Instability due to magnetically induced currents

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(Submitted 18 February 1986)
Zh. Eksp. Teor. Fiz. **91**, 332–342 (July 1986)

The instability, caused by magnetically induced currents, of a nonequilibrium conducting medium in zero external magnetic field is described. The mechanism of such an instability is, in a sense, universal. The conditions for the effect to be observable are obtained on the basis of an analysis of different physical situations, and the feasibility of experimental observation is discussed.

In the present paper we consider the macroscopic consequences of, and the microscopic mechanisms underlying, the following relation between a current and a magnetic field, a relation which is possible in homogeneous conducting media:

$$j^\alpha = \varkappa^{\alpha\beta} H^\beta. \tag{1}$$

Such a relation can be realized only in a nonequilibrium medium. Indeed, the Onsager symmetry relations are valid in the equilibrium case. As shown in the Appendix, a direct consequence of these relations and (1) would be a relation between the magnetic-moment density and the vector potential of the electromagnetic field:

$$M^\alpha = \hat{\varkappa}^{\beta\alpha} A^\beta / c, \tag{2}$$

which is not possible on account of gauge invariance. (In superconducting materials gauge invariance is violated, and we can have relations of the type (2).[1]) Accordingly, the results obtained by Éliashberg in Ref. 2 are incorrect. If the equilibrium medium possesses a magnetic structure, then $\hat{\varkappa}$ in (2) coincides with $\varkappa$ given in (1) for a medium with a time-reversed magnetic structure (i.e., a medium with oppositely directed spins), and gauge-invariance considerations again allows us to conclude that $\hat{\varkappa} \equiv 0$. Therefore, the effect predicted recently by Labzovskiĭ[3] is forbidden on the basis of these considerations.

In a nonequilibrium medium $\hat{\varkappa} \neq 0$ in the general case. Since $\varkappa^{\alpha\beta}$ is a pseudotensor, this requires that the symmetry group of the nonequilibrium medium, which is the intersection of the symmetry group of the equilibrium medium and that of the agent causing the state of nonequilibrium, not contain the operation of spatial inversion. Well-known examples of magnetically induced currents are the currents generated as a result of the Hall and Nernst effects; $\varkappa^{\alpha\beta}$ is then proportional to the electric field **E** and the temperature gradient ∇T, respectively. The currents responsible for the photoelectromagnetic effect[4] are also magnetically induced currents. The tensor $\hat{\varkappa}$ in this case is proportional to the intensity of the external radiation. If we assume that the original equilibrium medium is isotropic, then $\hat{\varkappa}$ has the simple tensorial structure

$$\varkappa^{\alpha\beta} = e^{\alpha\beta\gamma} N^\gamma \tag{3}$$

for all the three examples (in the case of the photoelectromagnetic effect the specified vector **N** is directed along the normal to the surface). The form (3) allows us to graphically interpret the magnetically induced current as the result of the action on the carriers of the Lorentz force, which is always perpendicular to the magnetic field. But in the general case the tensorial structure of $\hat{\varkappa}$ is more complicated than (3). In this sense a good example is the magnetically induced current predicted by Ivchenko and Pikus[5] should occur in a gyrotropic crystal with hot electrons. In this case the structure of $\hat{\varkappa}$ is completely determined by the symmetry of the noncentrosymmetric equilibrium medium, and is rather arbitrary.

The form of $\varkappa^{\alpha\beta}$ is important for the analysis of the stability of a homogeneous nonequilibrium medium against low-frequency, low-wave-number electromagnetic excitations. If $\hat{\varkappa}$ has the structure (3), then the medium is always stable. It is Éliashberg's[2] idea that this type of excitation is unstable in the case when $\hat{\varkappa}$ reduces to a pseudoscalar. In Ref. 6 the present author carried out a symmetry analysis of $\hat{\varkappa}$ in an anisotropic medium in the presence of a temperature gradient, and demonstrated the possibility of the convective instability of low-frequency electromagnetic excitations—thermomagnetic waves[7,8]—in such a medium.

This mechanism of instability development is universal, in the sense that, when certain symmetry requirements, which will be discussed below, are met, it can be realized in any conducting medium in an arbitrarily weak state of nonequilibrium.

In the first section we consider the stability of electromagnetic excitations at the phenomenological level, and obtain the criteria for instability. Further, we carry out consistent analyses of the magnetically induced currents in the particular cases when the nonequilibrium state is due to the presence of dissipative fluxes in the medium, external electromagnetic radiation, the passage of sound through the material, and the inequality of the carrier and lattice temperatures. Then we analyze within the framework of a simple model the question of the final phase of the development of the instability. In conclusion, we consider the feasibility of experimental observation of this effect.

1. Let us write the Maxwell equations describing the evolution of an electromagnetic field in a medium in the form

$$\mathrm{rot}_\alpha\, \mathbf{rot}\, \mathbf{A} = \frac{4\pi}{c} j^\alpha = \frac{4\pi}{c}\left(-\frac{\sigma^{\alpha\beta}\dot{A}^\beta}{c} + \varkappa^{\alpha\beta}\, \mathrm{rot}_\beta\, \mathbf{A} \right). \tag{4}$$

Here **A** is the vector potential of the electromagnetic field

 0038-5646/86/070193-07$04.00

and the terms on the right-hand side correspond to the normal conduction current ($\sigma^{\alpha\beta}$ is the conductivity tensor) and the magnetically induced current. These equations are valid at frequencies $\omega \ll \sigma$, $1/\tau$ and wave numbers $k \ll l^{-1}$ (τ is the time characterizing the momentum relaxation of the carriers and l is the mean free path). Let us determine the electromagnetic-excitation dispersion law. Assuming the conductivity to be isotropic, we obtain

$$i\omega(\mathbf{k})=(c/\sigma)[ik(\varkappa^{xy}-\varkappa^{yx})/2\pm k(\varkappa^{xx}\varkappa^{yy}-(\varkappa^{xy}+\varkappa^{yx})^2/4)^{1/2} -ck^2/4\pi]\equiv ikv_z\pm ku-Dk^2. \quad (5)$$

The signs $\pm$ pertain to the two different polarizations, which, in the general case, are elliptic, and the vector $\mathbf{v}$ and quantity u are respectively determined by the antisymmetric and symmetric—with respect to the indices—parts of $\varkappa^{\alpha\beta}$.

It can be seen from (5) that Eq. (4) with $k \gg \varkappa/c$ describes the usual "diffusion" of a magnetic field in a conducting medium. For $k \ll \varkappa/c$ the nature of the evolution of the excitations is essentially different. If u is imaginary, we have for a given k waves propagating with a constant velocity.[7,9] The polarizations are then linear, and $2iu$ has the meaning of the difference between the velocities of propagation of waves with different polarizations. If u is real, then instability develops at small k values: the perturbation of one of the elliptic polarizations grows exponentially, moving with velocity v. For one-dimensional wave packets this instability is convective in the case when $v_z^2 > u^2$ and absolute when $v_z^2 < u^2$.

Notice that u is real if

$$\eta^\alpha\varkappa_s{}^{\alpha\beta}\eta^\beta\neq 0 \text{ when } \eta\mathbf{k}=0, \quad \eta\neq 0; \quad \varkappa_s{}^{\alpha\beta}=[\varkappa^{\alpha\beta}+\varkappa^{\beta\alpha}]/2. \quad (6)$$

This condition is fulfilled at least for some directions of $\mathbf{k}$ if $\det\hat{\varkappa}_s \neq 0$. If $\hat{\varkappa}_s$ is a sign-variable matrix, then (6) is fulfilled for all directions; otherwise the $\mathbf{k}$ directions for which the excitations are unstable lie inside some elliptic cone. Allowance for the conductivity anisotropy does not alter the above presented instability criterion, namely, the requirement that $\det\varkappa_s \neq 0$. Indeed, for $k \ll \varkappa/c$ the disperison relation has a form similar to (5):

$$i\omega(\mathbf{k})=c[ik(K^{xy}-K^{yx})/2\pm k(K^{xx}K^{yy}-(K^{xy}+K^{yx})^2/4)^{1/2}] \equiv ikv_z\pm uk, \quad (5a)$$

where $K^{\alpha\beta} = (\hat{\sigma}^{-1})^{\alpha\gamma}\varkappa^{\gamma\beta}$. Instability obtains when $\det\hat{K}_s \neq 0$. In the absence of an external magnetic field and under conditions of a mild state of nonequilibrium the matrix is symmetric and nonsingular. The determinant $\det\hat{K}_s$ transforms like a pseudoscalar; therefore, in order for $\det\hat{K}_s \neq 0$, the symmetry group of the nonequilibrium medium should not contain reflection planes. When this symmetry condition is fulfilled, there is no reason for this determinant to be equal to zero.

Thus, electromagnetic excitations with small wave vectors $\mathbf{k}$ are unstable in any infinite reflection-plane-free medium in an arbitrarily weak state of nonequilibrium. In order to understand why this effect is not observed everywhere, we should turn to the analysis of the situation in a bounded medium.

Global instability can occur only if the smallest dimension of that region of the conducting medium where the state of nonequilibrium is maintained is greater than, or of the order of, $c/\varkappa$. This is due to the fact that the magnetic field inducing the currents should be produced by these same currents. To illustrate this point, let us consider the natural-frequency spectrum of electromagnetic excitations varying in the z direction in an isolated conducting plate of thickness d (the z axis is perpendicular to the plane of the plate), assuming $\hat{\sigma}$ and $\hat{\varkappa}$ are constants. The boundary conditions to (4) are the following: $\mathbf{H} = 0$ at the surfaces. Using these conditions, we obtain from (5) the spectrum:

$$i\omega_n=(u^2-v_z^2)/D\pm 2iuv_z/D-4\pi^2n^2D/d^2, \quad n=1, 2, \ldots.$$

Hence we have the criterion for instability

$$4[\varkappa^{xx}\varkappa^{yy}-(\varkappa^{xy2}+\varkappa^{yx2})/2]d^2/c^2\geq 1. \quad (6a)$$

If there is no global instability, we can, apparently, observe wave amplification in the medium in question. It follows from (5) that the excitations with frequency $\omega \lesssim \varkappa^2/\sigma$ will be amplified. The maximum growth rate $\mathrm{Im}\, k \sim \varkappa/c$, and the amplification will be substantial if

$$\varkappa d/c \gg 1. \quad (6b)$$

Thus, the fulfillment of condition (6b) is the minimum requirement for observation of the effect in question. In many problems the nonequilibrium medium can be considered to be infinite, and (6b) intuitively seems to be not too rigid, but analysis of specific mechanisms for the maintenance of the state of nonequilibrium shows that the criterion presented above for the onset of instability is very rigid.

2. Let us consider the situation when the state of nonequilibrium is maintained by the passage of electric current or heat flux through the medium. The symmetry conditions necessary for the occurrence of instability are fulfilled if the symmetry group of the equilibrium medium is discrete, and the flux vector does not lie in any of the reflection planes of this group. Thus, we shall be dealing with conducting crystals.

Weakly nonequilibrium experimental situations can be created in which the characteristic drift velocity of the carriers [$\sim v$, u in (5)] is much smaller than the rms velocity v_0. In this case the general form of $\hat{\varkappa}$ is

$$\varkappa^{\alpha\beta}=A^{\alpha\gamma\beta}E^\gamma+B^{\alpha\gamma\beta}\nabla_\gamma T.$$

It follows from the Onsager relations that $A^{\alpha\gamma\beta} = -A^{\gamma\alpha\beta}$. Analysis shows that, in the case of the passage of current, instability can occur in crystals of the C_n, C_s, C_{3h}, S_4, and C_{2v} classes and the classes obtained by the addition of a center of inversion. In the case of the passage of heat flux through the sample instability can occur in crystals of the C_{3v}, D_{3d}, and D_3 classes as well.

The coefficients $\hat{A}$ and $\hat{B}$ can be determined by solving the kinetic equation for the carriers:

$$v_{\mathbf{p}}{}^\alpha\left[eE^\alpha+\frac{(\varepsilon_{\mathbf{p}}-\mu)}{T}\frac{\partial}{\partial x^\alpha}T\right]\frac{\partial n_0}{\partial\varepsilon}+e^{\alpha\beta\gamma}H^\gamma\frac{e}{c}v_{\mathbf{p}}{}^\alpha\frac{\partial}{\partial p^\beta}f + \int[W(\mathbf{p},\mathbf{p}')f(\mathbf{p}')-W(\mathbf{p}',\mathbf{p})f(\mathbf{p})]\frac{d^3p'}{(2\pi)^3}. \quad (7)$$

Here $\mathbf{p}$ is the quasimomentum, $\varepsilon_{\mathbf{p}}$ is the energy, $v_{\mathbf{p}}^{\alpha} \equiv \partial\varepsilon/\partial p^{\alpha}$, $n_0 \equiv (1 + e^{(\varepsilon-\mu)/T})^{-1}$, $W(\mathbf{p}, \mathbf{p}')$ is the scattering probability, and one band is left. Let us introduce the operator $S(\mathbf{p}, \mathbf{p}')$ through the relation

$$\int [W(\mathbf{p},\mathbf{p}')S(\mathbf{p}',\mathbf{p}'')-W(\mathbf{p}',\mathbf{p})S(\mathbf{p},\mathbf{p}'')]\frac{d^3p'}{(2\pi)^3}=\delta^3(\mathbf{p}-\mathbf{p}''),$$

and express the coefficients of interest to us in terms of it:

$$\left\{\begin{matrix} A^{\alpha\gamma\beta} \\ B^{\alpha\gamma\beta}\end{matrix}\right\}=\frac{e}{c}\int\frac{d^3p'}{(2\pi)^3}\frac{d^3p''}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}$$
$$\times v_{\mathbf{p}'}{}^{\alpha}S(\mathbf{p}',\mathbf{p})v_{\mathbf{p}}{}^{\rho}e^{\rho\beta\delta}\frac{\partial}{\partial p^{\delta}}S(\mathbf{p},\mathbf{p}'')\frac{\partial n_0}{\partial\varepsilon}$$
$$\times v_{\mathbf{p}''}{}^{\gamma}\left\{\begin{matrix} e \\ (\varepsilon_{\mathbf{p}''}-\mu)/T\end{matrix}\right\}$$
$$\equiv\int\frac{d^3p}{(2\pi)^3}\varkappa^{\alpha\beta}(\mathbf{p})v^{\gamma}(\mathbf{p})\cdot\frac{\partial n_0}{\partial\varepsilon}\left\{\begin{matrix} e \\ (\varepsilon_{\mathbf{p}}-\mu)/T\end{matrix}\right\}. \quad (8)$$

It can be seen from (8) that, when the anisotropy is ~ 1, all the components of $A^{\alpha\gamma\beta}$ are of the same order of magnitude. For $\hat{\varkappa}$ the usual Hall estimate is valid:

$$\hat{\varkappa}\sim\sigma E e\tau/cm. \quad (9)$$

If we limit ourselves in (7) and (8) only to elastic scattering, then we find that $B^{\alpha\gamma\beta} = -B^{\gamma\alpha\beta}$. For the part $\hat{\varkappa}_1$ which has this symmetry we again have the usual estimate

$$\hat{\varkappa}_1\sim\sigma Q\nabla T e\tau/cm,$$

where Q is the thermoelectric coefficient. The expression for the other components $\varkappa_2$ in the case when elastic scattering predominates contains an additional factor which is small in comparison with the expression for $\hat{\varkappa}_1$. In crystals of the C_{3v}, D_{3d}, and D_3 classes the instability is governed precisely by $\hat{\varkappa}_2$. The most promising—from the experimental standpoint—crystals,[8] Bi and Sb, belong to the D_{3d} class.

Let us consider the criterion (6b). If current is passed through the medium, then from (6b) and (9) we find that, for instability to occur, it is necessary that $1 \lesssim \sigma Ed(e\tau/\mathrm{cm})$. Notice that the magnetic field produced by the transmitted current is then of the order of $cm/e\tau$, i.e., it substantially changes the conductivity over the cross section of the sample. The initial assumption that the sample is homogeneous is not valid. Under these conditions another instability mechanism, unrelated with the anisotropy, is also effective.[9-11] Nevertheless the mechanism described above operates in the inhomogeneous case as well. If it is more effective in this case than the mechanism considered in Refs. 8–11, a substantial increase in the critical current for the onset of instability should be experimentally observed in samples of the same thickness in those cases when the current vector lies in a symmetry plane of the crystal.

In the case of the passage of heat flux through a homogeneous isolated sample, no magnetic field is produced, and the criterion (6a) reduces to $\sigma Q\nabla T d e\tau/c^2\mathrm{m} \gtrsim 1$. Since $\nabla T d \ll T$, we should have $\sigma Q T e\tau/c^2\mathrm{m} \gg 1$, which coincides with the observability criterion for thermomagnetic waves, a criterion which is fulfilled in pure materials at low temperatures. Kopylov[8] has experimentally observed stable thermomagnetic waves under different conditions, including those in which the heat flux is not parallel to a symmetry plane of the crystal. It is difficult to say whether this is explained by the smallness of $\hat{\varkappa}_2$ noted above or by the characteristics of the experimental geometry.

3. Let the nonuniformity be maintained by external electromagnetic radiation. The expression for $\hat{\varkappa}$ should be constructed from the tensor characterizing the medium, the polarization vectors, and the wave vector of the radiation. But allowance for the wave vector in this expression is equivalent to allowance for the small quantities, v_0/c and e^2/c, since the electromagnetic radiation propagates much faster than the characteristic velocity of the carriers. Therefore, in the expression for $\hat{\varkappa}$ we retain only the terms of zeroth order in the wave vector: $\varkappa^{\alpha\beta} = F^{\alpha\beta\gamma\delta}E_{\gamma}E^{*}_{\delta}$; $F^{\alpha\beta\gamma\delta} \not\equiv 0$ if the crystalline medium does not possess a center of inversion. Under these conditions the magnetically induced currents are similar in the mechanism of their generation to the photoelectromagnetic current.[12,13]

Let us first consider a pure semiconductive material at low temperatues. The presence of free carriers in this case can be ignored. If the energy of the radiation quanta is higher than the threshold energy, then the production of pairs of free carriers occurs. We can determine $\hat{\varkappa}$ with the aid of the method used in Refs. 13 and 14 to compute the photoelectromagnetic current. The current in this case is determined with the aid of the correction to the carrier distribution function obtained by solving a kinetic equation of the type (7) with a particle source defined in terms of the characteristics of the radiation. For the source we have

$$\frac{\partial f}{\partial t}(n,\mathbf{p})=\frac{\omega}{2\pi}\sum_m D_{nm}{}^{\alpha}(\mathbf{p})E_{\omega}{}^{\alpha}D_{mn}{}^{\beta}(\mathbf{p})E_{\omega}{}^{*\beta}S[\omega-\varepsilon^n(\mathbf{p})$$
$$+\varepsilon^m(\mathbf{p})][n_0(\varepsilon_{\mathbf{p}}{}^n)-n_0(\varepsilon_{\mathbf{p}}{}^m)].$$

Here n and m number the bands, D^{α}_{nm} are the matrix elements of the dipole-moment operator, and ω is the radiation frequency. From this we immediately determine the rate of energy absorption by the material in a unit volume:

$$\dot{\varepsilon}=\int\frac{d^3p}{(2\pi)^3}\sum_n\varepsilon_{\mathbf{p}}{}^n\frac{\partial f}{\partial t}(n,\mathbf{p}).$$

For $\hat{\varkappa}$ we obtain [see (8)]

$$\varkappa^{\alpha\beta}=\int\frac{d^3p}{(2\pi)^3}\sum_n\varkappa_n{}^{\alpha\beta}(\mathbf{p})\frac{\partial f}{\partial t}(n,\mathbf{p}).$$

In a medium possessing a center of inversion $\partial f/\partial t$ is even in p, while $\varkappa^{\alpha\beta}(\mathbf{p})$ is odd. If there is no center of inversion, these quantities do not possess a definite parity with respect to $\mathbf{p}$ (Ref. 13): $\partial f/\partial t$ does not because of the asymmetry in the pair production processes; $\varkappa^{\alpha\beta}(\mathbf{p})$, because of the asymmetry in the scattering of the free carriers. In the general case this asymmetry is not formally small, but is, apparently, numerically small because of the structúre of real noncentrosymmetric crystals. We shall characterize the degree of asymmetry by the dimensionless parameter $\xi \lesssim 1$.

It is convenient, in estimating $\hat{\varkappa}$, to express the answer in terms of $\dot{\varepsilon}$:

$$\hat{\varkappa} \sim (e^2/cm) v_1 \tau^2 \dot{\varepsilon} \xi/\omega. \tag{10}$$

Here $v_1 \sim [m(\omega - \Delta)]^{1/2}$ is the characteristic velocity of the nascent carriers. In subsequent estimates we shall assume that v_1 and ω are of the order of the corresponding atomic quantities.

It is natural to assume that the radiation is exponentially damped as it propagates into the sample from the boundary, and that $\hat{\varkappa}(z) = \hat{\varkappa} e^{-z/\delta}$. Therefore, for the criterion (6a) to be fulfilled, it is necessary that $\varkappa\delta/c \gtrsim 1$ in any case. The estimate of $\hat{\varkappa}$ in the form (10) allows us to write this criterion in the form of a limitation on the radiation flux I, since $I = \dot{\varepsilon}\delta$:

$$I^{-1} \lesssim (e^2/c^2 m) v_1 \tau^2 \xi/\omega. \tag{11}$$

The inequality (11) depends weakly on the characteristics of the specific material (largely through τ). Therefore, to maximize the probability for the occurrence of the instability, we should use pure materials at low temperatures. For $\tau \sim 10^{-8}$ sec and $\zeta \sim 1$, $I \gtrsim 10^2$ W/cm^2. Such a large value of I will, apparently, not permit the observation of the instability under conditions of steady irradiation, since it will be difficult to remove this heat flux from the sample. The instability growth time (or the reciprocal frequency below which the waves are amplified in the sample) can be estimated, using (5) and the simplest estimates for the photoconductivity. We obtain $T_{\rm inst} \sim (\delta/l)\tau_{\rm rec} \sim 10^{-3}$ sec (where $\tau_{\rm rec}$ is the characteristic lifetime of the carriers) under the assumptions made above, and $\varkappa\delta/c \sim 1$. This allows us to hope that the instability can be detected in a pulsed irradiation regime.

Let us now consider the situation when the intraband mechanism of radiation absorption predominates (i.e., the case of low frequencies and high free-carrier densities). The radiation in this case can be taken into account by including the field term $eE^\alpha \partial f/\partial p^\alpha$ in the kinetic equation. Introducing the operator $S_\omega(\mathbf{p}, \mathbf{p}')$ defined by the relation

$$i\omega \hat{S}_\omega + \hat{S}^{-1} \hat{S}_\omega = \hat{1},$$

and expressing in terms of it the quadratic (in $\mathbf{E}$) correction to the distribution function, we obtain

$$\varkappa^{\alpha\beta} = \int \frac{d^3p}{(2\pi)^3} \left[\varkappa^{\alpha\beta}(\mathbf{p}) \frac{\partial f}{\partial t}(\mathbf{p}) + j^\alpha(\mathbf{p}) \frac{\partial}{\partial H^\beta} \frac{\partial f}{\partial t}(\mathbf{p}) \right],$$
$$j^\alpha(p) = \int \frac{d^3p'}{(2\pi)^3} v^\alpha(\mathbf{p}') S(\mathbf{p}', \mathbf{p}) e,$$
$$\frac{\partial f}{\partial t}(\mathbf{p}) = e^2 \operatorname{Re}\left(E_\omega^{*\alpha} E_\omega^\gamma \int \frac{\partial}{\partial p^\alpha} S_\omega(\mathbf{p}, \mathbf{p}') v^\gamma(p') \frac{\partial n}{\partial \varepsilon}(\mathbf{p}') \frac{d^3p'}{(2\pi)^3} \right), \tag{12}$$
$$\frac{\partial}{\partial H^\beta} \frac{\partial f}{\partial t}(\mathbf{p}) = \frac{e^3}{c} \operatorname{Re}\left(E_\omega^{*\alpha} E_\omega^\gamma \int \frac{\partial}{\partial p^\alpha} S_\omega(\mathbf{p}, \mathbf{p}') v^\delta(\mathbf{p}') e^{\delta\varepsilon\beta} \times \frac{\partial}{\partial p'^\varepsilon} S_\omega(\mathbf{p}', \mathbf{p}'') v^\gamma(\mathbf{p}'') \frac{\partial n}{\partial \varepsilon}(\mathbf{p}'') \frac{d^3p'}{(2\pi)^3} \frac{d^3p''}{(2\pi)^3} \right).$$

In this case

$$\dot{\varepsilon} = \int \varepsilon(\mathbf{p}) \frac{\partial f}{\partial t}(\mathbf{p}) \frac{d^3p}{(2\pi)^3}.$$

For $\omega \gg 1/\tau$ the second term in the expression (12) for $\hat{\varkappa}$ contains the small factor $(\omega\tau)^{-1}$ in comparison with the first term; for $\omega \lesssim 1/\tau$ both terms are of the same order of magnitude. The noncentrosymmetric nature of the crystal manifests itself here in the properties of $W(\mathbf{p}, \mathbf{p}')$ and $S(\mathbf{p}, \mathbf{p}')$. Estimating (12) in much the same way as was done above, we obtain

$$\hat{\varkappa} \sim (e^2/cm) v_0 \tau^2 \dot{\varepsilon}/\varepsilon_0 \sim (e^2/cm) \tau^2 \dot{\varepsilon}/p_0,$$
$$I^{-1} \lesssim (e^2/c^2 m) \tau^2/p_0. \tag{13}$$

Here ε_0 is the carrier energy measured from the bottom of the band and p_0 is the characteristic quasimomentum of the carriers. It should be noted that in form and in value (13) is very close to (10), (11) in spite of the significant difference in the physical situations, a difference such that the values of δ in the two situations can differ by several orders of magnitude. The estimates (10) and (13) can be obtained from the following crude qualitative arguments: a carrier, having absorbed an amount of energy $\Delta\varepsilon$, will change its velocity by an amount $\Delta v \sim (dv/d\varepsilon)\Delta\varepsilon$. This change in velocity is preserved during the time period τ; therefore, under the appropriate symmetry conditions in the case when $\dot{\varepsilon}$ is a constant a current $j_0 \sim e\dot{\varepsilon}\tau dv/d\varepsilon$ will flow in the medium. This current will change by an amount on the order of its value in a magnetic field of intensity $\sim$cm$/e\tau$; whence $\varkappa \sim e\tau j_0/$cm.

Note that magnetically induced currents also arise when, for a given crystal symmetry, the polarization of the radiation is such that the photoelectromagnetic current is forbidden on the basis of symmetry considerations. In this case the vector $\mathbf{v}$ in (5) is identically equal to zero, and only absolute instability can occur.

4. The arguments adduced above suggest that the estimate (13) can be improved if the nonequilibrium state is maintained not by electromagnetic radiation, but by sound, since for $v_0 \gg c_{\rm ac}$ the absorption of phonons more effectively changes the carrier velocity: $\Delta v \sim c_{\rm ac} \Delta\varepsilon/\varepsilon_0$. In contrast to the situation with electromagnetic radiation, the phonon wave vector $\mathbf{q}$ does not, when it is taken into account in the expressions for $\hat{\varkappa}$, introduce in the general case additional small factors. This allows us to use in the determination of $\hat{\varkappa}$ the simple model of an isotropic medium. The requirement that there will be no reflection planes will be met if elliptically polarized transverse sound propagates in such a medium.

Let us compute the tensor $\hat{\varkappa}$ to leading order in $c_{\rm ac}/v_0$. In this order we can take account of the interaction of the carriers with the lattice distortion by the standard method,[15,16] i.e., through the modification of the Hamiltonian for the carriers:

$$\varepsilon(\mathbf{p}) \to \varepsilon(\mathbf{p}) + (\lambda_{ik}(\mathbf{p}) + p_i \partial\varepsilon/\partial p_k) \partial u_k/\partial x_i$$
$$\equiv \varepsilon(p) + m^{-1}(\lambda + 1) p_i p_k \partial u_k/\partial x_i.$$

The computation of the correction to the distribution function which is quadratic in u allows us to determine $\hat{\varkappa}$. We obtain the estimate

$$\varkappa \sim (e^2/cm^2) \tau^2 \dot{\varepsilon}/c_{\rm ac}, \tag{14}$$

which is greater than (13) by a factor of $v_0/c_{\rm ac}$. But in this order the tensorial structure of $\hat{\varkappa}$ reduces to (e^α is the sound polarization vector)

$$\mathbf{j}=\varkappa_1 q^{-1}[\mathbf{qH}]+\varkappa_2 q^{-1}\,\mathrm{Re}\,[\mathbf{e}^*(\mathbf{e},\mathbf{H},\mathbf{q})]$$

and det $\varkappa_s = 0$. The magnetically induced current, which has the structure

$$\mathbf{j}=\varkappa_3\mathbf{H}q^{-1}\,\mathrm{Im}\,[(\mathbf{e}^*,\mathbf{e},\mathbf{q})]+\varkappa_4\,\mathrm{Im}\,[\mathbf{e}^*(\mathbf{e},\mathbf{H},\mathbf{q})]\,q^{-1}$$

and is responsible for the instability, appears in the next order in c_{ac}/v_0:

$$\varkappa_{3,4}\sim(e^2/cm)(\tau^2/p_0)\dot{\varepsilon}ql \quad \text{for} \quad ql\ll 1;$$

when $ql\sim 1$, the estimate (13) is valid. In the case of arbitrary elliptic polarization of the sound there is instability in the $\mathbf{k}\|\mathbf{q}$ directions.

The drift, determined by (14), of electromagnetic excitations in the $\mathbf{q}$ direction makes the onset of global instability impossible in this situation. The estimate for the observability of the amplification coincides with (13). These estimates are valid for both metals and semiconductors if $\dot{\varepsilon}$ is taken to be that part of the dissipated acoustic energy which is absorbed by the carriers.

5. Ivchenko and Pikus[5] have described the situation in which the carrier temperature differs from the lattice temperature in a gyrotropic conductor. Such a nonequilibrium state leads to a situation in which $\varkappa^{\alpha\beta}\not\equiv 0$, and the tensorial structure of $\varkappa^{\alpha\beta}$ is entirely determined by the crystal symmetry. The crystal must be enantiomorphic for instability to occur. If in this case the crystal does not belong to the pyroelectric class, then $\mathbf{v}\equiv 0$, and there can be only absolute instability. Let us estimate the magnetically-induced-current tensor, neglecting, unlike Ivchenko and Pikus,[5] the spin effects. In the case when the electron and phonon temperatures differ slightly from each other, the effect can be treated as the result of the interaction between the carriers and the nonequilibrium phonons, whose occupation numbers satisfy

$$\delta N_\mathbf{q}=-(\delta T/T)(\omega_\mathbf{q}/2T)\,\mathrm{sh}^{-2}(\omega_\mathbf{q}/2T).$$

The change in the electron-phonon collision integral, written in the standard[17] form, is given by

$$\delta St_\mathbf{p}\{f\}=\int\frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\delta N_\mathbf{q}\{n_{\mathbf{p}'}{}^0-n_\mathbf{p}{}^0\}[W(\mathbf{p}',\mathbf{q};\mathbf{p})\times\delta(\varepsilon_\mathbf{p}-\varepsilon_{\mathbf{p}'}-\omega_\mathbf{q})-W(\mathbf{p}';\mathbf{p},\mathbf{q})\delta(\varepsilon_\mathbf{p}-\varepsilon_{\mathbf{p}'}+\omega_\mathbf{q})].$$

Here summation over the types of phonon polarization is implied and $W(\mathbf{p}',\mathbf{q},\mathbf{p})$ is the probability for absorption by a carrier of a phonon with wave vector $\mathbf{q}$, followed by a transition into the state p:

$$W(\mathbf{p}',\mathbf{q};\mathbf{p})=w(\mathbf{p}+\mathbf{q}/2,\mathbf{q})\delta^3(\mathbf{p}'+\mathbf{q}-\mathbf{p}),\quad w(\mathbf{p},\mathbf{q})=w(\mathbf{p},-\mathbf{q}),$$

if the Umklapp processes are ignored. Similarly to (12), we obtain

$$\dot{\varepsilon}=\int\frac{d^3p}{(2\pi)^3}\varepsilon_\mathbf{p}\delta St_\mathbf{p}\{f\},$$

$$\varkappa^{\alpha\gamma}=\int\frac{d^3p}{(2\pi)^3}\varkappa_\mathbf{p}{}^{\alpha\gamma}\delta St_\mathbf{p}\{f\}+j_\mathbf{p}{}^\alpha\frac{\partial\delta St_\mathbf{p}}{\partial H^\gamma}. \qquad (14a)$$

In making estimates, we should take accurate account of the parity of the integrands with respect to q. This determines the order of the answer in c_{ac}/v_0, which is assumed to be small. Then for the first term in (14a) we again obtain the estimate (13). The order of magnitude of ξ is determined by the asymmetry in the scattering processes. If the scattering by the impurities predominates, then $\xi\sim 1$. The asymmetry is small in the case of scattering on the phonons[18,19]; in the region $T\gtrsim\omega_0$, for example, $\xi\sim\min\{\tau/\tau_{imp};amT/p_0\}$ (τ_{imp} is the characteristic time for the scattering on the impurities) if scattering on the phonons predominates. Assuming that the electronic and phonon bands are isotropic, that Fermi statistics is obeyed, and that $q/p_0\ll 1$, we obtain for the first term in (14a) the expression

$$\varkappa^{\alpha\beta}=\left\{\int_{\text{F.S.}}\frac{\partial}{\partial\varepsilon_\mathbf{p}}(\varkappa^{\alpha\beta}(\mathbf{p}))w(\mathbf{p})\frac{dS}{v_0}\Big/\int_{\text{F.S.}}w(\mathbf{p})\frac{dS}{v_0}\right\}\dot{\varepsilon}.$$

In the case of an arbitrary spectrum and arbitrary statistics, and for $q/p_0\ll 1$, (14) goes over into

$$\varkappa^{\alpha\beta}=\int\frac{d^3p}{(2\pi)^3}\frac{\partial\varkappa^{\alpha\beta}}{\partial p^\gamma}q^\gamma\omega_\mathbf{q}{}^2\delta N_\mathbf{q}w(\mathbf{p},q)\delta(\mathbf{qv})\frac{d^3q}{(2\pi)^3}\frac{\partial n_0}{\partial\varepsilon}(\mathbf{p}),$$

$$\dot{\varepsilon}=\int\frac{d^3p}{(2\pi)^3}\omega_\mathbf{q}{}^2\delta N_\mathbf{q}\frac{\partial n_0}{\partial\varepsilon}(\mathbf{p})w(\mathbf{p},q)\delta(\mathbf{qv})\frac{d^3q}{(2\pi)^3}.$$

The second term in (14) is the result of the allowance made for $\mathbf{H}$ in the collisional term. In general it gives a correction on the order of the small quantity $(ql)^{-1}$, but in this case, because of the change in parity with respect to $\mathbf{q}$ in the integrand, we gain the factor v_0/c_{ac}. We obtain the estimate

$$\hat{\varkappa}\sim(e^2/cm)(\tau/\omega)\dot{\varepsilon}/mc_{ac},\quad \omega=\min\{T,\omega_D\}. \qquad (15)$$

For $\omega\ll 1/\tau$ this contribution is greater than the contribution from the first term. For small $\mathbf{q}$ we obtain

$$\varkappa^{\alpha\beta}=\int\frac{d^3p}{(2\pi)^3}\frac{\partial j_\mathbf{p}{}^\alpha}{\partial p^\gamma\partial p^\delta}v_\mathbf{p}{}^\rho e^{\delta\rho\beta}q^\gamma\delta'(\mathbf{qv})w(\mathbf{p})\times\omega_\mathbf{q}\frac{\partial n_0}{\partial\varepsilon}(\mathbf{p})\delta N_\mathbf{q}\frac{\partial^3 q}{(2\pi)^3}.$$

Similar estimates carried out on the basis of the spin mechanism proposed by Ivchenko and Pikus[5] for the effect yield $\varkappa\sim(e^2/cm)a_B n\tau\dot{\varepsilon}/\varepsilon_0$, which is la_B times smaller than (13). This allows us to infer that the spin mechanism is not very effective here.

The observation of the magnetically induced currents in the situation described above is of interest in its own right. The estimates (13) and (15) allow us to hope that the effect will be measurable, especially as (15) increases with decreasing temperature. This estimate, like the entire procedure, is valid when $ql\gg 1$. When $ql\lesssim 1$, (15) goes over into (14). The instability criterion can also be written in the above form $I^{-1}\lesssim\varkappa/c\dot{\varepsilon}$, which does not explicitly depend on the sample geometry. Here I is the power flux absorbed by the electrons. The numerical estimates for I are better than the estimate given in Sec. 3 only at extremely low temperatures (specifically, at $T\lesssim 10^{-2}\,{}^\circ$K).

6. If the condition for global instability is fulfilled, then a weak electromagnetic perturbation grows exponentially, and the question arises of the solution of the nonlinear generalization of Eq. (4). In the case of a mild state of nonequilibrium the nonlinearity of (4) arises because $\hat{\sigma}$ and $\hat{\varkappa}$ tensors

depend on the magnetic field produced by the growing perturbations. This means that the characteristic scale of the magnetic fields that arise in the course of the development of the instability will be $mc/e\tau$, and in this case the magnetic field will have an appreciable effect on the carrier motion, the relative changes in $\hat{\sigma}$ and $\hat{\varkappa}$ being ~ 1.

It is not difficult to see that in the case of an unbounded medium, because of the development of the instability , a regime in which the magnetic field is periodic both in space and in time cannot be established. The system will not acquire a higher symmetry, and for the law of dispersion of electromagnetic perturbations about this regime in the case when ω and k are respectively much smaller than the frequency and spatial period of the magnetic field we obtain as before the relation (5a) with renormalized $\hat{\sigma}$ and $\hat{\varkappa}$. This indicates that such a regime is unstable. Apparently, the regime that will actually be established will be characterized by a magnetic field **H** that varies randomly in space and time.

In a bounded medium this cannot be so. As an illustration, let us consider a simple model of such an instability in an isotropic noncentrosymmetric medium. The tensor $\varkappa$ in this case reduces to a pseudoscalar, and depends only on $|\mathbf{H}|$. We shall ignore the dependence of σ on **H**. In dimensionless variables Eq. (4) has the form

$$\dot{\mathbf{H}}=\mathrm{rot\,rot}\,\mathbf{H}+\mathrm{rot}\,[\varkappa(H)\mathbf{H}],\quad \varkappa(0)=1. \tag{16}$$

Here the time and distance are measured in units of $\sigma/\varkappa^2$ and $c/\varkappa$, respectively, and we have taken the curl of (4). The steady state solutions to (16) are characterized by a wave vector **k**, and have the form of helicoidal structures ($\mathbf{k}\|z$):

$$H_x=H_0\sin kz,\quad H_y=H_0\cos kz,\quad k=\varkappa(H_0).$$

The stability of such a solution can be investigated analytically. The perturbation can be characterized by a vector $\mathbf{q}\|\mathbf{k}$. Then the dispersion relation has the form

$$i\omega=-q^2+\varkappa_1\varkappa_0/2\pm[\varkappa_1{}^2\varkappa_0{}^2/4+q^2\varkappa_0(\varkappa_0-\varkappa_1)]^{1/2}.$$

Here

$$\varkappa_0=\varkappa(H_0),\quad \varkappa_1=2[H^2\partial\varkappa(H)/\partial(H^2)]|_{H=H_0}.$$

It can be seen that all the solutions are unstable in the limit of an unbounded medium.

As an example of a bounded medium, let us consider a plane-parallel plate of thickness d. We shall consider solutions that vary in the direction of the normal to the plane of the plate; k and q can then assume quantized values: $k=(2\pi/d)(n+1/2)$, $q=2\pi m/d$. Instability occurs if $q\neq 0$ and $q^2<\varkappa_0^2$. Therefore, all the steady state solutions to (16), except the solution with $k=\pi/d=\varkappa(H_0)$, are unstable. The zeroth order solution is unstable if $d>\pi$. A stable solution results when $d=\pi+\varepsilon$. A stochastic regime apparently obtains when $d\gg\pi$.

7. In conclusion, let us discuss the possibility of observing the instability described above. In spite of the fact that this effect is a purely symmetry-related one and in an unbounded medium with the requisite symmetry the instability occurs no matter how small the deviation from equilibrium is, the condition (6b) for the occurrence of global or observable convective instability is a fairly stringent limitation, which explains why the effect is not generally observed.

Apparently, the best experimental conditions for the manifestation of the above-described mechanism can be achieved in conductors in the presence of a heat flux. In this respect, experiments with materials less symmetric than Bi are promising. But under such conditions there is always the possibility that the instability will be of a solely convective nature.

In the case when the instability is maintained by other means, there exists an almost universal limitation, (13), on the energy flux absorbed by the carriers. Then, even in matrials with the longest τ, the energy flux I is much higher than the energy fluxes normally used in low temperature physics, but, the removal of the heat produced by such fluxes evidently is a purely technical problem. As indicated above, of particular promise here is the study of noncentrosymmetric conductors.

In any case we can precisely specify the conditions under which, with the state of nonequilibrium maintained in a given way, we can observe the instability in a given sample by applying a magnetic field and measuring the tensor $\varkappa^{\alpha\beta}$.

The author is grateful to G. M. Éliashberg, L. S. Levitov, and E. L. Ivchenko for detailed and useful discussions related to the subject of the present paper.

APPENDIX

the most general form of the relation connecting the current density and the magnetic field is the following:

$$j^\alpha(\mathbf{x})=\int Q^{\alpha\beta}(\mathbf{x},\mathbf{x}')A^\beta(\mathbf{x}')d^3x'.$$

Let us consider the current repsonse to a homogeneous magnetic field. Let us choose the vector potential in the form $A^\alpha(x)=e^{\alpha\beta\gamma}H^\beta x^\gamma/2$. For the mean current density we obtain

$$j^\alpha=\frac{1}{2V}\int d^3x\,d^3x'Q^{\alpha\gamma}(\mathbf{x},\mathbf{x}')e^{\gamma\beta\delta}x'^\delta H^\beta\equiv\varkappa^{\alpha\beta}H^\beta.$$

Here V is the volume of the system. Let us now compute the mean magnetic-moment density in the presence of a homogeneous vector potential:

$$M^\alpha=\frac{1}{2cV}\int d^3x\,e^{\alpha\gamma\delta}x^\gamma j^\delta(\mathbf{x})$$
$$=\frac{1}{2cV}\int d^3x\,d^3x'\,e^{\alpha\gamma\delta}x^\gamma Q^{\delta\beta}(\mathbf{x},\mathbf{x}')A^\beta.$$

Under equilibrium conditions the Onsager relations impose on the kernel $Q^{\alpha\beta}(\mathbf{x},\mathbf{x}')$ the following constraint: $Q^{\alpha\beta}(\mathbf{x},\mathbf{x}')=Q^{\beta\alpha}(\mathbf{x}',\mathbf{x})$. Taking this into account, we obtain $M^\alpha=\varkappa^{\beta\alpha}A^\beta/c$.

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Translated by A. K. Agyei