Scattering of electrons by kinks on the dislocation line of a metal

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The interaction of the conduction electrons in a metal with dislocations containing kinks in the Peierls potential contour is theoretically investigated. In the field of random stresses arising on plastic deformation of the sample, the kinks on a dislocation line, which are in thermodynamic nonequilibrium at low temperatures, form a set of oscillators in the potential wells of various shapes. In the low-temperature regime, when the concentration of other defects is low, inelastic scattering of electrons by kinks, anisotropic with respect to angle, leads to a quadratic temperature dependence of the thermal conductivity of a metallic sample along the favored direction of the dislocation axes. In the plane perpendicular to the dislocation axes, large-angle elastic scattering of electrons predominates. The relationship of the results to experimental data[1,2] is discussed.

INTRODUCTION

The production of single crystals of metals of high purity and with a low concentration of lattice defects makes it possible in principle to investigate the interaction of conduction electrons with defects introduced in a controlled manner. Experimental investigation in recent years[1,2] has detected anomalies in the low-temperature kinetic properties of refined metallic samples after a relatively low number of dislocations (10^7–10^8/cm^2) has been introduced by means of plastic deformation. The behavior of the electronic thermal conductivity at helium temperatures indicates inelastic scattering of electrons by soft-mode vibrations, suppressed by subsequent annealing of the sample or by the additional introduction of a large number of dislocations (10^9/cm^2).[1,2] The resistivity of the sample at these temperatures always has an extrinsic character and changes monotonically as the concentration of dislocations increases.[1,2] An explanation of this effect in terms of the scattering of electrons by quasilocalized vibrations connected with the dislocations requires the assumption of an anomalously small magnitude of the linear strain of the dislocation line.[3]

In the present work another mechanism of inelastic scattering of electrons is proposed, due to the newly-introduced dislocations, in which the existence of the soft mode is unrelated to the smallness of the linear strain of the dislocation line.

In a real crystal the energy contour for a dislocation in a glide plane is an undulating surface due to the presence of the Peierls potential, connected with the lattice periodicity.[4] This leads to the existence of kinks in the dislocation line when different parts of the dislocation lie in different parallel valleys of the Peierls potential.[4] In metals such as copper in the absence of internal stresses isolated kinks may move along the dislocation line like particles with an effective mass determined by the height of the Peierls barrier, the linear density and the linear strain of the dislocation line.[5] The elastic energy of an isolated kink in such crystals is hundreds of degrees Kelvin.[4] Therefore, at helium temperatures kinks in thermodynamically equilibrium must be absent from a crystal without stresses. However, nonequilibrium kinks may exist, trapped in potential wells which are produced where the random stresses on the dislocation line change sign. At low temperatures the probability that a kink tunnel from such a well is exponentially small.[6]

In the following section the oscillations of isolated kinks are examined, as well as kinks on the portions of the dislocation lines not parallel to the direction of the valleys of the Peierls potential. In such portions it may be energetically favorable to form a sequence of kinks of one sign with a linear density n_k, making use of the condition[4,7] that $an_k \approx \theta$, where a is the repeat interval of the valleys in the Peierls potential, and θ is the angle between the direction along the valley and a given section of the dislocation line. Conduction electrons scatter inelastically on the deformation potential created by the oscillating kinks.

It is significant that, as shown in Section I, the kink oscillation frequencies ω_m (in temperature units) are at most on the order of a Kelvin. The oscillation frequency of a kink is determined by the absolute value of the gradient of the local stress. Closely situated kinks on oblique sections of dislocation lines have the highest frequencies. A frequency $\omega_m \sim 1$ K is attained for the densest "packet" of kinks when the stress field is of the order of the width l_k of an isolated kink along a dislocation line.[1)]

In Section II, we calculate the thermal conductivity of a metallic crystal with kinks on dislocation lines running along the preferred direction in the crystal in the temperature regime $T \gg \omega$, where ω is the upper limit of the kink oscillation frequency. In such conditions successive instances of inelastic scattering of an electron by kinks give rise to diffusion in energy. In the calculations the elastic scattering of electrons by the deformation potential of the kinks and the linear portions of the dislocations is also considered. We take the temperature to be sufficiently low that electron-phonon scattering may be neglected.

Comparison of the results with experimental data,[1,2] and an estimate of sample parameters for which the influence of the kinks should be most pronounced are in the final section.

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I. OSCILLATION OF KINKS ON A DISLOCATION LINE IN A CRYSTAL

To describe kinks on a dislocation line we use the model of an elastic line in a periodic potential.[5] For an (edge) dislocation along the z axis with the Burgers vector along the x axis (that is, the glide plane coincides with the xz plane) we have the following equation of motion:

$$E_0\partial^2 x/\partial z^2=(2\pi\alpha/a)\sin(2\pi x/a)-b\sigma_{xy}(z)+m_0\partial^2 x/\partial t^2, \qquad (1)$$

where $x(z,t)$ is the coordinate of the dislocation in the glide plane as a function of time and position along the z axis; E_0 is the dislocation energy per unit length (the "stiffness" of the dislocation line); m_0 is the linear density; $\sigma_{xy}(z)$ is the component of the random stress tensor in the neighborhood of the dislocation line (only its z-dependence is explicitly indicated); α is the height of the Peierls barrier (the dimension is energy per unit length); a is the repeat period of the valleys in the Peierls potential; and b is the modulus of the Burgers vector of the dislocation. In (1) the simplest approximation for the Peierls potential is used, valid for copper.[5] Under the condition $\sigma_{xy}=0$, the kink is described by the stationary solution of (1):

$$x(z)=\frac{2a}{\pi}\operatorname{arctg}\left\{\exp\left[\pm\frac{2\pi}{a}(z-z_0)\nu^{1/2}\right]\right\}, \qquad (2)$$

where $\nu=\alpha E_0^{-1}$ is a small quantity (for copper $\nu\sim10^{-4}$).[4,5] The energy functional for a dislocation in the Peierls potential in the model used is[5]

$$W=\int_{-\infty}^{+\infty}\left[\frac{m_0}{2}\left(\frac{\partial x}{\partial t}\right)^2+\frac{1}{2}E_0\left(\frac{\partial x}{\partial z}\right)^2+\alpha\left(1-\cos\frac{2\pi x}{a}\right)-b\sigma_{xy}x\right]dz, \qquad (3)$$

where a term in the stress field $\sigma_{xy}(z)$ is introduced. Taking the field $\sigma_{xy}(z)$ to be smoothly varying on scales of the order of the width $l_k\sim a/\nu^{1/2}$ of a kink, which is determined by expression (2), we get from (3) the effective Hamiltonian for a kink in the stress field $\sigma_{xy}(z)$ in the neighborhood of a point z_0 on the dislocation axis where $\sigma_{xy}(z)$ changes sign:

$$H=-\frac{1}{2M}\frac{\partial^2}{\partial z^2}+\frac{M\omega^2}{2}(z-z_0)^2,$$
$$M=(4am_0/\pi)\nu^{1/2},\quad \omega^2=abM^{-1}|\partial\sigma_{xy}(z_0)/\partial z|. \qquad (4)$$

Let us examine the situation in which dislocations are oriented predominantly along one of the directions of the valleys of the Peierls potential (taken to be the z axis of the system). In such a system a gradient in $\sigma_{xy}(z)$ is created by dislocations going at an angle to the z axis (the concentration of other defects in the crystal we assume to be small). If this angle is less than θ, then the minimum scale of variation in the field $\sigma_{xy}(z)$ (created by the other dislocations in the vicinity of the one under consideration) along the z axis has the order of magnitude $l\sim n_d^{-1/2}/\theta$, where n_d is the dislocation density. The characteristic size of σ_{xy} is[4] $\sigma_{xy}\sim Gbn_d^{1/2}$, where G is the elastic shear modulus of the crystal lattice. Therefore for the oscillation frequency of a single kink according to (4) we find the value

$$\omega\sim[M^{-1}Gab^2n_d\theta]^{1/2}. \qquad (5)$$

Stability of the dislocation line with kinks is achieved under the condition[4] $Gbn_d^{1/2}<\tau_p$, where τ_p is the Peierls stress (equal to $2\pi\alpha/ab$ in the model (1)–(4), so that $\tau_p/G\sim\nu$). Consequently, $n_d<\nu^2b^{-2}\sim10^8/\text{cm}^2$, which coincides with the experimental situation.[1,2] Now from (5) we find

$$\omega<\theta^{1/2}\nu^{1/4}\Theta_D\sim\theta^{1/2}0{,}1\ \text{K}, \qquad (6)$$

where Θ_D is the Debye frequency of the crystal lattice. As is clear from the Introduction, in a system of dislocations close to equilibrium, where the inclination from the direction of the Peierls valleys is connected only with the presence of kinks on the dislocation line, $\theta\leqslant\theta_m\sim a/l_k\sim\nu^{1/2}$.

Let us now examine the oscillation of a sequence of kinks of a given sign with a linear density n_k on a given section of a dislocation. From Eq. (3) we find the expression for the force F_z acting on a kink along the z axis in the region of a smooth change in the field $\sigma_{xy}(z)$:

$$F_z=\mp ab\sigma_{xy}(z), \qquad (7)$$

where the choice of sign depends on the sign of the exponent in Eq. (2). The repulsive force between kinks of the same sign on an edge dislocation is equal to[4]

$$F_{12}=Sa^2/2l_{12}^2,\quad S=Gb^2(1-2\sigma)/4\pi(1-\sigma),\quad l_{12}=|z_1-z_2|, \qquad (8)$$

where σ is the Poisson bracket. Using Eqs. (7) and (8), we obtain a system of equations for the equilibrium coordinates z_n of a sequence of kinks of one sign on the portion of a dislocation line where the magnitude of the field $\sigma_{xy}(z)$ produced by all the other dislocations in the neighborhood of the one under consideration passes through zero ($\sigma_{xy}(z)=\sigma_0 z/l$):

$$ab\sigma_0 z_n/l=Sa^2[(z_n-z_{n-1})^{-2}-(z_{n+1}-z_n)^{-2}]/2, \qquad (9)$$

where $\sigma_0\sim Gbn_d^{1/2}$, $l\sim n_d^{-1/2}/\theta$. In (9) the equation for the end kinks in the sequence ($n=-N,N$) is not written since it is assumed that $1\ll N\ll ln_k$. The solution of Eq. (9) has the form

$$z_n-z_{n-1}=z_0[1-(z_0^3\sigma_0 b/2Sal)n^2]^{-1},\quad 1\ll n\ll(Sal/z_0^3\sigma_0 b)^{1/2},$$
$$z_n-z_{n-1}=\tfrac{1}{2}z_0[\tfrac{2}{3}-(z_0^3\sigma_0 b/2Sal)^{1/2}|n|]^{-1/2},\quad n<\tfrac{2}{3}(2Sal/z_0^3 b\sigma_0)^{1/2}, \qquad (10)$$

where $z_0=\lim_{n\to0}|z_n-z_{n-1}|$. To evaluate the maximum size of N we put in (10) $z_0\gtrsim l_k$. Then we find

$$N_{max}\sim(Sal/l_k^3 b\sigma_0)^{1/2}\sim\nu^{3/4}/a(n_d\theta)^{1/2}>\nu^{-1/4}\theta^{-1/2}\sim10\theta^{-1/2}, \qquad (11)$$

where for n_d the value $n_d<\nu^2b^{-2}$ found above is used. The maximum length of an isolated series of kinks of one sign is thus shown to be

$$L_{max}\sim N_{max}l_k\sim n_d^{-1/2}\nu^{1/4}\theta^{-1/2}. \qquad (12)$$

In the case that the maximum angle of inclination of the dislocation axes from the z axis is $\theta_m\sim a/l_k$, we get from (11) and (12)

$N_{max}\sim\nu^{-1/2}\sim 10^2$, $L_{max}\sim n_d^{-1/2}$.

We note that the relations (11) and (12) confirm the correctness of the inequality $N\ll ln_k$, and thus permit the linearization of $\sigma_{xy}(z)$ on the left side of Eq. (9). From (10)–(12) it also follows that over the larger part of the sequence of kinks the distance between neighboring kinks can be considered constant, $z_0\sim n_k^{-1}$.

Linearizing (9) for small departures u_n of the kinks from their equilibrium positions, we get the wave equation:

$$M\frac{\partial^2 u_n}{\partial t^2}=\frac{Sa^2}{l_0^3}(u_{n-1}+u_{n+1}-2u_n)-\frac{ab\sigma_0}{l}u_n, \tag{13}$$

where $l_0=n_k^{-1}$, and M is determined by (4). From this we find the spectrum of oscillations of a long array of kinks:

$$\omega_k^2=\frac{2Sa^2}{Ml_0^3}(1-\cos k)+\frac{ab\sigma_0}{Ml}, \quad -\pi\leqslant k\leqslant\pi. \tag{14}$$

From Eq. (14), with use of (8), follows an estimate for the maximum kink oscillation frequency[2] ($n_k\sim l_k^{-1}$):

$$\omega_m\sim(Sa^2/Ml_k^3)^{1/2}\sim\nu^{1/2}\Theta_D\sim 1\ \text{K}. \tag{15}$$

In Section II it is demonstrated that when the condition $\omega\ll T$ holds for the oscillation frequencies of isolated kinks and arrays of kinks the contribution to the thermal conductivity resulting from their scattering of electrons depends only on the overall average linear density of kinks $\bar{n}_k$ on the dislocation line. This density is determined by the repeat frequency on the dislocation line of oblique sections of the dislocation, and so by the probability of appearance of an isolated kink. Furthermore $\bar{n}_k$ figures in the present work as a phenomenological parameter, the magnitude of which depends, obviously, on the preparation process and treatment of the sample.

II. THERMAL CONDUCTIVITY ALONG THE DISLOCATION AXIS AND IN THE TRANSVERSE PLANE

The kinetic equation for the distribution function $n_{\mathbf{p}}$ of the conduction electrons which scatter on the dislocations with kinks is, in the presence of a temperature gradient ∇T,

$$-\frac{\varepsilon_{\mathbf{p}}}{T}\nabla T\frac{\partial n_{\mathbf{p}}}{\partial\mathbf{p}}=\int\frac{d^3\mathbf{p}'}{(2\pi)^3}w(\mathbf{p},\mathbf{p}')K(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}'};q_x,q_z)\times\{n_{\mathbf{p}'}(1-n_{\mathbf{p}})-\exp[(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}'})/T]n_{\mathbf{p}}(1-n_{\mathbf{p}'})\}, \tag{16}$$

$$w(\mathbf{p},\mathbf{p}')=n_d a(\mathbf{p},\mathbf{q}), \quad \mathbf{p}'=\mathbf{p}+\mathbf{q}, \tag{17}$$

$$K(\omega;q_x,q_z)=L^{-1}\int dz\,dz'\,dt\exp[iq_z(z'-z)+i\omega t]\times\langle\langle\exp(-iq_x x(z,0))\exp(iq_x x(z',t))\rangle\rangle. \tag{18}$$

In (16)–(18) the following notation is used: $\varepsilon_{\mathbf{p}}$ is the electronic energy, measured from the chemical potential; $a(\mathbf{p},\mathbf{q})$ is determined by the Fourier transform of the deformation potential created by dislocations in a metal,[9] so that in the isotropic model for $q\ll p\sim p_F$, $a(\mathbf{p},\mathbf{q})$ has the form $F(\mathbf{p};q_x,q_y)/q^4$, where $F(\mathbf{p};q_x,q_y)$ is quadratic in q_x and q_y, the coefficients of which depend on the component of the unit vector $\mathbf{p}/p$. L is the length of the dislocation. The symbol $\langle\langle\ldots\rangle\rangle$ in (18) indicates an ensemble average and an average over the configurations of the kinks; the form of the dislocation line in the glide plane $x(z,t)$ depends on time due to the kink oscillations.

For an estimate of the electronic thermal conductivity of the crystal along the predominant direction of the dislocation lines, $\chi_\parallel$, and in the plane perpendicular to them, $\chi_\perp$, we linearize the right-hand side of (16) in the small correction of the electron distribution function $n^0(\varepsilon_{\mathbf{p}})$:

$$\delta n_{\mathbf{p}}=-[\partial n^0(\varepsilon_{\mathbf{p}})/\partial\varepsilon_{\mathbf{p}}]\Psi_{\mathbf{p}}.$$

Using the variational principle,[10] we get

$$\chi_i^{-1}=\Big\{m^2\int d\omega\,d\varepsilon\,dq_z\,dp_z\,d\varphi\,d\varphi'\,w(1-n^0(\varepsilon))n^0(\varepsilon-\omega)\times(\Psi_{\mathbf{p}}-\Psi_{\mathbf{p}+\mathbf{q}})^2K(\omega;q_x,q_z)\Big\}\times\Big\{m\int v_{\mathbf{p}}{}^i\varepsilon\Psi_{\mathbf{p}}\frac{\partial n^0(\varepsilon)}{\partial\varepsilon}2\,d\varepsilon\,dp_z\,d\varphi\Big\}^{-2}, \tag{19}$$

where $\chi_i^{-1}=\chi_\parallel^{-1},\chi_\perp^{-1}$. In the first case we select a trial function $\Psi_{\mathbf{p}}\sim\varepsilon_{\mathbf{p}}p_z$ and change $v_{\mathbf{p}}^i$ in (19) to $v_{\mathbf{p}}^z$; in the second case we use a trial function $\Psi\sim\varepsilon_{\mathbf{p}}p_x$ and substitute $v_{\mathbf{p}}^i=v_{\mathbf{p}}^x$. In (19) we also make the transformation from integration over $d^3\mathbf{p}d^3\mathbf{p}'$ to integration over $dp_z dq_z$, over the energies $\varepsilon,\varepsilon'=\varepsilon-\omega$, prior to the collision and afterward, and over the polar angles φ and φ' corresponding to the planes $\langle p_x p_y\rangle$ and $\langle p_x' p_y'\rangle$; $w=w(\mathbf{p},\mathbf{p}+\mathbf{q})$, and m is the effective mass of the electron on the Fermi surface in the isotropic model.

The inelastic contribution to $\chi_\perp^{-1}$ and $\chi_\parallel^{-1}$ we get from the general expression (19):

$$\chi_u^{-1}=\Big\{m^2\int d\omega\,d\varepsilon\,dq_z\,dp_z\,d\varphi\,d\varphi'\,w(1-n^0(\varepsilon))n^0(\varepsilon-\omega)\times p_z^2\omega^2K(\omega;q_x,q_z)\Big\}\Big\{\int p_z^2\varepsilon^2\frac{\partial n^0(\varepsilon)}{\partial\varepsilon}2\,d\varepsilon\,dp_z\,d\varphi\Big\}^{-2}. \tag{20}$$

To compute (20) it is convenient to first compute $\tilde{K}(\omega,q_x)$:

$$\tilde{K}(\omega,q_x)=\int K(\omega;q_x,q_z)\,dq_z=(2\pi/L)\int dz\,dt\exp(i\omega t)\times\langle\langle\exp(-iq_x x(z,0))\exp(iq_x x(z,t))\rangle\rangle. \tag{21}$$

In (21) the definition (18) is used. The expression for $x(z,t)$ under the condition $n_k\ll l_k^{-1}$ has the form

$$x(z,t)=\sum_n x_n(z-z_n(t)), \tag{22}$$

where the nth kink is described at time t, depending on its orientation on the dislocation line, by one of the functions (2), centered at the point $z_n(t)=z_{0n}+u_n(t)$, where $u_n(t)$ is the deviation from the equilibrium position. To evaluate (21) we put the expression (22) in the form

$$x(z,t)=\sum_n\int\frac{dk}{2\pi}\tilde{x}_n(k)\exp\{ik(z-z_{0n})-iku_n(t)\}, \tag{23}$$

where $\tilde{x}_n(k)$ is the Fourier transform of $x_n(z)$. The equations for $u_n(t)$ in the case of isolated kinks are derived from

(4), and for kinks in an array of a given sign are given by equations of the form (13). Below, $\tilde{K}(\omega,q_x)$ is calculated for two possible ranges of ω_0, the upper limit of oscillation frequency of kinks in a specific configuration: a) $\omega_0 \ll \omega_T \equiv (T/Ml_k^2)^{1/2}$ (~ 0.1 K for $T \sim 1$ K) and b) $\omega_T \ll \omega_0 \ll T$. In the range a) a given kink scatters an electron like a "free particle" with mass M, maximum transferred momentum $q_m \sim l_k^{-1}$, and transferred energy $|\delta\varepsilon| \sim \omega_T$. Therefore in case a) we obtain

$$\tilde{K}(\omega, q_x) = 2\pi^2 (M/2\nu^{1/2} n_k^a a q_x^2 T)^{1/2} \times \exp\left[-\frac{q_x^2 a n_k^a \nu^{1/2}}{2\pi MT} \left(\frac{\pi M \omega}{2\nu^{1/2} n_k^a a q_x^2} + 1 \right)^2 \right], \quad (24)$$

where n_k^a signifies the average linear density of kinks on the dislocation line, the oscillation frequency of which does not exceed the value satisfying the inequality a) (this accounts for both kinks in a series of one sign, and isolated kinks).

In case b) we have

$$\tilde{K}(\omega, q_x) = (2\pi)^2 \delta(\omega) + \frac{8\pi\nu^{1/2} q_x^2 a}{ML} \left\{ \sum_i n_i \int_0^{2\pi} \frac{1}{\omega_{ik}} \times [(N_{ik}+1)\delta(\omega_{ik}+\omega) + N_{ik}\delta(\omega_{ik}-\omega) - (2N_{ik}+1)\delta(\omega)] \frac{dk}{2\pi} \right\}. \quad (25)$$

In the expression (25) Σ_i signifies the summation over the different sequences of kinks of one sign on the dislocation line; n_i is the number of kinks in the ith sequence; ω_{ik} is given by Eq. (14), where l_0 is changed to l_i, the distance between neighboring kinks in the ith sequence; $N_{ik} \equiv N(\omega_{ik})$, where

$$N(\omega) = [\exp(\omega/T) - 1]^{-1}.$$

When $\tilde{K}(\omega,q_x)$ is substituted in the form (25) in Eq. (20) all the oscillation frequencies of the kinks enter in the result only in the combinations $\omega_{ik}/\sinh(\omega_{ik}/2T)$, so that under the condition $\omega_{ik} \ll T$, to leading order in ω_{ik}/T the terms in χ_u^{-1} from electron scattering on links of different sequences and of isolated kinks are distinguished only by the total number of kinks of these two types. Thus, we get

$$\chi_u^{-1} = A_0 n_d \bar{n}_k T^{-2}, \quad A_0 \sim a^2 (\nu)^{1/2}/M, \quad (26)$$

where $\bar{n}_k$ is the average linear density of kinks on the dislocation line. Besides (26), $\chi_\perp^{-1}$ contains one additional term, $\tilde{\chi}_\perp^{-1}$, which is derived directly from the general relation (19) and is a result of the large-angle scattering of electrons in the plane perpendicular to the dislocation axis:

$$\tilde{\chi}_\perp^{-1} = \left\{ m^2 \int d\omega\, d\varepsilon\, dq_z\, dp_z\, d\varphi\, d\varphi'\, w(1-n^0(\varepsilon))\, n^0(\varepsilon-\omega) \times q_x^2 \varepsilon^2 K(\omega; q_x, q_z) \right\} \left\{ \int (p_x^2/m)\, \varepsilon_p^2 (\partial n^0/\partial \varepsilon_p)\, 2\, d^3\mathbf{p} \right\}^{-2} = B n_d T^{-1} - A_0' n_d \bar{n}_k T^{-2}, \quad B \sim a^3, \quad (27)$$

where $A_0' \sim A_0$, $A_0'/A_0 < 1$. Expressions (24) and (25) for $\tilde{K}(\omega,q_x)$ were again used in deriving the result (27). Combining (26) and (27), we find

$$\chi_\perp^{-1} = B n_d T^{-1} + A n_d \bar{n}_k T^{-2}, \quad A = A_0 - A_0'. \quad (28)$$

At a temperature $T \sim 1$ K the ratio of the second term to the first in (28) is of the order $a\bar{n}_k \Theta_D^2/\varepsilon_F T < \nu^{1/2} \sim 10^{-2}$.

In the direction along the dislocation large-angle scattering is absent. The maximum change in the z-component of the electron momentum q_m is determined by the width of the kink along the dislocation line, $q_m \sim l_k^{-1}$. The corresponding term in $\chi_\parallel^{-1}$ also is derived from (19) and is equal to

$$\tilde{\chi}_\parallel^{-1} = \left\{ m^2 \int d\omega\, d\varepsilon\, dq_z\, dp_z\, d\varphi\, d\varphi'\, w[1-n^0(\varepsilon)]\, n^0(\varepsilon-\omega) \times q_z^2 \varepsilon^2 K(\omega; q_x, q_z) \right\} \left\{ \int p_z^2 \varepsilon^2 (\partial n^0/\partial \varepsilon)\, 2\, d\varepsilon\, dp_z\, d\varphi \right\}^{-2}. \quad (29)$$

For an estimate of $\tilde{\chi}_\parallel^{-1}$ we use the definition (18), in which we will consider the kinks motionless. Then we find

$$K_e(\omega; q_x, q_z) \approx 2\pi\delta(\omega) \frac{1}{L} \int dz\, dz' \exp[iq_z(z'-z)] \times \langle\langle \exp\{iq_x[x(z',0) - x(z,0)]\} \rangle\rangle. \quad (30)$$

Writing $x(z,0)$ in the form (23) and carrying out the configurational average, we find

$$K_e(\omega; q_x, q_z) \approx 2\pi\delta(\omega) \int_{-\infty}^{+\infty} \exp[-q_x^2 a^2 z^2 (\bar{n}_k^2 + 2\bar{n}_k \nu^{1/2}/a) - iq_z z]\, dz \approx 2\pi\delta(\omega)\, (\pi/2a\bar{n}_k q_x^2 \nu^{1/2})^{1/2} \exp[-q_z^2/8\nu^{1/2} a \bar{n}_k q_x^2]. \quad (31)$$

Putting (31) into (29), we determine

$$\tilde{\chi}_\parallel^{-1} = \tilde{B} n_d \bar{n}_k T^{-1}, \quad \tilde{B} \sim a^4 \nu^{1/2}. \quad (32)$$

The contribution $\Delta\chi_\parallel^{-1}$ to $\chi_\parallel^{-1}$ due to elastic scattering on dislocations with the density $\tilde{n}_d$, going through an angle θ, where $\theta_m < \theta \ll 1$, can be estimated if we use the result (27):

$$\Delta\chi_\parallel^{-1} \sim B \tilde{n}_d T^{-1} \theta^2.$$

The additional factor θ^2 originates from the small size of the change in the p_z-component of the electron momentum in a single scattering event on an oblique dislocation: $\Delta p_z \sim p_F \theta \ll p_F$, so that the process of changing p_z has a diffusive character. We estimate the relation between the contributions to $\chi_\parallel^{-1}$ found (here p_F, ε_F are the Fermi momentum and Fermi energy):

$$\Delta\chi_\parallel^{-1}/\chi_u^{-1} \sim \theta^2 \varepsilon_F T/\nu^{1/2} \Theta_D^2, \quad \tilde{\chi}_\parallel^{-1}/\chi_u^{-1} \sim \varepsilon_F T \nu^{1/2}/\Theta_D^2.$$

From these relations it follows that in the low-temperature region $T \lesssim 1$ K, where the maximum angle θ between the dislocation lines and the defined direction is small:

$$\theta \ll (\nu^{1/2} \Theta_D^2/\varepsilon_F T)^{1/2} \sim 10^{-1}, \quad (33)$$

the thermal conductivity along the dislocation line is determined by the "diffusive" change in the electron's energy in a range of width $\sim T$, due to inelastic scattering on the deformation potential of the oscillating kinks:

$$\chi_\parallel^{-1} \approx A_0 n_d \bar{n}_k T^{-2}. \quad (34)$$

III. DISCUSSION OF RESULTS

From (28) and (34) it follows that inelastic scattering of electrons from kinks substantially influences the thermal conductivity of a sample when large-angle scattering is weak. The presence of impurities, as well as the dislocations, leads to electron scattering. Let us estimate the sample purity necessary for (34) to hold. The inverse time for energetic relaxation of the electrons upon scattering on the deformation potential of the kinks, τ_ε^{-1}, can be derived from (26) or directly from (16):

$$\tau_\varepsilon^{-1} \sim n_d \bar{n}_k a^3 \Theta_D^2 / T. \tag{35}$$

The elastic relaxation time for electron scattering on impurities is $\tau_i \sim (n_i a^3 \varepsilon_F)^{-1}$, where n_i is the impurity concentration. The requirement $\tau_i^{-1} \ll \tau_\varepsilon^{-1}$ leads to the condition

$$n_i \ll n_d \bar{n}_k \Theta_D^2 / \varepsilon_F T \lesssim 10^{14} \text{ см}^{-3}, \tag{36}$$

where we take $T \sim 1$ K, $\bar{n}_k < l_k^{-1}$ and $n_d < \nu^2 b^{-2}$.

In Refs. 1 and 2 the observation of a dependence $\chi(T) \sim T^n$ ($n = 1.5$–2.7) is reported for the thermal conductivity of pure crystals of copper and silver with recently introduced dislocations in the region of temperatures and values of dislocation density dealt with in the present work, $T \sim 1$–5 K, $n_d \sim 10^4$–10^8 cm^{-2}. However, for a comparison of the experimental situation[1,2] with the model calculation described above and to overcome the important[9] discrepancies between the experimental and calculated values of the transport values due to electron scattering on dislocations, to which simple expressions such as (27) lead, we need information on the angular distribution of dislocations introduced on bending-and-straightening plastic deformation,[1,2] and also information on the spectrum of low-energy excitations arising as a result.

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[1)]The model of closely situated kinks on a dislocation line holds under the condition $n_k \ll l_k^{-1}$.[4]

[2)]In the equations of motion of the kinks used above, the periodic potential contour for movement of a kink along a dislocation line in a metal is not taken account of, since the kink oscillation frequency connected with it is extremely small[8]: $\omega_\parallel \sim \nu^{1/2} \omega_m$.

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