Nonlinear electromagnetic excitation of ultrasound in metals

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Nonlinear electromagnetic generation of longitudinal ultrasound under anomalous skin effect conditions is theoretically investigated. It is shown that the deforming force is the main source of double-frequency acoustic vibrations in the weak nonlinearity regime. This sound-generation mechanism is connected, on the one hand, with the strong spatial dependence of the electron distribution function and, on the other, with the presence of a Lorentz force related to the wave magnetic field. The amplitude of the ultrasound excited under these conditions in a metal with a spherical Fermi surface is calculated. It is found that the nonlinear-ultrasound amplitude under these conditions is greater by a factor of $(l/\delta)^2$ than the corresponding amplitude under normal skin effect conditions. The possibility of an experimental observation of this effect is discussed.

1. When an electromagnetic wave is incident at a metal boundary the electromagnetic energy is transformed into acoustic vibrations (contactless excitation of ultrasound). A detailed account of the experimental situation and the basic theoretical ideas about contactless excitation of ultrasound, as well as a fairly complete bibliography on this question can be found in Ref. 1.

There are two fundamentally different ultrasound excitation mechanisms in normal (nonmagnetic) metals, the induction and deformation mechanisms. These mechanisms can be differentiated by virtue of the existence of two forces of different natures: the ponderomotive force with density

$$F_i^u = \frac{1}{c}[\mathbf{j}, \mathbf{H}_0]_i \tag{1}$$

and the deformation force

$$F_i^d = -\frac{\partial}{\partial x_k}\langle \Lambda_{ik}\chi\rangle. \tag{2}$$

We are using standard notation.[1] Let us only note that Λ_{ik} is a renormalized deformation potential, $(\partial f_0/\partial\varepsilon)\chi$ is a correction to the local-equilibrium Fermi distribution function, and the angle brackets denote integration over the Fermi surface:

$$\langle\ldots\rangle = \frac{2}{(2\pi\hbar)^3}\int \ldots \frac{dS}{v}.$$

The overwhelming majority of the investigations of the electromagnetic excitation of ultrasound have been performed in the linear regime, when, naturally, the frequency ω of the generated acoustic vibrations coincides with the frequency of the incident elecromagnetic wave. As can be seen from (1), the induction mechanism then "operates" only if the conductor is located in a constant magnetic field $\mathbf{H}_0$ or a direct current flows through it.[1)] An electromagnetic wave incident at the surface of a metal excites a variable current in the skin layer. In a field $\mathbf{H}_0$ perpendicular to this current

$$F_1^u \sim \frac{1}{4\pi}\frac{H_0 H}{\delta}, \tag{1'}$$

where $\mathbf{H}$ is the intensity of the wave magnetic field and δ is the skin depth. The deformation mechanism of linear conversion operates in zero external magnetic field as well. The order of magnitude of the deformation-force density is given by

$$F_1^d \sim \frac{ne\omega\delta}{c}\frac{l^2}{\delta^2+l^2}H. \tag{2'}$$

This formulation emphasizes the nonlocal character of the deformation excitation of ultrasound. The conversion mechanism in this case consists in the following. Under the action of an alternating electric field $\mathbf{E}$, the electrons and ions in the skin layer acquire different and oppositely directed additions to their momenta. To the extent that the mean free path l is finite, locally these effects do not cancel each other out in collisions, and this gives rise to the appearance of strains in the lattice. Comparison of the expressions (1) and (2) shows that the induction and deformation forces are equal when

$$\omega_{c0} \approx \omega\left(\frac{\delta}{\delta_L}\right)^2\frac{l^2}{\delta^2+l^2}, \tag{3}$$

where ω_{c0} is the cyclotron frequency of the electrons in the field H_0 and δ_L is the plasma penetration depth. It can be seen that, under normal skin effect conditions (i.e., for $l \ll \delta$), the induction force is significantly stronger than the deformation force even in weak magnetic fields (fields that do not modify the carrier dynamics).

It follows from the expression (1) for the induction force that ultrasound with double frequency should also be excited in a metal as a result of the joint action on the lattice of the magnetic field of, and the current induced in the skin layer by, the wave. The amplitude of this force is proportional to the wave intensity, and has the form

$$F_2^u \approx \frac{1}{4\pi}\frac{H^2}{\delta}. \tag{4}$$

The principal source of nonlinearity in the deformation mechanism of energy conversion is the nonlinear correction $\chi \propto EH$ to the distribution function. The deformation force due to it has, as will be seen below, the following order of magnitude:

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$$F_2^d \sim \left(\frac{l}{\delta}\right)^2 \frac{H^2}{\delta}. \qquad (5)$$

Actually, this expression and all the subsequent ones are valid in the case of weak nonlinearity, the criterion for which is formulated in Sec. 2. Comparison of the last two expressions shows that, under anomalous skin effect conditions, the induction force is $(l/\delta)^2$ times weaker than the deformation force.

The induction mechanism of nonlinear excitation of double-frequency ultrasound under normal skin effect conditions is considered in Refs. 2 and 3.

In the present paper we construct a theory of contactless excitation of double-frequency ultrasound in the case of normal incidence of electromagnetic waves at the boundary of an isotropic semi-infinite metal. The generation is investigated in the absence of a constant magnetic field in the case when $\omega\tau \ll 1$ ($\tau = l/v_F$ is the electron relaxation time). The entire calculation is carried out under the simplest assumptions regarding the conduction electrons: isotropic dispersion law, the τ approximation for the collisiosn integral, specular reflection from the metal boundary. We have made this choice first, because, as far as we know, this is the first theoretical investigation of the nonlinear transformation of waves for an arbitrary relation between l and δ, and second, because the conversion factor is insensitive to both the electron dispersion law and the nature of the electron scattering by the surface. The latter assertion is explained by the fact that, in the most interesting case of the anomalous skin effect (i.e., the case $l \ll \delta$) the nonlinear response of the metal is found to be produced by the group of electrons grazing along the surface.

2. At the boundary of a metallic half-space $x > 0$ an electromagnetic wave whose electric **E** magnetic **H** components are aligned along the y and z axes, respectively, is assumed to be normally indicent. The kinetic equation for the nonequilibrium correction to the electron distribution function can be written in the form

$$v_x \frac{\partial \chi}{\partial x} + \frac{1}{\tau}\chi = -ev_y E_y - \frac{e}{c} H_z \tilde{\nabla}\chi, \qquad (6)$$

where $\tilde{\nabla} = v_y \partial/\partial v_x - v_x \partial/\partial v_y$. The fields entering into this equation are determined by the Maxwell equations with the current density

$$\mathbf{j} = -e\langle \mathbf{v}\chi\rangle. \qquad (7)$$

The weakness of the electromechanical interaction (in essence the smallness of the ratio m/M, where m is the electron mass and M is the ion mass) allows us to ignore the excited ultrasonic wave in the computation of the electronic and electromagnetic responses of the metal [Eqs. (6) and (7)].

A system of electrodynamic equations with nonlinear terms is solved by expanding all the quantities in terms of the harmonics of the fundamental frequency:

$$\begin{aligned} E &= E_1 \cos(\omega t + \Delta_{E1}) + E_2 \cos(2\omega t + \Delta_{E2}) + \ldots, \\ H &= H_1 \cos(\omega t + \Delta_{H1}) + H_2 \cos(2\omega t + \Delta_{H2}) + \ldots, \\ \chi &= \chi_0 + \chi_1 \cos(\omega t + \Delta_{\chi1}) + \chi_2 \cos(2\omega t + \Delta_{\chi2}) + \ldots, \end{aligned} \qquad (8)$$

with E_n, H_n, and $\chi_n \propto E_1^n$ (except the correction χ_0, which is $\propto E_1^2$). We must, in investigating the nonlinear electrodynamic effects in metals, distinguish between the cases of weak and strong nonlinearities.[4] In the present paper we consider the case of weak nonlinearity. In the case of the normal skin effect the condition for a weak nonlinearity is $\omega_c\tau \ll 1$, where ω_c is the electron cyclotron frequency in a magnetic field equal to the amplitude of the wave field. In the case of the anomalous skin effect the condition for a weak nonlinearity is more rigid:

$$\omega_c\tau \ll \delta/l,$$

i.e., the electron trajectories in the skin layer should be bent relatively litle by the alternating magnetic field, and their deviation along the normal to the surface should be substantially smaller than δ. It is found that in the case $\chi_0, \chi_2 \ll \chi_1$, and the method of successive approximations can be used to solve the kinetic equation (6). Substituting (8) into (6), and equating the corresponding harmonics, we obtain to first order in the nonlinearity a chain of equations of which the first three have the form[2)]

$$v_x \frac{\partial \chi_0}{\partial x} + \frac{1}{\tau}\chi_0 = -\frac{e}{2c}\,\mathrm{Re}(\hat{H}_1^* \tilde{\nabla}\hat{\chi}_1), \qquad (9)$$

$$v_x \frac{\partial \chi_1}{\partial x} + \frac{1}{\tau}\hat{\chi}_1 = -ev_y \hat{E}_1, \qquad (9')$$

$$v_x \frac{\partial \hat{\chi}_2}{\partial x} + \frac{1}{\tau}\hat{\chi}_2 = -\frac{e}{2c}\hat{H}_1 \tilde{\nabla}\hat{\chi}_1, \qquad (9'')$$

where

$$\hat{\chi}_{1,2} = \chi_{1,2}\exp(i\Delta_{\chi 1,2}), \quad \hat{E}_1 = E_1 \exp(i\Delta_{E1}),$$
$$\hat{H}_1 = H_1 \exp(i\Delta_{H1}), \quad \hat{H}_1^* = H_1 \exp(-i\Delta_{H1}).$$

Since the magnetic component of the electromagnetic wave in the metal is appreciably stronger than the electric component, we have dropped the term $-e\mathbf{v}\mathbf{E}_2$ from the right side of Eq. (9").

For the geometry specified, the ultrasound excited at the frequency ω of the electromagnetic wave in the absence of an external magnetic field possesses only transverse polarization—along the alternating-electric-field vector. The ultrasound of doubled frequency has only longitudinal polarization. Therefore, the elasticity equations for the first and second harmonics can be written as follows:

$$\frac{d^2u_1}{dx^2} + q_t^2 u_1 = -\frac{1}{\rho s_t^2} F_1^{\text{д}} = \frac{1}{\rho s_t^2}\frac{d}{dx}\langle \Lambda_{yx}\chi_1\rangle, \qquad (10)$$

$$\frac{d^2u_2}{dx^2} + q_l^2 u_2 = -\frac{F_2}{\rho s_l^2}, \qquad (10')$$

$$F_2 = F_2^{u} + F_2^{\text{д}} = \frac{j_1 H_1}{2c} - \left\langle \Lambda_{xx}\frac{d\chi_2}{dx}\right\rangle,$$

where $q_{l,t} = \omega/s_{l,t}$, the $s_{l,t}$ are respectively the longitudinal and transverse ultrasound velocities,

$$u_1(x) = u_{1y}(x), \quad u_2(x) = u_{2x}(x), \quad j_1 = -e\langle v_y \chi_1\rangle,$$

and

$$\Lambda_{ik}(\mathbf{p}) = \lambda_{ik}(\mathbf{p}) - \langle \lambda_{ik}(\mathbf{p})\rangle/\langle 1\rangle,$$

$\lambda_{ik}(\mathbf{p})$ being the deformation potential tensor.

3. The ultrasound excitation problem is solved by the Fourier method. Continuing the functions $u_2(x)$ and $F_2(x)$ into the half-space $x<0$ in such a way that the resulting functions are odd, and taking account of the boundary condition $(du_2/dx)_{x=0}=0$ at the free surface, we find from Eq. (10') that

$$[q^2-(2q_l)^2]u_2(q)=\frac{1}{\rho s_l^2}F_2(q)-2iqu_2(x=+0),$$

where $u_2(q)$ and $E_2(q)$ are the Fourier transforms of the functions $u_2(x)$ and $F_2(x)$.

The formal solution to the problem is obtained with the aid of the inverse Fourier transformation:

$$u_2(x)=\frac{1}{2\pi\rho s_l^2}\int_{-\infty}^{\infty}\frac{F_2(q)e^{iqx}}{q^2-(2q_l)^2}dq-\frac{\exp(2iq_lx)}{4\pi\rho\omega s_l}\int_{-\infty}^{\infty}\frac{qF_2(q)\,dq}{q^2-(2q_l)^2}. \tag{11}$$

The force $F_2(x)$ acting on the lattice, and determined by the currents and fields excited in the metal can be divided into two parts[6]: the "hydrodynamic" part, which is damped over distances of the order of δ, and the "kinetic" part, due to the effect whereby the field is drawn into the metal directly by the conduction electrons, and damped over distances[7,8] $\sim l$.

In the case $\omega\tau\ll 1$ under consideration the ultrasound attenuation distance is greater than both the skin depth δ and the mean free path l. Therefore, the "acoustic" pole $q=2q_l$ determines the behavior of the first term in (11) at distances $x\gg\max\{\delta,l\}$:

$$u_2(x)=u_{2\infty}\exp(2iq_lx),\quad x\gg\max\{\delta,l\},$$

where

$$u_{2\infty}=\frac{iF_2(2q_l)}{4\rho\omega s_l}-\frac{1}{4\pi\rho\omega s_l}\int_{-\infty}^{\infty}\frac{qF_2(q)\,dq}{q^2-(2q_l)^2}. \tag{12}$$

Since $\mathrm{Im}q_l$ is the smallest of the parameters of the same dimensionality as q that enter into the problem, the amplitude $u_{2\infty}$ is, to a good approximation, the limit of (12) for $\mathrm{Im}q_l\to 0$:

$$u_{2\infty}=-\frac{1}{2\pi\rho\omega s_l}\int_0^{\infty}\frac{qF_2(q)}{q^2-(2q_l)^2}dq. \tag{13}$$

We have taken account of the fact that $F_2(q)$ is an odd function.[3)] To compute it, we first of all solve the linear electrodynamics problem:

$$E_1(q)=-\frac{2i\omega}{c}H_1(0)r(q),\quad H_1(q)=-2iqH_1(0)r(q),$$
$$j_1(q)=-\frac{2i\omega}{c}H_1(0)\sigma(q)r(q), \tag{14}$$

where $H_1(0)$ is the magnetic field of the wave at the metal boundary,

$$r(q)=\left[q^2-\frac{4\pi i\omega}{c^2}\sigma(q)\right]^{-1}, \tag{15}$$

$$\sigma(q)=\frac{3\sigma_0}{4}\left[\left(1+\frac{1}{q^2l^2}\right)\frac{1}{iql}\ln\frac{1+iql}{1-iql}-\frac{1}{q^2l^2}\right]. \tag{16}$$

Here $r(q)$ is the Green function for the Maxwell equations and $\sigma(q)$ is the conductivity of the unbounded metal, with allowance made for the spatial dispersion (in the $\omega\tau\ll 1$ case).

Now solving Eq. (9"), and using the definition of the force $F_2(x)$ [see (10')], we find

$$F_2(q)=\frac{1}{2c}\int_{-\infty}^{\infty}H_1(q-q')[j_1(q')-\sigma(q,q')E_1(q')]dq', \tag{17}$$

or, substituting (14),

$$F_2(q)=-(2\omega H_1^2(0)/c^2)\int_{-\infty}^{\infty}r(q-q')(q-q')r(q')\times[\sigma(q')+\sigma(q,q')]dq', \tag{17'}$$

where

$$\sigma(q,q')=\frac{qe^2\tau^2}{\pi}\left\langle\frac{\Lambda_{xx}}{1+(qv_x\tau)^2}\left[\tilde{\nabla}\frac{q'v_x\tau}{1+(q'v_x\tau)^2}v_y+qv_x\tau\tilde{\nabla}\frac{1}{1+(q'v_x\tau)^2}v_y\right]\right\rangle_+. \tag{18}$$

In the formula (17) and (17') the integrand consists of two terms: the first term corresponds to the induction force; the second, to the deformation force. In (18) the average is taken over that part of the Fermi surface where $v_x>0$ [let us recall that $\Lambda_{ik}(-\mathbf{p})=\Lambda_{ik}(\mathbf{p})$]. The oddness of the Fourier transfirm of the force is guaranteed by the fact that $\sigma(-q,-q')=\sigma(q,q')$.

4. As noted in Sec. 1, different excitation mechanisms are responsible for the generation of double-frequency ultrasound in the limiting cases $l\gg\delta$ and $l\ll\delta$: the induction mechanism under normal skin effect conditions and the deformation mechanism under anomalous skin effect conditions. Indeed, from (17) and (18) we find that, in order of magnitude,

$$F_2^d(q)\sim(l/\delta)^2F_2^u(q).$$

This estimate is valid for wave vectors $q\sim 1/\delta$, which are the most important in the problem of the generation of the second harmonic of an ultrasonic wave.

The nonlinear generation of ultrasound under normal skin effect conditions is considered in Ref. 2 and 3. Let us give the corresponding formulas as obtained from the general expression (17). In the limit of local conductivity $(ql\to 0)$ we have

$$F_2(q)=-\frac{1+i}{4\pi}H_1^2(0)\frac{q\delta_n}{(q\delta_n)^2-8i}, \tag{19}$$

where $\delta_n=c[2\pi\sigma_0\omega]^{-1/2}$ is the normal skin depth. Substituting the expression (19) into the formula (13), we obtain

$$|u_{2\infty}|=\frac{H_1^2(0)}{32\pi\rho\omega s_l}\frac{1}{[1+(q_l\delta_n)^4/4]^{1/2}}. \tag{20}$$

5. Let us now consider double-frequency-ultrasound generation under anomalous skin effect conditions. We shall, in accordance with the foregoing, consider only the deformation mechanism of generation. The fact that the cor-

rection χ_1 to the electron distribution function for $ql \gg 1$ differs appreciably from zero only in a band, i.e., in the narrow interval of directions $v_x/v_F \lesssim \delta/l$, allows us to simplify the expression (18) for $\sigma(q,q')$ by discarding the term $v_x \partial/\partial v_y$ the operator $\tilde{\nabla}$:

$$\sigma(q,q') = \frac{2e^2\tau^3 qq'}{\pi m}\left\langle \Lambda_{xx} v_y^2 \frac{1-(q'v_x\tau)^2-2(qv_x\tau)(q'v_x\tau)}{[1+(qv_x\tau)^2][1+(q'v_x\tau)^2]^2}\right\rangle_+ . \tag{21}$$

In the case of an isotropic dispersion law the deformation potential tensor can be characterized by one scalar quantity $\tilde{m}$ having the dimensions of mass[9]:

$$\Lambda_{ik}(\mathbf{p}) = \tilde{m} v_F^2 (n_i n_k - {}^1/_3 \delta_{ik}), \quad \mathbf{n} = \mathbf{p}/p.$$

Therefore, $\Lambda_{xx} = \tilde{m} v_F^2 (v_x^2/v_F^2 - 1/3)$. Taking account of the smallness of v_x/v_F in the effective-electron band, we obtain

$$\sigma(q,q') = -\frac{1}{2\pi}\frac{\tilde{m}}{m}\sigma_0 ql \int_0^1 \frac{1-(q'l\varkappa)^2-2(ql\varkappa)(q'l\varkappa)}{[1+(ql\varkappa)^2][1+(q'l\varkappa)^2]} d\varkappa. \tag{22}$$

Assuming l is the largest parameter having the dimensions of length, we can rewrite the expression (22) in the form

$$\sigma(q,q') = -\frac{\tilde{m}}{m}\frac{\sigma_0 ql}{4}\frac{qq'}{(|q|+|q'|)^2}(\operatorname{sign} q - \operatorname{sign} q'). \tag{23}$$

Substituting (23) into (17), and going over to a new variable, we find

$$F_2^n(q) = -\frac{16H_1^2(0)}{3\pi^2}\left(\frac{l}{\delta_a}\right)^2 f(q\delta_a),$$
$$f(\eta) = \eta^2 \int_0^\infty \frac{\xi^2\, d\xi}{(\xi^3-8i)[(\eta+\xi)^3-8i]}. \tag{24}$$

Here $\delta_a = (32\delta_L^2 v_F/3\pi\omega)^{1/3}$ is the skin depth under anomalous skin effect conditions.

Let us give the asymptotic expressions for the function $f(\eta)$. To within logarithmic terms

$$f(\eta) \approx \begin{cases} (i/24)\eta^2, & \eta \ll 1 \\ \ln \eta/\eta, & \eta \gg 1 \end{cases}. \tag{24'}$$

Substituting the expression (24) into the formula (13) allows us to express the amplitude of the second ultrasound harmonic at infinity in the following form:

$$|u_{2\infty}| = \frac{H_1^2(0)}{\rho\omega s_l}\left(\frac{l}{\delta_a}\right)^2 \varphi(q_l\delta_a),$$
$$\varphi(q_l\delta_a) = \begin{cases} \beta_0 + \beta_1 (q_l\delta_a)^2 \ln(1/q_l\delta_a), & q_l\delta_a \ll 1 \\ \beta_\infty (q_l\delta_n)^{-1}, & q_l\delta_a \gg 1 \end{cases}. \tag{25}$$

The constants β_0, β_1, and β_∞ are computed in the Appendix.

It can be seen from the last formula that, as was to be expected, the amplitude of the second harmonic has a maximum at $q_l\delta_a \sim 1$. Its value is

$$|u_{2\infty}|^{max} \sim \frac{H_1^2(0)}{\rho\omega s_l}\left(\frac{l}{\delta_a}\right)^2. \tag{26}$$

6. In experiment the frequency of the incident electromagnetic wave is normally fixed, and the wave amplitude and sample temperature are varied. Using the formulas obtained above (into which the temperature enters through the mean free path l), let us find how the amplitude $|u_{2\infty}|$ of the generated ultrasound depends on l, assuming the wave amplitude $H_1(0)$ is a constant and satisfies the weak nonlinearity condition (see Sec. 2).

Let us begin with the case of relatively low frequencies

$$\omega \ll (s/v_F)^{1/2}(s/c)\omega_0 \sim (10^9-10^{10})\ \mathrm{c}^{-1}. \tag{27}$$

This inequality was obtained from the requirement that the maximum of the amplitude $u_{2\infty}$ as a function of the mean free path l fall within the domain of the normal skin effect. It is convenient for the purpose of deriving it to rewrite the formula (20) as follows:

$$|u_{2\infty}| = \frac{H_1^2(0)}{32\pi\rho\omega s_l} l\left[l^2 + \left(\frac{\omega v_F}{s_l^2}\delta_L^2\right)^2\right]^{-1/2}. \tag{20'}$$

Under anomalous skin effect conditions, the amplitude $|u_{2\infty}|$ increases like l^2 as l increases [see (25)]. The coefficient of l^2 increases in proportion to $\omega^{2/3}$, attaining its maximum value at the boundary of the interval (27). Notice that the function $\varphi(q_l\delta_a)$ has a maximum at $\omega \sim (s/v_F)^{1/2}(s/c)\omega_0$ [see (25)–(27)]. Figure 1 shows plots of

$$\zeta = \frac{|u_{2\infty}|}{H_1^2(0)/\rho\omega s_l}$$

as a function of l for two frequency values. Let us recall that these expressions are valid in the quasistatic case, i.e., up to $\omega\tau \lesssim 1$ or $l \lesssim v_F/\omega$. The value of ζ for $l \sim v_F/\omega$ decreases in proportion to $\omega^{-4/3}$, and at the boundary of the admissible interval (27)

$$\zeta^{max} \sim (v_F/s)^{1/2}.$$

At higher frequencies

$$\omega \gg (s/v_F)^{1/2}(s/c)\omega_0 \tag{27'}$$

the maximum in the normal-skin-effect region is not attained, and at $l \sim \delta_a$ the dependence of ζ on l changes from a linear [see (20')] to a quadratic (Fig. 2) dependence. According to (25), the coefficient of l^2 increases in proportion to $\omega^{2/3}$, attaining saturation at $q_l\delta_a \sim 1$. Its maximum value

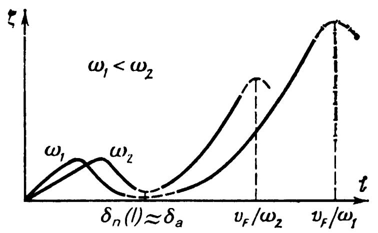

FIG. 1.

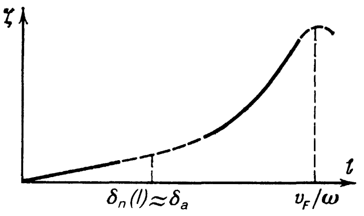

FIG. 2.

in this case is of the order of $s_l/\delta_L^2 v_F$. The highest ζ value, attained at $l \sim v_F/\omega$, is significantly greater than the corresponding value in the region of low frequencies (27):

$$\zeta^{max} \sim (v_F/s)^2.$$

7. In analysis carried out above the source of nonlinearity in the generation of double-frequency ultrasound is the action of the alternating magnetic field on the current it excites in the skin layer. In principle, however, other sources of nonlinearity can manifest themselves. For example, allowance for the nonlinear term in the deformation tensor[10] also leads to the appearance of a term

$$F_2'(x) = \frac{\rho s_t^2}{2} \frac{d}{dx} \left(\frac{du_1}{dx} \right)^2 \tag{28}$$

on the right-hand side of the elasticity equation. The amplitude u_1 of the first harmonic can easily be estimated, using the result obtained in the solution of the ultrasonic equation (10) by the Green's functions method. Under anomalous skin effect conditions we can have

$$u_{1\infty} \sim ne\delta^2 H_1(0)/\rho c s_t.$$

Substitution of $u_{1\infty}$ into (28) and comparison with F_2^d, as given by the formula (5), yield

$$\frac{F_2'}{F_2^d} \sim \frac{m}{M} \left(\frac{\delta^2}{\delta_L l} \right)^2 .$$

It can be seen from this that it is not important to allow for the terms quadratic in u_1 in the deformation tensor here.

The kinetic equation also contains sources of nonlinearity. Recently, Andreeve and Pushkarov[11] derived the exact nonlinear equations of the theory of elasticity of metals. They established that the field part of the kinetic equation contains terms quadratic in the lattice displacement and velocity. But in the problem of the electromagnetic generation of ultrasound these terms can be neglected on account of the weakness of the electromechanical interaction. Furthermore, the collision integral taking account of the inelastic scattering of the electrons contains a nonlinear term.[12] In a normal metal the source of the inelastic scattering of the electrons is their interaction with the phonons. But under anomalous skin effect conditions there is practically no scattering by phonons.

Second-harmonic generation also occurs because of the momentum nonlinearity,[13,14] which arises as a result of the distortion of the distribution function for the electrons effectively interacting with the acoustic first-harmonic wave, and manifests itself when the condition $\omega_0\tau \gtrsim 1$ is satisfied (ω_0 is the characteristic oscillation frequency of the electrons trapped by the acoustic-wave field). In the case of the electromagnetic generation of ultrasound the quantity $\omega_0\tau$ is small not only because it is proportional to the nonlinearity parameter $\omega_c \tau l/\delta$, but also because the expression for it contains the additional factor $(m/M)^{1/4}$:

$$(\omega_0\tau)^2 \sim \left(\frac{m}{M}\right)^{1/2} \frac{\omega_c\tau l}{\delta} (q_t\delta)^3 \ll 1.$$

The diffuseness of the electron scattering at the metal boundary plays an important role in the linear-wave-transformation processes.[15] According to Ref. 15, the second harmonic of the surface force satisfies $F_2^{\rm surf} \sim \langle p_x \chi_2 \rangle/\tau$. We can, by comparing this expression with F_2^d in the formula (5), show that

$$F_2^{\rm surf} \sim (\delta_a^2/l^2) F_2^d,$$

i.e., that the surface mechanism can be ignored in the problem of the nonlinear generation of ultrasound under anomalous skin effect conditions.

Although our entire analysis has been carried out under the assumption of an isotropic dispersion law, it is clear that allowance for the anisotropy of the Fermi surface will not change the results qualitatively if the normal to the surface of the metal coincides with a "good" direction in the crystal. The final formulas allowing for an aribtrary dispersion law should contain integrals over the band at the Fermi surface (cf., for example, Refs. 7 and 16).

8. Let us now make a few comments about the limits of applicability of the results obtained.

In computing $u_{2\infty}$ we assumed that $q_l l \gg 1$. It should be noted that the interaction between the ultrasound and the electromagnetic wave field should be stronger in the region $q_l l \sim 1$ because of the "kinetic" part of the field. True, this spatial resonance should be weak compared to the $q_l\delta \sim 1$ resonance because of the fact that the amplitude of the kinetic part of the field contains an additional smaller parameter ($l^2\delta_n^2$ in the case of the normal skin effect and δ_a^2/l^2 in the case of the anomalous skin effect).

In the "high-frequency" region $\tau \gg \omega^{-1}$ (practically beyond the limits of applicability of the formulas derived above) the mean free path l is replaced by $l^* = l/(1+\omega^2\tau^2)^{1/2}$, and then $l^* \to v_F/\omega$, while $\delta \to \delta_L$. Therefore, it is clear that, as $l = v_F\tau \to \infty$, the quantity ξ tends to $\xi_\infty(\omega)$, which is such that the higher the frequency ω is, the smaller is its value.

An important feature of our analysis is the assumption that the nonlinearity is weak. The electron dynamics in the skin layer is qualitatively different in the case of a fully developed nonlinearity, which results when the condition $\omega_c\tau \gtrsim \delta_a/l$ is satisfied. There appear groups of so-called grazing trapped electrons[4] effectively interacting with the wave, and causing an appreciable change in the linear response of the metal. The grazing electrons undergo multiple reflections from the surface in the course of their specular scattering, and the trajectories of the trapped electrons "wind" along the $H(x) = 0$ plane. It can be shown that, in the strong nonlinearity regime, the various harmonics of the correction to the equilibrium distribution function are comparable in order of magnitude: $\chi_0 \sim \chi_1 \sim \chi_2$. In this case we cannot use the interative procedure employed in the present paper. This means that the growth of the amplitude of the electromagnetic wave incident on the metal surface results in the approximately uniform distribution of the wave energy among all the frequencies that are multiples of ω, i.e., the amplitudes of the second and higher harmonics are comparable in order of magnitude to the amplitude of the first harmonic.

Thus, we have shown in the present paper that the amplitude of generated double-frequency ultrasound increases (as l increases) in proportion to $(l/\delta_a)^2$ when we go over from the normal to the anomalous skin effect. Upon the attainment of the critical value $\omega_c\tau \sim \delta_a/l$ the quadratic growth of the amplitude ceases. If we extrapolate the result (25) to values of $\omega_c\tau \sim \delta_a/l$, then the limiting double-frequency ultrasound amplitude $u_2^{\max}$ attains the value $u_{1\infty}$, the amplitude of the first ultrasound harmonic excited by the deformation force under anomalous skin effect conditions. The phenomenon of linear generation under these conditions has been experimentally observed by Maxfield and his co-workers[17,18] and Puskorius and Trivisonno.[19]

As the above-presented estimates show, even though the efficiency of the transformation processes considered is low, it is within the limits of present-day experimental possibilities. The formula (21) shows that the amplitude of the second ultrasound harmonic can serve as a source of information about the diagonal components of the deformation potential tensor. Of particular interest are the simultaneous measurements of both the diagonal and off-diagonal components of the tensor $\Lambda_{ik}(\mathbf{p})$ in terms of the second- and first-ultrasound-harmonic amplitudes, respectively.

APPENDIX

Substitution of the expression (24) into the formula (13) leads to the result (25), where the function

$$\varphi(a) = \frac{4}{3\pi^3}\int_0^\infty \frac{\xi^2\,d\xi}{\xi^3 - i}\int_0^\infty \frac{\eta^3\,d\eta}{(\eta^2 - a^2)[(\eta+\xi)^3 - i]}.$$

From this it follows that, for $a \ll 1$,

$$\varphi(a) \approx \beta_0 + \beta_1 a^2 \ln(1/a),$$

where

$$\beta_0 = \varphi(0) = \frac{4}{3\pi^3}\int_0^\infty \frac{\xi^2\,d\xi}{\xi^3 - i}\int_0^\infty \frac{\eta\,d\eta}{(\eta+\xi)^3 - i}, \qquad \beta_1 = i/3.$$

The expression for β_0 can be reduced to the form

$$\beta_0 = \frac{4}{3\pi^3}\int_0^\infty \frac{d\eta}{(1+\eta)^3\eta}\int_0^\infty \frac{\xi^4\,d\xi}{(\xi^3 - i/\eta^3)[\xi^3 - i/(i+\eta)^3]}.$$

In the inner integral the poles ξ_k have the arguments $\pi/6$, $5\pi/6$, and $3\pi/2$ ($\eta > 0$), i.e., there are no poles inside the segment $(-\pi/6, 0)$. Setting $\xi = \xi' e^{-i\pi/6}$, we obtain

$$\beta_0 = \frac{4}{3\pi^3}e^{i\pi/6}\int_0^\infty \frac{d\eta}{(1+\eta)^3\eta}\int_0^\infty \frac{\xi^4\,d\xi}{(\xi^3 + 1/\eta^3)[\xi^3 + 1/(1+\eta)^3]}.$$

The evaluation of the integrals yields

$$\beta_0 \approx 0.06 e^{i\pi/6}.$$

Since $\operatorname{Im}\beta_0 > 0$, it can be seen that, for $a \ll 1$, the function

$$|\varphi(a)| \approx [(\operatorname{Re}\beta_0)^2 + (\operatorname{Im}\beta_0 + {}^1/_3 a^2 \ln(1/a))^2]^{1/2}$$

increases with a in proportion to $a^2\ln(1/a)$.

For $a \gg 1$, it is convenient to divide the range of integration in (13) into two: from 0 to $1/\delta_a$ and from $1/\delta_a$ to ∞. Replacing $F_2(q)$ in the formula (13) by its asymptotic expressions, and using (24) and (24'), we obtain

$$\varphi(a) \approx \frac{8}{3\pi^3}\int_0^\infty \frac{\ln\eta\,d\eta}{\eta^2 - (2a)^2} \approx \frac{2}{3\pi a}, \qquad a \gg 1,$$

i.e., $\beta_\infty \approx \frac{2}{3}\pi$.

1) The induction mechanism presupposes an inhomogeneous distribution of the current or field over the cross section of the conductor.

2) That all the nonlinear terms discarded in the equations (9) are or order $\omega_c\tau l/\delta$ and therefore small can be deomonstrated, using the ineffectiveness concept,[5] i.e., the fact that χ_1 is substantially different from zero only for the electrons in the narrow layer with $v_x/v_F \lesssim \delta/l$.

3) We have chosen the natural manner of continuing the fields, in the sense that it holds in general for the complete system of Maxwell, Boltzmann, and elasticity equations used. In such an approach the even continuation of the function $u_1(x)$ should be used in the solution of Eq. (10) by the Fourier method. Notice that we then have $u_{1\infty} = iF_1(q_t)/2\rho\omega s_t$.

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