Scattering of light by nematic liquid crystals in cells with a finite energy of the anchoring of the director to the walls

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(Submitted 25 February 1986)
Zh. Eksp. Teor. Fiz. **91**, 851–860 (September 1986)

Theoretical and experimental investigations have been made of the influence of the anchoring of a nematic liquid crystal by the orienting surfaces of a cell on the wave-number spectrum of fluctuations of the director orientation in a planar nematic. The cross section for the small-angle scattering by such fluctuations is found as a function of the anchoring energy. The wave number spectrum of the scattering is used to estimate the polar anchoring energy of MBBA for several orienting surfaces.

INTRODUCTION

Light scattering by transverse fluctuations of the director **n** in a nematic liquid crystal has been investigated by many authors.[1–6] Studies have been made of the dynamics of such scattering[1,2] and of the influence of external fields,[3–5] the contribution of multiple scattering to this cross section has been considered,[2] an allowance has been made for the absorption of light,[6] etc. However, the influence of the bounding surfaces which orient a nematic liquid crystal on the scattering characteristics has not been investigated in detail. In view of the long-range nature of the intermolecular forces in nematic liquid crystals, we can assume that the interaction between the nematic molecules and the substrate should be manifested in the spatial spectrum of fluctuations of the director and, therefore, in the scattering of light.

The parameter which characterizes the substrate-nematic interaction is the anchoring energy W, the values of which vary from $\sim 10^{-5}$ to 10^{-1} erg/cm^2 (Ref. 7), depending on the nature of the surface and of the nematic liquid crystal. Obviously, the influence of W on the scattering characteristics is important if the surface energy of a nematic liquid crystal $F_s \approx Ws$ (s is the area of the bounding surface) is comparable with its volume energy $F_v \approx KQ^2V$ (K is the Frank constant, **Q** is the scattering vector, and V is the volume, i.e., $W \approx LKQ^2$ (L is the thickness of the cell with a nematic liquid crystal). The range of values of **Q** for which this condition is satisfied in the case of typical values of W in cells of length $L \sim 10$–$100\ \mu$m is 5×10^{-2}–10^{-4} cm^{-1} if $K \sim 10^{-7}$. At $\lambda = 0.63\ \mu$m these vectors correspond to scattering angles of $\theta = 12$–30 mrad. In practice, the values of the divergence of gas laser radiation are $\theta_d \sim 1$ mrad and, therefore, scattering at such angles can be detected quite readily. Preliminary experimental investigations have shown that these estimates are correct and that the small-angle scattering spectrum is indeed sensitive to changes in the conditions at the interface between a nematic liquid crystal and its substrate.[8]

We shall report a theoretical and experimental study of the influence of the orienting surface on the characteristics of light scattering in nematic liquid crystals. We shall use the one-constant approximation to solve the problem of the scattering in a cell with walls characterized by a finite anchoring energy. We shall show that the values of this energy W have a significant influence on the absolute cross section for *ee* scattering and on its angular dependence (the incident and scattered waves are assumed to be extraordinary relative to the nematic liquid crystal). We shall use the expressions obtained in this way to identify the experimental conditions under which the influence of the anchoring should be manifested most clearly and to estimate the polar anchoring energy of 4-methoxybenzylidene butylaniline (MBBA) for several orienting surfaces. A determination of the characteristics of the low-angle scattering may provide a new method for obtaining the anchoring energy. The experimental conditions in this method are preferable to those in which a nematic liquid crystal is disturbed from an equilibrium position by external agencies.

THEORY

The expression for the differential cross section of the scattering of light by fluctuations of the director is[1]

$$\sigma_\Omega=\pi^{-2}\lambda^{-4}\varepsilon_a^2\sum_\alpha (i_\alpha f_\parallel+i_\parallel f_\alpha)^2\langle\delta n_\alpha^2(\mathbf{Q})\rangle, \tag{1}$$

where $\varepsilon_a = n_e^2 - n_o^2$ is the anisotropy of the permittivity; n_e and n_0 are the refractive indices for the extraordinary (e) and ordinary (o) waves; $(f_\alpha f_\parallel + i_\alpha f_\parallel)$ is a geometric factor; $\alpha = 1, 2$; $i_\alpha = \mathbf{i}\mathbf{e}_\alpha$; $f_\alpha = \mathbf{f}\mathbf{e}_\alpha$; **i** and **f** represent the polarization of the incident and scattered light; $\mathbf{e}_1$ and $\mathbf{e}_2$ are unit vectors in a coordinate system (these vectors are perpendicular to the z axis, parallel to the director, and are oriented in such a way that the scattering vector **Q** lies in the $\mathbf{e}_1 z$ plane and $\mathbf{e}_2$ is perpendicular to this plane); $f_\parallel$ and $i_\parallel$ are the projections of the vectors **f** and **i** on the z axis; λ is the wavelength of light; and the angular brackets denote thermal averaging.

It follows from Eq. (1) that a calculation of the scattering intensity reduces to solution of the problem of the spectrum of fluctuations of the director which, after an allowance for the influence of the orienting coatings, should depend on the parameters W and L. In the case of extreme boundary conditions corresponding to $W\to\infty$ this problem is solved in Ref. 9.

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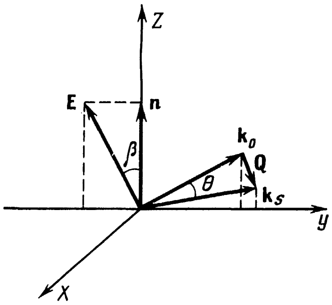

FIG. 1. Vector diagram explaining the theoretical analysis and experiments.

We shall calculate the spectrum $\delta n(\mathbf{Q})$ using a one-constant approximation and we shall ignore the change in the order parameter S near the walls compared with its value S_0 in the bulk of the nematic liquid crystal. The difference between the values of S_0 and S should alter the Frank constants $K(S^2)$. However, the maximum value of $\Delta S/S_0$ is found only in a layer of thickness $\sim 0.1\ \mu$m near the walls and it is already negligible at distances exceeding $0.2\ \mu$m (Ref. 10). Therefore, under the conditions of the one-constant approximation we can ignore the change in K because of the dependence $S(\mathbf{r})$.

The spectrum of fluctuations of the director will be found by writing down the expression for the free energy F of a nematic, allowing for the surface term. It follows from Ref. 11 that the contribution of the surface energy is

$$F_s = F_d + F_l,$$

where F_d is the surface density of the energy due to the inhomogeneity of the director on the surface given by the expression

$$F_d = {}^1/_2(\mathbf{nd})^2[\tilde{K}_1(d_i d_j \partial_i n_j)^2 + \tilde{K}_2(d_j \partial_i n_j)^2],$$

where $i,j = x,y,z$; d is the direction of easy orientation parallel or perpendicular to the surface; $\tilde{K}_1$ and $\tilde{K}_2$ are the surface elastic constants. The term F_l is the energy associated with a homogeneous distortion of the director relative to the easy orientation axis; it is given by

$$F_l = {}^1/_2[W(\mathbf{nd})^2 - {}^1/_3].$$

We shall consider a cell containing a nematic liquid crystal with the planar orientation. We shall select the z axis to coincide with the director and the y axis perpendicular to the cell walls (Fig. 1). Since the energy W necessary for the rotation of the director in the plane of the cell wall (azimuthal energy W_x) and at right-angles to the wall (polar energy W_y) may differ considerably,[7] we shall write down F_l in the form

$$F_l = {}^1/_2(W_x n_x^2 + W_y n_y^2).$$

In the selected geometry the contribution of the term F_d to the surface energy density is given by

$$F_d = {}^1/_2 n_z^2[\tilde{K}_1(\partial n_z/\partial z)^2 + \tilde{K}_2(\operatorname{grad} n_z)^2].$$

We can see that if the director fluctuations are small ($\delta\mathbf{n}\cdot\mathbf{n}_0 = 0$), the term F_d is of order $\delta n_z^2 \sim \delta n_x^4 \sim \delta n_y^4$, i.e., it is of higher order than $F_l \sim \delta n_x^2 \sim \delta n_y^2$, and F_d can be ignored.

Therefore, the total energy of a nematic can be written in the form

$$F = {}^1/_2 K \int dv[(\operatorname{div} n)^2 + (\operatorname{curl} n)^2] + {}^1/_2 \int_{s_1,s_2} ds(W_x n_x^2 + W_y n_y^2), \tag{2}$$

where s_1 and s_2 are the orienting surfaces perpendicular to the y axis. The operator on the right-hand side of Eq. (2) is self-adjoint.

Starting from the condition for the minimum of the elastic energy F and the identity $|n^2| = 1$ in the bulk and on the surfaces, we can obtain expressions describing the equilibrium distribution of the director of a nematic liquid crystal in a cell and at its walls. In particular, if the director fluctuations are small, the boundary conditions are

$$\begin{aligned} &\operatorname{div}\delta\mathbf{n} \pm (W_y/K)\delta n_y = 0,\\ &\partial(\delta n_x)/\partial y - \partial(\delta n_y)/\partial x \pm (W_x/K)\delta n_x = 0. \end{aligned} \tag{3}$$

The different signs correspond to the two bounding surfaces at $y = 0$ and L. In the limit $W \to \infty$ these expressions yield the boundary conditions for the strong coupling discussed in Ref. 9. For the Fréedericksz transition $[\delta\mathbf{n} = (0, \delta n_y, 0)]$ they transform into the boundary conditions given in Ref. 12.

Using Eq. (3), we find that the expression for fluctuations of the elastic energy can be reduced to

$$\delta F = -{}^1/_2 K \int \delta\mathbf{n}\Delta(\delta\mathbf{n})\, dv. \tag{4}$$

It takes the form of the average of the operator $-K\Delta/2$, which is self-adjoint when the boundary conditions of Eq. (3) are satisfied.

We shall expand fluctuations of $\delta\mathbf{n}(\mathbf{r})$ in terms of eigenfunctions of the operator $-K\Delta/2$ satisfying the conditions of Eq. (3) at the cell walls and forming, because of the self-adjoint nature, a complete system of orthogonal functions:

$$\begin{aligned} \delta n_x &= \sum_{q_y,q_\perp} \exp(i\mathbf{q}_\perp\mathbf{r}_\perp)[A_{q_y,q_\perp}\exp(iq_y y) + B_{q_y,q_\perp}\exp(-iq_y y)],\\ \delta n_y &= \sum_{q_y,q_\perp} \exp(i\mathbf{q}_\perp\mathbf{r}_\perp)[C_{q_y,q_\perp}\exp(iq_y y) + D_{q_y,q_\perp}\exp(-iq_y y)]. \end{aligned} \tag{5}$$

These expressions describe the modes of elastic deformations of the director in a resonator formed by a nematic liquid crystal and two plane infinite surfaces. In view of the boundary conditions, the values of the wave vectors q_y are discrete and the wave vectors $q_\perp$ lying in the plane of the surface may vary continuously.

Substituting the expansion of Eq. (5) into the boundary conditions of Eq. (3), we obtain a system of homogeneous linear equations for the coefficients $A_{q_\perp,q_y}$, $B_{q_\perp,q_y}$, $C_{q_\perp,q_y}$, and $D_{q_\perp,q_y}$, containing q_y as a parameter. Equating this determinant to zero, we shall write down the equation which should be satisfied by the wave vectors of the modes $\delta\mathbf{n}(\mathbf{q})$:

$$\begin{aligned} &[(t^2-\varepsilon_x^2)\operatorname{tg} t - 2t\varepsilon_x][(t^2-\varepsilon_y^2)\operatorname{tg} t - 2t\varepsilon_y]\\ &+ p^2[(t^2+p^2-2\varepsilon_x\varepsilon_y)\operatorname{tg}^2 t - 2t(\varepsilon_x+\varepsilon_y)\operatorname{tg} t] = 0. \end{aligned} \tag{6}$$

Equation (6) contains dimensionless quantities

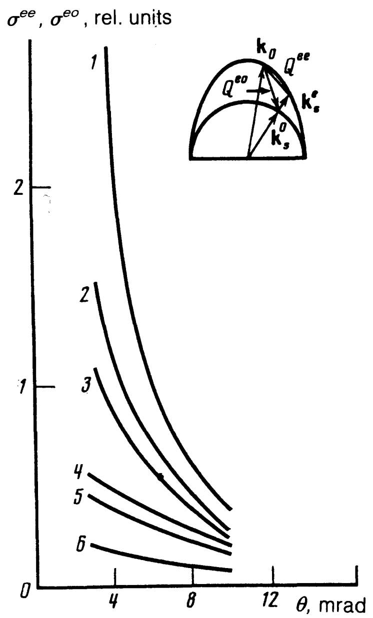

FIG. 2. Angular spectrum of the scattered light calculated from Eq. (10) for different values of the energy of anchoring of MBBA to a surface. Curves 1–5 correspond to the *ee* scattering and curve 6 corresponds to the *oe* scattering, calculated for different values of W_y (erg/cm^2): 1) 10^{-5}; 2) 5×10^{-5}; 3) 10^{-4}; 4) 5×10^{-4}; 5) 10^{-3}; 6) 10^{-5}–10^{-1}.

$p = q_x x$, $t = q_y L$, $\varepsilon_\gamma = W_\gamma L/K$, and $\gamma = x, y$. This last parameter is equal to the ratio of the cell thickness to the correlation length, and it represents the strength of the anchoring of the director to the substrate for a given value of L. If $q_x = 0$, then Eq. (6) splits into two equations and we then have

$$\operatorname{tg} t_\gamma = 2t\varepsilon_\gamma/(t^2-\varepsilon_\gamma^2). \tag{7}$$

In general, the solution of this equation can be found only numerically, but in the limiting case when $\varepsilon_\gamma \ll 1$, we obtain

$$t_\gamma = \pi m + 2\varepsilon_\gamma/\pi m, \qquad m=1, 2\ldots,$$
$$t_\gamma = (2\varepsilon_y)^{1/2}, \qquad m=0, \tag{8}$$

and if $\varepsilon_\gamma \gg 1$, then

$$t_\gamma = \pi m - 2\pi m/\varepsilon_\gamma, \qquad m=1, 2\ldots, \qquad m/\varepsilon_\gamma \ll 1.$$

It follows from these expressions that the spectrum of the $\delta\mathbf{n}(q_y)$ modes depends on W_γ. In the limit $W_\gamma \to \infty$ it is determined entirely by the thickness of the cell and it is characterized by a set of equidistant values $q_y = \pi m/L$, whereas in the case of weak anchoring the values of $q_{y,m}$ are shifted and the shift depends on W.

A complete description of the fluctuations $\langle|\delta\mathbf{n}(\mathbf{r})|^2\rangle$ can be provided by determining the explicit form of the coefficients $\langle|A_{q_\perp,q_y}|\rangle^2$, $\langle|B_{q_\perp,q_y}|\rangle^2$, $\langle|C_{q_\perp,q_y}|\rangle^2$, $\langle|D_{q_\perp,q_y}|\rangle^2$. This can be done by substituting the expressions from Eq. (5) into Eq. (4), and then using Eq. (6) and the theorem on the equipartition of energy among the degrees of freedom. If $q_x = 0$, then the coefficients $A_{q_\perp,q_y}$, and $B_{q_\perp,q_y}$, and also $C_{q_\perp,q_y}$ and $D_{q_\perp,q_y}$, are pairwise linearly dependent. Using Eq. (8), we find that this means that if $q_x = 0$ then the polar fluctuations δn_y and the azimuthal fluctuations δn_x, as well as the boundary conditions for these fluctuations are independent and are determined by W_y and W_x, respectively. Consequently, in the determination of the cross section it is possible to find independently the values of the anchoring energy. This can be done by selecting a geometry for which we have $q_x = 0$, i.e., by ensuring that the director $\mathbf{n}$ and the wave vectors of the incident and scattered light $\mathbf{k}_0$ and $\mathbf{k}$ lie in the yz plane. Then, in Eq. (1) we have $\alpha = x, y$ and

$$\sigma_\Omega^{ee} \approx \pi^{-2}\varepsilon_a^2\lambda^{-4}\sin^2(2\beta)\langle\delta n_y^2(W_y)\rangle,$$
$$\sigma_\Omega^{oe} \approx \pi^{-2}\varepsilon_a^2\lambda^{-4}\cos^2\beta\langle\delta n_x^2(W_x)\rangle, \tag{9}$$

where β is the angle between the director and the polarization vector of the incident light; the first index of σ_Ω gives the state of polarization of the incident light and the second describes the state of polarization of the scattered light.

We can find the explicit forms of σ_Ω^{ee} and σ_Ω^{oe} by Fourier transformation of the fluctuations $\delta n_{x,y}$ which are described by Eq. (5), using the explicit form of the coefficients $\langle A^2\rangle$, $\langle B^2\rangle$, $\langle C^2\rangle$, and $\langle D^2\rangle$, and substitution of the resultant expressions into Eq. (9). Then, if $q_x = 0$, we obtain

$$\sigma_\Omega^{ee} = \pi^{-2}\varepsilon_a^2\lambda^{-4}v\sin^2(2\beta)\sum_{q_y}\langle|C_{q_\perp q_y}|^2\rangle\Big[\operatorname{sinc}^2\Big(\frac{q_y-Q_y}{2}L\Big)$$
$$+\operatorname{sinc}\Big(\frac{q_y+Q_y}{2}L\Big)\Big]+\operatorname{sinc}\Big(\frac{q_y-Q_y}{2}L\Big)\operatorname{sinc}\Big(\frac{q_y+Q_y}{2}L\Big)$$
$$\times[2\operatorname{Re}(C_{q_y}D^*_{q_y})\cos(q_yL)-2\operatorname{Im}(C_{q_y}D^*_{q_y})\sin(q_yL)],$$
$$\langle|C_{q_y,q_\perp}|^2\rangle = \frac{k_BT}{vK(q_z^2L+t_y^2)}\Big\{2+\frac{2\varepsilon_y}{\varepsilon_y^2+t_y^2}$$
$$+\frac{1}{(\varepsilon_y^2+t_y^2)t_y}[(t_y^2-\varepsilon_y^2)\sin(2t_y)-4t_y\varepsilon_y\cos(2t_y)]\Big\}^{-1},$$
$$D_{q_yq_\perp} = C_{q_y,q_\perp}\frac{t_y^2-\varepsilon_y^2+2it\varepsilon_y}{\varepsilon_y^2-t_y^2}, \tag{10}$$
$$Q^{ee} = (0; -2k\sin\theta\sin\frac{\theta}{2}\cos\beta - 2k\cos\theta\sin\frac{\theta}{2}\sin\beta;$$
$$2k\sin\theta\sin\frac{\theta}{2}\sin\beta$$
$$+2k\cos\theta\sin\frac{\theta}{2}\cos\beta), \qquad k=2\pi/\lambda.$$

The expression for the *oe* scattering cross section is obtained from Eq. (10) by the substitutions $C\to A$, $D\to B$, $\varepsilon_y\to\varepsilon_x$, and $t_y\to t_x$ and replacing the factor $\sin^2(2\beta)$ with $\cos^2\beta$, and replacing $\mathbf{Q}^{ee}$ with $\mathbf{Q}^{oe}$.

The relationships in the system (10) describe the scattering cross section in a cell with a finite anchoring energy. Figure 2 shows the dependences $\sigma_\Omega^{ee}(\theta)$ obtained from Eq. (10) for several values of W_y. We can see that a change in the energy of anchoring to the substrate alters the angular scattering spectrum and the strongest effect is observed (as expected) at small scattering angles. Then, the anchoring energy has a strong influence on the *ee* scattering characteristics and a weaker influence on the scattering accompanied by a change in the state of polarization (*oe*). We can understand this result by writing down the simplified expressions for σ_Ω, which are valid in the case of weak coupling ($\varepsilon_\gamma \ll 1$) and angles $\theta \lesssim 10$ mrad:

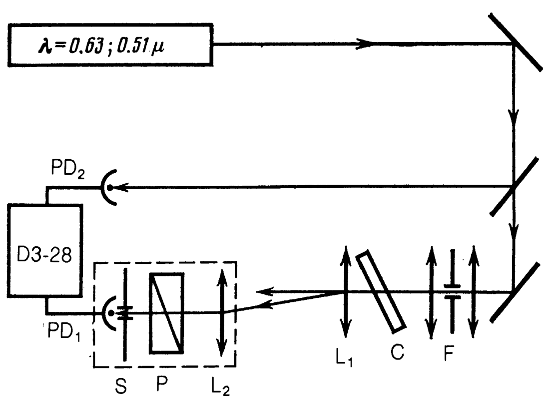

FIG. 3. Schematic diagram of the apparatus (explanations in text).

$$\sigma_\Omega^{oe} \approx \pi^{-2}\varepsilon_a^2\lambda^{-4} V k_B T \cos^2\beta \times\left\{\frac{2W_x}{L}+\frac{4K\pi^2}{\lambda^2}[n_e n_o\theta^2+(\Delta n)^2]\right\}^{-1},$$
$$\sigma_\Omega^{ee} \approx \pi^{-2}\varepsilon_a^2\lambda^{-4} V k_B T \sin^2(2\beta)\left(\frac{2W_y}{L}+\frac{4K\pi^2}{\lambda^2}n_e^2\theta^2\right)^{-1} \quad (11)$$

We can see that the feasibility of observing the influence of the anchoring energy W on the scattering characteristics is determined by the ratio of the specific surface energy (the first term in the denominator) to the specific volume energy (next term). For a given angle θ the wave vector $\mathbf{Q}^{oe}$ is always greater for the *oe* scattering than for the *ee* process (Fig. 2) and this gives rise to a large volume energy even in the case of zero scattering angles. Therefore, for all values of θ the influence of the anchoring energy is manifested less strongly in *oe* scattering than in the *ee* case. It also follows from Eq. (11) that an allowance for the anchoring energy removes the divergence of the scattering cross section[1)] in the limit $\theta \to 0$.

EXPERIMENTS

The above calculations allow us to identify the conditions under which we can observe experimentally the influence of the finite anchoring energy on the angular characteristics of light scattering and to estimate the value of this energy W.

1. We must investigate small-angle scattering ($\theta \lesssim 10$ mrad).

2. We must select an experimental geometry in which the vectors $\mathbf{n}$, $\mathbf{k}_0$, $\mathbf{k}_s$, and $\mathbf{Q}$ lie in the same plane.

3. We must detect the *ee* scattering.[2)]

All these requirments are satisfied by the experimental setup shown schematically in Fig. 3. Radiation from a cw laser [we used a helium-neon laser ($\lambda = 0.63\ \mu$m) and an argon laser ($\lambda = 0.54\ \mu$m) emitting the lowest transverse mode] passed through a spatial filter F and was directed at an angle β to a cell with a planar-oriented nematic liquid crystal, chosen to be MBBA. The polarization vector of the incident light made an angle of $\beta = 20°$ with the director. The observation (recording) plane was chosen to ensure that the vectors $\mathbf{n}$, $\mathbf{k}_0$, $\mathbf{k}_s$, and $\mathbf{Q}$ are coplanar. The scattered light was focused by a lens L_1 with a focal length 1 m. A short-focus lens L_2 increased the image of the focal plane and transferred it to a photodiode PD_1. In front of this photodiode there was a polarizer P, which selected the signal of the required polarization, and a stop S, the diameter of which was governed by the divergence of the radiation and the focal length of the lens L_1. The signal from the photodiode was passed on to a minicomputer and a fast plotter. The stop, the polarizer, and the photodiode were placed on a bench and the dependence $\sigma(\theta)$ was determined by moving these components on this bench. The laser radiation intensity was measured with a second photodiode PD_2.

The factors distorting the angular distribution of the scattered-light intensity were as follows: multiple scattering by fluctuations of the director, scattering due to static inhomogeneities of the distribution of the director on the cell walls, stray scattering by components of the optical system beyond the spatial filter, and the depolarization of the *oe* scattering signal in the polarizer.

The contribution of the stray scattering by the components of the system was estimated by recording the scattered-light intensity after heating the nematic liquid crystal to a temperature above its phase transition to the isotropic liquid state. It was found that this contribution was less than 1% throughout the investigated range of scattering angles. The contribution of depolarization of light in the polarizer was also less than 1%. Estimates carried out in accordance with the expressions obtained in Ref. 2 showed that multiple scattering of light was also negligible for the cell thicknesses and scattering angles used in our study.

The main distorting factor was the scattering by static inhomogeneities in the cell. We selected the constant component due to the scattering by ensuring that the diameter of the aperture in the stop was less than the characteristic size of a single speckle in the observation (recording) plane throughout the range of angles θ investigated. In this case the radiation passing through the stop contained an alternating component with a characteristic time of $\tau \sim 0.1$–1 sec due to fluctuations of the director and a constant component associated with static inhomogeneities of the director orientation. The constant component was calculated from the total intensity so that only the scattering by the director fluctuations was recorded.

The anchoring energy was determined for the nematic liquid crystal MBBA in glass cells of thickness $L = 20$–100 μm which had orienting coatings made of polyvinyl alcohol and polyacrylate lacquer or with substrates rubbed in one direction. The experimental results obtained in these experiments were compared with the prediction of Eq. (10) (Fig. 4). In the calculations we used the following parameters occurring in the formulas: $K = 5\times10^{-7}$ dyn, $n_e = 1.72$, $n_o = 1.54$, $\lambda = 0.63\ \mu$m, $\beta = 20°$. A comparison of the experimental and theoretical results enabled us to estimate the anchoring energy for an orienting coating made of polyacrylate lacquer [$W_y \approx (2 \pm 0.5)\times10^{-4}$ erg/cm^2], polyvinyl alcohol [$W_y \approx (5 \pm 2)\times10^{-5}$ erg/cm^2], and glass rubbed in one direction [$W_y \approx (1 \pm 0.5)\times10^{-4}$ erg/cm^2]. The experimental errors given in the graphs and in the text were governed by the accuracy of the measurements and the errors made in the determination of the constants used in the

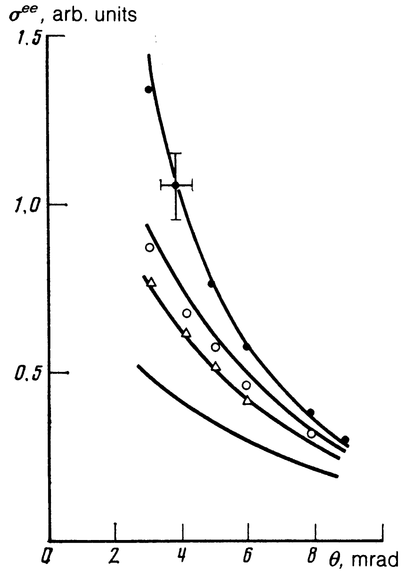

FIG. 4. Angular spectrum of the light scattered in MBBA with planar orientation in contact with several surfaces: ●) coated by polyvinyl alcohol; ○) rubbed glass; △) coating of polyacrylate lacquer. The continuous curves are theoretical dependences calculated from Eq. (10) assuming $W_y = 5\times10^{-5}$, 10^{-4}, 2×10^{-4}, and 5×10^{-4} erg/cm^2 (from top to bottom).

theoretical calculations. We also allowed for the errors asociated with the theoretical approximations.

An independent check of the values of W_y was made by measuring the dependences of the *ee* scattering cross section on the thickness of the cell for different angles θ. This dependence, which was obtained from Eq. (10), differed considerably from the linear relationship predicted by the de Gennes theory.[1] Figure 5 shows the theoretical dependences and the experimental results obtained for $\theta = 3$ mrad and cells with polyacrylate lacquer coatings. The experimental data were in satisfactory agreement with the measurements of the angular dependence $\sigma(\theta)$. Figure 6 shows the angular spectrum of the *ee* scattering cross section calculated for MBBA in contact with the surface characterized by $W_y = 5\times10^{-4}$ erg/cm^2 in cells of the same thickness. The experimental points represent measurements on MBBA in cells with polyacrylate lacquer coatings and with $L = 35$, 50, and 80μ. In agreement with the theory, the $\sigma_\Omega(\theta)$ dependences are flattened and they demonstrate a reduction in the absolute value of σ_Ω as the cell thickness is reduced.

DISCUSSION

The results of the theoretical calculation, confirmed by our experimental data, allow us to conclude that the anchoring of the nematic molecules to the bounding surfaces largely determines the angular spectrum of the scattering of light by transverse fluctuations of the director and ensures that the scattering cross section is finite for $\theta = 0$.

The influence of the anchoring energy W is manifested most strongly in the case of the *ee* scattering. The scattering cross section is most sensitive to changes in the values of W in the range $\sim10^{-5}$–10^{-2} erg/cm^2. This is due to the fact that—as demonstrated by Eq. (8)—the spectrum of the di-

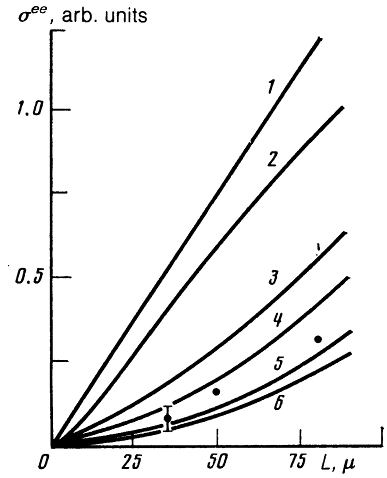

FIG. 5. Theoretical dependences of the *ee* scattering cross section on the thickness of MBBA layer at $\theta = 3$ mrad and different values of W_y (erg/cm^2): 1) 0–10^{-6}; 2) 10^{-5}; 3) 5×10^{-5}; 4) 10^{-4}; 5) 5×10^{-4}; 6) 10^{-3}. The points are the experimental values.

rector fluctuation modes obtained for high values of W is little different from the case of strong coupling to the bounding surfaces, whereas at low anchoring energies the contribution to the total distortion energy of a nematic liquid crystal is negligible.

We shall now consider the errors due to the simplifications made in our calculations. The most important of these are the neglect of a possible inhomogeneity of an equilibrium distribution of the director on a bounding surface and the one-constant approximation.

The variation of the distribution of the director on the cell walls $\mathbf{n}_0(\mathbf{r})$ may be ignored if the gradient is weak and the amplitude is small. The gradient length and the amplitude can be estimated from the contribution of the constant component to the total cross section σ. Experiments indicate that for the minimum angle $\theta_{\min} = 3$ mrad the constant

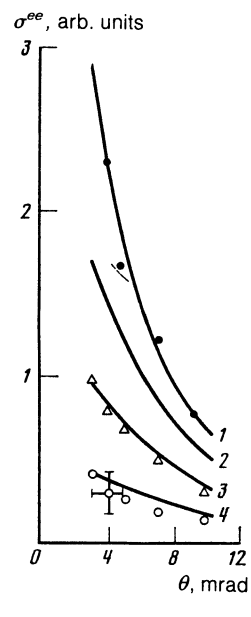

FIG. 6. Angular spectrum of the *ee* scattering in MBBA in cells of different thickness calculated from Eq. (10) for $W_y = 5\times10^{-4}$ erg/cm^2: 1) $L = 80\,\mu$; 2) 65 μ; 3) 50 μ; 4) 35 μ. The experimental points are plotted for MBBA in cells covered by polyacrylate acid lacquer: ●) $L = 80\,\mu$; △) 50 μ; ○) 35 μm.

component $\sigma_{\rm const}$ represents a few percent of the total value of σ, whereas at large angles it is completely negligible. This means that the scale of the distortions is indeed large (the characteristic size of an inhomogeneity is $\Lambda = \lambda/\theta_{\rm min} \approx 0.2$ mm, i.e., it is comparable with the diameter of the scattered beam) and the amplitude of the inhomogeneity is considerably less than the amplitude of the director fluctuations. We shall now estimate the error in our calculations due to the use of the one-constant approximation. Since the vectors $\mathbf{n}$, $\mathbf{k}_0$, $\mathbf{k}_s$, and $\mathbf{\Omega}$ lie in the same zy plane, fluctuations of the director δn_y in the same plane simply produce longitudinal and transverse bending, but no torsion.[1] The Frank elastic constants K_{11} and K_{33} corresponding to these deformations have similar values and, therefore, the use of the one-constant approximation in our calculations is fully justified. In quantitative estimates we shall use the fact that in the range of angles $\theta > 10$ mrad, when the influence of the walls in our experiments is small, the value of $(\sigma^{ee})^{-1}$ is proportional to $K_{11} q_y^2 + K_{33} q_z^2$ (Ref. 1). Bearing in mind that $K_{11}/K_{33} \approx 1.4$ and writing down the values of q_z for the experimental value of $\beta = 20°$ we find that $(\sigma^{ee})^{-1} \approx 1.15 K_1 q_z^2$. This means that in the range of angles investigated the error due to the use of the one-constant approximation amounts to $\sim 15\%$.

There have been several investigations in which various methods have been used to estimate the value of the energy representing the anchoring of MBBA to orienting surfaces.[13-18] Depending on the material of the substrate and the method for treating its surface, the value of W_y is found to vary from 10^{-4} to 10^{-2} erg/cm^2. The anchoring energy W_y found for the planar-oriented MBBA on a silane coating is 4×10^{-3} erg/cm^2 (Ref. 18), whereas in the case of quasiplanar-oriented MBBA on glass with obliquely evaporated SiO (Ref. 16) it is 10^{-4}–10^{-2} erg/cm^2. The values of W_y for planar-oriented MBBA on our substrates are similar: $W_y \sim 10^{-4}$ erg/cm^2.

It should be pointed out that a comparison of the results obtained by different research groups on the anchoring energy is very difficult at the present state of the technology of preparing the samples, because the anchoring energy W depends not only on the nematic liquid crystal and the orienting surfaces but also on other factors. For example, the values of W_y for a given nematic liquid crystal–surface pair may differ by an order of magnitude as a result of variations in the surface treatment technology,[17] they can decrease as a result of aging of a sample (it is reported in Ref. 16 that aging can reduce W_y by 20–30% in three days), variation with temperature may be more than one order of magnitude, there may be a reduction on approach to the bleaching point,[17] etc. Therefore, the values of W obtained by us and other authors are governed not only by selected nematic liquid crystal–surface pairs, but also by the characteristics of the actual cell and the experimental conditions.

It follows from the above discussion that the light scattering characteristics can be used to determine the polar anchoring energy in the range $W_y = 10^{-5}$–10^{-2} erg/cm^2. It seems to us that this method is very promising because it utilizes fluctuations $\delta\mathbf{n}$ always present in nematic liquid crystals and the role of light is to detect these fluctuations. A specific feature of the method is also the ability to carry out measurements using the same cell which need not be prepared in any special way. A shortcoming of the proposed method in the case of cells with a planar orientation of a nematic liquid crystal is the fact that in the present state of the development of the method it is difficult to determine the azimuthal anchoring energy W_x. It may be possible in the future to find W_x by detecting the forward-scattered light.

We shall conclude by noting that the small-angle scattering we have investigated is not an exotic form of scattering. Quite the reverse, it is this scattering that creates the stray background of laser beams transmitted by liquid crystal cells and displays, and by nonlinear-optical and holographic systems utilizing liquid crystals.

The authors are grateful to L. M. Blinov, O. D. Lavrentovich, S. G. Odulov, and P. P. Pinkevich for numerous valuable discussions and comments.

[1)]It was assumed earlier that the value of σ_Ω corresponding to $\theta = 0$ remains finite because of the finite absorption of light in a nematic.[6]

[2)]It is traditionally assumed that this type of scattering is absent in the linear approximation,[1] but a tilt of the director relative to the polarization vector $\mathbf{E}$ of the incident light gives rise to such scattering even in the linear approximation [see Eq. (9)], as shown in Ref. 5.

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Translated by A. Tybulewicz