Inflationary universe from the point of view of general relativity

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The geometry of the early universe in the process of transition from the vacuum-dominated stage characterized by the equation of state $p = -\varepsilon$ to the radiation-dominated stage is discussed. The description of the geometry of the universe in the $\varepsilon =$ const stage (Guth model) by the nonstationary form of the de Sitter metric with exponentially increasing scale factor $a(t)$, which is interpreted as the scale of the universe, is based on the use of Lagrangian coordinates and a synchronous reference system comoving with the vacuum as the source of the geometry. This is inconsistent with the relativistic definition of the vacuum based on the absence of a distinguished frame of reference associated with it. In inflationary scenarios with a very slowly varying vacuum energy density at the start of the transition the universe remains vacuum dominated, and in the continuation of the $\varepsilon \approx$const stage the quantity $a(t)$ describes mutual separation of test bodies, and in a vacuum-dominated universe this does not define the scale factor. This makes the estimates of the size of the universe at the end of the inflationary stage ambiguous. A study is made of a phenomenological cosmological model that describes the geometry of the universe in the course of the transition from the vacuum-dominated stage to the radiation-dominated stage. In this model it is not assumed that there is a stage with a very slow variation of the vacuum energy density at the beginning of the transition and in which the duration of the transition and the scale of the universe at the end of the transition stage are obtained by matching the transition metric to the metric of the standard Friedmann model and are expressed in terms of observational characteristics of the universe. Comparison of the results with the predictions of inflationary scenarios shows that the question of the determination of the scale factor for a universe that passes through a vacuum-dominated stage in the course of its evolution requires further investigation.

The problem of the initial state of the universe is one of the most intriguing problems of physics and astronomy. The standard Friedmann cosmology describes well the evolution of the universe, but there exist observational facts that it does not fully explain. First, there is the isotropy and homogeneity of the universe on large scales. This problem is usually formulated as the horizon problem. The homogeneous and isotropic Friedmann cosmology describes the geometry of the universe in a system of reference that is comoving with the matter, which moves at each point in a well-defined standard manner as a cosmological fluid with 4-velocity u_α. The time coordinate—the universal time—is chosen in such a way that at each instant of this time the metric is the same at all points and in all directions and is normalized in such a way that $g_{00} = 1$; then it determines the proper time of a "fundamental observer" moving together with the matter with velocity u_α. The absence of distinguished directions gives $g_{0\alpha} = 0$.[1,2]

In the synchronous and comoving system of reference chosen in this manner, the Friedmann metric, written in the Robertson-Walker form, is

$$ds^2=c^2dt^2-a^2(t)\left[dr^2(1-kr^2)^{-1}+r^2(d\vartheta^2+\sin^2\vartheta d\varphi^2)\right]. \tag{1}$$

Here, r, ϑ, φ are the Lagrangian coordinates of the moving matter; the scale factor $a(t)$ determines the distances between any pair of fundamental observers; and the values $k = 0, +1, -1$ correspond to the spatially flat, closed, and open Friedmann models. The metric (1) satisfies the Friedmann equations (Ref. 1, p. 47)

$$\ddot a=-(4\pi G/3c^2)a(\varepsilon+3p), \tag{2a}$$

$$\dot a^2=(8\pi G/3c^2)\varepsilon a^2-kc^2. \tag{2b}$$

The dot denotes differentiation with respect to the time, G is the gravitational constant, c is the velocity of light, ε is the energy density, and p is the matter pressure. At small t, the scale factor behaves as

$$a(t)\propto t^q,\quad q=2/3(1+\beta), \tag{3}$$

where β characterizes the equation of state of the matter, $p = \beta\varepsilon$, and takes values $0\leqslant\beta\leqslant 1$.[1-3] In accordance with (1), during the time from the beginning of expansion to any finite time, t_0 light can traverse only the finite distance

$$R_0=a(t_0)c\int_0^{t_0} a^{-1}(t)\,dt. \tag{4}$$

Because of this, sufficiently distant regions in the universe cannot exchange signals during the course of the expansion, and one must consider why the universe is so homogeneous and isotropic on large scales.

Second, there is the flatness problem, which is as follows. Given the very moderate difference between the currently observed density of the universe and the critical density corresponding to the spatially flat model, it follows that in the limit $t\to 0$ the matter density was close to the critical value with extremely high accuracy. For $k = 0$, the mean matter density in the universe satisfies the relation

$$\rho=\rho_c=3H^2/8\pi G,\quad H=\dot a/a \tag{5}$$

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during the course of the entire evolution of the universe. As follows from Eq. (2b), the deviation of the density of the universe from the critical value varies during the evolution in accordance with the law.

$$\zeta=(\rho_c-\rho)/\rho_c=-kc^2/\dot{a}^2. \tag{6}$$

If we extrapolate the behavior (3) of the scale factor to very small t, it is easy to see that $\zeta\to 0$ as $t\to 0$.

In 1981, to explain the flatness and horizon problems, Guth proposed the model of an inflationary universe[4] based on the properties of phase transitions in gauge theories with spontaneous breaking of the vacuum symmetry. In accordance with this model, there exists in the history of the early universe a stage in which the universe is, as a reuslt of strong supercooling, in a symmetric vacuum state in which the energy density of the relativistic particles is negligibly small compared with the vacuum energy density $\varepsilon = \Lambda c^2/8\pi G = \text{const}$, where Λ is the cosmological constant. In such a situation, the physical state of the universe is characterized by the equation[5,6]

$$p=-\varepsilon, \tag{7}$$

and the geometry is described by the de Sitter metric, which Guth writes in synchronous coordinates in the form (1) for $k = 0$. Then the gigantic exponential increase of $a(t)$ in accordance with the law

$$a(t)=a_0\exp(H_0t),\quad H_0=(\Lambda/3)^{1/2} \tag{8}$$

is interpreted as the inflation of a small causally connected region to a dimension that by many orders of magnitude exceeds the radius of the observable part of the universe, and this solves the horizon problem. After the inflation there must occur phase transitions with powerful energy release and production of entropy, these ensuring ultimately the transition to the radiation-dominated stage of the standard hot Friedmann model.

The flatness problem is solved in Guth's model by the choice from the very beginning of the spatially flat metric with $k = 0$. Generally speaking, the de Sitter metric produced by a vacuum with nonvanishing constant energy density ε can be written in the form (1) for all three values of k.[3,7] We consider the geometry produced by the vacuum with the equation of state (7) and described by the de Sitter metric in more detail.

A universe described by the metric (1) with $k = 0$ and $a(t) = a_0\exp(H_0t)$ is stationary (Ref. 1, p. 135) in the sense that all its local properties are independent of the time. A scale transformation in combination with a shift of the origin,

$$\overline{\overline{r}}=r\exp(H_0t_0),\quad \overline{\overline{t}}=t-t_0, \tag{9}$$

leaves the geometry unchanged. During the entire state described by the solution (8) the equation of state of the vacuum has the form (7) and remains unchanged. Therefore, during the entire stage of exponential inflation the metric (1) can be reduced to the static form described by the de Sitter interval (Ref. 8, p. 353 of the Russian translation)

$$ds^2=(1-\bar{r}^2/a_0^2)c^2dt^2-d\bar{r}^2(1-\bar{r}^2/a_0^2)^{-1}-\bar{r}^2(d\vartheta^2+\sin^2\vartheta d\varphi^2), \tag{10}$$

where

$$a_0^2=3c^2/\Lambda=3c^4/8\pi G\varepsilon_0, \tag{11}$$

by means of the Lemaître-Robertson coordinate transformation

$$r=\bar{r}(1-\bar{r}^2/a_0^2)^{-1/2}\exp(-\bar{t}c/a_0),\quad t=\bar{t}+{}^1/{}_2a_0\ln(1-\bar{r}^2/a_0^2).$$

The situation is not changed if from the very beginning the metric (1) is chosen with arbitrary value of k. As is shown in Ref. 7, the condition $p = -\varepsilon$ is a necessary and sufficient condition for one to be able to reduce the Robertson-Walker metric (1) to the form (10) for all k.

The de Sitter metric written in the form (1) formally satisfies the Friedmann equations written in the synchronous reference system comoving with the expanding matter that creates the given geometry. On the other hand, the equation of state (7) characterizes the vacuum, i.e., a form of matter which is determined by the absence of a distinguished system of reference associated with it. A vacuum or vacuumlike state characterized by the equation of state (7) is completely described from the macroscopic point of view within the framework of general relativity and corrresponds to one of the terms of the classification scheme of energy-momentum tensors in accordance with the types of the algebraic structures (Ref. 5, and Ref. 9, §19). The vacuum energy-momentum tensor

$$T_{ik}^{\text{vac}} = \varepsilon g_{ik} \tag{12}$$

is diagonal in any orthogonal frame, i.e., any system of reference is comoving for the vacuum, the concept of a velocity relative to the vacuum has no meaning, and in this respect a vacuum that possesses a nonvanishing energy density behaves in the same way as an ordinary vacuum with $\varepsilon = 0$. Indeed, for the motion of any test body in a vacuum the system of reference comoving with it is also a comoving system for the vacuum. Therefore, everything that happens in it is independent of its velocity, i.e., the principle of the relativity of the velocity is satisfied, as for motion in an ordinary vacuum with $\varepsilon = 0$.[5,7]

The question arises of whether it is correct to interpret the quantity $a(t)$ defined by the relation (8) as scale factor determining the size of the universe. Such an interpretation is based on the use of Lagrangian coordinates and the synchronous system of reference comoving with the vacuum as the source of the given geometry, but this, in accordance with what was said above, is inconsistent with the definition of the vacuum based on the principle of relativity of the velocity and on the absence for the vacuum of a distinguished comoving system of reference. Therefore, in the vacuum-dominated universe $a(t)$ does not characterize the expansion of the matter of the radius of the universe, as in ordinary cosmology, but only the choice of the coordinate system, a conclusion that is confirmed by the possibility of reducing the metric to the static form.[7] Formally, one can use a synchronous coordinate system (like any other), but questions arise: by what bodies can it be realized as a system of reference?[1)] To whom is it comoving? How can the matter produced in the process of the phase transition know that it has been produced in a universe that has already been inflated to a radius of 10^{70} cm (Ref. 4)? And how can this quantity be measured?

According to Ref. 4, during the entire state of exponential inflation nothing happens to the universe apart from the gigantic increase in $a(t)$. But to an empty (without matter) universe in which nothing occurs except an inflation that can be eliminated by a coordinate transformation it appears natural to apply Birkhoff's theorem,[8] according to which any sperhically symmetric gravitational field in vacuum (and in the presence of a cosmological constant $\Lambda = \text{const} \neq 0$) is static.

According to general relativity, the geometry of spacetime is generated by the motion and distribution of matter. How can matter to which nothing happens give rise to such grandiose geometrical changes? In inflating the vacuum at a constant value of the Hubble parameter H_0, Guth effectively uses the fact that nonstationarity is a characteristic property of the synchronous reference system itself (Ref. 2, p. 368). In such a case, the inflation of the vacuum is a coordinate effect, and the impossibility of associating with the vacuum a distinguished comoving system of reference suggests that this inflation is fictitious.

Thus, the possibility of describing the real universe in a vacuumlike state with $p = -\varepsilon$ and $\varepsilon = \text{const}$ by the metric (1) with an exponentially increasing scale factor $a(t)$ that is interpreted as the radius of the universe raises doubts. Rather, one should conclude that any de Sitter stage characterized by the equation of state $p = -\varepsilon$ when $\varepsilon = \text{const}$ is essentially static.

On the other hand, in the de Sitter universe the gravitational effect of a medium that satisfies the equation of state (7) leads to mutual separation of test bodies (in all coordinates, see Ref. 8, p. 357 of the Russian translation). This property makes the de Sitter universe unstable with respect to transition of the vacuum into matter.[7,10,11] It is this physically nonstationary process that can be meaningfully regarded as the cause of the transition from the static de Sitter universe to the nonstationary expanding Friedmann universe. In other words, if one can speak of a stage of evolution characterized by an expoential dependence of $a(t)$, then the duration and development of this stage must be determined by the produced matter, i.e., such a stage is possible, not before, but during the phase transition, when the vacuum decays and relativistic particles are produced and heated.

It is in this form that the problem is posed in modern scenarios of an inflationary universe (Ref. 12, p. 209). The results and prospects for the development of inflationary models and details of the modern formulation of the inflationary scenarios are presented in the review of Ref. 12. In these scenarios, the right-hand side of the Einstein equations is in fact the center of attention; for in these scenarios one investigates the physical conditions and puts forward definite mechanisms for the realization of a stage in the evolution of the universe in which its physical state is characterized for a certain time by an equation of state close to (7), while the geometry is described by a metric close to the nonstationary form of the de Sitter metric and having the form (Ref. 13, p. 961)

$$a = a_0 \exp\left[\int_0^t H(t)\,dt\right]. \tag{13}$$

At the present time it is assumed that a stage of exponential inflation in the early universe can be produced by quantum corrections to the gravitational field equations that are quadratic in the curvature tensor[3,11] and by various scalar fields (see Ref. 12 and the references given there). An investigation of multicomponent de Sitter stages produced by the joint influence of scalar fields and quantum-gravitational effects was made in Refs. 14 and 15. Questions related to the generality and conditions of realization of the inflationary stages of the expansion are investigated in detail in Ref. 16, where it is shown that for a large class of solutions the universe enters a regime in which the equation of state tends to Eq. (7).

A characteristic feature of modern inflationary scenarios is the existence of a stage of very slow variation of the vacuum energy density or the potential $V(\varphi)$ of the scalar field at the beginning of the transition of the vacuum into matter, the values of ε or $V(\varphi)$ hardly changing during this period. $V(\varphi) \approx V_0$, while the equation of state has the form $p = -V_0$ (see, for example, Ref. 12, pp. 195, 201, 204, and also Ref. 17, where this assertion is formulated directly in the abstract). During all this time the universe expands exponentially, just as before the onset of the transition (Ref. 12, p. 195), and it is precisely the fulfillment of the condition $V(\varphi) \approx \text{const}$ which ensures the inflationary regime. During the stage of the slow variation of the vacuum energy density or the scalar field potential $V(\varphi)$ the expression of the solution to the Einstein equations that determines the geometry of the universe effectively corresponds to fulfillment of the condition $\varepsilon = \text{const}$. For example, in the random inflation scenario $a(t)$ is determined by the expression

$$a = a_0 \exp\left\{\frac{\pi\varphi_1^2}{M_p^2}\left(1 - \exp\left[-\left(\frac{2\lambda}{3\pi}\right)^{1/2} M_p t\right]\right)\right\},$$

which for small t gives (Ref. 13, p.962).

$$a(t) \sim a_0 \exp[(2\pi\lambda/3)^{1/2}\varphi_1^2 t/M_p],$$

where φ_1 is the value of the scalar field at $t = 0$, and $\lambda/4$ is the constant coefficient of the term proportional to φ^4 in the scalar field potential $V(\varphi)$.

In such a case, we must again consider the question of the correctness of interpreting $a(t)$ as the scale factor determining the size of the universe (its radius in the case of the closed model), since as long as the equation of state is near $p = -\varepsilon$ the universe remains vacuum dominated and Eq. (2a) actually describes the mutual separation of test bodies and $a(t)$ characterizes their relative distances. If $a(t)$ is to be interpreted as a scale factor, one must choose as system of reference the space-filling matter itself (Ref. 2, p. 458; Ref. 18, §94; Ref. 19), and such matter still remains a vacuum in the stage $\varepsilon = V(\varphi) \approx \text{const}$. Since the mutual separation of test bodies in a vacuum-dominated universe does not define the scale factor, estimates of the scale of the universe at the moment at which it ceases to be vacuum dominated are ambiguous.

This can be seen by considering a phenomenological cosmological model that describes the geometry of the universe in the process of transition from a vacuum-dominated to a radiation-dominated stage in which one does not assume the presence of a stage of very slow variation of the vacuum energy density ε and the duration of the transition and the size of the universe at the end of the transition are obtained by matching the transition metric to the metric of the standard Friedmann model and they are expressed in terms of contemporary observational characteristics of the universe.

The possibility of transition from a vacuum-dominated de Sitter universe to a nonstationary expanding Friedmann universe due to instability of a vacuum de Sitter universe with respect to transition of the vacuum into matter was first considered in Ref. 7 as an alternative to the inescapability of a singularity in Friedmann cosmology. In Ref. 7, it was shown on the basis of an analysis of the equations of general relativity that a vacuum de Sitter universe 1) can under certain conditions be the final state of a medium that undergoes gravitational collapse, and 2) can be the initial state for any of the three Friedmann models. A cosmological model of the transition was proposed in Ref. 10. In this model, the physical state of the universe in the process of the transition is characterized by the phenomenological transitional equation of state

$$p+\varepsilon={}^4/_3\varepsilon_1(\varepsilon_0-\varepsilon)^\alpha/(\varepsilon_0-\varepsilon_1)^\alpha, \tag{14}$$

which in the limit $\varepsilon\to\varepsilon_0$ goes over into (7) but in the limit $\varepsilon\to\varepsilon_1$ into the ultrarelativistic equation of state

$$p=\varepsilon_1/3. \tag{15}$$

The parameter α can take on values in the interval $0\leqslant\alpha\leqslant1$ and characterizes phenomenologically the rate of transition (corresponding, for example, to the steepness with which the scalar field potential decreases). Although, on the one hand, this equation of state is not tied to any definite transition mechanism, it does, on the other, have a fairly general nature and gives a simple analytic model of the transition, permitting the obtaining of an analytic description of the geometry of the universe during the entire transition stage and the obtaining of predictions for the duration of the transition and the size of the universe at the end of the transition, relating these quantities by a matching of the transition metric to the standard Friedmann metric with observable characteristics of the universe. The only assumption that is made concerning the onset of the transition is that the created matter arises in a causally connected region bounded by a radius a_0 of the event horizon of the de Sitter universe determined by the expression (11). The conservation law $T^a_{k;a}=0$, written down for the vacuum and the created matter in the form[7]

$$T_{\text{mat}}{}^a{}_{k;a}=-\varepsilon_{,k}, \tag{16}$$

controls the exchange of energy and momentum between them. The transition of the vacuum energy into the energy of the ultrarelativistic particles is described by the equation

$$\varepsilon=\varepsilon_0-[4(1-\alpha)\varepsilon_1(\varepsilon_0-\varepsilon_1)^{-\alpha}\ln(a/a_0)]^{1/(1-\alpha)}. \tag{17}$$

In a system of reference comoving with the created matter, $\ddot{a}>0$, the matter is brought into a state of expansion, its density decreases, and by virtue of (16) and (17) the process of vacuum decay becomes irreversible. By this token, the reason for the onset of the expansion is to be found in the properties of the initial state.[7,10,20]

During the transition stage, the connection beween the scale factor and the energy density is characterized by the relation

$$a=a_0\exp[(\varepsilon_0-\varepsilon_1)^\alpha(\varepsilon_0-\varepsilon)^{1-\alpha}/4\varepsilon_1(1-\alpha)]. \tag{18}$$

Since the energy density of the radiation decreases during the subsequent expansion as a^{-4} (Ref. 1, p. 159), the quantity ε_1 satisfies at the end of the transition stage the equation

$$\varepsilon_1a_1^4=\varepsilon_{\gamma c}a_c^4,$$

where in accordance with (2b)

$$a_c^2=(3c^4/8\pi G\varepsilon_c)(1-3H_c^2c^2/8\pi G\varepsilon_c)^{-1},$$

which makes it possible to find the connection beween ε_1 and the contemporary values of the Hubble parameter H_c, the radius of the universe a_c, the energy density ε_c, and the radiation energy density $\varepsilon_{\gamma c}$,[10]

$$a_0^2\exp\left[\frac{\varepsilon_0-\varepsilon_1}{2\varepsilon_1(1-\alpha)}\right]=\frac{3c^4}{8\pi G\varepsilon_c}\left(\frac{\varepsilon_{\gamma c}}{\varepsilon_1}\right)^{1/2}\left(1-\frac{3H_c^2c^2}{8\pi G\varepsilon_c}\right)^{-1}. \tag{19}$$

Setting a_0 equal to the de Sitter radius of the horizon (11), we obtain an equation for the energy density ε_1 at the end of the transition:

$$\frac{\varepsilon_c}{\varepsilon_0}\exp\left[\frac{\varepsilon_0-\varepsilon_1}{2\varepsilon_1(1-\alpha)}\right]=\left(\frac{\varepsilon_{\gamma c}}{\varepsilon_1}\right)^{1/2}\left(1-\frac{3H_c^2c^2}{8\pi G\varepsilon_c}\right)^{-1}. \tag{20}$$

The metric of the transition stage for $\alpha=\frac{1}{2}$ has for the spatially flat model the form

$$a=a_0\exp[B\sin(tc/a_0B)], \tag{21}$$

where the parameter B depends on ε_1 and on the initial vacuum energy density as follows:

$$B=[\varepsilon_0(\varepsilon_0-\varepsilon_1)]^{1/2}/2\varepsilon_1. \tag{22}$$

In the spatially flat model, the initial expansion rate is very high: $H_0=(\Lambda/3)^{1/2}$. One can give arguments for making the choice $\langle\dot{a}_0\rangle=0$. Indeed, since a vacuum is the initial state, correlation of the velocities of the produced particles is impossible and it would be natural to set $\langle\dot{a}_0\rangle=0$.[10] It then follows from Eq. (2b) that $k>0$, which corresponds to a closed universe with $a_0=(3c^2/\Lambda)^{1/2}$. The dependence of the scale factor of the transition metric for the closed model for $\alpha=\frac{1}{2}$ is determined by the function

$$a(t)=a_0e^{f(t)},$$

where

$$f(t)=\begin{cases}2(c/a_0)^2t^2, & t\leqslant3a_0/c,\\ B\sin(tc/a_0B), & t\geqslant3a_0/c.\end{cases} \tag{23}$$

The behavior of the metric at small t is, apart from the coefficient of t^2, described by the same function as in Starobinskiĭ's model[3] at the end of the epoch of exponential inflation, when the vacuum decay and particle production commence (scalaron stage of Starobinskiĭ's model).

We note that in the considered phenomenological model an expanding Friedmann universe arises from part of the vacuum de Sitter universe bounded by the event horizon a_0. Since the event horizon in the de Sitter universe is an eliminable singularity, it does not limit the extension of the vacuum universe, and this indicates the possibility of multiple production of universes from a common initial state.[10]

In contrast to inflationary models in which the scalar 4-curvature of space-time (the unique invariant characteristic of the de Sitter geometry) remains constant during the entire stage of exponential inflation (see, for example, Ref. 3), in the considered transition model it decreases smoothly from the de Sitter value $R_0 = 12a_0^{-2}$ to the value $R_1 = 0$ corresponding to the equation of state (15).

The duration of the transition stage is determined by the relation

$$\varepsilon_0^{1/2} \sin(t_1 c/a_0 B) = (\varepsilon_0 - \varepsilon_1)^{1/2}. \tag{24}$$

For the initial vacuum energy $\mathscr{E}_0 \approx 5 \cdot 10^{14}$ GeV it is $t_1 \approx 2 \times 10^{-32}$ sec. The radius of the universe at the end of the transition depends weakly on the value of the transition parameter α and in the considered case is characterized by $a_1 \sim 10^2$ cm.

Comparison of this value with the predictions of the inflationary universe scenarios, $a_1 \sim 10^{800}$ cm,[12] shows that the question of determining the scale factor for a universe that passes through a vacuum-dominated stage during the course of its evolution requires further investigation.

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[1)] The possibility of associating the system of reference with test particles does not save the situation, since they, by definition, do not affect the metric, and the comoving system of reference in a cosmology that uses Lagrangian coordinates must be comoving with precisely the matter that creates the given geometry.

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