z',\ \bar{2}_x',\ \bar{2}_y'$	0	$A_0A_x(S_y)$	$A_0S_x(A_y)$
31	$1,\ \bar{1}',\ 2_z,\ \bar{2}_y,\ \bar{2}_x,\ 2_x',\ 2_y',\ \bar{2}_z'$	0	$A_0S_x(A_y)$	$A_0A_x(S_y)$

for the homogeneous phase. It is thus possible, for example, for the magnetization component M_α to contain a component of symmetry S_x in addition to one of symmetry A_x. The only requirement is that the symmetric component must vanish at distances $x \gg \Delta$.

Conditions I and II provide a generalized definition of a domain wall with a Bloch line. The above discussion enables us to immediately remove from consideration all classes containing the operations $1'$ and $\bar{1}$, as well as certain classes containing the elements $\bar{2}_z$ and $2_z'$.

We are then left with one class $U_0 \equiv 1$ of order 1, 13 classes $U_1 - U_{13}$ of order 2, 16 classes $U_{14} - U_{29}$ of order 4, and two classes U_{30}, U_{31} of order 8. The symmetry of the magnetization distribution determined by the second-order classes is listed in Table I ($k = 1 - 13$), while Table II gives the distributions for the higher-order classes ($k = 14 - 32$).

As an illustration, let us find the symmetry of M_x corresponding to the fourth-order class U_{14}: $1, 2_z, 2_x, 2_y$. We take the generators to be the elements 2_z and 2_y. By symmetry, we must have

$$M_x(x,\ y)=2_zM_x(2_zx,\ 2_zy)=2_yM_x(2_yx,\ 2_yy), \tag{16}$$

whence

$$M_x(x,\ y)=M_x(-x,\ -y)=-M_x(-x,\ y).$$

Thus, M_x has the symmetry $A_0S_x(S_y)$, and the information provided by A_0A_x (or A_xS_y) alone is sufficient for the classification (for this reason, the symmetry of the magnetization distribution with respect to y is indicated in parentheses in Table II for subgroups of order greater than 2).

It should be noted that the symbol S_0 in Table I can be interpreted in different ways—as $S_0S_x(S_y)$, as $S_0A_x(A_y)$, or as having no definite symmetry in the coordinates x and y taken separately. The situation is analogous for A_0.

3. ANALYSIS OF THE SYMMETRY OF THE MAGNETIZATION

We now examine the magnetization distributions in Tables I and II in greater detail. Table III lists the magnetic structures corresponding to the symmetry classes that admit Bloch lines. The second column lists the elements of the class, while the third enumerates the components M_α whose symmetry is consistent with condition I (specifying the direction of $\mathbf{M}$ in the domains). The fourth column lists the M_β for which condition II (necessary for a Bloch line to exist) is satisfied. The remaining nonzero components M_γ in the domain wall are listed in the next column, while the last column gives the international symbols for the classes in abbreviated form.

We note that several types of domain walls can be distinguished, depending on the symmetry elements determining the distribution $\mathbf{M}$ in the wall. For example, if the symmetry class of the domain wall contains at least one of the elements g_3 (14), the Bloch line can be said to have a center, i.e., the magnetization distribution has a plane of symmetry (or antisymmetry) with respect to the y coordinate. Walls whose symmetry group contains elements g_2 (13) have a central plane, i.e., $\mathbf{M}$ is symmetric (antisymmetric) under $x \to -x$. If the symmetry group contains elements g_4 (15) then the domain wall has a center with respect to which $\mathbf{M}$ is symmetric (antisymmetric). None of these features are present for a wall with a Bloch line if the symmetry group consists only of elements g_1 (12).

The following point should be noted. The magnetic classes U_{10}, $U_{14} - U_{18}$, and $U_{27} - U_{31}$ describe distributions $\mathbf{M}(x,y)$ for which $\mathbf{M}(0,0) = 0$. This means that Bloch lines described by these classes can exist either in FM (ferromagnets or ferrites) whose magnetic anisotropy energy exceeds the exchange interaction energy, or else near the Curie temperature. In ordinary FMs at low temperatures ($\mathbf{M}_2 = \text{const}$), 21 different types of Bloch lines are thus possible.

Figures 2–4 show some magnetization distributions corresponding to magnetic symmetry classes that admit Bloch lines. Figure 2a shows a Bloch wall (class U_{13}), while Fig. 2b shows a Néel wall (U_8). Figure 3 shows a head-to-head wall with a Bloch line (U_{17}), and Fig. 4 shows a wall in an FM for which the easy magnetization axis lies in the xy plane (U_{31}). This magnetic structure is reminiscent of the

TABLE III. Structure of domain walls with Bloch lines as determined by the various symmetry classes.

k	Symmetry elements	α	β	γ	International symbols
0	1	x, y, z	x, y, z		1
1	$1, 2_z$	x, y	x, y, z		2
2	$1, 2_x$	x	y, z		2
3	$1, 2_y$	x, z	x, y, z		2
4	$1, \bar{2}_z$	z	z		m
5	$1, \bar{2}_x$	y, z	x, y, z		m
6	$1, \bar{2}_y$	y	x, z		m
7	$1, 2_z'$	z	x, y, z		$2'$
8	$1, 2_x'$	y, z	x		$2'$
9	$1, 2_y'$	y	x, y, z		$2'$
10	$1, \bar{1}'$	x, y, z	x, y, z		$\bar{1}'$
11	$1, \bar{2}_z'$	x, y	x, y		m'
12	$1, \bar{2}_x'$	x	x, y, z		m'
13	$1, \bar{2}_y'$	x, z	y		m'
14	$1, 2_z, 2_x, 2_y$	x	y, z		222
15	$1, 2_z, \bar{2}_x, \bar{2}_y$	y	x, z		mm
16	$1, \bar{1}', 2_z, \bar{2}_z'$	x, y	x, y		$2/m'$
17	$1, \bar{1}', 2_x, \bar{2}_x'$	x	y, z		$2/m'$
18	$1, \bar{1}', 2_y, \bar{2}_y'$	x, z	y		$2/m'$
19	$1, 2_z, \bar{2}_x', \bar{2}_y'$	x	y	z	$m'm'$
20	$1, 2_x, \bar{2}_z', \bar{2}_y'$	x	x, y		$m'm'$
21	$1, 2_y, \bar{2}_z', \bar{2}_x'$	x	y		$m'm'$
22	$1, 2_z, 2_x', 2_y'$	y	x	z	$22'2'$
23	$1, 2_y, 2_z', 2_x'$	z	x	y	$22'2'$
24	$1, 2_z', \bar{2}_x, 2_y'$	z	y	x	mm'
25	$1, 2_x', \bar{2}_y, \bar{2}_z'$	y	x		mm'
26	$1, 2_y', \bar{2}_x, \bar{2}_z'$	y	x, y		mm'
27	$1, \bar{1}', \bar{2}_z, 2_z'$	z	z		$2'/m$
28	$1, \bar{1}', \bar{2}_x, 2_x'$	y, z	x		$2'/m$
29	$1, \bar{1}', \bar{2}_y, 2_y'$	y	x, z		$2'/m$
30	$1, \bar{1}', 2_z, 2_x, 2_y, \bar{2}_z', \bar{2}_x', \bar{2}_y'$	x	y		$m'm'm'$
31	$1, \bar{1}', 2_z, \bar{2}_x, \bar{2}_y, 2_x', 2_y', \bar{2}_z'$	y	x		mmm'

one for domain walls with a horizontal Bloch line (we note that horizontal Bloch lines in thin films have recently been shown to be unstable—they tend to migrate out to the surface of the material; see, e.g., Ref. 10).

Table III also lists magnetization distributions which describe magnetic structures that have not been discussed previously. As an example, consider the magnetic class U_{23}: $1, 2_y, 2_z', 2_x'$. Inside the domains, the dominant alignment of $\mathbf{M}$ is parallel to $\mathbf{e}_z$; in general, however, there is a nonzero component $M_y \in S_0 S_x$ (S_y) at the domain wall, i.e., the wall is similar in this respect to a simple Bloch wall. In addition, however, there is another component M_x with symmetry $S_0 A_x$ (A_y) which gives rise to a Bloch line as defined by conditions I and II. This component vanishes in the central plane (at $x = 0$) and at the center of the Bloch line ($y = 0$).

Such a structure can be explained as follows, for example. Consider a ferromagnet of class D_2; the easy axis coincides with the z axis, while the x axis is intermediate. As was shown in Ref. 5, ordinary Bloch lines cannot form in such a crystal, because right- and left-hand rotations of $\mathbf{M}$ are not energetically equivalent due to the presence of the invariants $M_x \partial M_z/\partial y - M_z \partial M_x/\partial y$ in the Hamiltonian for the system. Nevertheless, an "anomalous" Bloch line described by the U_{23} symmetry class may be present in such an FM.

Indeed, linear (pulsating) walls are known to be present near the Curie temperature T_c in FMs (see, e.g., Ref. 1); in these walls $\mathbf{M} = M_z(x)\mathbf{e}_z$ has only one nonzero component. However, because of the invariant $M_y \partial M_z/\partial x - M_z \partial M_y \partial x$ in our case, a component $M_y \sim \partial M_z/\partial x$ is necessarily induced in the domain wall. This implies that near T_c the wall is described by the class G_7 in the classification in Ref. 4, i.e., $\mathbf{M}$ rotates in the yz plane. However, since

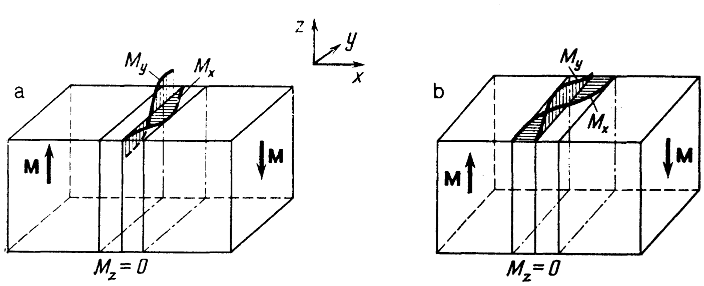

FIG. 2. Distribution of the magnetization in Bloch and Néel domain walls: a) for the class U_{13}; b) for the class U_8.

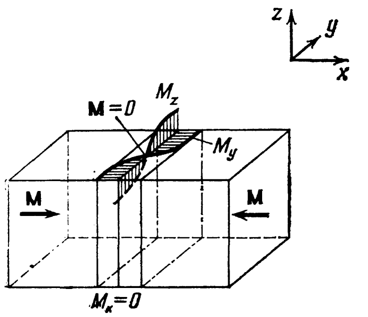

FIG. 3. Magnetization distribution in a "head-to-head" wall containing a Bloch line (class U_{17}).

the plane of easy magnetization for this FM is the xz plane, we expect that **M** will become realized and acquire a component M_x in the domain wall as the temperature drops. The **M** vector can leave the yz plane in two different directions. Such an FM can therefore support Bloch lines in the sense of conditions I and II.

It is important to find out which magnetic symmetry classes correspond to Bloch and Néel domain walls which admit Bloch lines in ordinary FMs. We consider a model rhombic FM with an energy density of the standard form

$$w=\frac{\alpha}{2}\left\{\left(\frac{\partial\theta}{\partial x}\right)^2+\left(\frac{\partial\theta}{\partial y}\right)^2+\sin^2\theta\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]\right\}+\frac{1}{2}K_z\sin^2\theta+\frac{1}{2}K_x\sin^2\theta\sin^2\varphi. \quad (17)$$

Here α is the inhomogeneous exchange interaction constant, K_z and K_x are the anisotropy constants, and θ, φ are the polar and azimuthal angles for **M**, with θ measured from the z axis and φ from the y axis. When $K_z>0$ and $K_z>K_x$, the predominant alignment of **M** in the domains is parallel to $\mathbf{e}_z$. The inequalities $K_x>0$ and $K_x<0$ correspond to stable Néel and Bloch walls, respectively.

If $K_z\gg|K_x|$, we have the following approximate solution for a domain wall with a Bloch line[1]:

for a Bloch wall

$$\cos\theta=-\operatorname{th}(x/\Delta),\quad \cos\varphi=-\operatorname{th}(y/\Delta_0), \quad (18)$$

for a Néel wall

$$\cos\theta=-\operatorname{th}(x/\Delta),\quad \cos\varphi=-\operatorname{ch}^{-1}(y/\Delta_0). \quad (19)$$

Here Δ and Δ_0 are the width of the wall and the Bloch line, respectively; $\Delta=(\alpha/K_z)^{1/2}$, $\Delta_0=(\alpha/K_x)^{1/2}$. The magnetization distribution (18) satisfies the conditions

$$M_x^*\in S_0S_x(S_y),\quad M_y^*\in A_0S_x(A_y),\quad M_z^*\in A_0A_x(S_y), \quad (20)$$

while the distribution (19) satisfies

$$M_x^*\in A_0S_x(A_y),\quad M_y^*\in S_0S_x(S_y),\quad M_z^*\in A_0A_x(S_y). \quad (21)$$

We readily see by consulting Tables I and II that there is no symmetry class that corresponds to this approximate solution, although classes U_{13} and U_8, which admit a lower degree of symmetry and describe Bloch and Néel walls, do exist. This indicates that one or more invariants admitting a rhombic symmetry must have been omitted from the energy (17) used in deriving the solution (18), (19). And in fact, it is easy to convince oneself that the expression for the energy can also contain terms

$$\tilde{w}=\beta_1\frac{\partial M_x}{\partial y}\frac{\partial M_y}{\partial x}+\beta_2\frac{\partial M_x}{\partial x}\frac{\partial M_y}{\partial y}, \quad (22)$$

where β_1 and β_2 are constants. The origin of the term $\tilde{w}$ can clearly be traced to the spatial dispersion of the anisotropy energy. One can show that when $\tilde{w}$ is included, the magnetization distribution **M** becomes

$$\mathbf{M}=\mathbf{M}^*+\mathbf{m}, \quad (23)$$

where **m** is given by the following expressions:

for a Bloch domain wall

$$m_x^{(B)}(x,y)\approx-4\frac{\beta}{\alpha}Q^{-1/2}\gamma(x)\operatorname{sh}\frac{x}{\Delta}\left(\operatorname{ch}\frac{x}{\Delta}\operatorname{ch}\frac{y}{\Delta_0}\right)^{-2},$$
$$m_y^{(B)}(x,y)\approx-8\frac{\beta}{\alpha}Q^{-1/2}\gamma(x)\operatorname{sh}\frac{x}{\Delta}\operatorname{sh}\frac{y}{\Delta_0}\left(\operatorname{ch}\frac{x}{\Delta}\operatorname{ch}\frac{y}{\Delta_0}\right)^{-2}, \quad (24)$$
$$m_z^{(B)}(x,y)=0,$$

where $\beta=\beta_1+\beta_2$ and $Q=\Delta_0^2/\Delta^2$ is the quality factor of the FM. The function $\gamma(x)$ behaves asymptotically as follows:

$$\gamma(x)\to2-(2\ln2)/\pi^2,\quad x\to0, \quad (25)$$
$$\gamma(x)\to1,\quad x\to\infty.$$

In our approximation $K_z\gg|K_x|$ we have for a Néel wall

$$m_x^{(N)}=-m_y^{(B)},\quad m_y^{(N)}=m_x^{(B)},$$
$$m_z^{(N)}=0. \quad (26)$$

It is easy to see that the additional terms (24)–(26) indeed reduce the symmetry of the solution (18), (19) to that described by the classes U_{13} and U_8 for Bloch and Néel domain walls, respectively.

We will now determine how many types of Bloch lines can exist in Bloch and Néel domain walls. If the symmetry class G_{bc} for the boundary conditions for the wall coincides with G_1, then the boundary condition symmetry class G_{bc}^{BL} for a Bloch line in the wall must be U_{18} for a Bloch wall and U_{28} for a Néel wall. As noted above, the symmetry class of a wall with a Bloch line coincides with U_{13} for a Bloch wall and U_8 for a Néel wall. The corresponding symmetry classes for a wall without a Bloch line are G_7 and G_9, respectively (Ref. 4).

Together with the discussion in Sec. 1, this implies that $N=4$ types of Bloch lines can exist in such a domain wall; the direction of rotation of **M** in the Bloch line inself and the relative alignment of the magnetic moments in the segments of the domain wall separat-

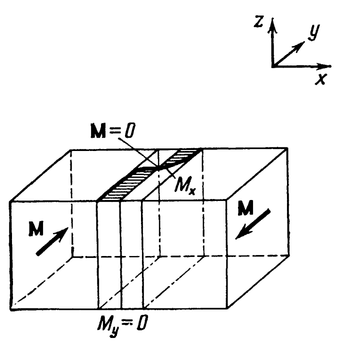

FIG. 4. Magnetic structure in an FM with easy axis in the xy plane (class U_{31}).

TABLE IV. Change in magnetic symmetry of a wall caused by motion of a Bloch line (the numbers k are given for classes U_k corresponding to the same domain wall with $V=0$ and $V\neq 0$).

Magnetic class	Kinematic class	Magnetic class	Kinematic class	Magnetic class	Kinematic class	Magnetic class	Kinematic class
0	0	8	8	16	10	24	24
1	0	9	0	17	10	25	8
2	0	10	10	18	18	26	5
3	3	11	0	19	13	27	27
4	4	12	0	20	13	28	28
5	5	13	13	21	3	29	10
6	0	14	3	22	8	30	18
7	7	15	5	23	23	31	28

ed by the Bloch lines both depend on the type. No Bloch lines can form in the walls if G_{bc} coincides with the symmetry classes G_7 or G_9 for Bloch and Néel walls without Bloch lines. This situation was considered in Ref. 5.

4. KINEMATIC SYMMETRY OF DOMAIN WALLS WITH BLOCH LINES

We now consider a domain wall with a Bloch line that moves uniformly with velocity $\mathbf{V} = V_y \mathbf{e}_y$. Following Ref. 4, we introduce the kinematic magnetic class in order to describe the magnetic symmetry of the wall. This symmetry class is obtained from U_k, the class for a wall with a Bloch line at rest, by eliminating from U_k all operations that alter the direction V. This of course neglects the change in the symmetry due to the applied forces causing the Bloch line to move.

Each of the kinematic classes coincides with one of the classes U_k in Tables I and II. To find the kinematic class, it therefore suffices to know the index of the magnetic class U_k. Tables I–IV can then be used to find the qualitative form of the magnetization distribution in a domain wall with a moving Bloch line. The change in wall symmetry due to the motion of the Bloch line is indicated in Table IV.

Among other things, Table IV shows that for Bloch lines in domain walls of the Bloch or Néel type, the kinematic symmetry classes coincide with the original classes U_{13} and U_8, i.e., the line motion does not change the wall symmetry in this case.

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