Integral equation for electron-electron scattering. Energy loss to optical phonons

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The kernel $W(\varepsilon,\varepsilon')$ for the collision integral due to electron-electron scattering is calculated with allowance for the static screening in the case of small disequilibrium. The kernel is used in the integral kinetic equation for the emission of optical phonons due to thermalized distribution. The range of validity of the integral equation turns out to be substantially larger than in the Landau-Fokker-Planck approximation.

I. INTRODUCTION

A study of electron-electron scattering in two-dimensional quantum wells[1] has revealed that the collision integral corresponding to this scattering cannot be reduced to the Landau-Focker-Planck (LFP) form. Another approach was developed, based on a calculation of the scattering probability and a solution of an integral equation. The integral-equation method is applied in the present paper to a three-dimensional gas. If the static screening is weak and the minimum energy transfer in electron scattering is small compared with other scales of the distribution function, the integral equation can be reduced to the LFP approximation. At the same time, this equation is just as suitable for the case of strong screening, for which electron-electron scattering has heretofore not been studied. The solution of the integral equation defines more precisely the limits of applicability of the LFP approximation.

It is assumed in the calculation of the electron-electron scattering probability that the Coulomb interaction is screened at impact distances of the order of the Debye radius r_D. The probability is calculated in the Born approximation. It describes the scattering of a test electron by the main group of electrons that have a Maxwellian distribution. The expression for $W(\varepsilon,\varepsilon')$ differs from that used in Refs. 2 and 3. It turns out, in addition, that an estimate of the minimum energy $\Delta\varepsilon_{\min}$ transferred in *ee* scattering in connection with scattering turns out to be different compared with the expression for $\Delta\varepsilon_{\min}$.[3–6] The Coulomb logarithm is also different from the logarithm used in Refs. 3–10.

The superiority of the integral equation is most fully manifested in solutions of threshold problems. We consider therefore the problem of optical phonons emitted in a thermalized distribution. Electrons with temperature $T_e \ll \hbar\Omega_0$ propel a small fraction of the electrons from the passive region $\varepsilon < \hbar\Omega_0$ through the threshold $\varepsilon = \hbar\Omega_0$, tending to make up for the depletion, due to phonon emission, of the distribution in the active region $\varepsilon > \hbar\Omega_0$ ($\hbar\Omega_0$ is the optical-phonon energy without allowance for dispersion). For a $2D$ gas, the kinetic equation was solved by the Wiener-Hopf method.[1] In the case of a $3D$ gas the probability of emission of an optical phonon near the threshold, which is proportional to the density of the states of the electron near the bottom of the band $\tau^{-1}(\varepsilon) \sim \varepsilon^{1/2}$, depends on the energy [$\tau(\varepsilon)$ is constant for a $2D$ gas]. The solution of the equation becomes therefore more complicated. The distribution functions $f(\varepsilon)$ at all energies of the heat power W lost to the lattice are calculated.

The problem described was investigated earlier,[2–5] and the LFP approximation was used in Refs. 4 and 5. To be able to use the LFP approximation, it was assumed in Refs. 4 and 5 that the minimum jump $\Delta\varepsilon_{\min}$ along the energy axis is small compared with the characteristic scales of $f(\varepsilon)$, such as T_e and the depth of penetration into the active region. The case when $\Delta\varepsilon_{\min}$ exceeds the electron penetration depth into the active region was investigated in Ref. 3.

To solve the problem, a more precise criterion was used for the applicability of the LFP approximation. This criterion is different for $f(\varepsilon)$ and Q. A different result is obtained for Q in the case treated in Ref. 3, where the power loss was calculated without solving the kinetic equation, by using a model assumption concerning the form of $f(\varepsilon)$.

We have thus investigated the competition between electron-electron scattering and emission of optical phonons in that part of the concentration-temperature plane where the pair-collision approximation can be used for *ee* scattering. Numerical estimates have been obtained for a number of semiconductors.

II. ELECTRON-ELECTRON SCATTERING

A. The probability $W(\varepsilon,\varepsilon')$

We calculated the probability of transition of a test electron from an energy ε to an energy ε' via scattering by the Maxwell-distributed bulk of the electrons. The law of interaction of two colliding electrons with distance $R = r_1 - r_2$ between them is given by

$$V(R) = (e^2/R)e^{-aR}, \tag{1}$$

where $a = r_D^{-1}$ is the modulus of the Debye wave vector; an effective electron charge is assumed, with allowance for the static dielectric constant. The matrix element of the scattering of two electrons ($\mathbf{k}, \mathbf{p} \to \mathbf{k}', \mathbf{p}'$) is easily calculated in the plane-wave Born approximation and is equal to

$$M_{\mathbf{k},\mathbf{p}\to\mathbf{k}',\mathbf{p}'} = \frac{4\pi e^4}{L^3}\frac{1}{q^2+a^2}\delta_{\mathbf{k}'+\mathbf{p}'-\mathbf{k}-\mathbf{p}}, \tag{2}$$

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where L^3 is the normalization volume, and $\mathbf{q} = \mathbf{k}' - \mathbf{k} = \mathbf{p} - \mathbf{p}'$ is the momentum transfer. Using it to write down the four-particle transition probability $W(\mathbf{k}, \mathbf{p} \to \mathbf{k}', \mathbf{p}')$ and averaging the latter over $\mathbf{p}$ and $\mathbf{p}'$ with the Maxwellian distribution function $f_{T_e}(p)$, we obtain the probability in momentum space:

$$W(\mathbf{k}, \mathbf{k}') = \frac{(8\pi)^{1/2}\pi n e^4}{\hbar^2 L^3}\left(\frac{m}{T_e}\right)^{1/2}\frac{1}{q(q^2+a^2)^2}\exp(-|\mathbf{k}_\parallel'|^2/2mT_e). \quad (3)$$

Here $k_\parallel' = (\mathbf{k}'\mathbf{q})/|\mathbf{q}|$. The kinetic equation for the distribution function along the energy axis contains the probability (3) averaged over the angles between the vectors $\mathbf{k}$ and $\mathbf{k}'$. Denoting the cosine of this angle by x, we get

$$W(\varepsilon, \varepsilon') = W_0 K(y, y'), \quad W_0 = 2\pi^{5/2} n e^4 \hbar^3/m^2 T_e^3 L^3,$$

$$K(y, y') = \int_{-1}^{1} \frac{dx}{\tilde{q}(\tilde{q}^2+y_0)^2}\exp\{-[y'-x(yy')^{1/2}]^2/\tilde{q}^2\}, \quad (4)$$

$$\tilde{q}^2 = y + y' - 2x(yy')^{1/2}.$$

We have introduced energies normalized to the temperature: $y = \varepsilon/T_e$, $y' = \varepsilon'/T_e$, $y_0 = \hbar^2 a^2/2mT_e$. The quantity y_0, which has the meaning of dimensionless energy with wave vector equal to the Debye vector, assumes the role of the screening parameter.

It will be seen from the solution of the kinetic equation that energy transfers $|\varepsilon - \varepsilon'| \ll \varepsilon, \varepsilon', \hbar\Omega_0$ are important near the threshold $\varepsilon = \hbar\Omega_0$. We shall use this circumstance and the condition $\varepsilon, \varepsilon' \gg T_e$ to simplify the kernel $K(y, y')$. For small transfers $\omega = (\varepsilon' - \varepsilon)/T_e$ the kernel depends only on $\gamma = \hbar\Omega_0/T_e \approx y \approx y'$ and ω. Small transfers correspond to scattering through small angles: $1 - x \ll 1$. The integral (4) can be expressed in terms of the variable $\xi = 2\gamma(1 - x)$ in the form

$$K(\omega) = \frac{e^{-\omega/2}}{2\gamma}\int_0^\infty \frac{d\xi}{\xi^{1/2}(\xi+y_0)^2}\exp\left\{-\frac{\omega^2}{4\xi} - \frac{\xi}{4}\right\}, \quad (5)$$

where the upper limit $\xi = 4\gamma$ is replaced by infinity in view of the rapid convergence of the integral. For arbitrary y_0 and ω (which are naturally small compared with γ by virtue of the transition from (4) to (5), although the integrand is not formally dependent on γ) the integral (5) is equal to

$$K(\omega) = \frac{\pi e^{-\omega/2}}{4y_0\gamma}\left[\frac{2y_0+\omega^2-y_0^2}{2y_0^{3/2}}\exp\left(\frac{\omega^2}{4y_0}+\frac{y_0}{4}\right)\right.$$
$$\left.\times\operatorname{erfc}\left(\frac{y_0+|\omega|}{2y_0^{1/2}}\right) + \frac{y_0-|\omega|}{\pi^{1/2}y_0}\exp(-|\omega|/2)\right], \quad (6)$$

where we have for small values of the screening parameter

$$K(\omega) = \frac{\pi^{1/2}}{\gamma}\frac{2+|\omega|}{|\omega|^3}\exp\left(\frac{-(\omega+|\omega|)}{2}\right), \quad (7)$$
$$|\omega| \gg y_0^{1/2}, \quad y_0 \ll 1 \text{ or } |\omega| \gg y_0, \quad y_0 \gg 1,$$

and for large ones

$$K(\omega) = \frac{\pi^{1/2}}{\gamma}\frac{1}{(y_0+|\omega|)^2}\exp\left(\frac{-(\omega+|\omega|)}{2}\right), \quad y_0 \gg 1. \quad (8)$$

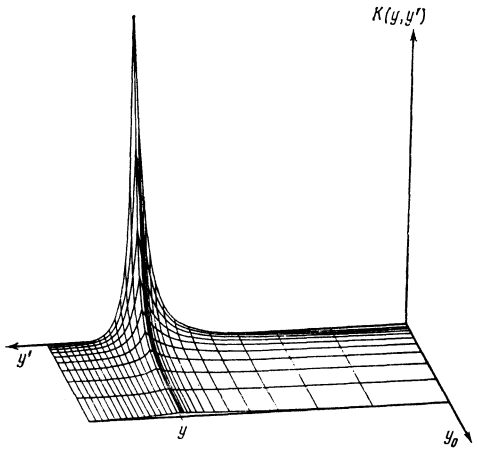

FIG. 1. Plots of the function $K(y,y')$ at fixed $y = 10$ for $0 \ll y' \ll 13$ in the range $0.1 \leqslant y_0 \leqslant 2$ of parameter y_0.

The regions where the asymptotes (7) and (8) are valid overlap at $|\omega| \gg y_0 \gtrsim 1$. Figure 1 shows the kernel $K(y, y')$ as given by Eq. (4). At small transfers $|\omega| \ll 1$ the screening becomes substantial at $|\omega| \lesssim y_0^{1/2}$. This important circumstance will be discussed in detail below. At large transfers $|\omega| \gg 1$ the screening comes into play when $|\omega| \lesssim y_0$. If the screening is insignificant, the representation (7) is valid for all transfers ($|\omega| \gtrsim 1$). We note also that

$$K(0) = \begin{cases} \pi^{1/2}/\gamma y_0^2, & y_0 \gg 1 \\ \pi/4\gamma y_0^{3/2}, & y_0 \ll 1 \end{cases}, \quad (9)$$

as follows from (6). The asymptotic form (7) turns out to be different from Eq. (23) of Ref. 3.

B. Minimum and maximum transfers. The Coulomb logarithm

The kernel (7) is precisely the one used to calculate the diffusion and dynamic-friction coefficients in the LFP approximation. For the diffusion coefficient we have

$$D(\varepsilon) = \frac{1}{2}\int d\varepsilon'\, g(\varepsilon')(\varepsilon-\varepsilon')^2 W(\varepsilon, \varepsilon'). \quad (10)$$

The usual integration limits in this equation are 0 and ∞, assuming rapid convergence of the integral. For Coulomb scattering with kernel (7), the integral (10) diverges not only logarithmically at small $|\varepsilon - \varepsilon'| \ll T_e$, but also in power-law fashion at large $|\varepsilon - \varepsilon'| \gg T_e$. In the FP approximation, however, the distribution function can be expanded only in powers of small transfers. Having among other properties a scale on the order of the electron temperature, the distribution function cannot be expanded in a series if $|\varepsilon - \varepsilon'| \gtrsim T_e$. Thus, besides the minimum energy transfer, which is determined by the screening, the integral (10) has also a maximum energy transfer $\Delta\varepsilon_{\max} \sim T_e$ that limits the applicability of the FP method itself.

We discuss now the minimum energy transfer. The following argument can be found in the literature.[6,3-5] Let the electron acquire as a result of scattering a momentum $\mathbf{q} = \mathbf{p}' - \mathbf{p}$. Then

$$2m\Delta\varepsilon = (\mathbf{p}+\mathbf{q})^2 - p^2 = 2p^2[1-(\mathbf{pp}'/p^2)] = 2p^2[1-\cos\varphi],$$

where φ is the angle between the vectors $\mathbf{p}$ and $\mathbf{p}'$. The last transition contains an error, since $|\mathbf{p}| \neq |\mathbf{p}'|$. The formula $\Delta\varepsilon_{\min} = \varepsilon\varphi_{\min}^2$ does not hold for small-angle scattering. The minimum energy transfer is obtained from the scattering kernel and amounts to

$$\Delta\varepsilon_{min} \sim \varepsilon y_0^{1/2}. \tag{11}$$

In the case of the transfer (11) the expressions (7) and (9) are of the same order of magnitude. This, on the average, is precisely the energy transfer when the scattering alters the momentum by $q \sim \hbar/r_D$ in the Born approximation.

In the Born approximation we have finally for the Coulomb logarithm

$$\Lambda_{ee} = \ln(\Delta\varepsilon_{max}/\Delta\varepsilon_{min}) = {}^1/_2 \ln(T_e^2\varepsilon_B/ne^6) = \ln(1/y_0^{1/2}) \tag{12}$$

in lieu of the usual $\frac{1}{2}\ln(\varepsilon\varepsilon_B T_e/ne^6)$. Just such a factor (T_e/ε) distinguishes also the Coulomb logarithm in the classical approximation from that given in the literature.

We note that it is incomparably simpler to derive Eq. (12) in momentum space. The maximum (for the FP approximation) scattering angle is $\varphi_{\max} \sim (T_e/\varepsilon)^{1/2}$, the minimum scattering angle is $\varphi_{\min} \sim q_{\min}/p \sim \hbar(m\varepsilon)^{1/2}/r_D$; the logarithm of their ratio is given by (12). It usually assumed that $\varphi_{\max} \sim 1$ (Ref. 10, p. 211, and elsewhere).

Therefore, if the condition

$$\ln(\varepsilon/T_e) \ll \Lambda_{ee} \tag{13}$$

does not hold for hot electrons, the usual Coulomb logarithm must be replaced by the logarithm (12).

We ultimately have for the diffusion coefficient, in view of (12),

$$D(\varepsilon) = 2^{3/2}\pi ne^4 T_e \Lambda_{ee}/m^{1/2}\varepsilon^{1/2}, \quad \varepsilon \gtrsim T_e. \tag{14}$$

We can use for the integral of the dynamic-friction coefficient

$$A(\varepsilon) = \int d\varepsilon'\, g(\varepsilon')\,(\varepsilon - \varepsilon')\, W(\varepsilon, \varepsilon') \tag{15}$$

a similar reasoning, and as a result we obtain, naturally, $A = D/T_e$.

Energy transfers $|\varepsilon - \varepsilon'| \gg T_e$ are not taken into account in the LFP approximation. The question whether such transfers are significant reduces to an estimate of integrals of the form $C_{ee}^{\pm}$, which make up the electron-electron collision integral $C_{ee} = C_{ee}^{+} + C_{ee}^{-}$, where

$$-C_{ee}^{-} = f(\varepsilon) \int_0^\infty d\varepsilon'\, g(\varepsilon')\, W(\varepsilon, \varepsilon'),$$

$$C_{ee}^{+} = \int_0^\infty d\varepsilon'\, g(\varepsilon') f(\varepsilon')\, W(\varepsilon', \varepsilon). \tag{16}$$

It can be shown that the contribution of the transfers $|\varepsilon - \varepsilon'| \gg T_e$ to the integral C_{ee}^{-} is small, but for the integral C_{ee}^{+} this must be verified separately in each problem after determining $f(\varepsilon)$ in the LFP approximation. We note in addition that if the scale of the distribution function is $\bar{\varepsilon} \ll T_e$, validity of the LFP approximation calls for $\ln(T_e/\bar{\varepsilon}) \ll \ln(T_e/\Delta\varepsilon_{\min}) \equiv \Lambda_{ee}$.

III. KINETIC EQUATION

A. Derivation of equation

Assume that external heating maintains the electron gas at constant low temperature T_e such that $\exp(-\hbar\Omega_0/T_e) \ll 1$. In view of the electron-electron scattering, a small fraction of the electron rises above the threshold $\varepsilon = \hbar\Omega_0$ and emits optical phonons.[3-5] We must calculate the electron distribution function and the power lost to optical phonons

$$Q = \int_0^\infty d\varepsilon\, g(\varepsilon)\, \varepsilon S(\varepsilon), \tag{17}$$

where

$$S(\varepsilon) = -f(\varepsilon)/\tau_0(\varepsilon), \quad 1/\tau_0(\varepsilon) = t^{1/2}\theta(t)/\bar{\tau}_0\gamma^{1/2},$$

$$t = (\varepsilon - \hbar\Omega_0)/T_e. \tag{18}$$

The distribution function satisfies the equation

$$C_{ee}^{+} + C_{ee}^{-} + S = 0, \tag{19}$$

where $C_{ee}^{\pm}$ are given by the integrals (16) of the probability (4) with kernel (6). The square-root dependence of $\tau_0(\varepsilon)$ is determined only by the density of states $g(\varepsilon) \sim \varepsilon^{1/2}$ on the bottom of the band and is valid for either a deformational or a polarizational interaction with the optical phonons.

We have introduced the characteristic scattering time $\bar{\tau}_0$ (Ref. 11). No arrival term will be taken into account for this type of scattering, since we are interested in the distribution function near the threshold $|\varepsilon - \hbar\Omega_0| \ll \hbar\Omega_0$ or $t \ll \tau$, and the lattice temperature T_L satisfies the equation $T_L \ll T_e$. Equation (19) does not conserve energy, so that the loss power is (17).

We transform in (19) to an argument t and to a new function $\varphi(t)$:

$$\varphi(t) = e^{-t} - f(\gamma + t)/f_{Te}(\gamma). \tag{20}$$

The equation takes the form

$$\rho\hat{K}[\varphi(t)] = \theta(t)t^{1/2}[e^{-t} - \varphi(t)], \tag{21}$$

where ρ is the second parameter of the problem (besides y_0):

$$\rho = \bar{\tau}_0/\bar{\tau}_{ee}, \quad 1/\bar{\tau}_{ee} = 2^{1/2}\pi ne^4/m^{1/2}T_e^{3/2},$$

and the operator

$$\hat{K}[\varphi(t)] = \frac{\gamma}{\pi^{1/2}} \int_{-\infty}^{\infty} dt'\, [K(t'-t)\varphi(t') - K(t-t')\varphi(t)]$$

contains the kernel (6). The integration extends over all possible energy transfers t in view of the convergence of the integral at values of t that do not exceed $|t| \sim \max(1, y_0)$.

With an aim at using for (21) the Wiener-Hopf method,[1] we introduce the functions $\varphi^{\pm}(t)$ defined by $\varphi^{\pm}(t) = \varphi(t)\theta(\pm t)$, and take the Fourier transform of (21). As a result we obtain a second integral equation:

$$\rho\bar{K}(k)[\bar{\varphi}^+(k)+\bar{\varphi}^-(k)]=\hat{S}[(1+ik)^{-1}]-\hat{S}[\bar{\varphi}^+(k)],$$

$$\hat{S}[\varphi(k)]=\frac{1}{4\pi^{1/2}i^{1/2}}\int_c dk'(k-k')^{-1/2}\varphi(k'), \tag{22}$$

with the contour drawn such that $\mathrm{Im}k<\mathrm{Im}k'<1$. The "transform" of the kernel is calculated from the equation

$$\bar{K}(k)=-\frac{\gamma}{\pi^{1/2}}\int_{-\infty}^{\infty}dt(e^{ikt}-1)K(t)$$

using the integral representation (5) and is equal to

$$\bar{K}(k)=xe^{y_0x}E_1(y_0x)|_{x=k(k-i)}, \tag{23}$$

where E_1 is the integral exponential function. The asymptotes are of the form

$$\bar{K}(k)=\begin{cases}x\ln(\beta/y_0x), & y_0x\ll 1\\ y_0^{-1}, & y_0x\gg 1\end{cases}, \tag{24}$$

where $(-\ln\beta)=C$ is the Euler constant, $\beta=0.561\ldots$. The functions $\bar{\varphi}^+(k)$ and $\bar{\varphi}^-(k)$ are analytic in the half-planes $\mathrm{Im}k<1$ and $\mathrm{Im}k>0$, respectively.

The plan for the analysis of Eq. (22) is the following (study Fig. 4 simultaneously). We draw in the quadrant $\rho>0, y_0>0$ the lines $\rho=1, \rho=y_0, y_0=1, \rho=y_0^{5/4}\Lambda_{ee}^{-1}$. The region $y_0\ll 1, \ln y_0^{-1}\gg\ln[\rho\Lambda_{ee}]^{-4/5}$ is called the weak-screening region and is investigated in the next subsection B. The region $\rho\ll 1$ is called the region of weak electron-electron scattering and is studied in Subsec. C. The region $y_0\gg 1$ is called the strong screening region and is analyzed in Subsec. D. Finally, the strong electron-electron scattering section $\rho\gg 1$ is not examined separately, since, accurate to small corrections, $f(\varepsilon)=f_{T_e}(\varepsilon)$ in the vicinity of the threshold, as will be seen later from (28) and (29) at $y_0\ll 1$ and $\rho\gg 1$, and from (51) and (52) at $y_0\gg 1$ and $\rho\gg y_0$. A method of solving (22) for large ρ can be borrowed from Ref. 1.

B. Weak screening

We begin with the case $y_0\ll 1$, when moreover $y_0x\approx y_0k^2\ll 1$ or $t\sim k^{-1}\gg y_0^{1/2}$. Consider the corresponding equation:

$$\rho k(k-i)\ln[\beta/y_0k(k-i)][\bar{\varphi}^+(k)+\bar{\varphi}^-(k)]$$
$$=\hat{S}[(1+ik)^{-1}]-\hat{S}[\bar{\varphi}^+(k)]. \tag{25}$$

The logarithmic dependence on k is insignificant if

$$\ln y_0^{-1}\gg\ln k^2. \tag{26}$$

Returning in this case to the variable t, we get from (25) and (26) the usual LFP equation

$$2\rho\Lambda_{ee}[\varphi''+\varphi']=\theta(t)t^{1/2}[e^{-t}-\varphi^+(t)], \tag{27}$$

which was solved in Refs. 4 and 5, where the distribution function

$$f(t)=f_{Te}(t)-(1-b)f_{Te}(0),\quad t<0 \tag{28}$$

was obtained below the threshold, and

$$f(t)=bf_{Te}(0)u(t/\lambda),\quad t>0 \tag{29}$$

above the threshold. Here

$$b=1+\frac{1}{u'(0)}\int_0^{\infty}dx\,x^{1/2}u(x), \tag{30}$$

and the function $u(x)$ satisfies the equation

$$u''+\lambda u'-x^{1/2}u=0,\quad u(\infty)=0,\quad u(0)=1. \tag{31}$$

The parameter λ is connected with ρ:

$$\lambda^{5/2}=2\rho\Lambda_{ee}. \tag{32}$$

The function $u(x)$ and the parameter b were investigated in the Appendix of Ref. 5 [see also Eq. (56)]. These equations describe the distribution function at not too small ρ. Namely, from (26), (29), and (32) we get

$$\ln y_0^{-1}\gg\ln[\rho\Lambda_{ee}]^{-4/5} \tag{33}$$

(in contrast to the criteria in Refs. 4 and 5).

C. Weak electron-electron scattering

A small ρ, Eq. (22) can be simplified by making the substitution

$$\bar{\varphi}^+(k)=(1+ik)^{-1}+\rho\bar{\varphi}^+(k) \tag{34}$$

followed by neglect of the term containing ρ^2:

$$\bar{K}(k)[(1+ik)^{-1}+\bar{\varphi}^-(k)]=\hat{S}[\bar{\varphi}^+(k)]. \tag{35}$$

By virtue of the derivation, Eqs. (27) and (35) should be simultaneously valid at $1\gg\rho\gg y_0^{5/4}/\Lambda_{ee}$. Neglect of the terms $\propto\rho^2$ violates, however, this validity at sufficiently large k; it turns out further that at small y_0 Eq. (35) is valid precisely when (27) is not. For large y_0, Eq. (35) is valid at $k\ll\rho/y_0$ [see (50) and (51)].

To solve (35) we use the fact that the integral in the right-hand side of (35) is an analytic function of k at $\mathrm{Im}\,k<1$. Introducing, in accordance with the Wiener-Hopf method, the functions

$$\bar{K}(k)=K^-(k)/K^+(k),$$

$$K^{\pm}(k)=\exp\left\{\int_{C^{\pm}}\frac{d\zeta}{\zeta-k}\ln\bar{K}(\zeta)\right\}, \tag{36}$$

$$C^{\pm}:\ \mathrm{Im}\,\zeta\gtrless\mathrm{Im}\,k,\quad 0<\mathrm{Im}\,\zeta<1,$$

we get

$$K^+(k)\hat{S}[\bar{\varphi}^+(k)]-K^-(k)\bar{\varphi}^-(k)=K^-(k)(1+ik)^{-1}, \tag{37}$$
$$\hat{S}[\bar{\varphi}^+(k)]=(1+ik)^{-1}K^-(i)/K^+(k), \tag{38}$$

whence

$$\bar{\varphi}^-(k)=(1+ik)^{-1}[K^-(i)/K^-(k)-1]. \tag{39}$$

From the integral equation (38) we obtain [see (34)]

$$\varphi^+(t)=e^{-t}+\rho\frac{K^-(i)}{2\pi t^{1/2}}\int\frac{dk\,e^{ikt}}{1+ik}\frac{1}{K^+(k)},\quad \mathrm{Im}\,k<1, \tag{40}$$

and from (39) we have

$$\varphi^-(t)=\frac{1}{2\pi}\int\frac{dk\,e^{ikt}}{1+ik}\left[\frac{K^-(i)}{K^-(k)}-1\right],\quad \operatorname{Im}k>0. \tag{41}$$

Together with the explicit form of the "transform" of the kernel (23), Eqs. (40) and (41) give the distribution function in accordance with (20). For the case of weak screening, if (26) is valid, the factorization of $K(k)$ is trivial:

$$K^-(k)=2k\Lambda_{ee},\quad K^+(k)=(k-i)^{-1}. \tag{42}$$

In this approximation we obtain for the functions φ and f directly from (40) and (41)

$$\varphi^+(t)=e^{-t},\quad f(t)=0,\quad t>0, \tag{43}$$

$$\varphi^-(t)=1,\quad f(t)=f_{Te}(t)-f_{Te}(0),\quad t<0, \tag{44}$$

which agree with (38) and (29) as $\rho\to 0$ if (56) is taken into account. For large y_0 we get $K^{\pm}=y_0^{\pm 1/2}$ whence we have for the distribution function

$$\varphi^-(t)=0,\quad f(t)=f_{Te}(t),\quad t<0, \tag{45}$$

$$\varphi^+(t)=e^{-t}[1-\rho y_0^{-1}t^{-1/2}],\quad f(t)=\rho y_0^{-1}t^{-1/2}f_{Te}(0)e^{-t},$$

$$t>0,\quad t\gg\rho y_0^{-1}. \tag{46}$$

We refine now Eq. (45), since the result $f(t)=0$ obtained in the logarithmic approximation (42) is not satisfactory. At sufficiently large t, when $|k-i|\ll 1$, we can neglect in (35) the function $\bar{\varphi}^-(k)$, which is regular at $k=i$, against the background $(1+ik)^{-1}$ and put $k=i$ everywhere except for the difference $k-i$ itself. We have

$$\bar{s}[\bar{\varphi}^+(k)]=\ln[iy_0(k-i)/\beta], \tag{47}$$

hence

$$f(t)=f_{Te}(0)\rho t^{-3/2}e^{-t},\quad t\gg t_0. \tag{48}$$

We can determine t_0 by comparing (48) with (39) at ρ, $\lambda\ll 1$. Equation (29) is valid at $t\ll t_0$ and (48) is valid at $t\gg t_0$:

$$t_0=\rho^{2/5}\Lambda_{ee}^{2/5}[\ln\Lambda_{ee}]^{4/5}. \tag{49}$$

Physically, t_0 is the depth of penetration that can be reached into the active medium with equal probability either by diffusion or by a single collision.

D. Strong screening

Equation (22) becomes much simpler in the case of strong screening $y_0\gg 1$. It turns out at the same time that $y_0k(k-i)\gg 1$, since we are interested in the solution at $|t|\lesssim 1$. Using the asymptotic form (24) for the "transform" of the kernel, we obtain

$$(\rho/y_0)[\bar{\varphi}^+(k)+\bar{\varphi}^-(k)]=\bar{s}[(1+ik)^{-1}]-\bar{s}[\bar{\varphi}^+(k)]. \tag{50}$$

Returning to the usual variables, we have an algebraic solution whose solution yields a distribution function

$$f(t)=e^{-t}f_{Te}(0)\,(\rho/y_0)[(\rho/y_0)+t^{1/2}]^{-1},\quad t>0, \tag{51}$$

that depends on one parameter ρ/y_0.

At $t<0$ we obtain in first-order approximation

$$f(t)=f_{Te}(t),\quad t<0. \tag{52}$$

Equations (51) and (52) yield the same distribution functions as Eqs. (45) and (46) in the overlap region $\rho\ll 1$, $y_0\gg 1$.

IV. POWER LOSS

The power loss to optical phonons can be represented in the form of the following integral of the distribution function [see (17), (19), and (20)]:

$$\frac{Q}{Q_0}=1-\frac{2}{\pi^{1/2}}\int_0^\infty dt\,t^{1/2}\varphi^+(t),\quad Q_0=\frac{n\hbar\Omega_0}{\tau_0}\exp\left(-\frac{\hbar\Omega_0}{T_e}\right). \tag{53}$$

In the case of strong electron-electron scattering, the correction $\varphi^+(t)$ is small and we have in first-order approximation

$$Q=Q_0,\quad \rho\gg\max(1,\ y_0). \tag{54}$$

In the case of weak screening when the LFP approximation is valid, the power loss was calculated in Refs. 4 and 5. In our notation,

$$Q/Q_0=4\pi^{-1/2}\rho\Lambda_{ee}(1-b),\quad y_0\ll\min(1,\ \rho^{4/5}\Lambda_{ee}^{4/5}), \tag{55}$$

with

$$b=\lambda(^2/_5)^{1/5}\Gamma(^2/_5)/\Gamma(^3/_5),\quad Q/Q_0=4\pi^{-1/2}\rho\Lambda_{ee}, \tag{56}$$

for $\rho\ll 1$ and

$$1-b=2^{-1}\pi^{1/2}\lambda^{-3/2},\quad Q/Q_0=1 \tag{56a}$$

for $\rho\ll 1$. The correct Coulomb logarithm in (56) is given by (12).

In the case of weak electron-electron scattering it follows from (40) and (53) that

$$Q/Q_0=2\pi^{-1/2}\rho\xi(y_0),\quad \xi(y_0)=K^-(i)/K^+(0),\quad \rho\ll 1. \tag{57}$$

Using the explicit form of the functions $K^{\pm}(k)$, we can represent ξ by the following integral:

$$\xi(y_0)=\exp\left\{\frac{1}{2\pi}\int_0^\infty\frac{d\zeta}{z}\ln[e^{zy_0}E_1(zy_0)]\right\},\quad z=\frac{1+\zeta^2}{4}. \tag{58}$$

A plot of the function $\xi(y_0)$ is shown in Fig. 2. The following asymptotic relations hold [see (24)]:

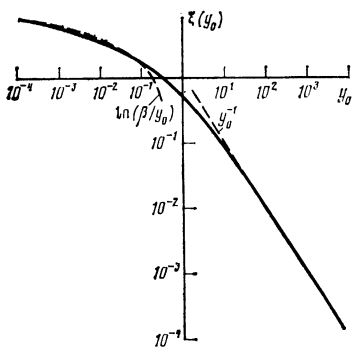

FIG. 2. Plot of the function $\xi(y_0)$ (58). The dashed lines show the asymptotes (59) and (69). Case of the weak electron-electron scattering.

$$\xi(y_0)=\ln(\beta/y_0), \quad y_0\ll 1, \tag{59}$$

$$\xi(y_0)=y_0^{-1}, \quad y_0\gg 1. \tag{60}$$

In the case of strong screening the integral (53) of Eq. (51) takes, if (20) is used, the form

$$Q/Q_0=\eta(\rho/y_0), \quad y_0\gg 1,$$

$$\eta(x)=1-\frac{2}{\pi^{1/2}}\int_0^\infty \frac{d\zeta\,\zeta e^{-\zeta}}{x+\zeta^{1/2}}. \tag{61}$$

This function has the asymptotic form

$$\eta(x)=\begin{cases} 2\pi^{-1/2}x, & x\ll 1 \\ 1-2\pi^{-1/2}x^{-1}, & x\gg 1 \end{cases}. \tag{61a}$$

A plot of the function $\eta(x)$ is shown in Fig. 3.

We have thus investigated the distribution function and the power loss in the entire (ρ, y_0) plane except in the region $\rho\sim 1, y_0\sim 1$, for which Eq. (22) or (21) must be solved exactly. At $\rho\sim 1$ and $y_0\sim 1$, we have, of course, $f(\varepsilon)\sim f_{T_e}(\varepsilon)$, $|\varepsilon-\hbar\Omega_0|\lesssim T_e, Q\sim Q_0$.

V. DISCUSSION

It is useful to map the (ρ, y_0) plane on the (T_e, n) plane of Fig. 4. The latter indicates the regions where the LFP approximation and the integral equation are valid. The coordinates of the point (T_e^*, n^*) determined from the equation $\rho = y_0 = 1$ are

$$T_e^*=\alpha^2\varepsilon_B, \quad n^*=\pi^{-1}\alpha^4 a_B^{-3}, \quad \alpha=\hbar/\bar{\tau}_0\varepsilon_B, \tag{62}$$

where a_B is the Bohr radius.

The physical meaning of the various lines in Fig. 4 is the following. The line $\rho = 1, n\propto T_e^{3/2}$ corresponds to the condition $\tau_0(2\hbar\Omega_0)=\bar{\tau}_{ee}(T_e)$ and indicates the arguments at which the scattering times must be compared. The line $y_0 = 1, n\propto T_e^2$ demarcates the regions where the minimum energy transfer $\Delta\varepsilon_{\min}$ in *ee* scattering is small ($y_0\ll 1$) and large ($y_0\gg 1$) compared with T_e. The line $\rho = y_0$ corresponds to the case when the departure into the passive region at the point $\varepsilon=\hbar\Omega_0+T_e$ proceeds equally rapidly as a result of *ee* scattering and of phonon emission (in the case of strong screening). Finally, the line $y_0=(\rho\Lambda_{ee})^{4/5}$ can be obtained from the condition that $\Delta\varepsilon_{\min}$ be equal to the diffusion depth of penetration into the active region.

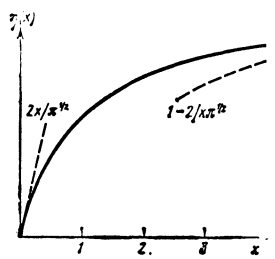

FIG. 3. Plot of the function $\eta(x)$ (61). The dashed lines show the asymptotes (61a). Case of strong screening.

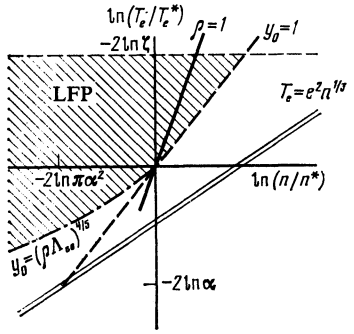

FIG. 4. The region on the (T_e, n) plane where the Landau-Fokker-Planck approximation is valid is hatched and marked by the letters LFP. At the point—$2\ln\zeta$ we have $T_e=\hbar\Omega_0$, at the point—$2\ln\alpha$ we have $T_e=\varepsilon_B$. The case $\varepsilon<T_e^*<\hbar\Omega_0$ is chosen. See Table I.

If condition (33) is met, the distribution function above the threshold is calculated in the LFP approximation at scales $t\ll t_0$ (49), while for scales $t\gg t_0$ the distribution is described by Eq. (48), which does not contain a Coulomb logarithm. The scale of the function (29) is the diffusion penetration depth into the active region in the presence of optical-phonon emission[5] with diffusion coefficient (14). In temperature units, this depth $t\sim\lambda\sim\rho^{2/5}\Lambda_{ee}^{2/5}$ (32) is somewhat less than t_0 (49). Since the probability of returning an electron to the passive region via electron-electron scatter is low at $t\gg\lambda$, the distribution (48) can be obtained directly from the relation $C_{ee}^{+}+S=0$, where C_{ee}^{+} was calculated with the function (44).

When y_0 increases, the condition (33) is violated, and at $y_0^{1/2}\gtrsim\rho^{2/5}\Lambda_{ee}^{2/5}$ the distribution in the active region is the following. Over scales $t\gg y_0^{1/2}$ with $y_0\ll 1$, the function $f(6)$ changes little by virtue of the properties of the kernel (9). At larger scales, $t\gg y_0^{1/2}$, the distribution is described as before by Eq. (48). Each electron is then propelled into the active region by a single collision and emits immediately a phonon. No Coulomb logarithm appears. The distribution is given in the passive region by (44), or by the more accurate (28) if (33) is valid.

Equation (55) for the power loss is equivalent at $\rho\ll 1$ and $\ln y_0^{-1}\gg 1$ to Eq. (57) with ξ from (59). It turns out that the LFP power loss is always valid at

$$\Lambda_{ee}\gg 1, \tag{63}$$

i.e., even when the distribution in the active region is not described by the LFP approximation. The point is that at $b = 0$ the power loss is the maximum possible for the given ρ. It can be obtained by solving in the passive region Eq. (27) with the boundary condition $\varphi(0) = 1$, i.e., $f(\varepsilon=\hbar\Omega_0)=0$. The solution described by Eq. (44) or (28) with $b = 0$ has a scale $t\sim 1$, from which (63) follows as the condition for the validity of the approximation.

The LFP equation can thus be used in threshold problems to find the distribution function in the passive region, as well as the power loss (i.e., the flux through the threshold) when condition (63) is satisfied. It can also be used to find the main contribution made to the distribution by active region when the more stringent condition (33) is met. Over

TABLE I.

	$\hbar\Omega_0$, K	ε_B, K	T_e^*, K	n^*, cm^{-3}	α
GaAs	420	65	51	$2\cdot10^{16}$	0.88
CdSe	307	216	480	$3\cdot10^{18}$	1.49
InSb	284	7	22	$1\cdot10^{15}$	1.80
InAs	350	17	76	$3\cdot10^{16}$	2.11
GaSb	347	31	10	$1\cdot10^{15}$	0.58

1) *Note*: The initial data were taken from Ref. 13 for CdSe and from Ref. 11 for the remaining materials.

scales substantially larger than the depth of diffusional penetration into the active region, the distribution function can be taken only from the integral equation.

In strong-screening case $y_0 \gg 1$ it follows from (52) that the distribution in the passive region does not differ from an equilibrium distribution in the first-order approximation in y_0^{-1}. A difference does appear in the active region at $t \gtrsim \rho/y_0$. This is clear also from qualitative considerations. The lifetime of an electron of energy ε relative to electron-electron scattering is given by

$$\frac{1}{\tau_{ee}} = \int_0^\infty d\varepsilon'\, g(\varepsilon')\, W(\varepsilon, \varepsilon') = \frac{1}{\bar\tau_{ee} y_0 \gamma^{1/2}}, \quad y_0 \gg 1$$

and is independent of energy. The phonon emission time (19) becomes equal to τ_{ee} at $t = \rho/y_0$. If $\rho/y_0 \gg 1$, a Maxwellian function can be substituted in the integral (17) and (54) is obtained; in the opposite case one must use the solution (51), from which (61) follows.

It was shown in the course of the solution of the kinetic equation that the distribution function above the threshold at a scale $\varepsilon - \hbar\Omega_0 \lesssim T_e$ is Maxwellian if

$$\rho \gg \max(1, y_0). \tag{64}$$

On the other hand, it is proposed in Ref. 12 to solve this problem by comparing the expressions (in our notation)

$$\left(\frac{dE}{dt}\right)_{ee} = \int_0^\varepsilon d\varepsilon'\, g(\varepsilon')\,(\varepsilon - \varepsilon')\, W(\varepsilon, \varepsilon'), \tag{65}$$

$$(dE/dt)_0 = \hbar\Omega_0/\tau_0(\varepsilon). \tag{66}$$

The explicit form for the integral (65) turns out to be quite unwieldy, but its asymptotes have the simple form

$$\left(\frac{dE}{dt}\right)_{ee} = \begin{cases} (T_e/\bar\tau_{ee})\,(\pi/4y_0\gamma), & y_0 \ll 1 \\ (T_e/\bar\tau_{ee})\,(1/y_0^2\gamma^{1/2}), & y_0 \gg 1 \end{cases}.$$

Calculating (66) at $(\varepsilon - \hbar\Omega_0) \sim T_e$ and comparing with (65), we see that $(dE/dt)_{ee} \gg (dE/dt)_0$ if

$$\rho \gg \begin{cases} \gamma y_0^{1/2}, & y_0 \ll 1 \\ \gamma y_0^2, & y_0 \gg 1 \end{cases}.$$

The criterion (64) above therefore does not agree with the criterion in Ref. 12.

We discuss now the region where the theory is valid. The problem of the interaction of a hot test electron with the surrounding gas was reduced here to pair collisions. It is known that this electron-electron scattering mechanism prevails over excitation of collective oscillations of the plasma is sufficiently ideal, i.e., the sphere r_D^2 contains many electrons.[6,8,10] The necessary condition is therefore

$$e^2 n^{1/3} \ll T_e. \tag{67}$$

This condition is quite stringent for semiconductors. Thus, a plasma at the point (T_e^*, n^*) is ideal if $\alpha \gg 1$ for $\alpha = \hbar/\bar\tau_O \varepsilon_B$. In binary semiconductors we usually have $\alpha \gg 1$, if $\bar\tau_O$ is taken to mean the characteristic time of emission of a polarization optical phonon

$$\hbar/\bar\tau_{PO} = \zeta(\varepsilon_B \hbar\Omega_0)^{1/2}, \quad \zeta = 2(\varkappa_0/\varkappa_\infty - 1), \tag{68}$$

where $\varkappa_0$ and $\varkappa_\infty$ are the static and dynamic dielectric constants. We can then write (see Fig. 4)

$$T_e^* = \zeta^2 \hbar\Omega_0 = \alpha^2 \varepsilon_B, \quad \alpha = \zeta(\hbar\Omega_0/\varepsilon_B)^{1/2}. \tag{69}$$

The numerical values for a number of semiconductors are listed in Table I.

We note finally that the employed Born approximation is valid if $\hbar\Omega_0 \gg \varepsilon_B$.

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