Magnetoelectric susceptibilities of atoms

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The susceptibilities γ_{kn} determining the energy of an atom in a constant magnetic (**B**) and alternating electric (**F**) fields are computed with the aid of perturbation theory up to the sixth order for an arbitrary orientation of the vectors **F** and **B**. The susceptibilities at **F**-field frequencies ω higher than the ionization threshold are complex, the imaginary part determining the magnetic-field-related correction to the photoionization cross section. The numerical values of the ground state susceptibilities for the hydrogen and alkali-metal atoms are computed in a broad range of frequencies ω, and the effect of the magnetic field on the photoionization cross section is analyzed.

1. INTRODUCTION

The effects of the simultaneous action of an electric and a magnetic field on an atom have been investigated only in the simplest cases: in the lowest orders in the intensities of the electric (F) and magnetic (B) fields and in parallel and mutually perpendicular static **F** and **B** fields in the case of the hydrogen-like atoms. The corrections of the order of FB and F^2B have been determined for the excited states of hydrogen[1,2] and many-electron atoms.[3] Higher order corrections to the ground-state energy of hydrogen are found with the aid of algebraic procedures, various modifications of which are used in Refs. 4–6. But these methods, effective in the case when the vectors **F** and **B** are parallel, are complicated in the case of arbitrary orientation of the vectors, and are also inapplicable in the case of alternating (monochromatic) fields $\mathbf{F}(t)$.

In the general case the magnetoelectric susceptibilities γ_{kn} of an atomic level with energy E_0, which are defined as the coefficients in the expansion of the energy $\mathscr{E}$ (or quasi-energy in the case of variable fields) of an atom in a field in powers of F and B 1):

$$\Delta\mathscr{E}=\mathscr{E}-E_0=\sum_{k,n}\gamma_{kn}F^k(\alpha B)^n, \tag{1}$$

can easily be analyzed within the framework of the standard schemes of perturbation theory. In the present paper we carry out such an analysis for terms with $k+n\leqslant 6$ in both constant and monochromatic $\mathbf{F}(t)$ fields.

As follows from symmetry considerations, the γ_{kn} terms with odd k are nonzero only in the excited hydrogen states with a constant dipole moment; therefore, in the case of the S states only the effects of even order in the amplitude F make a contribution. The index n can be either even or odd, and in the latter case $\Delta\mathscr{E}_{2k,2n+1}\neq 0$ only in an alternating field $\mathbf{F}(t)$ with a nonzero degree of circular polarization. In this case the energy corrections of odd orders in B arise as a result of the interaction with the magnetic field of the constant magnetic moment induced in the atom by the rotating electric field. As shown in Sec. 3, the quantities $\gamma_{2k,2n+1}$ are uniquely connected with the corrections to the Verdet constant of a gas in strong light and magnetic fields. The susceptibilities $\gamma_{2k,2n}$ and their connection with the Cotton-Mouton magnetic birefringence constant are discussed in Sec. 4. For the numerical calculations of the composite matrix elements determining the γ_{kn} in hydrogen and the alkali-metal atoms, we use the analytic continuation of the Sturm expansion of the radial Green function of the optical electron in the atom,[7] a procedure which allows us in the case of an alternating field $\mathbf{F}(t)$ to investigate the frequency (ω) regions below and above the ionization potential $|E_0|$. In the latter case the susceptibilities $\gamma_{2,n}$ have imaginary parts, which give rise to corrections $\sim(\alpha B)^n$ to the normal photoionization cross section. These corrections, which determine the effect of a constant magnetic field on the photoionization of atoms, are discussed in Sec. 5.

2. PERTURBATION THEORY FOR THE QUASI-ENERGY OF AN ATOM IN AN ALTERNATING ELECTRIC AND A CONSTANT MAGNETIC FIELD

Let us consider an atom located in a constant magnetic field **B** and the field of a plane electromagnetic wave with frequency ω and electric vector

$$\mathbf{F}(\mathbf{r},\ t)=F\,\mathrm{Re}\{\mathbf{e}\exp[i(\mathbf{kr}-\omega t)]\},$$

where F and $\mathbf{e}$ are the amplitude and polarization vector and $\mathbf{k}=\alpha\omega\mathbf{n}$ ($|\mathbf{n}|=1$) is the wave vector. We can adequately take the interaction with the light-wave field into account in the electric dipole approximation, neglecting the magnetic component $\mathbf{H}(\mathbf{r},t)=[\mathbf{n}\cdot\mathbf{F}(\mathbf{r},t)]$ and the effects of the spatial inhomogeneity of the field. Since we are interested only in the "interference" terms $\gamma_{kn}F^kB^n$, due to the joint action of the magnetic field **B** and the wave field, allowance for the magnetic component will give only small corrections $\sim\alpha^2$ to γ_{kn}, which, like the spatial field inhomogeneity effects ($\sim\alpha^2\omega^2\ll 1$), are negligible.

The operator describing the interaction of the atom with the **F** and **B** fields is

$$V(\mathbf{r},\ t)=V_F+V_{M1}+V_d, \tag{2}$$

where

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$$V_F=F\,\mathrm{Re}\{(\mathbf{re})e^{-i\omega t}\}, \tag{2a}$$

$\mathbf{r}$ is the radius vector of the valence electron,

$$V_{M1}=-\alpha\mathbf{B}(\mathbf{I}+\mathbf{S})/2, \tag{2b}$$

$\mathbf{I}$ and $\mathbf{S}$ are the total angular momentum and spin of the electron, and

$$V_d=\alpha^2[\mathbf{Br}]^2/8 \tag{2c}$$

is the diamagnetic interaction operator.

The expansion (1) of the quasi-energy $\mathscr{E}$ of the atom in powers of F and B and the computational formulas for the susceptibilities γ_{kn} can be obtained, using the general perturbation-theory formalism for the quasi-energy states[8,9]:

$$\mathscr{E}=E_0+\langle\langle 0|V|\Psi\rangle\rangle, \tag{3}$$

where E_0 is the energy, $|0\rangle\rangle$ is the ground-state vector of the unperturbed atom,

$$|\Psi\rangle\rangle=|0\rangle\rangle-\mathfrak{G}_{\mathscr{E}}V|\Psi\rangle\rangle \tag{4}$$

is the wave function of the atom in the field, and $\mathfrak{G}_{\mathscr{E}}$ is the quasienergy Green function for the unperturbed Schrödinger equation. The double brackets denote integration over the space, and averaging over the time, variable. In order to obtain an explicit expression for the correction of the order F^kB^n to E_0, we must have an expression for the quasi-energy $\mathscr{E}$ in the $(k+n)$th order in the interaction V. To obtain the latter expression, we must substitute into (3) the $(k+n-1)$th order expression for $|\Psi\rangle\rangle$, obtained by iteration of (4). Then expanding in a series in powers of V the parameter $\mathscr{E}$ in the Green functions $\mathfrak{G}_{\mathscr{E}}$, and grouping the resulting terms, we obtain the following expressions for the corrections $\Delta\mathscr{E}^{(q)}$ with $q\leqslant 6$ (cf. Ref. 9):

$$\begin{gathered}\Delta\mathscr{E}^{(1)}=\langle\langle 0|V|0\rangle\rangle,\quad \Delta\mathscr{E}^{(2)}=-T_1,\quad \Delta\mathscr{E}^{(3)}=T_{11}-\Delta\mathscr{E}^{(1)}T_2,\\ \Delta\mathscr{E}^{(4)}=-T_{111}+2\Delta\mathscr{E}^{(1)}T_{21}-(\Delta\mathscr{E}^{(1)})^2T_3-\Delta\mathscr{E}^{(2)}T_2,\\ \Delta\mathscr{E}^{(5)}=T_{1111}-\Delta\mathscr{E}^{(1)}[2T_{211}+T_{121}]+(\Delta\mathscr{E}^{(1)})^2[2T_{31}+T_{22}]\\ +2\Delta\mathscr{E}^{(2)}T_{21}-\Delta\mathscr{E}^{(3)}T_2-2\Delta\mathscr{E}^{(1)}\Delta\mathscr{E}^{(2)}T_3-(\Delta\mathscr{E}^{(1)})^3T_4,\\ \Delta\mathscr{E}^{(6)}=-T_{11111}+2\Delta\mathscr{E}^{(1)}[T_{2111}+T_{1211}]-(\Delta\mathscr{E}^{(1)})^2[2T_{311}\\ +2T_{221}+T_{131}+T_{212}]-\Delta\mathscr{E}^{(2)}[2T_{211}+T_{121}]+2\Delta\mathscr{E}^{(1)}\Delta\mathscr{E}^{(2)}[T_{22}\\ +2T_{31}]+2(\Delta\mathscr{E}^{(1)})^3[T_{41}+T_{32}]+2\Delta\mathscr{E}^{(3)}T_{21}-\Delta\mathscr{E}^{(4)}T_2\\ -[2\Delta\mathscr{E}^{(1)}\Delta\mathscr{E}^{(3)}+(\Delta\mathscr{E}^{(2)})^2]T_3-3(\Delta\mathscr{E}^{(1)})^2\Delta\mathscr{E}^{(2)}T_4\\ -(\Delta\mathscr{E}^{(1)})^4T_5.\end{gathered} \tag{5}$$

Here

$$T_{n_1\,n_2\,\ldots\,n_p}=\langle\langle 0|V\mathfrak{G}_{E_0}{}^{n_1}V\mathfrak{G}_{E_0}{}^{n_2}V\ldots\mathfrak{G}_{E_0}{}^{n_p}V|0\rangle\rangle \tag{6}$$

is the composite matrix element of the $(p+1)$st order in V, containing $(n_1+n_2+\cdots+n_p)$ unperturbed quasi-energy state (QES) Green functions $\mathfrak{G}_{E_0}$. By substituting into these formulas the explicit expressions for the perturbation operator V, (2), we can obtain the expansion of $\mathscr{E}$ in powers of F and B right up to the terms with $k+n=6$.

3. THE SUSCEPTIBILITIES $\gamma_{2k,2n+1}$ AND THE VERDET CONSTANT

The general expression for the susceptibility γ_{21} in a state with arbitrary angular momentum J is derived in Ref. 3. For the state with $J=0$

$$\gamma_{21}={}^1/_{24}A\delta_{11}^{101}(\omega\omega)\cos\beta, \tag{7}$$

where $-1\leqslant A\leqslant 1$ is the degree of circular polarization of the wave [$A=1(-1)$ for right-handed (left-handed) circular polarization and $A=0$ for linearly polarized radiation], β is the angle between the vectors $\mathbf{B}$ and $\mathbf{n}$,

$$\delta_{l_1l_2}^{n_1n_2n_3}(\omega_1\omega_2)=\rho_{l_1l_2}^{n_1n_2n_3}(\omega_1\omega_2)-\rho_{l_1l_2}^{n_1n_3n_3}(-\omega_1-\omega_2)$$

is the difference between radial composite matrix elements of the type

$$\rho_{l_1l_2\ldots l_p}^{n_1n_2\ldots n_{p+1}}(\omega_1\omega_2\ldots\omega_p)=\langle 0|r^{n_1}g_{l_1}^{\omega_1}r^{n_2}\ldots r^{n_p}g_{l_p}^{\omega_p}r^{n_{p+1}}|0\rangle, \tag{8}$$

where

$$g_l^{\omega}(r,r')=\sum_k R_{kl}(r)R_{kl}(r')/[E_k-(E_0+\omega)]$$

is the radial Green function for the valence electron.[8] Similar expressions can be obtained for γ_{41} and γ_{23}. Notice that these quantities are closely tied with the susceptibilities determining the Faraday effect in an atomic gas, as well as with the corrections to the Faraday effect in strong electric and magnetic fields.[10,11] If we write the Verdet gas constant with allowance for the corrections $\sim F^2$ and $\sim B^2$ in the form[11]

$$v(\omega)=\pi N_0\alpha^2\omega\{Y_0(\omega)+F^2Y_1(\omega)+(\alpha B)^2Y_2(\omega)\},$$

where N_0 is the number of atoms in a unit volume of the gas, then the susceptibilities $\gamma_{2k,2n+1}$ can be expressed in terms of the coefficient Y as follows:

$$\gamma_{21}={}^1/_8AY_0(\omega)\cos\beta,$$

$$\gamma_{23}={}^1/_8AY_2(\omega)\cos\beta,\quad \gamma_{41}={}^1/_8AY_1(\omega)\cos\beta.$$

The numerical computation and the characteristics of the frequency dependence of the quantities $Y(\omega)$ for the atoms of hydrogen, the alkali metals, and the inert gases are discussed in Refs. 10 and 11. Here we only note that the parameters Y are determined by the differences between composite matrix elements of the type (8), differences which go to zero as $\omega\to 0$, with the result that in a constant electric field (i.e., for $\omega=0$), as in the case when $F=0$, the corrections $\Delta\mathscr{E}_{kn}$ of odd order in B vanish.

4. THE SUSCEPTIBILITIES $\gamma_{2k,2n}$ AND THE COTTON-MOUTON CONSTANT

The corrections $\Delta\mathscr{E}_{2k,2n}$ are different from zero at all frequencies and for all polarizations of the electric field $\mathbf{F}(t)$, and also for arbitrary orientation of the vectors $\mathbf{e}$ and $\mathbf{B}$, irrespective of the quantum numbers of the atomic level. Of greatest interest here is the first nonvanishing correction

$$\Delta\mathscr{E}_{22}=F^2(\alpha B)^2\gamma_{22}(\omega,\theta),$$

which determines the change in the electric susceptibility in a magnetic field and, conversely, the change in the diamagnetic susceptibility in an electric field. With the aid of the expressions for $\Delta\mathscr{E}^{(3)}$ (with the operator V_d), and $\Delta\mathscr{E}^{(4)}$ (with the operator V_{M1}) we can, after integrating over the angle variable for the valence electron, and averaging over the time in (5) and (6), obtain

$$\gamma_{22}(\omega,\ \theta)=a_{22}(\omega)-b_{22}(\omega)P_2(\cos\theta),$$

$$a_{22}(\omega)={}^1/_{144}[2\sigma_{01}^{211}(0,\omega)+\sigma_{11}^{121}(\omega\omega)$$
$$-\langle r^2\rangle\sigma_{11}^{101}(\omega\omega)-2\sigma_{111}^{1001}(\omega\omega\omega)],$$

$$b_{22}(\omega)={}^1/_{360}[2\sigma_{21}^{211}(0\omega)+\sigma_{11}^{121}(\omega\omega)-5\sigma_{111}^{1001}(\omega\omega\omega)],\quad (9)$$

where

$$\sigma_{l_1\ldots l_p}^{n_1\ldots n_{p+1}}(\omega_1\ldots\omega_p)=\rho_{l_1\ldots l_p}^{n_1\ldots n_{p+1}}(\omega_1\ldots\omega_p)$$
$$+\rho_{l_1\ldots l_p}^{n_1\ldots n_{p+1}}(-\omega_1\ldots-\omega_p)$$

is the sum of the radial composite matrix elements of the $(p+1)$st order, (8), θ is the angle between the vectors **e** and **B**, and $P_2(x)$ is the Legendre polynomial [the field $\mathbf{F}(t)$ is assumed to be linearly polarized]. If the electric field is circularly polarized, then $P_2(\cos\theta)$ in (9) should be replaced by $-P_2(\cos\beta)/2$. And if the field **F** is a constant field, then the right member of (9) should be multiplied by 2.

Notice that the correction $\Delta\mathscr{E}_{22}$ can also be expressed in terms of the atom's magnetoelectric susceptibility $\eta_{ijkl}(-\omega;\omega00)$, which governs the Cotton-Mouton effect in an atomic gas[12]:

$$\Delta\mathscr{E}_{22}=-\frac{F^2}{8}\sum_{ijkl}\eta_{ijkl}(-\omega;\ \omega00)e_ie_jB_kB_l.$$

Evaluating here the sum with allowance for the symmetry properties of the tensor η_{ijkl}, we obtain for the case of the nondegenerate states of the atom the expression

$$\gamma_{22}(\omega,\theta)=-\frac{1}{24\alpha^2}[2\eta_{xxzz}+\eta_{zzzz}+2(\eta_{zzzz}-\eta_{xxzz})P_2(\cos\theta)].\quad (10)$$

The higher-order susceptibilities $\gamma_{2k,2n}$ determine both the corrections to the energy and the higher-order corrections to the susceptibility η_{ijkl}. In particular, the dependence of the components of the tensor η_{ijkl} on the intensities F and B can be represented in the form

$$\eta_{zzzz}(-\omega;\ \omega00)=-8\alpha^2[\gamma_{22}(\omega,\ 0)+F^2\gamma_{42}(\omega,\ 0)$$
$$+(\alpha B)^2\gamma_{24}(\omega,\ 0)].\quad (11)$$

This same expression with $\theta=0$ replaced by $\theta=\pi/2$ gives the component η_{xxzz}.

As in (9), the dependence of $\gamma_{2k,2n}(\omega,\theta)$ on θ has the form

$$\gamma_{2k,2n}(\omega,\ \theta)=a_{2k,2n}(\omega)-b_{2k,2n}(\omega)P_2(\cos\theta),\quad (12)$$

with $P_2(\cos\theta)$ replaced by $-P_2(\cos\beta)/2$ in a field with circular polarization.[2)] Similarly to (9), the quantities $a_{2k,2n}(\omega)$ and $b_{2k,2n}(\omega)$ can be represented in the form of linear combinations of the composite matrix elements σ of (higher) orders right up to the $(2k+2n)$th.

Notice that the operator V_{M1} makes a contribution to $\gamma_{2k,2n}(\omega,\theta)$ only when $\theta\neq 0$, and this gives rise to a significant difference in the frequency dependences of $\gamma_{2k,2n}(\omega,0)$ and $\gamma_{2k,2n}(\omega,\pi/2)$. Thus, for example, $\gamma_{22}(\omega,0)$ has second-order resonance poles at the frequencies for the transitions into the P states, remaining sign-constant and nonzero everywhere, whereas $\gamma_{22}(\omega,\pi/2)$ has third-order poles at these same frequencies and changes sign on going through a resonance, going to zero within each interresonance interval. In view of this characteristic of the dispersion dependence of the $\gamma_{2k,2n}$ tensor components, it is convenient to consider not the quantities $a_{2k,2n}(\omega)$ and $b_{2k,2n}(\omega)$, which are the irreducible parts of the $\gamma_{2k,2n}$ tensor, but their linear combinations.

$$\gamma_{2k,2n}^{\parallel}(\omega)\equiv\gamma_{2k,2n}(\omega,0)=a_{2k,2n}(\omega)-b_{2k,2n}(\omega),$$

$$\gamma_{2k,2n}^{\perp}(\omega)\equiv\gamma_{2k,2n}(\omega,\pi/2)=a_{2k,2n}(\omega)+b_{2k,2n}(\omega)/2.$$

As an example, in Table I we depict the dispersion dependence of the quantities $\gamma_{22}^{\parallel}(\omega)$ and $\gamma_{22}^{\perp}(\omega)$ for the hydrogen atom in the ground state. In the region $\omega>|E_0|=\frac{1}{2}$ these quantities are complex, a situation which corresponds to the possibility of ionization of the atom; therefore, in the table we give both Re γ and Im γ for $\omega>\frac{1}{2}$. It should be noted that $\gamma_{22}^{\parallel}(\omega)$ is of constant sign, but that $\gamma_{22}^{\perp}(\omega)$ changes sign in each interresonance interval. This behavior of $\gamma_{22}(\omega)$ is universal for the s states of atoms. Figure 1 shows plots of the functions $\gamma_{22}^{\parallel}(\omega)$ and $\gamma_{22}^{\perp}(\omega)$ for cesium in the $6s$ ground state. As can be seen from the figure, the γ tensor for this state has the same structure and frequency dependence as the corresponding tensor for the hydrogen atom. As has already been noted above, the change in sign of $\gamma_{22}^{\perp}$ is caused by the contribution of the dipole interaction V_{M1}, which, for $\theta\neq 0$, leads to the Zeeman splitting of the virtual p states with angular momentum components $m=\pm 1$. The contribution of this splitting in second order in **B** is given by the matrix element σ_{111}^{1001} in (9), which can also be written in terms of the second frequency derivative of the normal dynamical polarizability $\alpha(\omega)$ of the atom. The difference in the dispersion formulas for $\gamma^{\parallel}$ and $\gamma^{\perp}$ leads to a situation in which longitudinal and transverse magnetic fields have in the region $\omega<|E_0|$ qualitatively different effects on the electric dipole polarizability $\alpha(\omega)$, which, with allowance for the terms $\sim B^2$, has the form

$$\alpha(\omega,\ B)=\alpha(\omega)-4(\alpha B)^2\gamma_{22}(\omega,\ \theta).\quad (13)$$

In particular, in the region $0\leqslant\omega<\omega_r$ (ω_r is the ground state—first excited p state transition frequency) $\gamma^{\parallel}$ only makes the polarizability $\alpha(\omega,B)$ smaller than $\alpha(\omega)$, while $\gamma^{\perp}$ changes the sign in this ω range and substantially increases $\alpha(\omega,B)$ as ω approaches ω_r.

Let us point out the weakening effect of a magnetic field on the static polarizability of atoms (in the case of the hydrogen atom this fact has been noted before by Turbiner[5]). In this case the effect of a longitudinal magnetic field is weaker

TABLE I. Magnetoelectric susceptibilities of the hydrogen atom in the ground state $(n) \equiv 10^n$.

ω	$\gamma_{22}^{\parallel}$	$\gamma_{22}^{\perp}$
0.05	2.58(0)	4,20(0)
0.10	2,89(0)	4.33(0)
0,15	3,56(0)	4.45(0)
0.20	4,98(0)	4,10(0)
0,25	8,57(0)	5.10(−1)
0.30	2.23(1)	−3.42(1)
0.35	2.22(2)	−1,73(3)
0.3825	3.46(3)	8.97(4)
0.4025	4.36(2)	2,50(3)
0.4225	4.92(2)	7,77(2)
0.4325	1,27(3)	−5.46(2)
0.4375	3,63(3)	−9.16(3)
0,60	1,72(−1)−i·5,27(−1)	1.78(0)−i·5,12(0)
0.75	9.57(−2)−i·2,26(−1)	1,07(0)−i·1,53(0)
1.00	4,33(−2)−i·7,56(−2)	3.92(−1)−i·3.26(−1)
1.25	2.24(−2)−i·3,20(−2)	1.66(−1)−i·1,01(−1)
1,50	1.28(−2)−i·1,58(−2)	8,04(−2)−i·3.94(−2)
2,00	5.08(−3)−i·5,12(−3)	2.55(−2)−i·9.44(−3)
2.50	2,43(−3)−i·2,12(−3)	1,05(−2)−i·3.26(−3)

than that of a transverse field $(\gamma_{22}^{\parallel}(0) < \gamma_{22}^{\perp}(0))$. To corroborate this fact, let us give the numerical values of $\gamma_{22}(0)$ for a number of atoms:

Atom:	H	Na	Cs
$\gamma_{22}^{\parallel}$:	159/32	1048	1627
$\gamma_{22}^{\perp}$:	797/96	1445	4779

The results for the hydrogen atom coincide with the data reported in Ref. 4, while the $\gamma_{22}^{\parallel}$ values coincide with the data reported in Refs. 5 and 6. The value $\gamma_{22}^{\perp} = 731/96$, obtained in Ref. 5, is incorrect (only the case of parallel fields is considered in Ref. 6).

Let us note that, for the hydrogen atom in static **F** and **B** fields, the computation of the composite matrix elements (8) is trivial when use is made of the Sturm expansion of the reduced Coulomb Green function[8] and the orthogonality properties of the Laguerre polynomials. Below we present the thus computed values of the coefficients $a_{2k,2n}(0)$ and $b_{2k,2n}(0)$ in (12) for higher-order susceptibilities of hydrogen in the ground state.

$$a_{24}=-256619/5184, \quad b_{24}=-724687/41472;$$
$$a_{42}=5598895/4608, \quad b_{42}=826075/1152.$$

For $\gamma_{24}^{\parallel}(0)$ and $\gamma_{42}^{\parallel}(0)$ these numbers give the same result as Ref. 6; the $\gamma_{24}(0)$ also coincide with the data reported in Ref. 4, in which the susceptibilities γ_{42} are not computed. The numerical values obtained in Ref. 5 for $\gamma_{24}(0)$ coincide neither with the data presented for the **F**‖**B** case, nor with those presented for the **F**⊥**B** case, and are incorrect.

5. CORRECTIONS TO THE PHOTOIONIZATION CROSS SECTION IN A MAGNETIC FIELD

The question of the change in the photoionization of atoms under the action of a constant magnetic field is important from both theoretical and practical standpoints. On the one hand, this question is, from the methodological standpoint, interesting in respect of the elucidation of the general laws governing the effect of a magnetic field on the quantum transition probabilities, and, on the other, it provides information for the quantitative description of the elementary photoprocesses that occur in gases in the presence of magnetic fields.

Within the framework of the standard perturbation-theory scheme allowance for the effect of a weak magnetic field on photoionization would require the consideration of

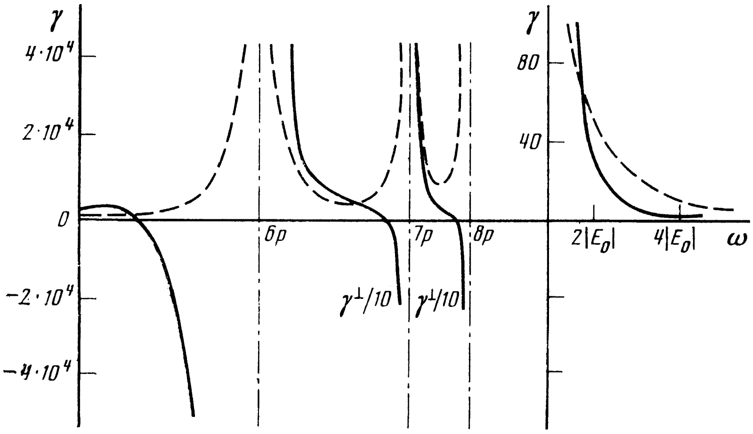

FIG. 1. The real parts of the longitudinal ($\gamma_{22}^{\parallel}(\omega)$, dashed curves) and transverse ($\gamma_{22}^{\perp}(\omega)$, continuous curves) components of the magnetoelectric susceptibility of cesium atoms. The resonances on the 6*p*, 7*p*, and 8*p* states and the behavior in the region above the photoionization threshold are shown.

the effect of the **B** field up to second order in the energies and wave functions of the discrete and continuous spectra of the atomic electron, a procedure which, in the latter case, is quite difficult because of the singularities that arise in the computation of the corrections to the continuum wave functions. But in the magnetoelectric-susceptibility formalism the effect of a magnetic field on the photoionization cross section comes out naturally in the consideration of the frequency region $\omega > |E_0|$, where $|E_0|$ is the binding energy of the valence electron. For such frequencies the susceptibilities $\gamma_{k,n}(\omega,\theta)$ are complex, and the imaginary parts of $\gamma_{2,n}(\omega,\theta)$ determine the corrections $\sim B^n$ to the photoionization cross section, in accord with the general ideology of the complex-quasi-energy formalism.[9]

Taking account of the effects of only the first two nonvanishing orders in **B**, we can represent the total cross section for photoionization of the s state in the form

$$\sigma(\omega, B)=\sigma_0(\omega)-\sigma_1(\omega)\alpha B-\sigma_2(\omega)(\alpha B)^2. \tag{14}$$

Here $\sigma_0(\omega)$ is the cross section for photoionization of the atom for $\mathbf{B}=0$,

$$\sigma_1(\omega)=16\pi\alpha\omega \operatorname{Im}\gamma_{21}(\omega, \beta)$$

is the coefficient determining the correction of first order in **B**, a quantity which is nonzero only when the degree of circular polarization is nonzero and the direction of propagation of the field $\mathbf{F}(t)$ makes an angle $\beta \neq \pi/2$ with the vector **B** [see (7)]. The second-order correction, determined by the coefficient

$$\sigma_2(\omega)=16\pi\alpha\omega \operatorname{Im}\gamma_{22}(\omega, \theta), \tag{15}$$

is nonzero for any polarization of the alternating field and any mutual orientation of the vectors **e** and **B**.

It is clear that, for $\gamma_{21}\neq 0$, the dominant contribution to the constant-magnetic-field induced change in the cross section for ionization of an atom is made by the term linear in **B**, which is easily computed from (7):

$$\operatorname{Im}\gamma_{21}={}^1/_{24}A\cos\beta \operatorname{Im}\rho_{11}^{101}(\omega\omega).$$

Next, using $\sigma_0(\omega)$ and $\operatorname{Im}\rho_{11}^{101}$ explicitly expressed in terms of the radial matrix element $\rho_{E1,ns}$ for the bound-free dipole transition into the state with energy E and electron orbital angular momentum $l=1$, we obtain

$$\sigma(\omega, B)=[1-{}^1/_2\alpha BA\cos\beta(d/dE)]\sigma_0(\omega),$$
$$\sigma_0(\omega)={}^4/_3\pi\alpha^2\omega|\rho_{E1,ns}|^2. \tag{16}$$

In particular, for the 1s state of hydrogen this expression assumes the form

$$\sigma_{1s}(\omega,B)=\sigma_{1s}(\omega)\left[1+\frac{1}{2}\alpha BA\cos\beta\left(\frac{4+10k^2}{k^2(1+k^2)}-\frac{4}{k^3}\operatorname{arctg}k+\frac{2\pi}{k^3(1-e^{2\pi/k})}\right)\right],$$

where

$$\sigma_{1s}(\omega)=\frac{4\pi^2\alpha\omega}{3}\frac{256}{(1+k^2)^5}\frac{\exp(-4k^{-1}\operatorname{arctg}k)}{1-e^{-2\pi/k}}.$$

and $k^2/2=E=\omega-1/2$ is the kinetic energy of the photoelectron. Thus, depending on the sign of the product $A\cos\beta$, a circularly polarized field can intensify or abate photoionization. And if for hydrogen the derivative $(\partial/\partial E)|\rho_{E1,ns}|^2$ is negative for all $E>0$, for many-electron atoms this quantity can change sign, going to zero at some E values. The meaning of the result (16) becomes quite clear if we take into account the fact that, in the linear approximation, the effect of a magnetic field manifests itself only in the splitting of the levels by an amount $\Delta E_{m_l}=\frac{1}{2}m_l\alpha B$ (the Zeeman effect). Therefore, the photoelectron energy $E=E_0+\omega$ in the case of ionization from the s state in a field B is changed to $E_0+\omega+\Delta E_{m_l}$, $m_l=\pm 1$, and (16) gives the first two terms of the expansion of the cross section σ_0 for ionization ($E=E_0+\omega\pm\frac{1}{2}\alpha B$) into states with $m_l=\pm 1$ in a series in powers of B. It is clear that the linear (in **B**) terms can enter into the cross section only in the form of a true scalar, which in our problem is the quantity $[\mathbf{ee}^*]\cdot\mathbf{B}$. This explains the presence of the factor $A\cos\beta$ in (7) and (16) and the vanishing of the effect in the case of linear polarization of the field $\mathbf{F}(t)$ and when $\mathbf{k}\perp\mathbf{B}$ in the case of elliptic polarization. Notice that the quadratic terms that arise from further expansion of $\sigma_0(E_0+\omega+\Delta E_{m_l})$ in powers of B are nonzero in these cases as well (see below).

A similar calculation can be carried out for the second-order (in B) correction determined by the imaginary part of the susceptibility $\gamma_{22}(\omega)$ in (15). The results obtained in Sec. 4 show that, for hydrogen, $\operatorname{Im}\gamma_{22}<0$ at all electric-field frequencies and for all values of the angle θ, i.e., the cross section for photoionization by linearly polarized radiation is always greater in a **B** field. In this case, as follows from Table I, $|\operatorname{Im}\gamma_{22}^{\perp}|>|\operatorname{Im}\gamma_{22}^{\parallel}|$ (by almost an order of magnitude at the ionization threshold), i.e., a transverse **B** field more effectively promotes ionization than a longitudinal field. It should also be noted that, in the frequency region ($\omega>|E_0|$) in question, the real part of γ_{22} is positive, and this leads to an increase (in absolute value) in the real part of the polarizability (13) ($\alpha(\omega)<0$ for $\omega>|E_0|$), and, consequently, in the ground-state quasi-energy shift in the resultant field; in this case $\operatorname{Re}\gamma_{22}^{\perp}>\operatorname{Re}\gamma_{22}^{\parallel}$. This result does not depend on the field frequency in the region $\omega>|E_0|$, since all the susceptibilities monotonically decrease in absolute value as ω increases.

Thus, the nature of the effect of a magnetic field on the dynamical polarizability $\alpha(\omega)$ of hydrogen in the frequency region $\omega>|E_0|$ is the direct opposite of what obtains in the static case. As is well known, in static **F** and **B** fields the magnetic field has a stabilizing effect on the bound state, decreasing both the electric-field induced energy shift and the probability for tunneling ionization in the **F** field.[13,6] The situation is different in a high-frequency electric field, as can easily be seen in the case of the level shift from the different characters of the effects that a static and a linearly polarized high-frequency field $\mathbf{F}(t)$ have on an atom in the absence of a magnetic field. Whereas in a constant (or low-frequency) **F** field the level E_0 is lowered, in fields with $\omega>|E_0|$ it is shifted upwards (by an amount equal to the mean vibrational energy of the electron in the wave field in the case when $\omega\gg|E_0|$).

TABLE II.

ω	Im σ				
	σ_{01}^{211}	σ_{21}^{211}	σ_{11}^{121}	σ_{11}^{101}	σ_{111}^{1001}
0,6	−131,8	−84,2	−32.3	−46.5	180.1
1,0	−18.18	−9.98	9,90	−4.18	10.02

The application of a magnetic field leads, as a result of the localizing effect on the bound states, to a decrease in the deformability of the electron shell by the electric field, i.e., it makes a negative contribution to the polarizability ($\gamma_{22}(0)$ and Re γ_{22} ($\omega > |E_0|$) are positive). But if in a constant field this leads to a decrease in the total level shift ($\alpha(0) > 0$), in fields with $\omega > |E_0|$, in which Re $\alpha(\omega) < 0$, the level shift only increases.

The effect of a magnetic field on the ionization is also not the same for static and alternating **F**(t) fields. In an alternating field the corrections $\sim B^2$ to the photoeffect cross section arise as a result of the **B**-field induced changes in the ground-state and continuum energies and wave functions of the atom. The contributions of each channel can be analyzed by investigating the imaginary parts of the individual matrix elements σ entering into γ_{22} in (9). The term taking account of the change in the ground-state energy is given by the imaginary part of σ_{11}^{101} (ω,ω), and can, similarly to (16), be represented in the form of a product of the dynamical shift $\Delta E_d = (1/12)\langle r^2\rangle(\alpha B)^2$ and the derivative $d\sigma_0/dE$. This effect makes $\sigma(\omega,B)$ smaller than $\sigma_0(\omega)$, since $d\sigma_0/dE < 0$ (in hydrogen $\sigma_0(\omega)$ decreases monotonically with increasing ω). In contrast, Imσ_{111}^{1001} can be expressed in terms of $d^2\sigma_0/dE^2 > 0$, and increases the cross section as a result of the above-discussed Zeeman splitting of the continuum states (this effect vanishes in the case **F**$(t)\|$**B**, when the component of the orbital angular momentum of the photoelectron in the direction of **B** is equal to zero). The imaginary parts of σ_{01}^{211} and σ_{21}^{211} arise from the "diamagnetic" corrections, $\sim B^2$, to the ground-state wave function, and lead to an increase in the photoeffect cross section. This corresponds to the fact that the effective localization of the bound state by the magnetic field leads to an increase in the overlap integral involving the oscillating continuum wave function. Finally, Im σ_{11}^{121} makes to the cross section a contribution that arises from the effect of the **B** field on the continuum wave function. Here we cannot draw an unequivocal conclusion about the sign of the effect, since the result depends on the energy of the photoelectron. The numerical calculations fully corroborate the foregoing, and show that only Im $\sigma_{11}^{121}(\omega,\omega)$ behaves nonmonotonically, changing sign in the frequency range from $\omega = 0.6$ to $\omega = 0.65$. For comparison of the contributions of the various effects, in Table II we present the values of the imaginary parts of the individual matrix elements in (9) for $\omega = 0.6$ and 1.0. Numerically, the contributions of Im σ_{11}^{101} and Im σ_{11}^{121} to Im γ_{22} are insignificant. Thus, the monotonic character of the frequency dependence of the correction, $\sim B^2$, to the cross section for the photoeffect in hydrogen is largely accidental, and is due to the fairly smooth frequency dependence of the composite matrix elements for the transitions into the continuum in the case of the purely Coulomb potential.

For many-electron atoms the frequency dependence of Im γ_{22} is more complicated, and does not allow us to draw a general conclusion about the nature of the effect of a magnetic field on photoionization in the entire frequency region $\omega > |E_0|$. Thus, for example, in the case of the alkali metals there exist characteristic frequencies at which the ionization cross section σ_0 is close to zero (the so-called Cooper minima). The imaginary parts of the susceptibilities γ_{22} vanish or also attain their minimum values roughly at these same frequencies. As an example, in Fig. 2 we show plots of the functions $\sigma_0(\omega)$ and $\sigma_2^{\perp,\|}(\omega)$ for the ground state of the sodium atom, as computed with the use of the technique of reexpansion of the Sturm series for the optical-electron Green function[7] in the approximation of the model-potential method.[8] As can be seen from the figure, σ_0 has a characteristic minimum at a frequency $\omega \approx 1.5|E_0|$, while $\sigma_2^{\|}$ changes sign in this frequency region. Such oscillations of the imaginary part of the susceptibility $\gamma_{22}(\omega,\theta)$ are clearly a result of the electron-core interaction, which essentially depends on the photo-electron energy. A similar type of interaction is also responsible for a number of other effects in the photoionization of the alkali-metal atoms: a change in the direction of polarization of the photoelectrons (see, for example, Ref. 14), a change in the direction of the drag current,[15] etc.

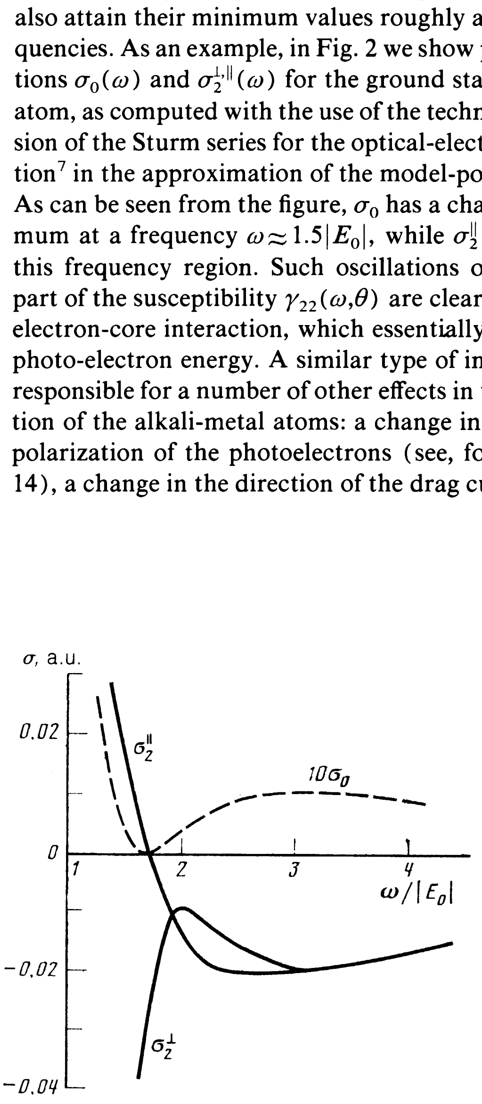

FIG. 2. Photoionization cross section $\sigma_0(\omega)$ (dashed curve) and the second-order corrections in a longitudinal and a transverse magnetic field ($\sigma_2^{\|}(\omega)$, $\sigma_2^{\perp}(\omega)$, continuous curves) for sodium atoms. The abscissas are the values of the ratio of the photon energy to the ionization potential.

6. LIMITS OF APPLICABILITY OF THE PERTURBATION THEORY AND THE POSSIBILITY OF EXPERIMENTAL OBSERVATION OF THE MAGNETOELECTRIC EFFECTS

The above-performed analysis of the tensor structure and dispersion properties of the magnetoelectric susceptibilities γ_{kn} allows us to obtain new information about atomic spectra in variable electric and static magnetic fields. Although all the quantitative data were obtained for only the ground states of hydrogen and the alkali-metal atoms, it is clear that we can easily carry out in similar fashion calculations for the excited states of nonhydrogenic atoms. In particular, numerical data for the susceptibilities γ_{21} of the excited nP_J states of a number of atoms are obtained in Ref. 3. For the excited states of hydrogen the situation is complicated by the accidental Coulomb degeneracy. In static **F** and **B** fields it is possible in a number of cases to obtain an analytic solution to the problem of the hydrogen spectrum on the basis of a group-theoretical analysis[1,2] or special modifications of the semiclassical perturbation theory.[16,17] But for optical $\mathbf{F}(t)$ fields, it is doubtful whether the indicated approaches can be effective, and quantitative results can, apparently, be obtained only through a numerical diagonalization of the matrices formed by the composite matrix elements for the degenerate states of the nth shell.

The extension of the calculations to the case of arbitrarily polarized and oriented variable electric fields, and also to the case of the easily polarizable alkali-metal atoms makes it possible for the theory to be quantitatively verified in experiments with metallic vapor located in a magnetic field and irradiated by intense light radiation. There arises here the question of the field intensity (F and B) regions where we can limit ourselves to terms $\sim F^2B$ and F^2B^2 in the expression (1) for $\Delta\mathscr{E}$. Since the expansion (1) is a perturbation theory series, it is clear that F and $B\alpha$ should, in any case, be small compared to the characteristic "intra-atomic" fields F_0 and B_0 ($E_0 = 5.14\times10^9$ V/cm and $B_0 = 1.71\times10^7$ G for the ground state of hydrogen). But the specific estimates $F_{\rm lim}$ and $B_{\rm lim}$ for the intensities of the fields in which the terms $\sim F^2B^4$ and F^4B^2 are still insignificant depend on the magnitudes of the corresponding susceptibilities, and are different for different atoms. Furthermore, the situation is not the same for constant and variable **F** fields, since the series (1) is asymptotic in the first case, but has a finite radius of convergence with respect to F in the second.[18]

Let us first consider the case of a static electric field or a linearly polarized $\mathbf{F}(t)$ field. The results of Sec. 4, as well as the results of numerous susceptibility calculations carried out for purely electric and magnetic fields (see, for example, Ref. 6) allow us to write down the following expansion of $\Delta\mathscr{E}$ for the 1s state of hydrogen right up to terms of sixth order:

$$\Delta\mathscr{E}_{1s}=-{}^1/{}_2\alpha(F)F^2-{}^1/{}_2\chi(B)(\alpha B)^2+\gamma(B,F)F^2(\alpha B)^2, \tag{17}$$

$$\alpha(F)=4{,}5+111{,}1F^2+9816F^4+\ldots, \tag{17a}$$

$$\chi(B)=-0{,}5+0{,}55(\alpha B)^2-2{,}422(\alpha B)^4+\ldots, \tag{17b}$$

$$\gamma(B,F)=(8{,}302-3{,}333\cos^2\theta)+(1574-1076\cos^2\theta)F^2 -(58{,}24-26{,}21\cos^2\theta)(\alpha B)^2+\ldots, \tag{17c}$$

θ being the angle between **F** and **B**. Here $\gamma(B,F)$ can be regarded as the magnetic-field related corrections to the electric polarizability $\alpha(F)$, or, conversely, as the corrections to the magnetic susceptibility $\chi(B)$ in the **F** field. As can be seen, the electric-field induced corrections increase significantly faster than the magnetic-field induced corrections ($\gamma_{k+2,n}/\gamma_{k,n} \gg \gamma_{k,n+2}/\gamma_{k,n}$), so that the perturbation theory in terms of the electric field is the first to break down as F and B increase. As to the critical field intensities $F_{\rm lim}$ and $B_{\rm lim}$ up to which we can use the results of the first nonvanishing order in **B** and **F**, they turn out, as can be seen from (17a)–(17c), to be quite high, and in laboratory experiments, in which the fields F and αB are several orders of magnitude smaller than F_0 and B_0, we can, with a high degree of accuracy, set $\gamma(B,F)\approx\gamma_{22}(\omega=0,\theta)$. In a linearly polarized alternating $\mathbf{F}(t)$ field the expansion of the quasienergy $\Delta\mathscr{E}_{1s}$ again has the form (17), with the only difference that $\alpha(F)$ and $\gamma(B,F)$ are now frequency dependent. The known values of the dynamical polarizabilities and hyperpolarizabilities of hydrogen,[8] as well as the $\gamma_{24}(\omega)$ and $\gamma_{42}(\omega)$ estimates show that in this case also the quantities $F_{\rm lim}$ and $B_{\rm lim}$ do not differ essentially from the estimates obtained in the case of static fields (the case of resonance $\mathbf{F}(t)$ fields, which requires a separate analysis, being the only exception). A similar investigation can be carried out for the ground states of the alkali-metal atoms. The critical fields in this case turn out to be lower than the corresponding fields for hydrogen. Nevertheless the approximation $\gamma(B,F)\approx\gamma_{22}(\omega,\theta)$ is fairly good in light-wave fields with $F\lesssim10^6$ V/cm and magnetic fields with intensities right up to 10^6 G.

In the case of circularly polarized $\mathbf{F}(t)$ fields the terms with odd powers of B in the expansion of the type (17) are also nonzero. The expression for $\Delta\mathscr{E}$ up to the fifth-order term is, when only the cross terms are taken into account, given by (for the meanings of the symbols, see Sec. 3)

$$\begin{aligned}\Delta\mathscr{E}^{BF}={}^1/{}_8A\alpha BF^2\{&Y_0(\omega)\cos\beta+F^2Y_1(\omega)\cos\beta\\&+8A\alpha B[a_{22}(\omega)+{}^1/{}_2b_{22}(\omega)P_2(\cos\beta)]\\&+(\alpha B)^2Y_2(\omega)\cos\beta+\ldots\},\end{aligned} \tag{18}$$

where β is the angle between **B** and the wave vector of the light field and $A=\pm1$. As an example, let us give the numerical values of the coefficients in (18) for hydrogen at the frequency $\omega=0.2$ a.u. (the values of $Y_{0,1,2}$ were taken from Refs. 10 and 11; those of $a_{22}=\frac{1}{3}(\gamma^{\parallel}+2\gamma^{\perp})$ and $b_{22}=\frac{2}{3}(\gamma^{\perp}-\gamma^{\parallel})$, from Table I): $Y_0=19.7$; $Y_1=1.8\times10^4$; $Y_2=3\times10^2$; $a_{22}=4.4$; $b_{22}=-0.59$. It can be seen from this that the higher-order corrections to Y_0 are important only in fields of intensities close to F_0,B_0. In the alkali-metal atoms the higher-order susceptibilities γ_{kn} increase more rapidly with increasing k and n. For example, for cesium at the neodymium laser frequency $\omega_N=0.043$ a.u. we have $Y_0=9.7\times10^4$; $Y_1=-7.5\times10^9$, $Y_2=2.64\times10^8$, $a_{22}=-6\times10^4$, and $b_{22}=-8\times10^4$, so that $\alpha B_{\rm lim}$ is two orders of magnitude smaller than B_0. The critical frequency dependence of the higher-order susceptibilities, which is typical of atomic susceptibilities in the optical-frequency region,[8] should be noted. Thus, at the rubidium laser radiation frequency (ω_R

$= 0.0656$ a.u.) we have in the case of cesium $Y_0 = 7.2\times 10^4$, $Y_1 = 2.6\times 10^{10}$, $Y_2 = 9.7\times 10^7$, $a_{22} = 7\times 10^4$, and $b_{22} = 6\times 10^4$. It can be seen that $Y_1(\omega_N)$ and $Y_1(\omega_R)$ have different signs, but that both are large in absolute value. Although because of the large Y_1 value the corrections $\sim F^2$ in (18) are comparable to Y_0 even in fields with intensities $F \gtrsim 10^{-3}F_0$, for the alkali-metal atoms such optical fields are extremely strong, and lead to the breakdown of the gas as a result of many-photon ionization.[8] Therefore, sources with $F < 10^{-3}F_0$ should be used in experiments on the magnetoelectric effects; in that case the higher-order (in F) effects in (18) are unimportant.

The corrections in (14) to the photoionization cross section in the lowest orders in B are determined by the imaginary parts of γ_{21} and γ_{22}. Analysis of the higher-order contributions leads to the same qualitative results obtained for the real parts of the susceptibilities. Thus, we can, in investigations of the effects of the joint action on an atom of electric and magnetic fields that are weak compared to the intratomic fields, limit ourselves to the consideration of only the first nonvanishing orders of the perturbation theory. The most promising experiments are then those with circularly polarized radiation, when the cross terms are linear in **B**, and can be appreciable.

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1) Below we use the atomic system of units; $\alpha = e^2/\hbar c$ is the fine structure constant.

2) Strictly speaking, the expression (12) is valid for arbitrary k and n only in a static (i.e., $\omega = 0$) field. In a variable ($\omega \neq 0$) field it is valid only for $k = 1$. As k increases, the structure of the tensor $\gamma_{2k,2n}$ becomes more and more complicated (it is sufficient to note that $\gamma_{40}(\omega) = \gamma_{ijkl}(-\omega;\omega,-\omega,\omega)$, the dynamical dipole hyperpolarizability, while not dependent on θ, already has two linearly independent components[8]).

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