Zero-phonon friction mechanism for linear defects in quantum crystals

S. E. Korshunov

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR
(Submitted 15 November 1985)
Zh. Eksp. Teor. Fiz. **90**, 2118–2136 (June 1986)

A study is made of the dynamical properties of a one-dimensional quantum-mechanical solid-on-solid (SOS) model used to describe linear defects in quantum solids (steps, edge dislocations). The case considered is one in which the relationship among the parameters corresponds to a delocalized state of the defect at zero temperature. It is shown that the motion of the defect as a whole inevitably involves an energy dissipation that can be described phenomenologically as the onset of a frictional force. At a finite temperature one is dealing with a viscous (proportional to the velocity) friction, and the coefficient of friction has a power-law dependence on temperature. The exponent of the power law is determined by the relationship of the model parameters. The frictional force in question is not due to the interaction of the linear defect with the phonon gas of the medium but to the discreteness of the atomic structure of the crystal. The conclusions reached in this study also apply to the quantum XY model, to the isomorphic quantum SOS model, and to a model describing a regular chain of Josephson junctions.

1. INTRODUCTION

As was first noted by Andreev and Parshin,[1] a step on the surface of a quantum crystal at zero temperature can be in two different states: atomically smooth (localized) and atomically rough (delocalized). In the atomically smooth state the jogs on the step are present only in paired form, as quantum fluctuations. The formation of a free (unpaired) jog involves the expenditure of a finite amount of energy. The amplitude of the fluctuations of the step with respect to some fixed position has a finite value, so that the translational symmetry in the direction perpendicular to the step is broken. Upon an increase in the quantum fluctuations, a phase transition to the atomically rough state can occur,[1] restoring this symmetry. This state is a conglomerate of delocalized jogs, so that the addition of one jog (say, by a change of boundary conditions) no longer requires a finite expenditure of energy. The square of the zero-point vibrations in this case diverges logarithmically with the length of the step, and to leading order the correlation functions are of the same form as for a free string moving in a plane.

The present study is intended to elucidate the differences between the properties of the atomically rough step and those of a free string. Our primary focus is on the question of whether it is possible to have a nondissipative motion of the step as a whole; this question is examined both at zero and at finite temperatures (of course, we will always be talking about a step on a face that is found in an atomically smooth state). It is shown that the macroscopic motion of a step found in an atomically rough state at zero temperature is always dissipative, and expressions are found for the velocity and temperature dependence of the force of friction.

If discussion is limited to jogs of minimum size, the Hamiltonian describing the step can be reduced to the Hamiltonian of a chain of spin-1 quantum spins.[2] For studying the questions of interest to us, however, it will be more convenient to use a one-dimensional version of the quantum solid-on-solid (SOS) model, which was introduced in Refs. 3 and 4 and studied in detail in Ref. 5. This model, which is the natural quantum-mechanical generalization of the classical discrete Gaussian model,[6,7] is used in the two-dimensional case to describe interfaces and planar defects in quantum crystals.[3–5] In the one-dimensional case its Hamiltonian can be written

$$\hat{H}=\sum_j\left[\frac{J}{2}(n_j-n_{j+1})^2-\frac{Y}{2}(\hat{a}_j^{+}+\hat{a}_j^{-})\right]. \tag{1}$$

Here the integer variables n_j represent the distance of the step from some fixed position (see Fig. 1), while the operators $\hat{a}_j^{+}$ and $\hat{a}_j^{-}$, respectively, describe the change in the configuration of the step upon the tunneling transition of an atom from the liquid to the crystal and the change due to the inverse process. The only nonzero matrix elements of the operators $\hat{a}_j^{+}$ and $\hat{a}_j^{-}$ are $\langle n_j+1|\hat{a}_j^{+}|n_j\rangle=\langle n_j-1|\hat{a}_j^{-}|n_j\rangle=1$.

The first term in braces in Eq. (1) represents the energy of a jog on the step and depends on the size of the jog. Of course, it would be more realistic to use a dependence of the form $J|n_j-n_{j+1}|$, but that would preclude the use of the

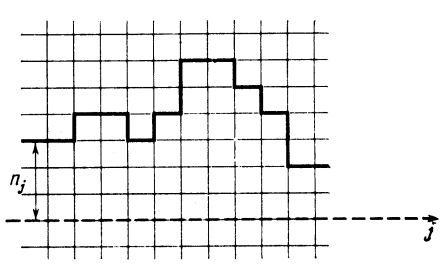

FIG. 1. Schematic diagram of a step on the basal plane of a crystal with a simple cubic lattice (view from above). In the SOS approximation it is assumed that the configuration of the step can be specified by a set of integer variables n_j which give the distance of the step from some fixed position.

 0038-5646/86/061242-10$04.00

analytical methods of study available to us. A numerical study[8] of two-dimensional classical SOS models, which are analogous in their properties to the one-dimensional quantum SOS models, shows that the transition to the atomically rough state occurs at parameter values such that the differences between the values of n_j at adjacent sites are almost everywhere equal to 0 or ± 1, so that the dependence of the potential energy on the this difference at large values of this difference is in certain respects unimportant. Furthermore, a renormalization-group analysis of the two-dimensional SOS models shows that on the transition to large scales the interaction becomes close to Gaussian even if the "bare" interaction is of a different form.[9]

Let us assume that Hamiltonian (1) is supplemented by the cyclic boundary condition

$$n_0 = n_N \tag{2}$$

(in the thermodynamic limit $N \to \infty$). Hamiltonian (1) is invariant with respect to the simultaneous shift of all the variables n_j.

An edge dislocation in a quantum crystal is an object that is very similar in its properties to a step on the surface of such a crystal. Its motion in the slip plane can also be described schematically with the aid of Hamiltonian (1). In this case the variables n_j should be taken to mean the number of the Peierls valley in which the given segment of the dislocation is located. In the rest of this paper we shall for the sake of definiteness discuss only the step.

By introducing the phase variables φ_j, which are the Hamiltonian conjugates of the n_j, we can rewrite (1) in the form[3-5]

$$\hat{H} = \sum_j \left[\frac{J}{2} (\hat{n}_j - \hat{n}_{j+1})^2 - Y \cos \varphi_j \right], \tag{3}$$

where $\hat{n}_j \equiv -i(\partial/\partial\varphi_j)$; the φ representation turns out to be considerably more convenient for analysis of the case of interest here, viz., $\varkappa = (Y/J)^{1/2} \gg 1$.

Hamiltonian (3) admits a transformation to yet another equivalent formulation, with an independent domain of application. If we make the change of variables

$$\varphi_j = \varphi'_{j-1} - \varphi'_j, \tag{4}$$

then the first term in the Lagrangian

$$L(\varphi, n) = \sum_j \dot{\varphi}_j n_j - H(\varphi, n) \tag{5}$$

with allowance for (2) can be rewritten

$$\sum_j \dot{\varphi}_j n_j = \sum_j (\dot{\varphi}'_{j-1} - \dot{\varphi}_j') n_j = \sum_j \dot{\varphi}_j' (n_{j+1} - n_j), \tag{6}$$

implying that the variables conjugate to φ_j', are the variables $n_j' = n_{j+1} - n_j$ [in Eq. (5) and everywhere below we assume $\hbar = 1$]. Hamiltonian (3) can be rewritten in terms of the variables φ_j' as

$$\hat{H} = \sum_j \left[\frac{J}{2} \hat{n}_j'^2 - Y \cos(\varphi_j' - \varphi'_{j-1}) \right], \tag{7}$$

where $\hat{n}_j' \equiv -i(\partial/\partial\varphi_j')$.

Hamiltonian (7) is the quantum-mechanical generalization of the classical XY model (for the case of an infinite spin in units of $\hbar$), and we shall call it the quantum XY model. The two-dimensional analog of (7) was first introduced in Ref. 10 and was used in Ref. 11 for a qualitative description of the behavior of films of a superfluid Bose liquid with allowance for quantum effects. Under certain conditions (the presence of a strong capacitive coupling with a conducting substrate[12]) Hamiltonian (7) can be used to describe a regular chain of Josephson junctions.

States (3) with nonzero average velocity correspond to states (7) with a nonzero current along the chain. Therefore, in terms of the quantum XY model the question of whether the macroscopic motion of a step is dissipative or nondissipative goes over to the problem of assessing the possibility of the decay of the current states and the character of this decay, a question of undoubted interest. In view of the strict isomorphism of Hamiltonians (3) and (7), from here on we shall couch the exposition in terms of the one-dimensional quantum SOS model (3) and return to (7) only in interpreting the results.

2. INSTANTONS AND PHASE TRANSITIONS

The partition function of model (3) can be expressed in the form of the functional integral[4,5]

$$Z = \int \prod_j d\varphi_j(\tau) \exp(-S^E), \tag{8}$$

where

$$S^E = \int_{-\beta/2}^{\beta/2} d\tau \, L^E \tag{9}$$

is the Euclidean form of the action, $\beta = 1/T$ is the inverse temperature (in energy units), and

$$L^E = \frac{1}{2} \sum_{j,l} U^{-1}_{j-l} \dot{\varphi}_j \dot{\varphi}_l - Y \sum_j \cos \varphi_j + \frac{U_\infty^{-1}}{2} \left(\sum_j \dot{\varphi}_j \right)^2 \tag{10}$$

is the Euclidean form of the Lagrangian, expressed in the conventional way in terms of φ_j and $\dot{\varphi}_j$. Here

$$U^{-1}_{j-l} = -\int_{-\pi}^{\pi} \frac{dk}{2\pi} \frac{[1 - \exp ik(j-l)]}{U(k)} = \frac{-|j-l|}{2J},$$

$$U_\infty^{-1} = \int_{-\pi}^{\pi} \frac{dk}{2\pi} U^{-1}(k) = \infty, \qquad U(k) = 4J \sin^2(k/2).$$

We have used the lattice constant as the length unit, so the values of the momentum k range from $-\pi$ to π. The integration in (8) is over the functions $\varphi_j(\tau)$, which are continuous in time and take on values on the circumference. For $T \neq 0$ one should imposes on them cyclic (in the imaginary time τ) boundary conditions. We shall use different notation for the actual (real) time t and the fictitious (imaginary) time τ used in evaluating the partition function. Accordingly, in discussing two-dimensional [i.e., (1 + 1)-dimension-

al] space-time, we shall use two terms: Rt space and $R\tau$ space, depending on which time we are talking about.

For $J,T \ll Y$ the leading contribution to functional integral (8) is from the classical trajectories (stationary points of the action) and small fluctuations about them. The absolute minimum of action (9) is reached on the trajectory for which all the $\varphi_j(\tau)$ are zero. If we expand the second term in (10) to second order, Lagrangian (10) decomposes into the sum of the Lagrangians of harmonic oscillators having the spectrum

$$\omega_k = (JY)^{1/2} 2|\sin(k/2)| \approx \Omega k, \quad \Omega \equiv (JY)^{1/2}.$$

In calculating partition function (8) one must also take into account trajectories on which the action has a local minimum. Such trajectories contain instantons, i.e., points at which some of the variables pass through maxima of the periodic potential $-Y\cos\varphi_j$. For a classical trajectory containing a single instanton, the action diverges. In the case of two instantons of opposite sign, the expression for the action in the approximation introduced in Refs. 4 and 5 (corresponding to the approximation of a periodic potential near the minima of the parabolas) is of the form

$$S_2^E(R,\tau) = T\sum_{\omega_s=2\pi T s}\int_{-\pi}^{\pi}\frac{dk}{2\pi}\frac{(2\pi)^2 Y[1-\cos(kR-\omega_s\tau)]}{\omega_k^2+\omega_s^2}$$

$$= 2\pi^2 Y\int_{-\pi}^{\pi}\frac{dk}{2\pi}\frac{\operatorname{ch}(\beta\omega_k/2)-\cos(kR)\operatorname{ch}[(\beta/2-\tau)\omega_k]}{\omega_k \operatorname{sh}(\beta\omega_k/2)}. \quad (11)$$

Here the integer R is the separation of the instantons in space, and τ is their separation in imaginary time.

From now on in this section we study the interaction of the instantons only for $T=0$. For $\beta=\infty$ expression (11), after integration over k, becomes

$$S_2^E(R,\tau) = 2\pi\varkappa\Big[2\sum_{r=0}^{R-1}\frac{1}{2r+1} + \frac{\pi}{2}\int_0^{2\Omega\tau} dx[(-1)^R I_{2R}(x) - iE_{2R}(ix)]\Big], \quad (12)$$

where $\varkappa = (Y/J)^{1/2}$, $I_{2R}(x)$ is modified Bessel function of order $2R$, and $E_{2R}(x)$ is the Weber function of order $2R$ (see, e.g., Ref. 13).

Since expression (11) is itself approximate, valid only for $R,\tau\to\infty$, we shall not use its exact value (12) but instead evaluate it by taking the k dependence of ω_k in the "hydrodynamic" approximation ($\omega_k = \Omega|k|$), obtaining as a result

$$S_2^E(R,\tau) = \pi\varkappa\{\mathrm{Ein}[\pi(\Omega\tau+iR)]+\mathrm{Ein}[\pi(\Omega\tau-iR)]\}, \quad (13)$$

where

$$\mathrm{Ein}(z) = \int_0^z dz'\frac{1-e^{-z'}}{z'}$$

is the exponential-integral function.

The leading term in the asymptotic expansion of (13) for large values of R and τ is

$$S_2^E(R,\tau) \sim 2\pi\varkappa \ln[(\Omega\tau)^2+R^2]^{1/2}. \quad (14)$$

If in evaluating the integral over the momenta in (11) we replace the lattice cutoff by the continuous cutoff

$$\int_{-\pi}^{\pi}\frac{dk}{2\pi}\ldots \to \int_{-\infty}^{\infty}\frac{dk}{2\pi}\exp\Big(-\frac{|k|}{\pi}\Big)\ldots, \quad (15)$$

then in place of (13) we get

$$S_2^E(R,\tau) = \pi\varkappa\ln[(1+\pi\Omega\tau)^2+(\pi R)^2], \quad (16)$$

which also reduces to (14) for $R,\tau\to\infty$.

Thus, the interaction of instantons at large separations in $R\tau$ space is logarithmic, i.e., they form a two-dimensional Coulomb gas. The properties of such a system are well known.[6,11,14] For $\varkappa=\varkappa_c\approx 2/\pi$ the system undergoes a phase transition, whereas for $\varkappa>\varkappa_c$ the instantons are bound in neutral pairs of small size (a "dielectric" phase), and for $\varkappa<\varkappa_c$ they form a plasma with a finite screening radius. In terms of the original quantum SOS model (1) these phases are atomically rough and atomically smooth, respectively.[5]

At room temperature for any $\varkappa$ the step is found in the atomically rough state, since even for $Y=0$ it will have unpaired jogs which have formed as thermal fluctuations. It is clear that the dynamical properties can be different in the cases $\varkappa\ll 1$ (when we have a dilute gas of delocalized jogs, see Ref. 15) and $\varkappa\gg 1$ (when the n representation is inconvenient for analysis, and the dilute gas is now the instanton gas introduced in the φ representation). The methods used in the present paper permit study of the second case only. The dynamical properties of linear defects in quantum crystals (viz., the absorption of sound by dislocations) have been studied by Markelov[16] in the approximation of a dilute gas of jogs. The mobility of dislocations in quantum crystals was first studied by Petukhov and Pokrovskiĭ[17] for a model with continuous variables. Those authors, however, worked in the parameter region corresponding to localized dislocations.

In terms of the one-dimensional quantum XY model (7), the phase transition at $T=0$ between the atomically smooth and atomically rough states of the SOS model corresponds to a transition between normal and superfluid phases, i.e., between phases having exponential and power-law decays of the one-dimensional correlator $\langle\exp i(\varphi_j'-\varphi_l')\rangle$ as a function of $R=j-l$. At $T>0$ this correlator falls off exponentially for any relationship of the parameters.[12]

3. NONCONSERVATION OF THE MACROSCOPIC VELOCITY OF THE STEP

By commuting Hamiltonian (3) with the operators $\hat{\varphi}_j$ and $\hat{n}_j$, we find the equation of motion

$$\hat{v}_j \equiv \partial\hat{n}_j/\partial t = Y\sin\hat{\varphi}_j, \quad (17)$$

$$\partial\hat{\varphi}_j/\partial t = J(\hat{n}_{j+1}-2\hat{n}_j+\hat{n}_{j-1}). \quad (18)$$

We will be interested in whether there can exist states for

which the velocity of the step as a whole (the macroscopic velocity) is nonzero:

$$v=\frac{1}{N}\sum_j v_j=\frac{Y}{N}\sum_j\langle\sin\varphi_j\rangle. \tag{19}$$

It follows from the form of (19) that in such a state the average values of φ_j must be nonzero.

By differentiating (18) with respect to time and substituting (17) into the result, we obtain a second-order equation of motion containing only φ:

$$\partial^2\hat{\varphi}_j/\partial t^2=JY(\sin\hat{\varphi}_{j+1}-2\sin\hat{\varphi}_j+\sin\hat{\varphi}_{j-1}). \tag{20}$$

If we consider only purely classical solutions, i.e., if we treat (20) as an algebraic equation rather than an operator equation, then we easily see that in addition to the solution corresponding to a stationary step (all the $\varphi_j=0$), it also has solutions which correspond to a moving step:

$$\varphi_j=\Phi=\text{const}, \tag{21}$$

where the velocity is $v=Y\sin\Phi$.

The existence of classical trajectories of form (21) with $v\neq 0$ can serve as a starting point for the hypothesis that Hamiltonian (3) has eigenstates with $v\neq 0$. To check this hypothesis, we must study the role of quantum fluctuations on top of trajectories of form (21). Obviously, these trajectories are metastable, since they do not give an absolute minimum of the Euclidean action (9), (10).

Let us first consider quantum fluctuations of the small-oscillation type. In all of the subsequent analysis it is important that we have the following conservation law, obtained by summing all the equations of form (18) with allowance for boundary condition (2):

$$\frac{\partial}{\partial t}\sum_j\varphi_j=0. \tag{22}$$

Conservation law (22) exists because of the invariance of the initial Hamiltonian (1) with respect to the simultaneous shift of all the variables n_j (the translational symmetry in n space). Consequently, the zeroth Fourier component for φ_j ($\Sigma_j\varphi_j$) enters Lagrangian (10) with an infinite mass, so that conservation law (22) holds not only for the average values but also for all the trajectories which contribute to the functional integral [we have therefore dropped the operator sign in (22)].

In considering small-amplitude quantum fluctuations, which do not include transitions to other minima of the periodic potential $-Y\cos\varphi_j$, we need keep only the first terms of the Fourier expansion of this potential about the trajectory of interest, $\varphi_j=\Phi$, setting, for example,

$$-Y\cos\varphi_j\approx -Y\cos\Phi+Y\sin\Phi(\varphi_j-\Phi)+\frac{Y\cos\Phi}{2}(\varphi_j-\Phi)^2 \tag{23}$$

(we are considering the case $\cos\Phi>0$). If we assume that the constant Φ is chosen so as to satisfy the initial condition

$$\frac{1}{N}\sum_j\varphi_j=\Phi, \tag{24}$$

then upon the substitution of (23) into (10) the terms linear in $\varphi_j-\Phi$ cancel. In approximation (23) it turns out that the absolute minimum of the action is reached on trajectory (21) [with allowance for restriction (24)]. Here

$$\langle\varphi_j\rangle=\Phi,\quad \langle v\rangle=\tilde{Y}\sin\Phi,$$

where $\tilde{Y}=Y\exp(-1/2\langle(\varphi_j-\Phi)^2\rangle)$. In the present paper we shall be interested in the region of parameter values corresponding to the applicability condition for the instanton approximation $\langle(\varphi_j-\Phi)^2\rangle\ll 1$ (which for $\cos\Phi\sim 1$ is equivalent to $J,T\ll Y$). In this case we can neglect the difference between Y and $\tilde{Y}$.

The addition to (23) of the low-order anharmonicities (e.g., the third-order and fourth-order), which can be taken into account by perturbation theory, does not alter the finite character of the motion of each of the variables φ_j. Here, in the state with the lowest energy of the states which satisfy condition (24), the values of $\langle\varphi_j\rangle$ is only insignificantly different from Φ and, together with $\langle v\rangle$, remains finite.

Thus if we consider only quantum fluctuations of the vibrational type, the macroscopic motion of the step is nondissipative, i.e., the states with nonzero average velocity are eigenstates. The kinetic (in n space) energy of the step, $\tilde{K}=-Y\Sigma_j\cos\varphi_j$, is thus conserved. Let us now consider the influence of processes in which the variables φ_j tunnel into the neighboring minima of the periodic potential (we emphasize that these tunneling processes are not related to the tunneling processes described by the operators $\hat{a}_j^+$ and $\hat{a}_j^-$).

Let us consider a trajectory on which $\varphi_j=\Phi$ for $t\to-\infty$ and on which one of the variables (for example, φ_{j0}) changes its value by 2π over a certain finite time. In order for condition (24) and equation of motion (20) to hold as $t\to\infty$, the values of the variables φ_j must satisfy the following as $t\to+\infty$:

$$\varphi_j=\begin{cases}\Phi-2\pi/N, & j\neq j_0\\ \Phi+2\pi-2\pi/N, & j=j_0\end{cases}.$$

The total kinetic energy $\tilde{K}$ here changes by $-2\pi Y\sin\Phi$ (we have dropped the terms which vanish in the thermodynamic limit $N\to\infty$), while the average velocity v changes by $2\pi Y\cos\Phi/N$. The energy released should go over into the vibrational degrees of freedom.

We see that the instantonic trajectories can lead to a loss of kinetic energy. Granted, for a single instanton the action diverges. For pairs of instantons of unlike sign, however, separated from each other by an imaginary time τ (but located, say, at the same lattice site), the action is (to leading order)

$$S_2^E(\tau)=2\pi\varkappa\cos^{1/2}\Phi\ln(\Omega\tau)-2\pi Y(\sin\Phi)\tau \tag{25}$$

and with increasing τ it increases only up to a finite limit, and then it decreases. This means that in the functional space of trajectories, the classical trajectories $\varphi_j=\Phi$ and $\varphi_j=\Phi-2\pi/N$, although they represent different local minima of the Euclidean action, are separated by a barrier of only finite height and cannot be treated as belonging to different independent states. Therefore, the step velocity v does

not remain constant (as would be the case when only small-amplitude quantum fluctuations are taken into account), but its evolution is determined to a large extent by tunneling processes between neighboring minima of the potential $-Y\Sigma_j \cos\varphi_j$. Let us now turn to a study of these processes.

4. SELF-CONSISTENT "SINGLE-PARTICLE" APPROXIMATION

For a preliminary analysis, let us assume that the tunneling processes occur independently at each of the lattice sites. We keep the explicit form of the potential $-Y\cos\varphi_j$ for some one of the lattice sites (e.g., for the site with $j=0$), while for the other sites we use expansion (23) (with arbitrary Φ). After such a substitution, action (9), (10) becomes quadratic in all the variables except φ_0, so that almost all the integrations in functional integral (8) can be done exactly. As a result, we have to within a constant factor

$$Z=\int d\varphi_0(\tau)\exp(-S_{eff}^E), \tag{26}$$

where

$$S_{eff}^E=\int_{-\beta/2}^{\beta/2} d\tau V_{eff}(\varphi_0(\tau))+T\sum_{\omega_s=2\pi Ts} G_{eff}^{-1}(\omega_s)(\varphi_0)_{\omega_s}(\varphi_0)_{\omega_s}^*. \tag{27}$$

Here

$$V_{eff}(\varphi_0)=-Y\cos\varphi_0-(Y\sin\Phi)\varphi_0 \tag{28}$$

is the effective potential for the coordinate φ_0, and

$$G_{eff}(\omega_s)=Y^{-1}\left[\left(\frac{\omega_s}{2\Omega}\right)^2+\left|\frac{\omega_s}{2\Omega}\right|\left(1+\left(\frac{\omega_s}{2\Omega}\right)^2\right)^{1/2}\right]^{-1} \tag{29}$$

is the propagator for its motion in the absence of a potential ($\Omega=(JY)^{1/2}$).

The problem thus reduces to a familiar one. An analogous effective action is obtained when the interaction of a particle with a medium is simulated by an interaction of the particle with an infinite set of harmonic oscillators (heat bath) by the Feynman-Vernon method.[18] In this case the total Lagrangian of the system is assumed to be of the form

$$L^E=V(\varphi_0)+\frac{m_*}{2}\dot\varphi_0^2+\varphi_0\sum_\alpha C_\alpha x_\alpha+\sum_\alpha\frac{\gamma_\alpha}{2}(\dot x_\alpha^2+\omega_\alpha^2 x_\alpha^2). \tag{30}$$

The integration of the oscillator variables leads to a nonlocal effective action for φ_0 of the form in (27), with

$$V_{eff}(\varphi_0)=V(\varphi_0)-\frac{1}{2}\varphi_0^2\sum_\alpha\frac{C_\alpha^2}{\gamma_\alpha\omega_\alpha^2}$$

and

$$G_{eff}^{-1}(\omega_s)=\frac{m_*}{2}\omega_s^2+\frac{\omega_s^2}{\pi}\int_0^\infty\frac{d\omega}{\omega}\frac{\rho_{eff}(\omega)}{\omega_s^2+\omega^2},$$

where

$$\rho_{eff}(\omega)=\frac{\pi}{2}\sum_\alpha\frac{C_\alpha^2}{\gamma_\alpha\omega_\alpha}\delta(\omega-\omega_\alpha)$$

is the effective density of the oscillator distribution.[19]

One can introduce a coefficient of viscous friction η for describing slow motions under the condition

$$\lim_{\omega_s\to 0}[|\omega_s^{-1}|G^{-1}(\omega_s)]=\eta/2, \tag{31}$$

which can be satisfied[19] if the function $\rho_{\text{eff}}(\omega)$ at small ω is of the form $\eta\omega$. Such an approach ("nonlocal dissipation") was first used by Caldeira and Leggett[19] for studying the influence of dissipation on the tunneling decay of metastable states into the continuum and has since come into widespread use. In formulation (30) our propagator (29) corresponds to a particle of mass

$$m_*=\lim_{\omega_s\to\infty}[\omega_s^{-2}G^{-1}(\omega_s)]=(2J)^{-1},$$

interacting with a set of oscillators distributed with an effective density

$$\rho_{eff}(\omega)=\begin{cases} Y\dfrac{\omega}{2\Omega}\left[1-\left(\dfrac{\omega}{2\Omega}\right)^2\right]^{1/2}, & \omega<2\Omega \\ 0, & \omega\geqslant 2\Omega \end{cases}.$$

Here, according to (31), the coefficient of viscosity is $\eta=\varkappa/2$. Thus effective action (27)–(29) describes the motion of a particle with mass $m_*=(2J)^{-1}$ and viscosity $\eta=\varkappa/2$ in a periodic potential $-Y\cos\varphi_0$ under the influence of a force $f=Y\sin\Phi$.

Returning to the original problem, we note that when approximation (23) is used for the kinetic (in n space) energy, the consistent definition of the velocity v_j is not (17) but

$$v_j=Y[\sin\Phi+\cos\Phi(\varphi_j-\Phi)].$$

Here expression (19) for the average (over sites) velocity goes over to

$$v=Y\left\{\sin\Phi+\frac{1}{N}[(\sin\varphi_0-\sin\Phi)-\cos\Phi(\varphi_0-\Phi)]\right\}. \tag{32}$$

In Eq. (32) we must keep the terms proportional to $1/N$, since we shall later want to take into account the contribution to $\partial v/\partial t$ from the tunneling processes at all N sites.

It follows from the form of (32) that the evolution of the average velocity v is uniquely determined by the character of the motion of the coordinate φ_0. The motion of a quantum particle in a periodic potential with dissipation has been studied by Schmid,[20] Bulgadaev,[21,22] and Grabert and Weiss.[23] In the case when the periodic potential has deep minima, this motion can be treated as consisting of a series of tunneling processes between adjacent minima, and its properties can be explained on the basis of the properties of tunneling in a double-well potential.

Tunneling with dissipation in a double-well potential has been studied by various methods in a number of papers.[24–30] It has been shown that if the value of the so-called dimensionless viscosity $\alpha=\eta q_0^2/2\pi$ (where q_0 is the distance between the minima of the potential) is greater than unity (in our case $q_0=2\pi$ and $\alpha=\pi\varkappa$), then the individual tunneling events occur practically independently of one another, and the process as a whole is incoherent (purely exponential relaxation).[25–27] For wells of equal depth, the relaxa-

tion rate $2\nu_T$ (twice the tunneling frequency) goes to zero as $T\to 0$ in accordance with the law[26,27]

$$2\nu_T \propto T^{2\alpha-1}. \tag{33}$$

At $T=0$, under the same assumption $\alpha>1$, tunneling is possible only if the wells are of different depth and occurs in one direction only (corresponding to a decrease of energy), with a frequency

$$\nu_\varepsilon \propto \varepsilon^{2\alpha-1}, \tag{34}$$

where ε is the difference of the well depths.[28-30]

Dependence (34) can be found[28] by the conventional method of calculating the rate of decay of a metastable state from the imaginary part of the free energy[31,32] by a procedure analogous to that which was used in Ref. 19 for decay into the continuum. In the given case the action on the two-instanton trajectory

$$S_2^E(\tau)=2\alpha \ln(1+\omega_c\tau)-\varepsilon\tau$$

reaches a maximum (for $\tau>0$) at $\tau=\tau_*=2\alpha\varepsilon^{-1}-\omega_c^{-1}$, so that $\exp(-S_2^E(\tau_*))\propto\varepsilon^{2\alpha}$. The additional factor ε^{-1} in (34) arises in the Gaussian integration in the neighborhood of τ_*. This same result has been obtained using the instanton technique in real time.[29,30]

The independence of the individual tunneling events permits the conclusion that for $\alpha>1$ and $\Phi=0$ the motion of the coordinate φ_0 in neglect of small-amplitude vibrations is a purely diffusive process, which can be described with the aid of a diffusion coefficient[23]

$$D=q_0^2\nu_T$$

(recall that in our case $q_0=2\pi$). For finite Φ this motion occurs against the background of a drift at the constant velocity $u_0=\langle\partial\varphi_0/\partial t\rangle$, which at small Φ is linearly related to the "force" $f=Y\sin\Phi$ through the mobility λ:

$$u_0=\lambda f,$$

where λ can be expressed in terms of D with the aid of the Einstein relation:

$$\lambda=D/T=q_0^2\nu_T/T. \tag{35}$$

A more formal derivation of the temperature dependence of the mobility can be found in Ref. 22.

In terms of the original SOS model, f coincides with the average velocity v of the step, while according to (32) the drift velocity $u_0=\langle\partial\varphi_0/\partial t\rangle$ determines the average contribution to $\partial v/\partial t$ from tunneling processes occurring at the site under consideration (the zeroth site):

$$\partial v/\partial t=-Y\lambda v/N. \tag{36}$$

Summing over all N sites under the assumption that the tunneling processes occur independently at the different sites, we get

$$dv/\partial t=-K^{-1}v, \quad K^{-1}=4\pi^2 Y\nu_T/T. \tag{37}$$

We stress that because we are taking into account the contributions from a macroscopic number of independent processes, relation (37) holds exactly, unlike relation (36), which was obtained by considering the tunneling at one of the sites and holds only on the average.

Thus the combined influence of tunneling processes on the macroscopic motion of a step can be reduced to the appearance of a viscous frictional force, with a coefficient of friction that depends on temperature as $T^{2\pi\varkappa-2}$ and goes to zero at $T=0$.

In an analogous way, but using (34) instead of (33), we find that for $T=0$ the macroscopic motion is also dissipative, but in this case the force of friction has a nonlinear dependence on the velocity even at low velocities: $\partial v/\partial t\propto v^{2\pi\varkappa-1}$.

Let us now turn to a more formal description that does not involve any *a priori* assumptions about the independence of the processes at different sites.

5. TRANSITION TO INSTANTON GAS IN *Rt* SPACE

Quantum tunneling with dissipation in a double-well potential has been studied[27,29,30] by the construction of a functional integral describing the evolution of the density matrix (see also Ref. 33). The variables on which the density matrix is assumed to depend (it is averaged over all the remaining variables) in this case are simply the number of the well, which here, of course, takes on only two values. It turns out that this functional integral is of the form of the partition function of a one-dimensional instanton gas (distributed along the real time axis t) with a binary instanton interaction $S_2(t)$ which can be obtained as the analytical continuation of the instanton interaction in imaginary time and is no longer purely real.[27]

For $\alpha\gg 1$ the typical trajectories giving the main contribution to the functional integral under study contain comparatively long segments on which the system is found in a diagonal state, separated by short "blips" into nondiagonal states. The short duration of these latter segments is due to the fact that the real part of the interaction of their ends (instantons), $\mathrm{Re}\,S_2(t)$, increases logarithmically with increasing t (for $T>0$ the interaction grows even faster). One can say that the instantons are bound into pairs of small size.

Each tunneling event (between diagonal states) corresponds to a pair of instantons, each of which corresponds to a change of one of the arguments of the density matrix. Since different bound pairs of instantons are far apart and interact weakly, the different tunneling events occur practically independently, so that the tunneling is incoherent. If we neglect the interaction between instanton pairs, the tunneling frequency is determined directly by the interaction of the instantons[27]:

$$\nu=\frac{\Delta^2}{4\omega_c}\int_{-\infty}^{\infty} dt\exp[-S_2(t)].$$

Such an approach can be extended to the quantum SOS model considered in the present paper. The evolution of the density matrix in this case can be described with the aid of evolution operators expressed in terms of the functional integrals:

$$\rho(\{\varphi(t)\},\{\varphi'(t)\})$$
$$=\int_{-\pi}^{\pi} D\{\varphi(t_0)\}D\{\varphi'(t_0)\}T_t(\{\varphi(t)\},\{\varphi(t_0)\})$$
$$\times T_t^*(\{\varphi'(t)\},\{\varphi'(t_0)\})\rho(\{\varphi(t_0)\},\{\varphi'(t_0)\}), \tag{38}$$

where $\{\varphi\}$ denotes the whole set of variables φ_j, which here range from $-\pi$ to π, and

$$T_t(\{\varphi(t)\},\{\varphi(t_0)\})=\lim_{P\to\infty}\prod_{p=0}^{P-1}\Big[\sum_{\{m(t_p)\}}\int_{-\infty}^{\infty}D\{n(t_p)\}\Big]$$
$$\times\prod_{p=1}^{P-1}\Big[\int_{-\pi}^{\pi}D\{\varphi(t_p)\}\Big]\exp\Big\{-i\sum_{p=0}^{P-1}\sum_j\Big[\frac{J\Delta t}{2}(n_j(t_p)$$
$$-n_{j+1}(t_p))^2+n_j(t_p)(\varphi_j(t_{p+1})$$
$$-\varphi_j(t_p)-2\pi m_j(t_p))+Y\Delta t(1-\cos\varphi_j(t_p))\Big]\Big\} \qquad (39)$$

(cf. Ref. 5). Here $\Delta t=(t-t_0)/P$, $t_p=t_0+p\Delta t$. The variables $n_j(t_p)$ are continuous and assume values from $-\infty$ to $+\infty$, while the variables $m_j(t_p)$ are integers. The variables appearing in the functional integral that determines $T_t^*(\{\varphi'(t)\},\{\varphi'(t_0)\})$ will be denoted by the same letters as in (39), but with a prime. Since φ_j runs between the limits $-\pi$ and π in the sum over $m_j(t_p)$, we can assume that the only nonzero terms are those with $m_j(t_p)=0;\ \pm 1$.

For $v\ll Y$

$$\langle v\rangle=\frac{Y}{N}\Big\langle\sum_j\sin\varphi_j\Big\rangle\approx\frac{\tilde{Y}}{N}\sum_j\varphi_j,$$

where for $J,T\ll Y$ the quantity $\tilde{Y}$ differs only insignificantly from Y. Since we have gone over to the reduced variables φ_j defined on the interval $(-\pi,\pi)$, $\Sigma_j\varphi_j$ can change by jumps of $\pm 2\pi$; here, for $M=(1/2\pi)\Sigma_j\varphi_j$ we will have

$$M(t)-M(t_0)=\sum_j\sum_{p=0}^{P-1}m_j(t_p).$$

In analyzing the behavior of v, we will be interested in a density matrix (38) in which only the dependence on $\Sigma_j\varphi_j(t)$ and $\Sigma_j\varphi_j'(t)$ is kept and an average is taken over the remaining variables. With allowance for substitutions of the type in (39), expression (38) becomes a sum over m and m' and a functional integral over n, n', φ and φ'. The integration over n and n' is Gaussian and can be done in a trivial way. The integration over φ and φ' will be done by the method of steepest descent after $Y(1-\cos\varphi_j(t_p))$ in (39) is replaced by $(Y/2)\varphi_j^2(t_p)$ and $\rho(\{\varphi(t_0)\},\{\varphi'(t_0)\})$ is replaced by the equilibrium thermalized density matrix of the set of harmonic oscillators to which the system goes when an analogous substitution is made in the Hamiltonian:

$$\rho(\{\varphi(t_0)\},\{\varphi'(t_0)\})=\rho_0\exp\Big\{-\frac{1}{2}\int_{-\pi}^{\pi}\frac{dk}{2\pi}\Big(\frac{Y}{U(k)}\Big)^{1/2}$$
$$\times[(\varphi_k\varphi_{-k}+\varphi_k'\varphi_{-k}')\operatorname{cth}\beta\omega_k-2\varphi_k\varphi_{-k}'\operatorname{sh}^{-1}(\beta\omega_k)]\Big\}.$$

Here ρ_0 is a normalization constant, and φ_k and φ_k' are the respective Fourier transforms (along the chain) consecutively of $\varphi_j(t_0)$ and $\varphi_j'(t_0)$.

If in the calculation of the integrals by the method of steepest descent we keep only the action which enters the system on the steepest-descent trajectory and drop the pre-exponential factors, then we obtain

$$\rho(2\pi M,2\pi M')=\sum_{\substack{m_j(t_p)=0,\pm1\\ m_j'(t_p)=0,\pm1}}\exp\Big\{+\frac{i\pi}{2}\sum_{j,p}(m_j(t_p)-m_j'(t_p))$$
$$-2\pi^2\sum_{j,l}\Big[\sum_{0\leqslant p,q<P}(m_j(t_p)-m_j'(t_p))$$
$$\times G'_{j-l}(t_p-t_q)(m_l(t_q)-m_l'(t_q))$$
$$-i\sum_{0\leqslant q<p<P}(m_j(t_p)-m_j'(t_p))G''_{j-l}(t_p-t_q)$$
$$\times(m_l(t_q)+m_l'(t_q))\Big]\Big\}, \qquad (40)$$

where the sum over $m_j(t_p)$ and $m_j'(t_p)$ includes only those terms which satisfy the conditions

$$\sum_{j,p}m_j(t_p)=M,\qquad\sum_{j,p}m_j'(t_p)=M',$$

and where

$$G_R'(t)+iG_R''(t)$$
$$=(Y/2)\int_{-\pi}^{\pi}\frac{dk}{2\pi}\omega_k^{-1}\cos kR[\operatorname{cth}(\beta\omega_k/2)\cos\omega_k t+i\sin\omega_k t].$$

Expression (40) (in which one easily perceives the similarity to the Feynman-Vernon influence functional[18]) is of the form of the partition function of a two-dimensional gas of instantons with charges $Q_j(t_p)=m_j(t_p)-m_j'(t_p)$ and with an interaction

$$2\pi^2[G_R'(t)\pm iG_R''(t)],$$

where the sign of the imaginary part also depends on $m_j(t_p)+m_j'(t_p)$.

The action corresponding to a solitary instanton diverges; it is finite only in the case of a pair of instantons having charges of opposite sign or for some set of such pairs. The expression for the action corresponding to a configuration consisting of two instantons with charges $Q=\pm 1$, separated from each other by a distance R (in space) and t (in time),

$$S_2(R,t)=4\pi^2[G_0'(0)-G_R'(t)\mp iG_R''(t)]$$

agrees here with the analytical continuation of the expression for the interaction of instantons in imaginary time (11):

$$S_2(R,t)=S_2^E(R,\pm it). \qquad (41)$$

The incoherent tunneling regime corresponds to an instanton-gas phase (in Rt space) in which the instantons are bound into pairs of small size, located far from one another (cf. Ref. 27). To see that this is actually the case for $\varkappa\gg 1$, we should study the interaction of the instantons in real time or, more precisely, its real part $\operatorname{Re}S_2(R,t)$. Let us do this at zero temperature first.

Considering the asymptotic expression for the interaction of the instantons in $R\tau$ space (14) and substituting the imaginary value τ into it, we can see that the most problematical element is the existence of a sufficiently strong attraction of the instantons in the case of the "diagonally" oriented

(in Rt space) pairs ($R \approx \Omega t$, even if the distance between instantons $X \equiv (R^2 + \Omega^2 t^2)^{1/2}$ is large then).

Therefore, for analysis of the case $R \approx \pm \Omega t$ we must turn to the more accurate approximation (13). Substituting into (13) $\tau = \pm iR/\Omega$, we obtain

$$\operatorname{Re} S_2(R, \pm R/\Omega) = \pi\varkappa \operatorname{Cin}(2\pi R), \tag{42}$$

where

$$\operatorname{Cin} z = \int_0^z dz' \frac{1-\cos z'}{z'}$$

is the cosine-integral function. For $R \gg 1$ we have $\pi\varkappa \operatorname{Cin}(2\pi R) \approx \pi\varkappa \ln R$. This same asymptotic form for $\operatorname{Re} S_2(R, \pm R/\Omega)$ is found by using expression (16), which was obtained in the approximation of a continuous momentum cutoff, in which case

$$\begin{aligned}\operatorname{Re} S_2(R, t) &= (\pi/2)\varkappa \ln \{[1+(\pi R)^2-(\pi\Omega t)^2]^2+(2\pi\Omega t)^2\} \\ &\geqslant (\pi/2)\varkappa \ln [1+2\pi^2(R^2+\Omega^2 t^2)]. \end{aligned} \tag{43}$$

It follows from (42) and (43) that $\operatorname{Re} S_2(R,t)$ satisfies the condition

$$\operatorname{Re} S_2(R, t) > \pi\varkappa \ln [R^2+\Omega^2 t^2]^{1/2},$$

the right-hand side of which differs from expression (14) for $S_2^E(R,t)$ only by a factor of 1/2.

Since for $\varkappa > \varkappa_c \approx 2/\pi$ the instanton gas in $R\tau$ space consists of bound pairs, the instanton gas in Rt space for $\varkappa > 2\varkappa_c$ will certainly break up into pairs of instantons, with the interaction between pairs being of practically no importance. It is extremely probable that this approach applies in a qualitative way for $\varkappa_c < \varkappa < 2\varkappa_c$ as well.

Using (11) and (41), one can show that a nonzero temperature will lead to an additional positive correction to $\operatorname{Re} S_2(T,t)$ that at low T grows as

$$(\pi^2 J/T) \max(|R|, \Omega|t|)$$

and is dominant at large R and t. This clearly will only improve the compliance with the incoherent tunneling conditions.

6. CALCULATION OF THE FORCE OF FRICTION

Thus, for $\varkappa \gg 1$ one can calculate (40) by taking into account independently the contributions from the different instanton pairs, just as in Ref. 27. Each tunneling event for the variable M corresponds to a bound pair of instantons. The physical meaning of the condition that the instantons forming a pair have zero total charge is that on both sides (in time) of the bound pair the density matrix is in a state that is diagonal in the variable M.

We stress that the instantons forming a pair can belong to different lattice sites, i.e., if in a given tunneling event the change of M by unity is due to a change of φ_j by 2π, then the change of M' can be due to a change of the φ_l' with $l \neq j$. This is due to the equivalence of values of φ and φ' which differ by 2π. The tunneling probability (per unit time) is

$$\nu = \Omega_c^2 \sum_R \int_{-i\infty}^{i\infty} d\tau \exp[-S_2^E(R, \tau)], \tag{44}$$

where $\Omega_c = \Omega g(\varkappa, T)$ is a characteristic frequency which can be calculated only with allowance for the pre-exponential factors in (40). At low temperatures we have to leading order $g(\varkappa, T) = g(\varkappa)$.

In the approximation considered in Sec. 4, each tunneling event is assumed to be localized at some site. This corresponds to taking into account only those pairs of instantons which are located at the same site, i.e., to allowance for only one term of the sum over R in (44). Since here too we are working in an approximation in which we are ignoring the mutual influence of the individual tunneling events, we can as before use formulas (37) for the coefficient of friction, but now with (44) substituted in instead of (33).

At a finite temperature the integration over τ in (44) is conveniently done after first shifting the path of integration in such a way that it crosses the real τ axis at the point where $S_2^E(R,\tau)$ has a maximum, i.e., at the point $\tau = \beta/2$.

In the expression for

$$\begin{aligned} S_2^E(R, \beta/2+it) &= 2\pi^2 Y \int_{-\pi}^{\pi} \frac{dk}{2\pi} \left[\frac{\operatorname{th}(\beta\omega_k/4)}{\omega_k} + \frac{1-\cos kR \cos \omega_k t}{\omega_k \operatorname{sh}(\beta\omega_k/2)} \right], \end{aligned}$$

where we shall henceforth assume $\omega_k = \Omega|k|$, it is only in integrating the first term, which does not depend on R and t, that we must take into account the presence of a quasimomentum cutoff, which we take to be of form (15); we get

$$\begin{aligned} &2\pi^2 Y \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{\operatorname{th}(\beta\Omega k/4)}{\Omega k} e^{-|k|/\pi} \\ &= 2\pi\varkappa \left[2\ln \frac{\Gamma[1/\pi\beta\Omega]}{\Gamma[1/2+1/\pi\beta\Omega]} - \ln \frac{1}{\pi\beta\Omega} \right] \approx 2\pi\varkappa \ln(\beta\Omega) \end{aligned} \tag{45}$$

(really it is only the coefficient in front of the logarithm of β that is important for us, and it does not depend on the form of the cutoff). Here and below we assume $T \ll \Omega$. In integrating the second term we can shift the limits of integration to infinity and get

$$\begin{aligned} &2\pi^2 Y \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{1-\cos(kR)\cos(\Omega kt)}{k \operatorname{sh}(\beta\Omega k/2)} \\ &= \pi\varkappa \ln \left[\frac{1}{2} \operatorname{ch} \frac{2\pi t}{\beta} + \frac{1}{2} \operatorname{ch} \frac{2\pi R}{\beta\Omega} \right]. \end{aligned} \tag{46}$$

After substituting for $S_2^E(R, \beta/2+it)$ the sum of (45) and (46) and replacing the summation over R by an integration, we get

$$\begin{aligned} \nu_T &= \Omega_c^2 \int_{-R}^{R} dR \int_{-\infty}^{\infty} dt \exp[-S_2^E(R, \beta/2+it)] \\ &= \frac{1}{2\pi} \left[\frac{\Gamma(\pi\varkappa/2)}{\Gamma((\pi\varkappa+1)/2)} \right]^2 \frac{\Omega_c^2}{\Omega} \left(\frac{T}{\Omega} \right)^{2\pi\varkappa-2}. \end{aligned} \tag{47}$$

Assuming, as is commonly believed,[34] that the dimensionless factor $g(\varkappa)$ in the pre-exponential factor Ω_c is proportional to the square root of the action on the instantonic

trajectory, $g(\varkappa) \propto \varkappa^{1/2}$, we find that the coefficient of friction in (37), given to within a numerical factor by

$$K^{-1} \propto Y(T/\Omega)^{\theta}, \quad \theta = 2\pi\varkappa - 3, \tag{48}$$

is only weakly dependent on $\varkappa$.

We can also calculate the tunneling probability at $T = 0$, but for a nonzero velocity v. Adding to (16) a term linear in t due to the difference in the depths of the adjacent wells [cf. Eq. (25)] and replacing the summation over R by an integration, we obtain

$$\nu_v = \Omega_c^2 \int_{-\infty}^{\infty} dR \int_{-\infty}^{\infty} dt [(1+i\pi\Omega t)^2 + (\pi R)^2]^{-\pi\varkappa} \exp(2\pi i v t)$$

$$= \frac{2\Gamma(\pi\varkappa - 1/2)}{\pi^{1/2}\Gamma(\pi\varkappa)\Gamma(2\pi\varkappa - 1)} \frac{\Omega_c^2}{\Omega} \left(\frac{2v}{\Omega}\right)^{2\pi\varkappa - 2} \exp\left(-\frac{2v}{\Omega}\right), \tag{49}$$

which implies that at low velocities

$$\partial v/\partial t \propto -(v/\Omega)^{2\pi\varkappa - 2}. \tag{50}$$

We emphasize that for $R \gg 1$ the integrals over t in (47) and (49) cannot be done by the method of steepest descent.

The exponents in (48) and (50) differ by 1 from those found in Sec. 4 in the cruder "single-particle" approximation. Relation (50) also holds at low but finite temperatures for velocities satisfying the condition $T \ll v \ll \Omega$.

7. CONCLUSION

We have found that in the framework of the quantum SOS model (1) the macroscopic motion of delocalized linear defects in crystals is dissipative. From a phenomenological standpoint the influence of irreversible processes can be described as the appearance of a force of friction, which for $T = 0$ depends on the velocity in a nonlinear way but which for finite temperatures is linear in the velocity at low velocities (viscous friction). The coefficient of friction then has a power-law dependence on temperature [see Eq. (48)], with an exponent θ that depends on the relationship of the parameters J and Y and, in the case of dislocations, for example, can change with pressure.

The calculation in Sec. 6 was done under the assumption that $T \ll \Omega$. In this case the force of friction under consideration is small in the sense that it determines a rate of relaxation that is small compared to the characteristic frequency Ω; this is consistent with the necessary condition for applicability of the method used.

It was assumed in the derivation that the mutual influence of the instanton pairs is unimportant; this becomes a better assumption as $\varkappa$ increases. It is extremely probable that a power-law dependence of the form in (48) will hold not only for $\varkappa \gg 1$ but also in the physically more interesting region $\varkappa \sim 1$ (most likely all the way to $\varkappa_c$), with some renormalization of the exponent θ. In the self-consistent approximation, without allowance for renormalizations, $\varkappa_c = 2\pi^{-1}$ and $\theta > 1$. The combination $2\pi\varkappa$ appearing in the expression for θ is the pre-logarithmic factor in the interaction of the instantons. If we assume that when the renormalizations are taken into account the same relation will still hold, but with renormalized values of $2\pi\varkappa$ and θ, then for $\varkappa > \varkappa_c$ we will, as before, have $\theta > 1$.

The frictional force under consideration is due to the discreteness of the atomic structure of the crystal, which is manifested in the discreteness of the variables n_j which determine the position of the step (dislocation). When the discrete variables n_j in (1) are replaced by continuous variables and the kinetic energy is changed accordingly, we obtain the Hamiltonian of a discrete free string, whose motion is nondissipative and whose velocity as a whole is a conserved quantity. The discreteness of the variables n_j leads to the possibility of tunneling processes which violate the conservation of the velocity of the macroscopic motion, so that the kinetic energy of the motion of the step as a whole is irreversibly converted into vibrational degrees of freedom. Since the variables φ_j are the quasimomenta (in n space), these tunneling processes are nothing but umklapp processes. In this sense the dissipative nature of the motion of the step has the same cause as the low-temperature electrical resistance of an ideal crystal according to Landau and Pomeranchuk.[35]

A competing dissipation mechanism is friction against the phonon gas. In the case of dislocations the anharmonicity-caused scattering of phonons by the slowly decaying stress field of the dislocation gives rise to a force of friction proportional[36] to $(T/T_D)^5$ (T_D is the Debye temperature), while the reemission of phonons by induced vibrations of the dislocation (fluttering) gives rise to a force of friction proportional[37] to $(T/T_D)^3$. Comparison with (48) shows that there is a region of parameter values in which our "zero-phonon" part of the force of friction should dominate at low temperatures.

The Granato-Lücke[38] theory of the absorption of sound by dislocations in crystals is based on the assumed existence of a temperature-independent coefficient of viscous friction for the motion of dislocations. Our analysis can thus serve (to the extent that the SOS model applies) as justification for applying the Granato-Lücke theory to quantum crystals (in the case $\varkappa > \varkappa_c$).

Experimental studies of the velocity[39,40] and absorption[41–43] of sound in ^{4}He crystals, with the data processed in accordance with the Granato-Lücke theory on the assumption that the coefficient of friction depends on temperature as T^{θ}, give values for the exponent θ which lie basically in the range $\theta = 1.5$–3. The theory proposed in the present paper can explain both the difference of the exponent θ from 3 and its change with pressure.

Wolf *et al.*[44] made an experimental study of the rate of growth of ^{4}He crystals at low temperatures. The observed character of the dependence of the rate of growth on the applied chemical-potential difference permitted those authors to conclude that for faces parallel to the sixfold axis the growth is due to the motion of steps terminating at screw dislocations. This in turn permitted determination of the temperature dependence of the mobility of the step from the temperature dependence of the rate of growth. The observed temperature dependence of the mobility ($T^{-\theta}, \theta \approx 2$) cannot be attributed to friction against the phonon gas, but indicates the presence of another dissipation mechanism such as that considered in the present paper, for example.

It should be kept in mind, however, that the quantum SOS model considered here applies only in a schematic sense

to the description of linear defects in quantum crystals, since it does not take into account their interaction with the crystal through the elastic stress field or the motion of the liquid that invariably accompanies the motion of a step along an interface between a quantum crystal and quantum liquid of different densities. An attempt at a more detailed description of such objects will, generally speaking, require a modernization of the theory. In this sense model (1) is most realistic for describing a step on a planar defect in a crystal (e.g., on a domain wall in a magnet or on a twinning boundary).

As we mentioned in the Introduction, the quantum SOS model (1) is isomorphic to the quantum XY model (7), which is used to describe a regular chain of Josephson junctions. In terms of the XY model the conclusions of this study reduce to the absence of undamped current states in such a system (the current corresponds to the velocity of the step as a whole). At a finite temperature the volt-ampere characteristic is linear at small currents, and the resistance has a power-law dependence on temperature. In terms of the XY model the tunneling processes responsible for energy dissipation represent the usual phase slip, which occurs in this regime as quantum fluctuations, and evidently have a more transparent meaning than in the quantum SOS model.

If each of the junctions in a regular chain of Josephson junctions is assumed to be shunted by a finite Ohmic resistance, then this leads only to a certain increase in the exponent in the power-law temperature dependence of the resistance of the chain as a whole.

In the quantum XY models of higher dimensionality (e.g., the two-dimensional case), which are used to describe granulated superconductors,[45] a decrease of the current due to the analogous localized phase slips is impossible for purely geometric reasons. Similarly, it is also impossible in the quantum SOS model for the free surface of a crystal,[3,46] where the potential (in the φ representation) energy is of the same form as in the XY model. Therefore, in such systems, even at finite temperature, there can exist a real superfluid phase in which the current (or the mass current) is nondissipative.

I wish to thank S. V. Iordanskiĭ and S. V. Meshkov for discussion of this study and helpful comments.

[1]A. F. Andreev and A. Ya. Parshin, Zh. Eksp. Teor. Fiz. **75**, 1511 (1978) [Sov. Phys. JETP **48**, 763 (1978)].
[2]L. A. Bol'shov, V. L. Pokrovskiĭ, and G. V. Uĭmin, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 145 (1984) [JETP Lett. **39**, 173 (1984)]; L. A. Bol'shov, V. L. Pokrovsky, and G. V. Uimin, J. Stat. Phys. **38**, 191 (1985).
[3]E. Fradkin, Phys. Rev. **B28**, 5338 (1983).
[4]S. V. Iordanskiĭ and S. E. Korshunov, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 542 (1983) [JETP Lett. **38**, 655 (1983)].
[5]S. V. Iordanskiĭ and S. E. Korshunov, Zh. Eksp. Teor. Fiz. **87**, 927 (1984) [Sov. Phys. JETP **60**, 528 (1984)].
[6]V. L. Berezinskiĭ, Dissertation [in Russian], L. D. Landau Inst. of Theor. Phys., Academy of Sciences of the USSR (1971).
[7]S. T. Chui and J. D. Weeks, Phys. Rev. **B14**, 4978 (1976).
[8]J. D. Weeks and G. H. Gilmer, in: Advances in Chemical Physics, Vol. 40 (ed. by I. Prigogine and S. A. Rice) (1979), p. 157.
[9]A. A. Migdal, Zh. Eksp. Teor. Fiz. **69**, 1457 (1975) [Sov. Phys. JETP **42**, 743 (1975)]; J. V. Jose, L. P. Kadanoff, S. Kirkpatrick, and D. R. Nelson, Phys. Rev. **B16**, 1217 (1977).
[10]V. G. Vaks and A. I. Larkin, Zh. Eksp. Teor. Fiz. **49**, 975 (1965) [Sov. Phys. JETP **22**, 678 (1966)].
[11]V. L. Berezinskiĭ, Zh. Eksp. Teor. Fiz. **61**, 1144 (1971) [Sov. Phys. JETP **34**, 610 (1972)].
[12]R. M. Bradley and S. Doniach, Phys. Rev. **B30**, 1138 (1984).
[13]M. Abramowitz and I. A. Stegun (eds.), Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables, Dover, New York (1964) [Russian translation: Nauka, Moscow (1979), p. 316].
[14]J. M. Kosterlitz and D. J. Thouless, J. Phys. **C5**, L124 (1972); **6**, 1181 (1973); J. M. Kosterlitz, J. Phys. **C7**, 1046 (1974).
[15]A. F. Andreev, Zh. Eksp. Teor. Fiz. **68**, 2341 (1975) [Sov. Phys. JETP **41**, 1170 (1975)]; Usp. Fiz. Nauk **118**, 251 (1976) [Sov. Phys. Usp. **19**, 137 (1976)].
[16]A. V. Markelov, Zh. Eksp. Teor. Fiz. **88**, 205 (1985) [Sov. Phys. JETP **61**, 118 (1985)].
[17]B. V. Petukhov and V. L. Pokrovskiĭ, Zh. Eksp. Teor. Fiz. **63**, 634 (1972) [Sov. Phys. JETP **36**, 336 (1973)].
[18]R. P. Feynman and F. L. Vernon, Jr., Ann. Phys. (N.Y.) **24**, 118 (1963).
[19]A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981); Ann. Phys. (N.Y.) **149**, 374 (1983); **153**, 445E (1984).
[20]A. Schmid, Phys. Rev. Lett. **51**, 1506 (1983).
[21]S. A. Bulgadaev, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 264 (1984) [JETP Lett. **39**, 315 (1984)]; Phys. Lett. **A104**, 215 (1984).
[22]S. A. Bulgadaev, Zh. Eksp. Teor. Fiz. **90**, 634 (1986) [Sov. Phys. JETP **63**, 369 (1986)].
[23]H. Grabert and U. Weiss, Phys. Lett. **A108**, 63 (1985).
[24]S. Charkavarty, Phys. Rev. Lett. **49**, 681 (1982).
[25]A. J. Bray and M. A. Moore, Phys. Rev. Lett. **49**, 1545 (1982).
[26]W. Zwerger, Z. Phys. **B53**, 53 (1983); **54**, 87 (1983).
[27]S. Charkavarty and A. J. Leggett, Phys. Rev. Lett. **52**, 5 (1984).
[28]U. Weiss, P. Riseborough, P. Hanggi, and H. Grabert, Phys. Lett. **A104**, 492E (1984).
[29]H. Grabert and U. Weiss, Phys. Rev. Lett. **54**, 1605 (1985).
[30]M. P. A. Fisher and A. T. Dorsey, Phys. Rev. Lett. **54**, 1609 (1985).
[31]J. S. Langer, Ann. Phys. (N.Y.) **41**, 108 (1965); **54**, 258 (1969).
[32]C. Callan and S. Coleman, Phys. Rev. **D16**, 1248 (1978).
[33]L. D. Chang and S. Chakravary, Phys. Rev. **B31**, 154 (1985).
[34]A. I. Vaĭnshteĭn, V. I. Zakharov, V. A. Novikov, and M. A. Shifman, Usp. Fiz. Nauk **136**, 553 (1982) [Sov. Phys. Usp. **25**, 195 (1982)].
[35]L. D. Landau and I. Ya. Pomeranchuk, Zh. Eksp. Teor. Fiz. **7**, 379 (1937).
[36]A. D. Brailsford, J. Appl. Phys. **43**, 1380 (1972).
[37]T. Ninomiya, J. Phys. Soc. Jpn. **36**, 399 (1974).
[38]A. V. Granato and K. Lücke, in: Physical Acoustics, Vol. IV-A (ed. by W. P. Mason), Academic Press, New York (1966), p. 225.
[39]R. Wanner, I. Iwasa, and S. Wales, Solid State Commun. **18**, 853 (1976).
[40]I. D. Calder and J. P. Franck, Phys. Rev. **B15**, 5262 (1977).
[41]Y. Hiki and F. Tsuruoka, Phys. Lett. **A56**, 484 (1976); **62**, 50 (1977); F. Tsuruoka and Y. Hiki, Phys. Rev. **B20**, 2702 (1979).
[42]I. D. Calder and J. P. Franck, J. Low Temp. Phys. **30**, 579 (1978).
[43]V. L. Tsymbalenko, Zh. Eksp. Teor. Fiz. **74**, 1507 (1978) [Sov. Phys. JETP **47**, 787 (1978)].
[44]P. E. Wolf, F. Gallet, S. Balibar, E. Rolley, and P. Nozieres, J. Phys. (Paris) **46**, 1987 (1985).
[45]K. B. Efetov, Zh. Eksp. Teor. Fiz. **78**, 2017 (1980) [Sov. Phys. JETP **51**, 1015 (1980)].
[46]S. V. Iordanskiĭ and S. E. Korshunov, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 466 (1984) [JETP Lett. **39**, 567 (1984)]; S. V. Iordansky and S. E. Korshunov, J. Low Temp. Phys. **58**, 425 (1985).

Translated by Steve Torstveit