On the domain structure of two-dimensional ferromagnets

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It is shown that in two-dimensional ferromagnets a special type of nonperiodic domain structure, characterized by an anomalous susceptibility $m \propto \ln^{-\nu}(H_0/H)$, is possible.

1. Usually, the domain structure of ferromagnets in conditions of equilibrium possesses a definite period. For a sufficiently large sample thickness L, a certain complicated branched domain structure, with period $a \propto L^{2/3}$ (Refs. 1–3) is realized.[1)]

It is customary to assume that with decrease of the thickness the $L^{2/3}$ law is replaced by an $L^{1/2}$ law (for $L \gg a$). In fact, however, this is not so. For a small anisotropy constant ($\beta \ll 4\pi$), a candidate for a structure with $a \propto L^{1/2}$ could be the Landau-Lifshitz structure[4] (Fig. 1a), but, as Lifshitz showed,[5] it is unstable at $L_c = 64\delta$, $a_c = 16\delta$ (formulas (32) in Ref. 5 with $\beta \to 0$). By making use of the ideas of Lifshitz,[5] it is not difficult to verify that the structure is absolutely unstable against the appearance of small wedges (see Fig. 1c). For simplicity, we shall assume the wedge boundary to be planar. For a small wedge angle (2ψ), it follows from the condition of closure of the magnetic flux that the angle of deviation of the magnetization in the region ABCO is equal to 2ψ, and $\sphericalangle ABC = \psi$. The change of the energy is determined primarily by two contributions: a loss – the energy $2\sigma h$ of the wedge boundaries, where $\sigma = \beta\delta M^2$ is the energy per unit are of the boundary[4,3] and M is the magnitude of the magnetization, and a gain $-2\beta M^2 h^2 \psi$ associated with the change of direction of the magnetization in the shaded regions. Thus, when the width of the wedge at the base ($2\psi h$) is greater than 2δ, i.e., in all cases when the width of the domains is appreciably greater than δ and the macroscopic problem has meaning, the formation of wedges is favored.

Privorotskiĭ[2] proposed a structure ($a \propto L^{1/2}$) with an energy lower than that of the Landau-Lifshitz structure. However, as in Ref. 4, in determining the emergent surface structure he minimized only the anisotropy energy and did not take into account the energy of the magnetic field or the energy associated with the nonuniformity. Let us assume, however, that the nonuniformity energy need not be taken into account, i.e., the characteristic distances over which the direction of the magnetization changes substantially are much greater than δ. Then, in order that the angle of deviation of the magnetization be appreciable, fields $H \sim \beta M$ are necessary, and the quantity $\mathbf{M}\cdot\mathbf{H} \sim \beta M^2 > 0$. By virtue of the Maxwell equations in the absence of an external field, we have

$$\int \mathbf{BH}\, dV = 0$$

(the integration is over all space) or

$$-4\pi \int \mathbf{MH}\, dV = \int H^2\, dV \propto \beta^2;$$

consequently, since there are regions with dimensions of order a in which $\mathbf{M}\cdot\mathbf{H} \sim \beta M^2$ is positive, there should also be regions of the same size in which $\mathbf{M}\cdot\mathbf{H} \sim \beta M^2$ is negative. In these regions the magnetization direction (against the field) corresponds to a metastable state, but, as was established by Privorotskiĭ,[6] in a field $\sim \beta M$ the size of a critical nucleus (for $\beta \ll 4\pi$) is of the order of δ. Consequently, it is not possible to disregard the nonuniformity energy and field energy.

We recall here that for $\beta < 4\pi$ a domain structure does not exist at all in wafers that are too thin, and appears continuously at $L = L_k = \pi\delta$ (Ref. 7) ($\beta \ll 4\pi$), while the $L^{2/3}$ law is observed[1] up to small thicknesses, when $a \sim L$.

For a large anisotropy constant $\beta \gg 4\pi$, in a narrow range of thicknesses $\beta\delta \lesssim L \lesssim 15\beta\delta$, the Kittel structure[8] is realized[1] (Fig. 1b). At $L = L_c = 15\beta\delta$ (then $a \cong 0.2L$), the stability of the planar shape of the interdomain boundaries in the neighborhood of their emergence at the surface is lost, and for large thicknesses one observes[1] a complicated emergent structure in which the period of the domains in the depth of the sample is $a \propto L^{2/3}$. On the other hand, for $L \cong \beta\delta$ one observes a hexagonal structure of cylindrical domains.[9] The formula $a \cong 0.77(\beta\delta L)^{1/2}$ obtained for the period of the structure by Kittel[8] in the limit $L \gg a$ also works quite well in the above-indicated region, where $1 \lesssim L/a \lesssim 5$. In fact, the exact energy density of the Kittel structure for an arbitrary ratio a/L is equal to

$$\sigma L a^{-1} + 16\pi^{-2} M^2 a \sum_{n=0}^{\infty} (2n+1)^{-3}\left[1 - e^{-(2n+1)\pi L/a}\right] \qquad (1)$$

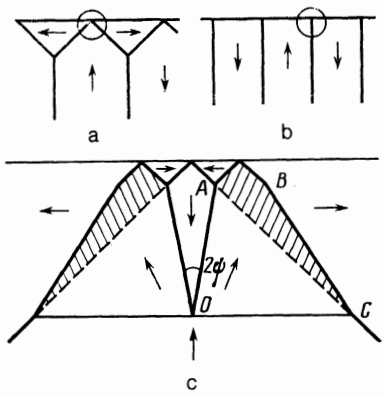

FIG. 1.

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(cf. the problem in Sec. 44 of Ref. 3). Retaining here the first correction for $a \ll L$, it is not difficult to convince oneself that the error in the Kittel formula for $L/a = 1$ is about 5%, and for $L/a = 2$ is already only 0.5%.

In the present paper we shall investigate the domain structure of extremely thin ($L \ll \beta\delta$) wafers for ferromagnets with $\beta > 4\pi$, in which a uniform state with the magnetization along the anisotropy axis (perpendicular to the plane of the wafer) is stable under small deviations. It is found that in such a ferromagnet a fundamentally new nonperiodic domain structure is possible.

2. In formula (1) we take the limit $a \gg L$:

$$\sigma L a^{-1} + 2\pi M^2 L - 4L^2 M^2 a^{-1} \ln(a/L). \qquad (2)$$

Here the second term is the magnetic-energy density of the uniform state (then, the field inside the wafer is $H = -4\pi M$), and the third term gives (with logarithmic accuracy) the magnetic-energy gain associated with the appearance of domains. The minimum of (2) corresponds to the domain width

$$a \sim L \exp(\beta\delta/4L), \quad \beta\delta \gg L. \qquad (3)$$

For further investigation of this essentially two-dimensional situation it is convenient to represent the energy as follows. We note that in the case when the period is appreciably greater than the thickness of the wafer, the energy of the magnetic dipole-dipole interaction reduces to the integral

$$U = \tfrac{1}{2} \iint |\mathbf{r}_1 - \mathbf{r}_2|^{-3} m(\mathbf{r}_1)\, m(\mathbf{r}_2)\, dS_1\, dS_2.$$

Here $|\mathbf{m}| = m_0 = ML$ is the magnetization density per unit area of the wafer ($\mathbf{m} \perp \mathbf{r}_1 - \mathbf{r}_2$). Integrating this expression twice by parts and omitting the formally divergent integral determining the energy of the uniform state [the second term in (2)], we obtain

$$U = \frac{-1}{2} \int\int |\mathbf{r}_1 - \mathbf{r}_2|^{-1} \frac{\partial \mathbf{m}_1}{\partial r_{1i}} \frac{\partial \mathbf{m}_2}{\partial r_{2i}} dS_1\, dS_2.$$

The gradients of the magnetization are nonzero only on the boundaries between the domains, and therefore

$$U = -2m_0^2 \iint |\mathbf{r}_1 - \mathbf{r}_2|^{-1} d\mathbf{l}_1\, d\mathbf{l}_2, \qquad (4)$$

where the integration is taken over lines on the boundaries, the vector $d\mathbf{l}$ having a direction such that, e.g., on the right the magnetization points downward.

By making use of the representation (4), we shall find the energy density of the above-considered [formulas (2), (3)] striped structure in an external field perpendicular to the plane of the wafer [cf. Ref. 10, formula (4)]:

$$\frac{2\alpha}{a} - \frac{8m_0^2}{a} \ln\left(\frac{a \sin \pi c}{\pi\Delta}\right) - (1-2c) m_0 H. \qquad (5)$$

Here c is the concentration of domains in which the direction of the magnetization is against the field H, $\alpha = \sigma L$, and a is the period of the striped structure (for $H = 0$ it is twice the width of one domain). The integral that diverges logarithmically at short distances is cut off at a length Δ which, obviously, is of the order of L if $L > \delta$ or of the order of δ if $L < \delta$.

Minimizing the energy (5) with respect to a and c, we find the following expressions for the period and the magnetization $M = (1-2c)m_0$ of the structure:

$$a = a_0 [1 - (H/H_k)^2]^{-1/2}, \quad M = m_0 \arcsin(H/H_k), \qquad (6)$$

where

$$H_k = (4m_0/e\Delta) \exp(-\alpha/4m_0^2), \quad a_0 = \pi e \Delta \exp(\alpha/4m_0^2), \qquad (7)$$

where e is the base of the natural logarithms.

We now show that the striped structure is unstable. For this, for $H = 0$ we shall find the change of energy upon a small deviation (of the form $y = f\cos kx$) of one of the boundaries for a fixed rectilinear arrangement of all the others. The intrinsic energy of the chosen boundary will change, in the approximation quadratic in f, by the amount

$$\int dx \left\{ \frac{\alpha}{2} \left(\frac{dy}{dx}\right)^2 + 2m_0^2 \int dx' \left[\frac{[y(x) - y(x')]^2}{2|x-x'|^3} - \frac{1}{|x-x'|} \frac{dy(x)}{dx} \frac{dy(x')}{dx'} \right] \right\}$$
$$= f^2 \left[\frac{\alpha k^2}{4} + m_0^2 k^2 \ln\left(\frac{\mathbf{C} k \Delta}{2}\right) \right] \int dx, \qquad (8)$$

where $\mathbf{C} \approx 0.577$ is the Euler constant. The energy of the interaction with the other boundaries will change by the amount

$$8m_0^2 \sum_{n=1}^{\infty} (-1)^{n+1} \iint dx\, dx'\, y^2 \left\{ \frac{3}{2} a^2 n^2 [(x-x')^2 + a^2 n^2]^{-5/2} - [(x-x')^2 + a^2 n^2]^{-3/2} \right\} = \frac{\pi^2 m_0^2 f^2}{3a^2} \int dx. \qquad (9)$$

We shall express the quantity Δ in terms of $a = a_0$ from (7) and substitute the result into (8), when, for the total energy change (8) + (9), we obtain

$$(\pi^2 m_0^2 f^2/3a^2) [1 + 3(ka)^2 \ln(\mathbf{C} k a / 2\pi e)] \qquad (10)$$

per unit length of the boundary. For this result, in (5) and (8) the calculations were performed to within the constants under the logarithm that arise in the integration over distances large in comparison with Δ. The energy (10) is negative in a certain range of wave vectors.

Thus, it is necessary to consider certain more-complicated structures. In principle, there are two essentially different structures. First, the real structure can retain the basic property of the striped structure—the symmetry τR: translation by a half-period with time reversal (i.e., $\mathbf{m} \to -\mathbf{m}$). By virtue of this symmetry the average magnetization of the structure is then equal to zero. Secondly, a structure of the bubble-lattice type (Fig. 2) is possible, and is observed when $L \sim \beta\delta$ (Ref. 9). A feature of this structure is the absence of a symmetry-based exclusion of the existence of an average magnetization $m_1 = \lambda m_0$ ($\lambda < 1$). Upon time reversal this structure is transformed into another with the opposite direction of the magnetization, and in this way the initial situation (with degenerate states $\pm m_0$) is repeated with a new magnetization value $\pm m_1$. Therefore, a superstructure

must necessarily appear—the system stratifies into regions with opposite values $\pm m_1$. Here again we have two possibilities: The superstructure is either of the striped type, or of the lattice type. Because of the anisotropy that arises in the energy of the boundaries between the states $\pm m_1$ on account of the particular symmetry of the bubble lattice, even a striped structure with straight boundaries can turn out to be stable. We shall assume, however, that, again, a superlattice of bubbles turns out to be favored. The characteristic length scale of such a superlattice is of the order of

$$a_1 \sim a_0 \exp(\mu\alpha/4\lambda^2 m_0^2),$$

where the parameter μ determines the average energy[2] $\alpha_1 = \mu\alpha$ of the boundary between the states $\pm m_1$.

Next, it is clear from scaling considerations that if $\mu/\lambda^2 > 1$ (this is the most interesting case), the procedure for the systematic construction of superstructures is in no way limited, and at the n-th step we shall have $m_n = \lambda m_{n-1} = \lambda^n m_0$, $\alpha_n = \mu^n \alpha$, and

$$a_n = \varkappa a_{n-1} \exp\left(\frac{\mu^n \alpha}{4\lambda^{2n} m_0^2}\right) = a_0 \varkappa^n \exp\left\{\frac{\alpha}{4m_0^2} \frac{(\mu/\lambda^2)^n - 1}{1-\lambda^2/\mu}\right\},$$

where $\varkappa$ is a certain constant.

We shall elucidate the behavior of the considered structure in an external field. Beginning with large fields, we note that a domain structure is absent in fields above a certain $H_0 \sim H_k$ (7) but appears at the point at which the energy

$$\varepsilon(R, H) = 8\pi m_0^2 R \ln(8R/e^2\Delta) + 2\pi R\alpha + 2\pi R^2 m_0 H \qquad (11)$$

of a circular domain of radius R and with magnetization opposed to the field becomes negative. Solving the system of equations $\varepsilon(R,H) = 0$ and $\partial\varepsilon/\partial R = 0$, we find

$$H_0 = (8/e^2) H_k > H_k, \quad R_0 = (e^3\Delta/8) \exp(\alpha/4m_0^2). \qquad (12)$$

In fields a little smaller than H_0, a lattice of domains with radius practically equal to R_0 should arise. Since the domains repel each other as magnetic dipoles ($\propto r^{-3}$), and the energy of the domains is linear in the field, i.e., $\varepsilon \propto (H - H_0)$, the lattice period a becomes infinite in accordance with the law $a \propto (H_0 - H)^{-1/3}$. From symmetry considerations, a hexagonal and a square lattice are singled out. The square lattice with an r^{-3} interaction law is found, however, to be unstable. We note that the energy of the square lattice is greater than that of the hexagonal lattice by about 1% in total.

Thermodynamically, the phase transition considered is undoubtedly a second-order phase transition. The kinetics of this transition upon decrease of the field should have at least two distinctive stages. In the first stage, nuclei (domains with reversed magnetization) are formed, and the situation is fully analogous to the nucleation stage in first-order phase transitions. In the second stage, the interaction between the domains should lead to the formation of the lattice. This stage and the kinetics of the reverse transition with increase of the field require special investigation.

Upon further decrease of the field the period of the structure becomes of the order of R_0, and in a field $H_1 \ll H_0$, when the magnetization of the structure is $M = m_1 = \lambda m_0$, a superlattice of domains of radius R_1 and with magnetization $m_1 = -\lambda m_0$ arises. The quantities R_1 and H_1 are obtained from R_0 and H_0 by the replacements $\lambda m_0 \to m_0$ and $\mu\alpha \to \alpha$:

$$H_1 \sim \lambda H_0 \exp(-\mu\alpha/4\lambda^2 m_0^2), \quad R_1 \sim R_0 \exp(\mu\alpha/4\lambda^2 m_0^2).$$

The process of the appearance of superlattices with decrease of the field will then be repeated, so that, when the magnetization of the structure at the nth step becomes equal to $\lambda^n m_0$ in the field

$$H_n \infty H_0 \exp\left\{-\frac{\alpha}{4m_0^2} \frac{(\mu/\lambda^2)^n}{1-(\lambda^2/\mu)}\right\} \qquad (13)$$

(here we have kept the leading term in the exponential as $n \to \infty$), a new superlattice appears. Using (13) to express the index n, we obtain the following dependence of the magnetization $M = \lambda^n m_0 \operatorname{sign} H$ on the magnetic field in the limit $H \to 0$:

$$M = m_0 \operatorname{sign} H \left\{\frac{4m_0^2(1-\lambda^2/\mu)}{\alpha} \ln\frac{H_0}{H}\right\}^{-\nu}, \qquad (14)$$

where the index ν is equal to

$$\nu = (2 - \ln\mu/\ln\lambda)^{-1}.$$

We note that a completely analogous pattern of critical behavior should be observed, instead of a first-order phase transition, on the surface of crystals and liquids, where, because of the striction[11] and electrocapillary[10] effects, we have the same logarithmic behavior of the energy of the interphase boundaries as above. The role of the external field in these cases is played by the temperature, and the characteristic analogous to the magnetization is the polarization P of the surface. Then

$$P(T) - P(T_c) \infty (P_1 - P_2) \operatorname{sign}(T - T_c) [\ln(T_c/|T - T_c|)]^{-\nu},$$

where P_1 and P_2 are the polarizations of the surface in the coexisting states 1 and 2 (see Refs. 10 and 11).

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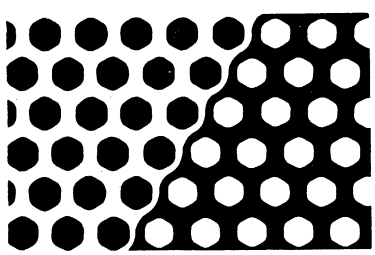

FIG. 2.

[1)]It is scarcely possible to calculate the coefficient in this formula, since the observed pattern[1] does not reduce to a two-dimensional problem, as is usually assumed,[2] and, moreover, it is difficult to take into account the nonuniformity of the field outside the sample over distance of the order of a period. This nonuniformity should be determined by the competition between the nonuniformity energy and the energy of the domain-emer-

gence structure, i.e., must make an important contribution.

[2)]The quantities λ and μ, like the lattice parameters, can hardly be found analytically.

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