# New possibilities of generation of pump-conjugate beams by stimulated scattering of opposed light waves

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It is shown that an absolute instability of stimulated Rayleigh scattering may develop when oppositely propagating multimode uncorrelated beams travel in a nonlinear isotropic medium. For scattering at small angles this process may generate light beams with complex amplitudes which are the conjugates of the complex amplitudes of the exciting (pump) beams. A theory of this effect is developed allowing for the self-interaction of high-power optical radiation. Estimates are obtained to show that it should be possible to observe experimentally generation of conjugate beams in a medium with a thermal nonlinearity.

**1.** Propagation of high-power optical radiation in an isotropic medium is known to be accompanied by stimulated scattering processes. Among those which have the lowest thresholds for long laser pulses are the Rayleigh, temperature, and Brillouin scattering, and scattering in the wing of a Rayleigh line.[1–4]

Stimulated scattering processes exhibit specific features in the field of oppositely propagating light waves when both waves can participate in the simultaneous excitation of those perturbations of the refractive index of the medium which cause the scattering. Under certain conditions the intensity of the scattered light can grow exponentially with time, i.e., an absolute instability of stimulated scattering may develop. Such an instability is typical of, for example, stimulated Brillouin backscattering in the fields of planar oppositely directed waves.[5,6] It is also known that an absolute instability can develop for other types of stimulated scattering in the fields of plane (both parallel or almost parallel) oppositely directed light waves.[7–9]

A study has also been made[10] of the absolute instability in the field of two light waves with large gradients in their transverse cross section (multimode waves) in the case when their amplitudes are complex conjugates of one another.[10] However, the possibility of absolute instability has not been investigated in the case of stimulated scattering involving arbitrary multimode light beams uncorrelated with one another over the transverse cross section.

We shall show that the oppositely directed beams need not be complex conjugate for absolute instability in stimulated scattering. We shall find the conditions under which perturbations of the refractive index appear in a nonlinear medium and cause simultaneously both stimulated scattering and absolute instability of the waves, even though their transverse structures are not correlated with one another. We shall show that in the case of small-angle stimulated scattering such effects may result in the generation of light beams with complex amplitudes which are conjugate to the complex amplitudes of the exciting (pump) beams. In stimulated backscattering these effects can result in the transfer of energy from one light beam to another and can alter their frequency spectra.

We shall concentrate our attention on the stimulated scattering of oppositely directed waves when the angle between the directions of their propagation is small. This case is interesting from the practical point of view because it is the codirected scattering that has the lowest threshold in the case of long ($\tau_p \gtrsim 10^{-7}$ s) pulses of wide-band laser radiation.[11]

**2.** We shall consider the qualitative pattern of the growth of instabilities in a medium with a cubic nonlinearity in the field of multimode light beams. We shall first consider the process of codirected stimulated scattering of light in the field of one light wave. In this case the interference between two waves—a strong pump wave $\varepsilon_0^+$ and a seed wave $\varepsilon_1^+$ traveling at an angle relative to one another—excites spatially inhomogeneous perturbations (gratings) of the refractive index:

$$\delta n^+ \propto (\varepsilon_0^+)^* \varepsilon_1^+ + \text{c.c.}$$

The Bragg diffraction of the $\varepsilon_0^+$ waves by the $\delta n^+$ perturbations can sometimes result in the growth of the wave $\varepsilon_1^+$ within a nonlinear medium (convective instability).

In the presence of one more oppositely directed pump beam $\varepsilon_0^-$ the convective instability may becomes absolute. We shall demonstrate this as follows. The opposed wave $\varepsilon_0^-$ may, like the wave $\varepsilon_0^+$, interfere with the weak wave $\varepsilon_1^-$ in the course of its propagation and it may excite the refractive index grating

$$\delta n^- \propto (\varepsilon_0^-)^* \varepsilon_1^- + \text{c.c.}$$

These $\delta n^\pm$ gratings may scatter not only the $\varepsilon_0^\pm$ waves which have excited the gratings, but also the $\varepsilon_0^\mp$ waves. Obviously, such parametric scattering of both waves, $\varepsilon_0^+$ and $\varepsilon_0^-$, by the $\delta n^+$ and $\delta n^-$ gratings is most effective when the spatial structures of the gratings are identical. In this case a common grating of the refractive index perturbations $\delta n = \delta n^+ + \delta n^-$ forms in the medium, which establishes feedback between the processes of stimulated scattering of the opposed waves. When the phase relationships satisfy certain conditions, this feedback results in the generation of the $\varepsilon_1^\pm$ waves (absolute instability of stimulated scattering). However, if the spatial structures of the $\delta n^\pm$ gratings are


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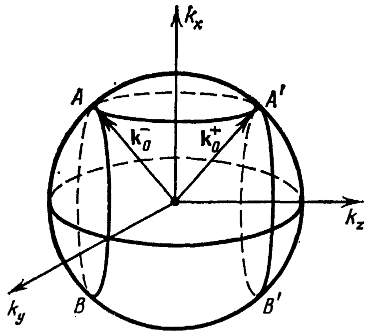


FIG. 1. Circles $AA'$, $AB$, and $A'B'$ represent the geometric loci of the ends of the wave vectors of the scattered radiation which satisfy the conditions for an absolute instability in stimulated scattering when the pump wave vectors $\mathbf{k}_0^+$ and $\mathbf{k}_0^-$ are fixed [see Eq. (1)]. The circle $AA'$ corresponds to the condition $\mathbf{k}_0^+ - \mathbf{k}_1^+ = \mathbf{k}_1^- - \mathbf{k}_0^-$, and the circles $A'B'$ and $AB$ correspond to the condition $\mathbf{k}_0^+ - \mathbf{k}_1^+ = \mathbf{k}_0^- - \mathbf{k}_1^-$.

independent, parametric scattering is negligible and an absolute instability does not develop.

We shall now consider when the spatial structures of $\delta n^+$ and $\delta n^-$ are identical. In the case of plane waves the refractive index perturbations are harmonic functions of the coordinates. Therefore, the required conditions are equivalent to the conditions of the four-wave phase matching:

$$\mathbf{k}_0^+ - \mathbf{k}_1^+ = \mathbf{k}_1^- - \mathbf{k}_0^- \quad \text{or} \quad \mathbf{k}_0^+ - \mathbf{k}_1^+ = \mathbf{k}_0^- - \mathbf{k}_1^-. \tag{1}$$

For a fixed direction of propagation of the pump waves the condition (1) determines the direction of generation of the scattered waves. For example, in the case of the degenerate interaction $|\mathbf{k}_{0,1}^\pm| = k$ it is found that when the pump waves have fixed wave vectors $\mathbf{k}_0^+$ and $\mathbf{k}_0^-$ the geometric loci of the ends of the wave vectors of the scattered radiation $\mathbf{k}_1^+$ and $\mathbf{k}_1^-$ satisfying Eq. (1) are circles representing the intersections of a sphere of radius $k$ (Ewald sphere[12]) by planes passing through the ends of the vectors $\mathbf{k}_0^+$ and $\mathbf{k}_0^-$ and perpendicular to the vectors $\mathbf{k}_0^+ \pm \mathbf{k}_0^-$ (Fig. 1). These conditions apply to the interaction of two pairs of opposed waves. Similar four-wave phase matching conditions have been discussed earlier[13–15] for the codirected interaction of two pairs of waves.

In the case of multimode pump beams $\varepsilon_0^\pm \propto \psi_0^\pm(\mathbf{r})$ and seed waves $\varepsilon_1^\pm \propto \psi_1^\pm(\mathbf{r})$ with large transverse gradients the perturbations of the refractive index in a medium excited because of their interference have a complicated spatially varying speckle structure:

$$\delta n^+ \propto (\psi_0^+)^* \psi_1^+ + \text{c.c.}, \quad \delta n^- \propto (\psi_0^-)^* \psi_1^- + \text{c.c.}$$

The spatial structures of the gratings $\delta n^+$ and $\delta n^-$ are identical if

$$(\psi_0^-)^* \psi_1^- = \psi_0^+ (\psi_1^+)^* \tag{2a}$$

or

$$(\psi_0^-)^* \psi_1^- = (\psi_0^+)^* \psi_1^+. \tag{2b}$$

It follows from the condition (2a) that the spatial gratings are identical (matched) when the pump beams are complex-conjugate: $\psi_0^- = (\psi_0^+)^*$. In the field of such beams we can expect effective growth of the waves with mutually conjugate wavefronts: $\psi_1^- = (\psi_1^+)^*$. It is this case that has been considered earlier in Ref. 10. We wish to draw attention to the possibility that an absolute instability can appear under conditions such that the backwards pump wave $\varepsilon_0^-$ is complex-conjugate not with the strong wave but with the seed wave, i.e., we shall assume that the conditions of Eq. (2b) are satisfied:

$$\psi_1^+ = (\psi_0^-)^*, \quad \psi_1^- = (\psi_0^+)^*. \tag{3}$$

The condition (3) may be obeyed when the seed waves are not imposed, but form from the noise. In this case we can expect excitation of waves $\varepsilon_1^\pm$ with spatial structures which are complex-conjugate to $\psi_0^\pm$. Seed waves with other transverse structures $\tilde{\psi}_1^\pm$ also interfere with $\varepsilon_0^\pm$ and excite gratings $\delta\tilde{n}^\pm$, but the opposed waves are not scattered by these gratings so that the gratings grow much more slowly (in this case an absolute instability changes to a convective one).

We shall consider in detail the instability of oppositely directed beams in the presence of perturbations with complex-conjugate wavefronts. At this stage we shall simply note that our discussion applies to the case of spatially local coupling between the interference fields $(\varepsilon_0^+)^*\varepsilon_1^+ + (\varepsilon_0^-)^*\varepsilon_1^-$ and perturbations of the refractive index of the medium $\delta n^+$. We shall concentrate on this case. It is realized in processes such as transient stimulated thermal and Brillouin scattering (when the waves interact at angles much greater than the divergence of the initial light beams) and also in the case of scattering in the wing of a Rayleigh line.

**3.** We now provide a quantitative description of an absolute instability in the field of oppositely directed multimode light beams. We adopt the simplest model and consider a layer $0<z<l$ of a medium with a local inertial nonlinearity in which the refractive index perturbation induced by the electric field $\mathbf{E}$ of a light wave is described by the equation

$$\left(\frac{\partial}{\partial t} + \frac{1}{\tau_r}\right)\delta n = \frac{n_2}{\tau_r}|\mathbf{E}|^2, \tag{4}$$

where $\tau_r$ is the relaxation time of the perturbations and $n_2$ is a nonlinear coefficient dependent of the properties on the medium. For example, in the case of stimulated thermal scattering of type II, when Eq. (4) describes a perturbation of the refractive index due to isobaric (constant-pressure) thermal expansion of the medium, we have

$$n_2 = (\tau_r \beta / c_p \rho_l)(\partial n / \partial T)_p, \quad \tau_r = \Lambda^2/\chi,$$

where $\beta$ is the absorption coefficient of light; $c_p$ is the specific heat of the medium at a constant pressure; $\rho_l$ is the unperturbed value of the density; $\chi$ is the thermal diffusivity; $\Lambda$ is the characteristic spatial scale of the interference pattern of $|\mathbf{E}|^2$.

Let us assume that spatially coherent beams $\mathbf{E}^+$ and $\mathbf{E}^-$ are incident from both sides of this layer. In the quasioptic approximation the total electric field in the layer can be represented by

$$\mathbf{E}=(\mathbf{x}_0/2)[\varepsilon^+\exp(i\omega t-ikz)+\varepsilon^-\exp(i\omega t+ikz)+\text{c.c.}],$$

where $\varepsilon^+$ and $\varepsilon^-$ satisfy the equations

$$\left[\frac{1}{v}\frac{\partial}{\partial t}\pm\frac{\partial}{\partial z}+\frac{i}{2k}\nabla_\perp^2+\frac{ik\delta n_l(\mathbf{r})}{n_0}\right]\varepsilon^\pm=-\frac{ik\delta n}{n_0}\varepsilon^\pm \quad (5)$$

subject to the boundary conditions

$$\varepsilon^+(z=0,\mathbf{r}_\perp,t)=a_0^+(0)\psi_0^+(0,\mathbf{r}_\perp)f^+(t),$$
$$\varepsilon^-(z=l,\mathbf{r}_\perp,t)=a_0^-(l)\psi_0^-(l,\mathbf{r}_\perp)f^-(t).$$

Here, $n_0$ is the homogeneous part of the linear refractive index of the medium and $\delta n_l$ is the inhomogeneous part; $v$ is the velocity of light in the medium; $\psi$ and $f$ are the normalization functions representing the spatial and temporal structures of the beams $\varepsilon^+$ and $\varepsilon^-$; $\nabla_\perp^2=\partial^2/\partial x^2+\partial^2/\partial y^2$; $\mathbf{r}_\perp=\mathbf{x}_0x+\mathbf{y}_0y$.

If the characteristic width $\Delta\omega$ of the spectrum of the pump wave and the divergence $\theta$ of this wave at the entry to nonlinear medium satisfy the condition $\Delta\omega(\theta^2/v)z_n\lesssim\pi/2$ (where $z_n$ is the characteristic nonlinear interaction length), then the spatial coherence of the radiation does not change during propagation in the nonlinear medium. The solution of Eq. (5) can then be found in the form

$$\varepsilon^\pm=[a_0^\pm(z,t)\psi_0^\pm(z,\mathbf{r}_\perp)+b^\pm(z,t)\tilde{\psi}^\pm(z,\mathbf{r}_\perp)]f^\pm(t\pm z/v), \quad (6)$$

where the functions $\psi_0^\pm(z,\mathbf{r}_\perp)$ describe how the transverse structures of the waves $\varepsilon^\pm$ vary in space during their linear propagation [the functions $\psi_0^\pm$ satisfy Eq. (5) when the right-hand side is equated to zero] and the functions $\tilde{\psi}^\pm$ describe formation of new modes (transverse structures) orthogonal to $\psi_0^\pm$ (in the sense of an integral over the transverse cross section of the nonlinear interaction region):

$$\iint_{S_\perp}[\psi_0^\pm(z,\mathbf{r}_\perp)]^*\tilde{\psi}^\pm(z,\mathbf{r}_\perp)d^2\mathbf{r}_\perp\equiv\langle\psi_0^\pm\tilde{\psi}^\pm\rangle=0.$$

We shall assume that the structures of the beams $\varepsilon_0^+$ and $\varepsilon_0^-$ are uncorrelated: $\langle\psi_0^+\psi_0^-\rangle=0$. It follows from qualitative considerations in Sec. 2 that in a field of uncorrelated opposed pump beams the concurrent interaction of these beams with seed waves may give rise to an absolute instability. The beams $\varepsilon_1^\pm$ which are then generated have the spatial structures $\psi_1^\pm$ that satisfy Eq. (3). Hence, we can identify in the functions $b^\pm\tilde{\psi}^\pm$ the projections on $(\psi_0^\pm)^*$:

$$b^\pm\tilde{\psi}^\pm=a_1^\pm(\psi_0^\mp)^*+\tilde{\psi}_1^\pm. \quad (7)$$

The functions $\tilde{\psi}^\pm$ represent the difference between the modes $\tilde{\psi}^\pm$ and $(\psi_0^\pm)^*$: $\langle\tilde{\psi}_1^\pm(\psi_0^\mp)^*\rangle=0$.

We shall also assume that the interaction of the oppositely directed waves is unimportant compared with the codirected interaction. The situation arises, for example, in processes such as stimulated thermal scattering because of the difference between the diffusion spreading times of the refractive index perturbations: small-scale (with a period $\Lambda=\pi/k$) excited by the interaction between oppositely directed waves and large-scale [$\Lambda=(k\theta)^{-1}$] responsible for the codirected stimulated scattering. The gratings with the period $\pi/k$ may also be suppressed if the polarizations of the opposed pump waves are mutually orthogonal. In this case Eq. (4) becomes

$$(\partial/\partial t+1/\tau_r)\delta n=(|\varepsilon^+|^2+|\varepsilon^-|^2)n_2/\tau_r. \quad (8)$$

Using the expansion of the fields $\varepsilon^\pm$ given by Eqs. (6) and (7), we shall describe the refractive index perturbations $\delta n$ satisfying Eq. (8) by

$$\delta n=\{\rho_0^+(z,t)|\psi_0^+|^2+\rho_0^-(z,t)|\psi_0^-|^2+\rho_1(z,t)(\psi_0^+\psi_0^-)^*$$
$$+[\tilde{\rho}^+(z,t)\tilde{\psi}_1^++\tilde{\rho}^-(z,t)\tilde{\psi}_1^-][(\psi_0^-)^*+(\psi_0^+)^*]+\text{c.c.}\}\frac{n_0}{k}, \quad (9)$$

where the complex coefficients $\rho(z,t)$ represent the amplitudes of the refractive index perturbations excited by individual spatially inhomogeneous structures of the light beams.

We shall consider the interaction of the beams $a_0^\pm\psi_0^\pm$ and $a_1^+(\psi_0^\mp)^*$ with the gratings (proportional to $\rho_1$ and $\rho_0^\pm$) they excite on the assumption that the influence of the remaining terms of the expansions of Eq. (8) and of the corresponding gratings (proportional to $\tilde{\rho}^\pm$) on this interaction is negligible. This hypothesis is typical of the mode theory of stimulated scattering.[16–18] The main condition for validity of this assumption is that the nonlinear interactions in the characteristic diffraction spreading length of one microinhomogeneity of a pump beam be weak: $z_d\propto(k\theta^2)^{-1}\ll z_n$. Applying the mode theory approximation to waves with an average intensity constant over the transverse cross section, we obtain

$$\begin{aligned}
&\pm\partial a_1^\pm/\partial z=ia_1^\pm(\sigma_0\rho_0^\mp+\sigma_1\rho_0^\pm)-ia_0^\pm\rho_1\sigma_1,\\
&\pm\partial a_0^\pm/\partial z=ia_0^\pm(\sigma_0\rho_0^\pm+\sigma_1\rho_0^\mp)+ia_1^\pm\rho_1^*\sigma_1,\\
&(\partial/\partial t+1/\tau_p)\rho_0^\pm=g(|a_0^\pm|^2+|a_1^\mp|^2),\\
&(\partial/\partial t+1/\tau_p)\rho_1=g[(a_0^+)^*a_1^++(a_0^-)^*a_1^-],
\end{aligned} \quad (10)$$

where

$$g=n_2k/n_0\tau_p,\quad \sigma_0=\langle|\psi_0^\pm|^4\rangle(\langle|\psi_0^\pm|^2\rangle)^{-1},$$
$$\sigma_1=\langle|\psi_0^+\psi_0^-|^2\rangle(\langle\psi_0^\pm|^2\rangle)^{-1}.$$

In the case of strongly inhomogeneous beams we have $\sigma_0=2$, whereas for plane waves we have $\sigma_0=1$ and $\sigma_1=1$.

The initial condition for the system (10) is the absence of the refractive index perturbations at $t=0$, i.e., $\delta n(\mathbf{r},0)=0$, and the boundary conditions for the complex amplitudes of the electric fields are

$$a_0^+(z=0)=A^+,\quad a_0^-(z=l)=A^-. \quad (11)$$

We shall also assume that the amplitudes $a_1^\pm$ differ from zero in the planes $z=0$ and $z=l$, but their "seed" values are small: $a_1^+(z=0)\ll A^+$, $a_1^-(z=l)\ll A^-$.

4. We shall first study the case of steady-state scattering when the duration $\tau_p$ of the light pulse is much greater than the relaxation time $\tau_r$ of the refractive index perturbations. The expression for the amplitudes of the refractive index gratings can then be written in the form

$$\rho_0^\pm=g\tau_p(|a_0^\pm|^2+|a_1^\mp|^2),$$
$$\rho_1=g\tau_p(1+i\delta\omega\tau_p)^{-1}[(a_0^+)^*a_1^++(a_0^-)^*a_1^-],$$

where $\delta\omega$ is the frequency mismatch between the interacting waves $\varepsilon_0^\pm$ and $\varepsilon_1^\pm$.

The system (10) has three independent first integrals:

$$C_1=|a_0^+|^2+|a_1^+|^2,\quad C_2=|a_0^-|^2+|a_1^-|^2,$$
$$C_3=a_0^+a_1^-+a_0^-a_1^+\approx A^+R$$

[we have introduced here a coefficient representing the conversion of a pump wave into a scattered wave: $R = a_1^-(0)/A^+$]. Using these integrals we can reduce the system (10) considered in the steady-state approximation to equations for the modulus $|V|$ and the phase $\Phi$ of a function

$$V=[(a_0^+)^*a_1^++(a_0^-)^*a_1^-]|A^+|^{-2}.$$

These equations are

$$d|V|^2/d\bar{z}=2|V|^2W\alpha,\quad d\Phi/d\bar{z}=[1+(1+\alpha^2)(\sigma_0-\sigma_1)]W, \quad (12)$$

where

$$W=(|a_0^+|^2+|a_1^-|^2-|a_0^-|^2-|a_1^+|^2)|A^+|^{-2}$$
$$=[|R|^2+(C_1-C_2)^2/4|A^+|^4-|V|^2]^{1/2},$$
$$\alpha=\delta\omega\tau_r,\quad \bar{z}=Gz/(1+\alpha^2)l,\quad G=g\tau_r|A^+|^2l.$$

The boundary conditions for the system (12) at $z=0$ are as follows:

$$|V(0)|^2=|R|^2(C_2|A^+|^{-2}-|R|^2),$$
$$\Phi(0)=\arg\varepsilon_1^+(0)-\arg\varepsilon_0^+(0). \quad (13)$$

The solutions of the system (12) subject to the boundary conditions (13) are

$$|V(z)|^2=2BD\exp[z\alpha BG/l(1+\alpha^2)]$$
$$\times\{1+D^2\exp[2\alpha GBz/l(1+\alpha^2)]\}^{-1},$$
$$\Phi(z)-\Phi(0)=(1/2\alpha)\ln(|V(z)|^2/|V(0)|^2)$$
$$\times[1+(1+\alpha^2)(\sigma_0-\sigma_1)], \quad (14)$$

where

$$B=[(C_1-C_2)^2/4|A^+|^4+|R|^2]^{1/2},$$
$$D=[B+{}^1/_2(C_1-C_2)|A^+|^{-2}+|R|^2]|V(0)^{-2}.$$

An analysis of the above solution shows that in the case of a positive mismatch $\delta\omega$ an increase in the coordinate $z$ near the boundary $z=0$ increases the amplitude and phase of the function $V$ and, consequently, the complex amplitude $\rho_1$. This results in conversion of the pump energy into a scattered wave ($R\neq0$). The solution given by the system (14) and the second boundary condition (at $z=l$) can be used to find the conversion coefficient $R$ and the frequency mismatch $\delta\omega$. We shall introduce the notation

$$a_{0,1}^-(l)/a_{0,1}^+(l)=R_{0,1}\exp(i\varphi_{0,1}).$$

[In the case of a specularly reflecting boundary the coefficients $R_{0,1}\exp(i\varphi_{0,1})$ represent the reflection coefficients of the pump and scattered waves.] Then, the boundary conditions at $z=l$ become

$$|V(l)|^2=R^2R_0^2F(R_0,\ R_1,\ \varphi_0-\varphi_1),$$
$$\Phi(l)-\Phi(0)=H(R_0,\ R_1,\ \varphi_0-\varphi_1)$$
$$+\ln[|V(l)|^2/|V(0)|^2](1+\alpha^2)(\sigma_0-\sigma_1)/2\alpha,$$

where

$$F(R_0,\ R_1,\ \varphi_0-\varphi_1)=[1+R_0^2R_1^2+2R_0R_1\cos(\varphi_0-\varphi_1)]\{R_0^2[R_0^2+R_1^2+2R_0R_1\cos(\varphi_0-\varphi_1)]\}^{-1},\quad H(R_0,\ R_1,\ \varphi_0-\varphi_1)$$
$$=\arg[R_0(R_1^2+1)+R_1(R_0^2+1)\cos(\varphi_0-\varphi_1)$$
$$+iR_1(R_0^2-1)\sin(\varphi_0-\varphi_1)].$$

It follows from Eqs. (12)-(14) that

$$R_0^2F(1+D^2)^{-2}\{1+D^2\exp[2\alpha GB/(1+\alpha^2)]\}^2$$
$$=\exp[2\alpha GB/(1+\alpha^2)](C_2|A^+|^{-2}-|R|^2), \quad (15)$$
$$2\alpha H=\ln[FR_0^2/(C_2|A^+|^{-2}-|R|^2)].$$

An analysis of the relationships (15) shows that there is a threshold or critical value $G_{cr}$ (or a corresponding value of the pump intensity, since $|A^+|^2\propto G$), below which no pump energy is converted into energy of the scattered wave ($R=0$):

$$G_{cr}=[{}^1/_8\ln^2F+2H]/H(1-R_0^2). \quad (16)$$

Then, the frequency mismatch of the interacting waves becomes

$$\delta\omega_{cr}=(2\tau_rH)^{-1}\ln F. \quad (17)$$

In the case of linear specular reflection by the $z=l$ boundary ($R_1=R_0$, $\varphi_0=\varphi_1=0$) the expression for the threshold simplifies to

$$G_{cr}=\{\ln^2[(1+R_0^2)/2R_0]+4\pi^2N^2\}/2\pi N(1-R_0^2),$$

where $N$ is an integer. The dependence $G_{cr}(R_0)$ is plotted in Fig. 2. The minimum threshold $G_{cr}=7.5$ corresponds to $R\approx0.3$. We shall show later that conversion of the pump energy into energy of the scattered radiation (when $R\neq0$) the scattered wave initially grows exponentially with time. Therefore, $G_{cr}$ represents the threshold of an absolute instability of stimulated scattering.

A specularly reflecting rear boundary need not be present for pump energy to be converted into scattered wave energy. It follows from Eq. (16) that in the absence of a mirror ($R_1=0$), we find that for any ratio of the intensities of the opposed pump beams the expression for the threshold of such conversion becomes

$$G_{cr}=(4\ln^2R_0+4\pi^2N^2)/2\pi N(1-R_0^2).$$

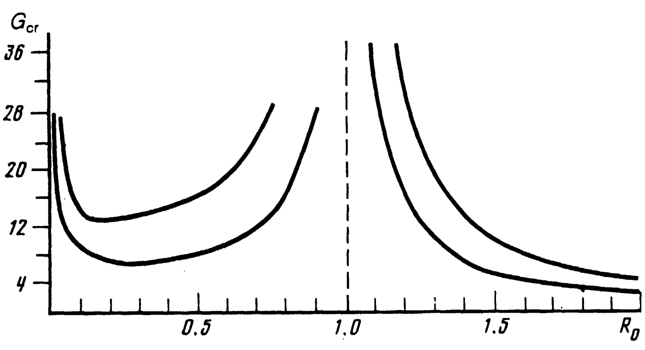


FIG. 2. Dependence of the threshold or critical gain $G_{cr}$ on $R_0$, which is the ratio of the amplitudes of the pump waves at the $z=l$ boundary of a nonlinear layer.

The dependence of $G_{cr}$ on $R_0$ is analogous to that shown in Fig. 2. It should be pointed out that if $G > G_{cr}$ ($R \neq 0$), then two scattered beams are generated and they are conjugate to the oppositely directed pump beams.

When the pump intensity exceeds the threshold for generation of such beams, the coefficient of conversion to the scattered wave rises rapidly [see Eq. (15)] from $|R|^2 = 0$ at $G = G_{cr}$ to its maximum value

$$|R|^2_{max} = {}^1/_4[R_0^2 + R_1^2 + 2R_0R_1\cos(\varphi_0 - \varphi_1)] \quad (18)$$

for $G \gg G_{cr}$.

It follows from Eq. (18) that in the case of specular reflection we can expect total transfer of the pump energy to the scattered wave (when $R_0 = R_1 = 1$ and $\varphi_0 - \varphi_1 = \pi/2$, we have $|R|^2_{max} = 1$).

It follows from the above discussion that under steady-state conditions the intramodulation perturbation of the refractive index (proportional to $\rho_0^\pm$) due to the interference between the components of the pump waves with one another does not affect the conversion of pump energy into scattered wave energy.

5. Under transient conditions ($\tau_p \lesssim \tau_r$) the dynamics of the wave interaction depends strongly on the degree of mutual correlation of the spatial structures of the pump radiation. This is due to the fact that the intramodulation perturbations of the refractive index with complex amplitudes

$$\rho_0^\pm = g\int_0^t [|a_0^\pm(z,t')|^2 + |a_1^\mp(z,t')|^2]\exp\left(\frac{-t+t'}{\tau_r}\right)dt' \quad (19)$$

give rise to additional transient phase corrections to the interacting waves:

$$a_0^\pm \propto \exp\left[\pm i\int(\sigma_0\rho_0^\pm + \sigma_1\rho_0^\mp)\,dz'\right],$$

$$a_1^\pm \propto \exp\left[\pm i\int(\sigma_1\rho_0^\pm + \sigma_0\rho_0^\mp)\,dz'\right].$$

Then, the period of the interference pattern proportional to $(a_0^+)^*a_1^+ + (a_0^-)^*a_1^-$ varies with time, which results in dispersal of the scattering grating $\rho_1$.

For constant pump wave intensities $|a_0^+|^2 = |A^+|^2$ and $|a_0^-|^2 = |A^-|^2$ the system (10) reduces to

$$\partial V/\partial z = iG\rho_1(1-R_0^2)l,$$

$$\left[\frac{\partial}{\partial t} + iG\frac{z}{\tau_r l}(1-R_0^2)(\sigma_1-\sigma_0)\exp\left(-\frac{t}{\tau_r}\right)\right]\rho_1 = V - \frac{\rho_1}{\tau_r}. \quad (20)$$

The dispersal of the scattering grating described by the second term on the left-hand side of the equation for $\rho_1$ is absent in the following cases: 1) when the spatial structures of the oppositely directed beams are completely correlated so that $\sigma_1 = \sigma_0$; 2) when the intensities of the pump waves are equal, $R_0^2 = 1$; 3) under steady-state conditions $t/\tau_r \gg 1$ when an intramodulation grating remains constant in time. In the case of dispersal of the grating $\rho_1$ a study of the system (20) for the stability of $\rho_1 \propto V \propto \exp(\lambda t + i\delta\omega t)$ gives the following dispersion equation (which is valid when $R_0^2 \neq 1$):

$$iG(1-R_0^2)/(1+\tau_r\lambda + i\delta\omega\tau_r) = iH + \ln F^{1/2}. \quad (21)$$

Substituting $\lambda = 0$ in Eq. (21), we can obtain expressions for the threshold values $G_{cr}$ and $\delta\omega_{cr}$, which can easily be shown to be identical with Eqs. (16) and (17).

Above the threshold the total growth rate of the scattered wave is given by

$$\lambda t = (G/G_{cr} - 1)(t/\tau_r), \quad (22)$$

i.e., when the threshold is exceeded the scattered radiation grows exponentially with time (absolute instability).

Dispersal of the grating $\rho_1$ reduces the increment representing an absolute instability. Let us consider, for example, the case of transient ($t/\tau_r < 1$) stimulated scattering of multimode uncorrelated opposed light beams ($\sigma_0 = 2$, $\sigma_1 = 1$). In this case a stability analysis of the system (20) gives the following dispersion equation which is different from Eq. (21):

$$G\{R_0R_1\sin(\varphi_0-\varphi_1) + i[1+R_0R_1\cos(\varphi_0-\varphi_1)]\} = 1 + \lambda\tau_r + i\delta\omega\tau_r. \quad (23)$$

It follows from Eq. (23) that the scattered wave increment is again described by Eq. (22), where

$$G_{cr} = [R_0R_1\sin(\varphi_0-\varphi_1)]^{-1}. \quad (24)$$

The relationship (24) shows that under transient conditions there is no absolute instability in the case of simple reflection by a mirror (if $\varphi_0 = \varphi_1$, then $G_{cr} \to \infty$). An absolute instability is possible, however, if the phase corrections to the pump and scattered waves due to reflection at the $z = l$ boundary are different, i.e., if $\varphi_0 \neq \varphi_1$. In this case because of the constant phase mismatch ($\varphi_0 - \varphi_1 \neq 0$) between the interference patterns proportional to $(a_0^+)^*a_1^+$ and $(a_0^-)^*a_1^-$ the combined scattering grating does not disperse completely. The best conditions for an absolute instability in stimulated scattering are obtained for $\varphi_0 - \varphi_1 = \pi/2$ (selective reflection of optical waves from a mirror).[1)]

We shall point out one more case in which an absolute instability may be realized as a result of multimode pumping under transient conditions. Let us assume that a specularly reflecting boundary ($R_0 = R_1$, $\varphi_0 = \varphi_1$) is located at a large distance $L$ from a nonlinear medium. Then, the boundary conditions at the boundary $z = l$, allowing for the finite time $\tau = 2L/c$ which light requires to travel a distance $2L$, become

$$a_0^-(l,t) = R_0a_0^+(l,t-\tau), \quad a_1^-(l,t) = R_1a_1^+(l,t-\tau). \quad (25)$$

A stability analysis of the system (10), subject to the boundary conditions (25) at the boundary $z = l$, in the case when $R_0 = R_1$ and $\varphi_0 = \varphi_1$ then gives the following dispersion relationship:

$$iG\{1+R_0^2\exp[-(\lambda+i\delta\omega)\tau + iG(1-R_0^2)\tau/\tau_p]\} = 1 - \tau_r\lambda + i\delta\omega\tau_r. \quad (26)$$

Hence, in the case of low values of $\tau$ we can obtain expressions for the total growth rate of the scattered radiation intensity:

$$2\lambda t=(2t/\tau_p)[2(GR_0^2)^2\tau/\tau_r-1][1+(GR_0^2\tau/\tau_r)^2]^{-1}. \qquad (27)$$

Therefore, our analysis demonstrates that an absolute instability may develop in stimulated scattering of opposed multimode beams when they propagate in a nonlinear medium.

**6.** In Secs. 4 and 5 we have considered the process of the interaction between beams with spatial structures satisfying the relationships of Eq. (3). However, in addition to the components $\psi_1^\pm$ of the seed radiation satisfying Eq. (3) at the boundaries of a nonlinear medium, we have included also the orthogonal components $\tilde{\psi}_1^\pm$. We now consider whether self-matched amplification in a field of two oppositely directed multimode beams can occur in the case of such orthogonal structures. This is possible if the projection (integral over the cross section) of a grating $\delta\tilde{n}^+$ on a grating $\delta\tilde{n}^-$ in an arbitrary $z=$ const plane differs from zero and if the sign of this projection is constant along the $z$ axis, i.e., if

$$\iint(\delta\tilde{n}^+)^*\delta\tilde{n}^-\,d^2\mathbf{r}_\perp\propto\iint(\psi_0^+)^*\psi_0^-\tilde{\psi}_1^+(\tilde{\psi}_1^-)^*\,d^2\mathbf{r}_\perp$$

$$\equiv\langle\psi_0^+\psi_0^-\tilde{\psi}_1^+\tilde{\psi}_1^-\rangle$$

has a constant sign. In the case of multimode beams with $\psi_0^+$ and $\psi_0^-$ and $\tilde{\psi}_1^+$ and $\tilde{\psi}_1^-$, when the inhomogeneity statistics can be regarded as Gaussian, the projection of the grating $\delta n^+$ on $\delta n^-$ can be represented by

$$\langle\psi_0^+\psi_0^-\tilde{\psi}_1^+\tilde{\psi}_1^-\rangle=\langle\psi_0^+\tilde{\psi}_1^+\rangle\langle\psi_0^-\tilde{\psi}_1^-\rangle$$
$$+\langle\psi_0^+\psi_0^-\rangle\langle\tilde{\psi}_1^-\tilde{\psi}_1^+\rangle+\langle\psi_0^+(\tilde{\psi}_1^-)^*\rangle\langle(\psi_0^-)^*\tilde{\psi}_1^+\rangle.$$

Since

$$\langle\psi_0^\pm\tilde{\psi}_1^\pm\rangle=\langle\psi_0^+\psi_0^-\rangle=\langle\psi_0^\pm(\tilde{\psi}_1^\mp)^*\rangle=0,$$

it follows that in this case we have

$$\iint(\delta\tilde{n}^+)^*\delta\tilde{n}^-\,d^2\mathbf{r}_\perp=0.$$

Consequently, in the case of a beam with the $\tilde{\psi}_1^+$ structure and one with the $\tilde{\psi}_1^-$ structure we cannot expect amplification in the field of the two pump waves simultaneously. This means that in the case of the components $\tilde{\psi}_1^\pm$ we can have only an independent convective instability for each of the pump waves.

Under steady-state conditions ($\tau_p/\tau_r\gg1$) the gain representing a convective instability of the wave $\tilde{\psi}_1^+$ in a pump field $\psi_0^+$ is governed by the quantity $G$ [see Eq. (12)]. Comparing it with the growth rate of Eq. (22), which governs an absolute instability of the beams in the presence of the perturbations $a_1^+(\psi_0^-)^*$ and $a_1^-(\psi_0^+)^*$, we find that if

$$G>G_{cr}(\tau_r/\tau_pG_{cr}-1)^{-1}$$

the rate of growth of the conjugate component exceeds the rate of growth of the perturbation uncorrelated with the pumping.

Under transient conditions ($\tau_p/\tau_r<1$) each multimode pump wave is separately stable against perturbations.[19,20] Therefore, in this case the absolute instability dominates in the case of two oppositely directed multimode beams.

In the case of a plane pump wave the gain in the case of convective transient stimulated scattering is described by the expression $(2Gt/\tau_r)^{1/2}$. Consequently, the condition for the gain of an absolute instability of a plane wave to be greater than the gain of a convective instability under transient conditions is

$$G>2(\tau_r/\tau_p)G_{cr}^2.$$

In determining the conditions for the components of the scattered radiation conjugate with the pump waves $[\psi_1^\pm=(\psi_0^\mp)^*]$ to dominate the uncorrelated components $\tilde{\psi}_1^\pm$ at the exit from a nonlinear medium we must allow for the difference of the seed values of these components. It is known (see, for example, Ref. 18) that among all the types of noise in stimulated scattering of multimode pump radiation with a divergence $\theta$ and a diameter $d$ the fraction of the power of the component which is conjugate with the pumping is $(\theta/\theta_d)^{-2}$, where $\theta_d$ is the divergence of a single-mode beam with a diameter $d$. Therefore, at the exit from a nonlinear medium the structures conjugate with the pump waves may be distinguishable against the background of the other types of stimulated scattering noise only if the difference between the total growth rates of absolute and convective instabilities exceeds the value of $\ln(\theta/\theta_d)^2$.

**7.** We shall discuss the possibility of experimental observation of the effect predicted in the present study. We note that the adopted model corresponds to several nonlinear mechanisms. One of the possible mechanisms is a thermal nonlinearity in a light-absorbing medium. The thermal nonlinear coefficient of liquids (acetone, alcohol, etc.) is described by the expression

$$g=(0.3\text{–}0.8)\,[\mathrm{cm^2/mW}]\,\beta\,[\mathrm{cm^{-1}}]/\tau_{rb}\,[\mathrm{sec}],$$

where $\tau_{rb}$ is the relaxation time for backscattering ($\tau_{rb}\approx50$ nsec). The relaxation time of the refractive index perturbations caused by heating of the medium when light is absorbed is $\tau_r=\Lambda^2/\chi$. In the case of large-scale gratings $[\Lambda\propto(k\theta)^{-1}]$ responsible for the codirected stimulated scattering this relaxation time can be quite long (for example, if $\theta\approx10^{-2}$ rad, we have $\tau_r\approx5\times10^{-4}$ s). Therefore, in the case of light pulses of $10^{-8}$–$10^{-4}$ s duration such a grating is transient and the absolute instability threshold is given by Eq. (24).

For codirected stimulated scattering of multimode pump beams a significant intensity of the scattered radiation can be expected only if the coefficient of its exponential growth exceeds the value $\ln(\theta/\theta_d)^2$. This condition determines the minimum energy density of the pump radiation at which it should still be possible to observe the effect. If $\theta/\theta_d=30$ and $\tau_p/\tau_r\ll1$, this energy density is given by $w\approx5G_{cr}/gl$. In the case of a layer of a medium with a thermal nonlinearity ($\beta l\approx0.05$, $\tau_{rb}\approx50$ nsec) and selective ($\varphi_0-\varphi_1=\pi/2$) reflection from a specular boundary when the critical increment of the effect given by Eq. (24) is $G_{cr}\approx5$ and the reflection coefficients obey $R_0^2\approx0.2$, this energy density amounts to 50 J/cm$^2$.

This value of the energy density necessary for the observation of scattered radiation structures conjugate with the pump radiation indicates the possibility of observing of this effect. However, one should point out that the need to satisfy very specific conditions for realization of the effect (such as, for example, selective reflection by a specular boundary) and the possibility of competition from other nonlinear effects (such as optical breakdown) are the reasons why there have yet been no reports of experimental realization of generation of conjugate structures in codirected stimulated scattering of oppositely directed light beams in real liquids and gases. Moreover, such effects have been already achieved in the case of the so-called photorefractive nonlinearity.[21] The relevant experiments have been carried out using cw radiation and the appropriate theory,[21] different from the theory in the present study, has been developed in the approximation of steady-state process.

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[1)] Such selective reflection may result from the use of, for example, nonreciprocal polarization elements.

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