Condensation of Alfvén waves into vortices in an inhomogeneous plasma

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It is shown that in an inhomogeneous plasma exponentially localized vortex tubes may form near intersections of Alfvén and drift branches. Since the plasma is not in thermodynamic equilibrium, such objects have negative energy, i.e., their formation is energetically favorable. In the presence of dissipation, their size and rotation rate increases. They may be observed in laboratory and space plasmas, and give rise to convective mixing of the plasma and to an increase in its thermal conductivity.

1. INTRODUCTION

Lasting fluctuations of large amplitude are possible in media which are not in thermodynamic equilibrium. For example, in a supercooled vapor, moderately large drops of fluid persist indefinitely. An inhomogeneous plasma is likewise a thermodynamically nonequilibrium system. It has been shown in Ref. 1 that a plasma can have waves with "negative energy," the amplitude of which grows in the presence of dissipation. However, in packets of such waves, the rate of dispersive spreading may exceed the rate of growth of their amplitudes. Therefore packets of small dimensions with large dispersion do not develop in the linear approximation. But large packets cannot arise because of magnetic shear and other stabilizing factors. Recently it has been shown that self-localization of wave packets is possible for sufficiently large amplitudes.[2] Nonlinearity may delay dispersive spreading as a result of which particle-like excitations like solitons, vortex tubes, etc., can form.

The present paper shows that Alfvén waves packets of finite amplitude may form solitary vortex tubes which vanish exponentially at infinity. In an inhomogeneous plasma their energy may become negative. In other words, the formation of Alfvén vortices is energetically favored. This effect may be called condensation, in analogy with the formation of fluid drops in a supercooled vapor. The formation of such vortices in a laboratory plasma causes convective mixing of particles across the magnetic field which may be the cause of anomalous thermal conductivity which has been observed.[3] The existence of such vortices in the magnetosphere of the Earth is indicated by observations of electromagnetic discontinuities registered at higher latitudes[4] and experiments on artificial excitation of Alfvén waves of large amplitude with the aid of surface explosions.[5] Note that the exponential decay in the vortices arises from the dispersion of Alfvén waves (analogous to solitons in dispersive media). Without inclusion of dispersion, the vortex vanishes according to a power law. In an inhomogeneous plasma, the vortex energy associated with a power-law decay diverges, and therefore such solutions are of no interest. Besides, vortices with power-law decay may erode under the influence of magnetic shear. It is also important to include dispersion, because it gives rise to coupling between Alfvén modes and drift waves.

2. MODEL EQUATIONS FOR DRIFT-ALFVÉN WAVES

We consider a low-beta plasma: $m_e/m_i \ll \beta \ll 1$ ($\beta \equiv 8\pi p/B_0^2$, where p is the plasma pressure and $B_0^2/8\pi$ is the pressure of the stationary magnetic field), in a magnetic field $\mathbf{B}_0$, directed along the z axis. We assume the plasma to vary as a function of x. We are interested in the case of diffusion of a low-frequency wave packet $\omega/\omega_B \ll 1$ (ω_B is the ion gyrofrequency and ω is the characteristic frequency close to the drift frequency) at a large angle to the magnetic field: $k_\parallel/k_\perp \ll 1$. The phase velocity of the disturbance along $\mathbf{B}_0$ is taken to be larger than the ion thermal velocity and smaller than the electron thermal velocity. The characteristic dimension $\lambda_\perp = 2\pi/k_\perp$ of the packet transverse to the magnetic field lies in the interval $r_s \ll \lambda_\perp \ll \varkappa^{-1}$ ($\varkappa^{-1}$ is the characteristic dimension of the inhomogeneous plasma, $r_s = c_s/\omega_B$, $c_s^2 = T_e/m_i$). In such perturbations, the oscillations in the magnetic field along the z axis may be neglected.[6] The transverse perturbations of the magnetic field are manifested in the z-component of the vector potential: $\mathbf{B}_\perp = [\nabla A_z, \boldsymbol{\zeta}]$, where $\boldsymbol{\zeta}$ is the unit vector along $\mathbf{B}_0$. The electric field parallel to the magnetic field $\mathbf{B} = \mathbf{B}_\perp + \mathbf{B}_0$ yields the relation

$$E_\parallel = -(\mathbf{B}\nabla\varphi)/B_0 - c^{-1}\partial A_z/\partial t, \tag{1}$$

while the transverse component $\mathbf{E}$ may be assumed to be electrostatic and equal to $\mathbf{E}_\perp = -\nabla_\perp\varphi$. In order to derive the nonlinear equations for Alfvén waves in an inhomogeneous plasma, we use the current continuity equation:

$$\operatorname{div}\mathbf{j} = \operatorname{div}\mathbf{j}_\perp + \operatorname{div}\mathbf{j}_\parallel = 0. \tag{2}$$

In our approximation

$$\operatorname{div}\mathbf{j}_\parallel = -(c/4\pi)(\mathbf{B}\nabla)\Delta A_z/B_0. \tag{3}$$

Only ions contribute to the divergence of the transverse flow.[6] In contrast to what happens in ordinary waves, in the case of drift-Alfvén waves the ion energy contains not only a contribution from the electric drift $\mathbf{v}_E = c[\mathbf{E}\mathbf{B}]/B_0^2$, but also a diamagnetic contribution $\mathbf{v}_L = [\boldsymbol{\zeta}\nabla p_i]/nm_i\omega_B$ (p_i is

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the ion pressure and n is the number density of the plasma). Then we have

$$\operatorname{div}\mathbf{j}_\perp=\frac{en_0}{\omega_B}\operatorname{div}\left\{\left(\frac{\partial}{\partial t}+\mathbf{v}_E\nabla\right)[\boldsymbol{\xi},\mathbf{v}_E+\mathbf{v}_L]\right\},\tag{4}$$

where $-e$ is the charge of the electron, n_0 the constant part of the density of the plasma.

In the equation for the longitudinal motion of the electrons, one may neglect the inertia, whereupon it reduces to a balance between the longitudinal electric field and the pressure gradient of the electrons:

$$eE_\parallel+(\mathbf{B}\nabla p_e)/n_0B_0=0.\tag{5}$$

Incorporating the electron pressure p_e leads to dispersion. The contribution of the density oscillations to (4) and (5) may be neglected, since it yields negligibly small corrections of order ω/ω_B, while in (4) and (5) there are nonlinear corrections which do not contain this parameter. This system of equations must be supplemented by the continuity equation for the electrons:

$$(\partial/\partial t+\mathbf{v}_E\nabla)n-\operatorname{div}(\mathbf{j}_\parallel/e)=0,\tag{6}$$

and the equation for the ion pressure:

$$(\partial/\partial t+\mathbf{v}_E\nabla)p_i=0.\tag{7}$$

It will be convenient to proceed to nondimensional variables:

$$\begin{gathered}\omega_Bt\to t,\quad r_s^{-1}\mathbf{r}_\perp\to\mathbf{r}_\perp,\quad z\omega_B/c_A\to z,\quad e\varphi/T_e\to\Phi,\\ eA_zc_A/cT_e\to A,\quad n/n_0\to N+1,\quad p_i/n_0T_e\to P+T_{i0}/T_e\end{gathered}\tag{8}$$

(c_A is the velocity of the Alfvén waves). In these variables, the above equations assume the form[7]

$$d\Delta\Phi/dt=-dJ/dz-\operatorname{div}\{P,\nabla\Phi\},\tag{9}$$

$$\partial A/\partial t=d(N-\Phi)/dz,\quad J=\Delta A,\tag{10}$$

$$dN/dt+dJ/dz=0,\quad dP/dt=0,\tag{11}$$

where

$$\{P,\Phi\}\equiv\frac{\partial P}{\partial x}\frac{\partial\Phi}{\partial y}-\frac{\partial P}{\partial y}\frac{\partial\Phi}{\partial x},\qquad\Delta\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},$$
$$d/dt\equiv\partial/\partial t+\{\Phi,\ldots\},\quad d/dz\equiv\partial/\partial z-\{A,\ldots\}.$$

The system (9)–(11) describes a packet localized in the neighborhood of the $x=0$ plane. Far from this plane we assume that

$$N\to\varkappa x,\quad P\to\varkappa_Px,\quad A\to A_0(x),\quad\varphi\to\varphi_0(x).$$

In what follows, it will be assumed that $\varkappa$ and $\varkappa_P$ are constant, while A_0 and φ_0 are zero, which corresponds to the absence of magnetic shear and an ambipolar electric field. One may take into consideration the curvature of the steady magnetic field by adding a term of the form $g\partial(N+P)/\partial y$ (g is the dimensionless effective force of gravity) on the left-hand side of Eq. (9). This effect has been taken into account in Refs. 8 and 9 for the case of flutelike perturbations. In the linear approximation, the system (9)–(11) yields the dispersion relation of drift-Alfvén waves[10]:

$$\left(1+\frac{\varkappa}{\omega}\right)\left[1-\frac{\omega^2}{k_z^2}\left(1-\frac{\varkappa_P}{\omega}\right)\right]+k_\perp^2\left(1-\frac{\varkappa_P}{\omega}\right)=0.$$

It is clear that the effect of the ions on the dispersion is neglected in (9)–(11). It will be convenient to introduce the perturbed part of the density $N_1=N-\varkappa x$. Taking into consideration that

$$\frac{\partial}{\partial t}\int d^3x\,N_1=0$$

it is readily verified that the system (9)–(11) has the following energy integral:

$$E=\int d^3x[(\nabla_\perp\Phi)^2+(\nabla_\perp A)^2+N_1^2+2\varkappa xN_1].\tag{12}$$

The last term in (12) describes "energy exchange" between the wave and the plasma. It exists only in an inhomogeneous plasma ($\varkappa\neq0$). It will be shown below that this term leads to negative vortex energy.

3. VORTEX TUBES

We will show that Eqs. (9)–(11) have a steady, two-dimensional, exponentially localized solution. Let all quantities depend only on x and $\eta=y+\alpha z-ut$, where α is the angle of inclination of the vortex to the magnetic field, u its velocity of propagation. Then the system (9)–(11) assumes the form

$$\begin{gathered}\Delta\Phi=[f_e(\tilde A)+f_\Phi(\tilde\Phi)]/(1+b_P),\\ \Delta A=f_A(\tilde A)-f_e'(\tilde A)\tilde\Phi,\quad\tilde\Phi\equiv\Phi-ux,\\ P=b_P\tilde\Phi,\quad N=\tilde\Phi+f_e(\tilde A),\quad\tilde A\equiv A-\alpha x,\end{gathered}\tag{13}$$

where b_P is a constant, f an arbitrary function and the prime denotes differentiation with respect to the argument. Analogously to Refs. 7 and 11, we represent f in the form of piecewise linear functions:

$$f_e=b_e\tilde A,\quad f_A=b_A\tilde A,\quad f_\Phi=b_\Phi\tilde\Phi.\tag{14}$$

Then one has instead of (13)

$$\Delta\Phi=B(b_\Phi\tilde\Phi+b_e\tilde A),\quad B\equiv(1+b_P)^{-1},\tag{15}$$

$$\Delta A=b_A\tilde A-b_e\tilde\Phi,\tag{16}$$

$$P=b_P\tilde\Phi,\quad N=\tilde\Phi+b_e\tilde A.\tag{17}$$

In polar coordinates $r^2=x^2+\eta^2$, $\tan\theta=\eta/x$, we introduce a circle of radius r_0. In contrast to Ref. 7, we assume the coefficients b_P and b_Φ to be constant everywhere, while b_e and b_A assume different constant values inside and outside the circle. Then, as will be shown below, one obtains a solution which decays exponentially to zero, while in Ref. 7 the decay was according to a power law.

Outside the circle $r=r_0$, the solution has the form

$$\begin{gathered}\Phi=e_1K_1(sr)\cos\theta,\\ A=a_1K_1(sr)\cos\theta,\quad r\geqslant r_0,\end{gathered}\tag{18}$$

where K_1 is the modified Bessel function of the second kind. Since $P\to\varkappa_Px$, $N\to\varkappa x$ at infinity, one obtains from (17) the values of the coefficients:

$$\begin{gathered}b_e=-(\varkappa+u)/\alpha,\quad r>r_0;\\ b_P=-\varkappa_P/u,\quad 0<r<\infty.\end{gathered}\tag{19}$$

It follows from the condition that Φ and A be localized and from (15) and (16) that

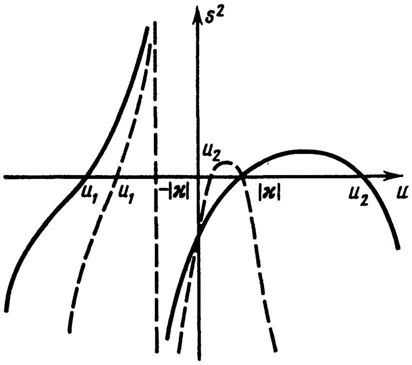

FIG. 1. Dependence of the square of s, the coefficient of exponential decay in vortices, on the velocity u. The solid line corresponds to the case $\alpha^2 > 2\varkappa^2$, the hatched line to $\alpha^2 < 2\varkappa^2$ ($\varkappa = \varkappa_P$).

$$b_A = b_e u/\alpha = -u(\varkappa + u)/\alpha^2, \qquad r > r_0;$$

$$b_\Phi = 1 + \varkappa/u, \quad 0 < r < \infty. \tag{20}$$

Substituting (18) into (15), (16) and taking into consideration (19) and (20), we obtain an algebraic system for determining the coefficients e_1 and a_1. The condition that this system be soluble is

$$s^2 = (\varkappa + u)[1/(u - \varkappa_P) - u/\alpha^2]. \tag{21}$$

The region of admissible values of u is determined from the condition that s^2 be positive (cf. Fig. 1). We look for solutions of the system (15), (16) in the region $r \leqslant r_0$ in the form

$$\begin{gathered} \Phi = [e_2 J_1(k_1 r) + e_3 J_1(k_2 r)]\cos\theta, \\ \tilde{A} = [a_2 J_1(k_1 r) + a_3 J_1(k_2 r)]\cos\theta, \end{gathered} \tag{22}$$

where J_1 is the Bessel function of the first kind. Substituting (22) into (15) and (16), one arrives at a homogeneous system of equations the condition of solubility of which determines the values of the coefficients b_e and b_A inside the circle in terms of the coefficients $k_{1,2}$, b_Φ and b_P:

$$Bb_e^2 = (k_1^2 + Bb_\Phi)(k_2^2 + Bb_\Phi), \qquad r < r_0, \tag{23}$$

$$b_A = -k_1^2 - k_2^2 - Bb_\Phi, \qquad r < r_0, \tag{24}$$

where

$$B = (1 - \varkappa_P/u)^{-1}, \quad Bb_\Phi = (u + \varkappa)/(u - \varkappa_P), \quad 0 < r < \infty.$$

The solutions (18) and (22) must be matched on the boundary $r = r_0$. The expressions (18) and (22) may be assumed to be solutions, if on substitution into the system (9)–(11) or (15)–(17), all terms at every point are finite. Discontinuities in the form of finite steps are admissible. For this purpose, it is sufficient, for a given choice of coefficients in the solutions of the equations (15)–(17), to demand continuity of Φ and $\nabla_\perp \Phi$. However, in view of the discontinuity of b_e and b_A, one must impose on A the more stringent condition $\tilde{A} = 0$ at $r = r_0$ together with the requirement that ∇A be continuous. Then, as is readily seen from Eq. (15), $\Delta\Phi$ is continuous, and the longitudinal flow, which is proportional to ΔA, has a discontinuity at the boundary, which is admissible. These conditions determine the coefficients in the solutions (18) and (22):

$$a_1 = \alpha r_0/K_1(\mu), \qquad e_1 = \alpha a_1/(u - \varkappa_P), \qquad r \geqslant r_0, \tag{25}$$

$$e_2 = \frac{k_1^2 + b_A}{b_e} a_2, \qquad e_3 = -\frac{k_2^2 + b_A}{b_e} \frac{J_1(l_1)}{J_1(l_2)} a_2, \qquad r \leqslant r_0, \tag{26}$$

$$a_3 = -\frac{J_1(l_1)}{J_1(l_2)} a_2, \qquad a_2 = \frac{\alpha^2 \mu^2 b_e}{r_0 (k_1^2 - k_2^2)(\varkappa + u) J_1(l_1)}, \qquad r \leqslant r_0, \tag{27}$$

where

$$\mu \equiv s r_0, \quad l_{1,2} \equiv k_{1,2} r_0.$$

The quantities r_0, u, α, and $k_{1,2}$ remain arbitrary for the time being.

The conditions for matching ∇A and $\nabla\Phi$ and the condition $\tilde{A} = 0$ on the inside of the boundary. Yield the dispersion relations with for $k_{1,2}$, which differ from the dispersion relations of geostrophic vortices[1]:

$$\frac{a_2 J_1(l_1)}{b_e}\{l_1^2 g(l_1) - l_2^2 g(l_2) - b_A r_0^2 [g(l_1) - g(l_2)]\}$$

$$= \frac{\alpha^2 r_0^3}{u - \varkappa_P}(1 - h) - u r_0^3, \tag{28}$$

$$a_2 J_1(l_1)[g(l_1) - g(l_2)] = -\alpha r_0 h, \tag{29}$$

where

$$h(\mu) = \mu K_2(\mu)/K_1(\mu), \quad g(l) = 1 - l J_2(l)/J_1(l). \tag{30}$$

Taking into consideration the dispersion relations (28) and (29), only the parameters r_0, u, and α remain undetermined. According to the initial approximations, one has the necessary conditions $\varkappa \sim \varkappa_P \sim u \sim \alpha \ll 1$. The case $\alpha = u$ corresponds to propagation of a vortex along $\mathbf{B}_0$ with the Alfvén velocity. From the condition that the dimension of the vortex be much larger than r_s, we obtain the inequalities $r_0 \gg 1$, $k_{1,2}, s \ll 1$. We note that α (the propagation in the nondimensional space (8) corresponds in dimensional space to the angle $\alpha c_s/c_A$.

We now prove that the dispersion relations (28)–(30) have solutions. We note that when l_1 and l_2 lie near a zero of J_1, these equations simplify greatly. It is known from the theory of Bessel functions that

$$g(l) = 1 + 2l^2 \sum_{n=1}^{\infty} \frac{1}{l^2 - \gamma_n^2}, \tag{31}$$

where γ_n satisfies $J_1(\gamma_n) = 0$. With the aid of this formula and the expansion of K_1 for small values of its argument ($\mu \ll 1$), one easily obtains solutions of the dispersion relations (28) and (29) in the vicinity of the poles $|l_1 - \gamma_1| \ll \gamma_1 \approx 3.8$, $|l_2 - \gamma_2| \ll \gamma_2 \approx 7.0$, $|u - u_1| \ll |u_1|$:

$$\frac{\gamma_1}{l_1 - \gamma_1} \approx \frac{2\alpha^2}{(u - u_1)(u_1 - u_2)} \times \left\{1 + \left[\frac{u_1 u_2}{\alpha^2} \frac{\gamma_2^2 u_2 - r_0^2 (u_1 + \varkappa)}{-\gamma_1^2 u_2 + r_0^2 (u_1 + \varkappa)}\right]^{1/2}\right\},$$

$$\frac{\gamma_2}{l_2 - \gamma_2} \approx \frac{2\alpha^2}{(u - u_1)(u_1 - u_2)} \times \left\{1 + \left[\frac{u_1 u_2}{\alpha^2} \frac{-\gamma_1^2 u_2 + r_0^2 (u_1 + \varkappa)}{\gamma_2^2 u_2 - r_0^2 (u_1 + \varkappa)}\right]^{1/2}\right\}, \tag{32}$$

where (cf. Fig. 1)

$$u_{1,2}={}^1/_2[\varkappa\pm(\varkappa^2+4\alpha^2)^{1/2}],\quad -\varkappa<u<u_1,$$

with $\alpha^2>2\varkappa^2$. This means that the vortices under consideration are Alfvénian (their propagation velocity along the z axis is less than the Alfvén speed). The presence of roots establishes the existence of Alfvén vortex tubes which decay exponentially. All parameters of the tubes are expressible in terms of α, u, and r_0. The tube constitutes a dipole with positive and negative vorticities.

With the aid of the relations derived above one can readily estimate that

$$\Phi\sim A\sim u_1 r_0, \tag{33}$$

whence we find that the characteristic rotation velocity v and the perturbation of the magnetic field in the vortex δB in dimensional quantities are of order

$$v/c_A\sim\delta B/B_0\sim\beta^{1/2}r_s/a, \tag{34}$$

where a is the length of the gradient dimensional quantities.

4. ENERGY OF VORTEX TUBES

We will now explain how the energy of the vortex tube (12) depends on its parameters. Since the tube is infinite in length, it is natural to speak of energy per unit length. According to Eq. (12), the energy per unit length of the tube has the form

$$W=\int dx\,d\eta[-A\Delta A-\Phi\Delta\Phi+N_1^2+2\varkappa x N_1]. \tag{35}$$

Substituting the solution (18) and (22) into (35) and executing the integration, one finds W as a function of the vortex parameters:

$$\begin{aligned}W=&-2\pi r_0^4\frac{u^2}{\mu^2}h(1-b_\Phi)\left(\frac{\alpha^2}{u^2}B-b_\Phi\right)+\frac{\pi\alpha^2r_0^4}{2}\\&\times\left[\frac{h^2}{\mu^2}-\frac{2h}{\mu^2}-1\right]\left[\frac{u^2}{\alpha^2}\left(\frac{\alpha^2}{u^2}B-b_\Phi\right)^2\right.\\&\left.+b_\Phi\left(1-B\frac{\alpha^2}{u^2}\right)\left(\frac{u^2}{\alpha^2}+B^2\right)\right]\\&+{}^1/_2\pi a_2^2r_0^2J_1^2(l_1)[F(l_1,l_2)+F(l_2,l_1)]\\&+\pi a_2r_0J_1(l_1)[M(l_1,l_2)-M(l_2,l_1)]\\&+\pi a_2^2r_0^2J_1^2(l_1)L(l_1,l_2),\end{aligned} \tag{36}$$

where

$$\begin{aligned}F(l_1,l_2)=&\left\{1-\frac{2}{l_1^2}[1-g(l_1)]+\frac{1}{l_1^2}[1-g(l_1)]^2\right\}\\&\times\left\{B\frac{l_2^2+r_0^2Bb_\Phi}{l_1^2+r_0^2Bb_\Phi}\left[\left(\frac{1}{B}\left(\frac{l_1^2}{r_0^2}+Bb_\Phi\right)-1\right)^2+\frac{l_1^2}{r_0^2}\right]+\frac{l_1^2}{r_0^2}\right\},\end{aligned} \tag{37}$$

$$M(l_1,\ l_2)=[1-g(l_1)][\alpha-u(l_2^2+r_0^2Bb_\Phi)/b_e r_0^2], \tag{38}$$

$$\begin{aligned}L(l_1,l_2)=&\frac{g(l_2)-g(l_1)}{l_1^2-l_2^2}\left[-\frac{2}{Br_0^4}(l_1^2+r_0^2Bb_\Phi)(l_2^2+r_0^2Bb_\Phi)\right.\\&\left.+(l_1^2+l_2^2)\frac{1-B}{r_0^2}-2B(1-2b_\Phi)\right].\end{aligned} \tag{39}$$

In the case of the solution (32), i.e., near the zeros of J_1, this expression may be expanded in powers of the quantity $\mu\ll1$. Then (36), with consideration of (21) and (32), reduces to the form

$$W=-4\pi r_0^2\varkappa^2/s^2,\qquad s^2\ll1. \tag{40}$$

It is seen that in this case the vortex energy in an inhomogeneous plasma ($\varkappa\neq0$) is negative. Thus, if there exists dissipation in the system, $|W|$ will grow. Hence it follows that in an inhomogeneous plasma Alfvén waves are energetically favored to condense in a structure of the form of a vortex tube.

5. AMPLIFICATION OF VORTICES UNDER THE ACTION OF DISSIPATION

We consider the behavior of vortex tubes under the influence of dissipation. Magnetic viscosity is the simplest form of dissipation; it is caused by finite conductivity. In order to take this effect into account, Eq. (10) will be rewritten in the form,[6]

$$\partial A/\partial t+d(\Phi-N)/dz=\nu\Delta A, \tag{41}$$

where ν is the coefficient of magnetic viscosity (generally speaking, in ν one must include the effect of electrons trapped in the vortex.[12] Then the energy E is no longer constant and varies with the time according to the formula

$$\partial E/\partial t=-2\nu\int d^3x(\Delta A)^2. \tag{42}$$

In the dissipation approximation one may substitute in (42) the solution in the form of the vortex found above, assuming that its parameters r_0, u and α are slowly varying functions of the time. It is seen from (42) that, because E is negative [cf. (39) and (40)], under the action of dissipation the vortex becomes stronger (its dimension and rate of rotation grow).

6. CONCLUSION

Alfvén waves are the most widely occurring mode of oscillation in laboratory and space plasma with $\beta>m_e/m_i$. They play an important role in particle acceleration processes in the earth's magnetosphere, turbulent mixing of plasma, etc. When dispersion is taken into consideration, this mode couples to the drift mode, which, as a result of the inhomogeneity, leads to wave-plasma interaction. As a consequence, the free energy of the plasma associated with the inhomogeneity, changes under the influence of dissipation into vortex motion. In regions of where modes cross, finite ion Larmor radius effects may be neglected since they do not influence the coupling, and one must only take into account the influence of the finite longitudinal electric field. In Ref. 7, equations are derived which make treat such effects. With the aid of these equations, it has been shown above that Alfvén waves become exponentially localized vortex tubes. The present study demonstrates that their energy may become negative in an inhomogeneous plasma. Therefore their formation is favored energetically, somewhat like condensation. Such vortices may exist and grow in a plasma which is linearly stable. They may develop out of strong fluctuations. Such fluctuations arise during HF heating, in explosive instability, during particle injection, etc. Consequently, it must be expected that large accumulations of such tubes may occur in a plasma. Their dimension is only restricted by magnetic shear. In view of their finite length, they may induce convective mixing of plasma. This may explain observed high thermal conductivity and diffusion in plasmas, significantly in excess of classical values.

We note that solutions in the form of Alfvén vortices have been studied previously.[13–16] However, in our view, these solutions are impermissible because they are not adequately matched to the boundaries. In view of this, the Jacobian $\{A,\Delta A\}$ in these papers becomes infinite at the boundary, which is incompatible with the initial approximations.

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