Theory of three-wave acoustooptic interaction

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Analytic and numerical studies are reported of transient pulsed regimes of nonlinear acoustooptic interaction of two collinear electromagnetic waves with sound. It is shown that soliton envelopes may form from boundary perturbations. The pumping regimes at the highest and intermediate frequencies are very different. The threshold condition for the conversion of a breather into solitons is found for the latter case. The soliton velocity is governed by pump modulation. An analysis of the soliton velocity is made on the basis of the general Whitham relationships.

Three-wave parametric interactions ($\omega_1 = \omega_2 + \omega$) of electromagnetic waves with vibrations or oscillations of equilibrium media (crystals, plasma, etc.) have been investigated sufficiently thoroughly, both theoretically and experimentally, on the assumption of a constant amplitude of the pump wave.[1,2] The theoretical predictions in respect to acoustooptic interactions can be formulated briefly as follows: laser pumping at a frequency $\omega_1 > \omega_2$ can make a system absolutely (or convectively) unstable under backscattering conditions (stimulated Brillouin backscattering) and convectively unstable in the case of forward scattering. An instability appears when the pump intensity exceeds a certain threshold associated with dissipation, phase mismatch, or escape of waves out of the interaction region, and it is manifested by an exponential increase in the intensity of the Stokes radiation and of sound. However, the approximation of a constant pump wave intensity ceases to be valid after the transient time, when the amplitude of signal waves reaches high values because of the development of the instability. Then all three waves are equivalent (the only distinctive feature of the pump wave being the constant supply of energy from the boundary) and the interaction between them becomes nonlinear. It should be pointed out that the problems of establishment of a steady state is bounded systems have not been investigated sufficiently thoroughly. Whereas the evolution of a system of opposite waves into a spatially inhomogeneous time-independent state is sufficiently well-grounded,[3] in the case of concurrent waves there are still a number of unsolved problems relating to the drift nature of their interaction: firstly this applies to the stabilization of a convective instability allowing for depletion of the pump intensity and formation of steady-state soliton-like pulses; secondly, we have to consider the probability of excitation of solitons and control of their dynamics by external signals. These problems have now become important because of experiments on fiber-optics waveguides in which a system of concurrent waves may be at resonance and the length of the interaction region may be considerable.

A nonlinear acoustooptic interaction (stimulated Brillouin backscattering) and the formation of solitons due to an electrodynamic nonlinearity at very low threshold fields[4] have both been observed in fiber-optics waveguides.

The possibility of propagation of steady-state pulses in an unbounded three-wave system has been demonstrated in Refs. 5 and 6. The integrability of the equations describing this system has been demonstrated on the basis of the inverse scattering problem.[7,8] The results of these investigations generally do not overlap because in Refs. 5 and 6 one of the amplitudes [$\sim \tanh b(x-vt)$] is not localized.

We shall report analytic and numerical studies of transient pulsed regimes of a nonlinear acoustooptic interaction of two parallel electromagnetic waves with sound. We shall demonstrate the possibility of formation of soliton envelopes from boundary perturbations (signals). In the case of pumping at the highest frequency we can expect soliton-like entities to form from a small perturbation because of the development of a convective instability. If the pump frequency is moderate, then solitons are formed only above a certain threshold of the boundary signal amplitude.

We shall assume that an electromagnetic pump wave of frequency ω_1 is incident on the entry to the system and that this wave interacts parametrically with a concurrent electromagnetic wave of frequency ω_2 and an acoustic wave with the difference frequency $\Omega = \omega_1 - \omega_2$. In the optical range of this interaction the frequency of sound ~ 100 MHz is much less than in stimulated Brillouin backscattering. Consequently, the absorption of sound in the length of a system $L \sim 100$ cm may be negligible. In view of the smallness of the difference between the refractive indices Δn, we can ignore also the difference between the velocities of electromagnetic waves in accordance with the inequality $\Delta nL/c\tau_p \ll 1$, where τ_p is the pulse duration. In the case of an exact resonance the system of reduced equations for the wave amplitudes is (the phases are assumed to have reached steady-state values)

$$\left(\frac{\partial}{\partial z}+\frac{n}{c}\frac{\partial}{\partial t}\right)E_{1,2}=\mp\beta_s E_{2,1}U, \quad \left(\frac{\partial}{\partial z}+\frac{1}{s}\frac{\partial}{\partial t}\right)U=\beta_p E_1 E_2,$$
$$\beta_s=(aKk/2\varepsilon)|G|, \quad \beta_p=(a/8\pi\rho s^2)|G|, \tag{1}$$

where a is the photoelastic constant; ε is the permittivity; s is the velocity of sound; K and k are the propagation constants of acoustic and electromagnetic waves; ρ is the density of the

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system; $|G| \leqslant 1$ is a dimensionless factor allowing for the transverse structure of the waves (overlap integral). Here and later we shall ignore the quantities $\sim (s/c) \sim 10^{-5}$ compared with unity. The boundary conditions for the system (1) are

$$E_{1,2}(0,\ t)=E_0 f_{1,2}(t), \quad U(0,\ t)=U_0 f_3(t), \tag{2}$$

where $f_i(t)$ are the given boundary conditons. The values of f_i for the signal and idler waves are assumed to vanish or to decrease sufficiently rapidly on increase in t.

We shall first consider the case of pumping at $\omega_1 > \omega_2$. We shall introduce the following dimensionless quantity

$$A_{1,2}=E_{1,2}/E_0, \quad A_3=(\beta_s U^2/\beta_p E_0^2)^{1/2}, \quad x=z/l_l, \quad \tau=st/l_l,$$

where $l_n = (\beta_s \beta_i E_0^2)^{-1/2}$ is the spatial scale associated with the investigated nonlinearity; $\nu = c/s$. In the simplest case of a one–soliton regime the solution of the system (1) is easily shown to be

$$A_1=-g(\tau-x/\nu)\ \mathrm{th}\,\chi, \quad A_2=g(\tau-x/\nu)\ \mathrm{sech}\,\chi,$$

$$A_3=b\ \mathrm{sech}\,\chi, \quad \chi=b\left[x-\tau+\frac{1}{b^2}\int^{\tau-x/\nu} g^2(\tau')\,d\tau'\right], \tag{3}$$

where $g(\tau - x/\nu)$ is an arbitrary function which—subject to Eq. (2)—is given by $g(\tau) = [f_1^2(\tau) + f_2^2(\tau)]^{1/2}$, and b is an arbitrary constant.

The instantaneous velocity of the center of an acoustic soliton in Eq. (3) (the physical meaning of this velocity will be discussed later) is related to the amplitude b by

$$v=1-g^2(b^2-g^2/\nu)^{-1}<1. \tag{4}$$

since $g = g(\tau - x/\nu)$, the velocity v is a function of the modulation of the pump amplitude so that in the case of sufficiently wide pulses it is possible to control effectively the soliton dynamics. For example, if $f_1(\tau) = \exp(-\alpha^2\tau^2)$ and $b \sim 1$, we have $v = 1 - \exp[-\alpha^2(\tau - x/\nu)^2]$, which makes it possible to delay an acoustic pulse. In the case of the unmodulated pumping corresponding to $g \equiv 1$ the system of equations (3) describing the solution agrees with Ref. 5.

The solution (3) demonstrates that solitons can exist against the background of a pump pulse in an unbounded medium. In practice it is more important to consider the possibility of formation of solitons in bounded media and to determine the dependences of their amplitudes and velocities on the parameters of external signals. [From this point of view the solution (3), which requires special boundary conditions, is of limited interest.] The characteristics of formation of solitons in a semibounded region $x \geqslant 0$ on injection of an acoustic signal of the type

$$A_3(0,\ \tau)=2B\ \mathrm{sech}\ 2B\tau, \quad B=(4\beta_s U_0^2/\beta_p E_0^2)^{1/2}$$

and of unmodulated pumping at the entry $A_1(0,\tau) = 1$ will be illustrated by the following exact solution of the system (1), taken from Ref. 9:

$$A_1=\frac{1-\psi^2}{1+\psi^2}, \quad A_2=\frac{2\psi}{1+\psi^2}, \quad A_3=\frac{2}{1+\psi^2}\frac{\partial\psi}{\partial x}, \tag{5}$$

$$\psi=B(1+B^2)^{-1/2}\ \mathrm{sh}\ [(1+B^2)^{1/2}x]\ \mathrm{sech}\ B(2\tau-x).$$

The Stokes wave A_2 is excited here as a result of the interaction described by $A_2(0,\tau) = 0$. We shall analyze the solution (5) in greater detail for the case when $B \ll 1$. We can easily see that the expressions in Eq. (5) describe the following three stages of formation of excitation. If $x \ll 1$ and $2B\tau \ll 1$ are small, then $\psi \ll 1$; consequently, the expressions in the system (5) become

$$A_1=1, \quad A_2=2B\ \mathrm{sh}\ x, \quad A_3=2B\ \mathrm{ch}\ x$$

and represent the solution known from the theory of constant pumping.[2] However, if $x \gg 1$, then the quantity $\psi \propto \exp\{x - B|2\tau - x|\}$ is exponentially large and in the limit $x - 2\tau \gg B^{-1}$ we have a soliton described by the solution (3) with $b \approx 1$, $\chi = x - v\tau$, and $v = -2B < 0$ (with the additive constant omitted), which describes the formation, in the region defined by $x \gg 1$, of a pump depletion zone and its displacement toward the boundary. The system then behaves nonlinearly. During the third stage when $x \gg 1$ and $2\tau - x \gg B^{-1}$, a soliton of Eq. (3) with an amplitude $b \approx 1$ and velocity $v \approx 2B > 0$, traveling in the positive direction, is established in the system. It should be pointed out that if $B \ll 1$, then the soliton amplitude is $b \approx 1$ and it is independent of the boundary "seed" B. In general, the length and the time of formation of a soliton are inversely proprotional to B and are given by, respectively,

$$l_l(1+B^2)^{-1/2}, \quad l_l[B^{-1}+(1+B^2)^{-1/2}]/2s.$$

At first sight it seems surprising that there are excitations traveling from the interaction region to the boundary in a system of concurrent waves. Therefore, the above result was checked by a numerical experiment involving direct solution of the system (1) for various input signals $f_i(\tau)$. A typical pattern of the evolution of the amplitudes A_i is represented in Fig. 1 for

$$A_1(0,\ \tau)=1, \quad A_2(0,\ \tau)=0{,}2\ \mathrm{sech}\ 2(\tau-3), \quad A_3(0,\ \tau)=0$$

and zero initial conditions. We can see that transient processes occur initially: small perturbations grow and their amplification is limited by depletion of the pump amplitude. The region of characteristic depletion of the pump amplitude travels from the right to the left reaching the entry at $x = 0$, which corresponds to a soliton traveling in the opposite direction, as predicted above. After a time interval $\tau \sim 8$ representing the transient stage, a pulse with a steady-state amplitude travels in the system and it is superimposed on a background of nonsoliton corrections, which are not always small. A soliton pulse travels at a velocity $v \approx 0.8$ related to the amplitude $b = 2.4$ by a characteristic "soliton" relationship (4). An allowance for the attenuation in the system has practically no effect on the solition amplitude, but reduces considerably nonsoliton oscillations. A similar pattern is observed also in the case of optical and acoustic excitations, and it is on the whole in agreement with Eq. (5).

The soliton nature of the solution discussed above will now be demonstrated. Substituting in the system (1) the variables

$$p=\frac{1}{b}\int^{\tau-x/\nu} g^2(\tau')\,d\tau', \tag{6}$$

$$q=b(x-\tau)$$

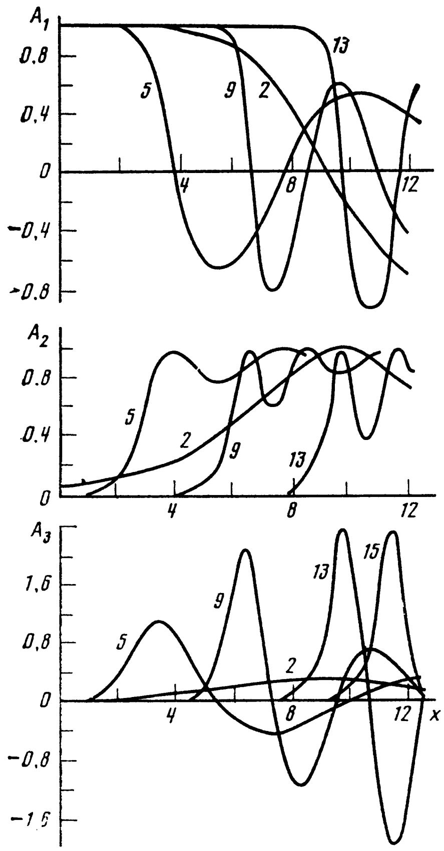

FIG. 1. Results of a numerical calculation of the distribution of the wave amplitudes A_i in a semibounded medium ($x \geqslant 0$) obtained for various moments in time τ (given by numbers alongside the curves) in the case of a nonsoliton input signal.

and the functions

$$A_1 = g\cos\frac{\Phi}{2}, \quad A_2 = g\sin\frac{\Phi}{2},$$
$$A_3 = \left(\frac{\partial}{\partial x} + \frac{1}{v}\frac{\partial}{\partial\tau}\right)\frac{\Phi}{2}, \tag{7}$$

where b is an arbitrary constant and $g = g(\tau - x/v)$ is an arbitrary function. Then, Φ is described on the basis of the system (1) by the sine-Gordon equation

$$\partial^2\Phi/\partial p\,\partial q = \sin\Phi. \tag{8}$$

Equation (8) has been investigated thoroughly in the literature, particularly for systems with distributed Josephson contacts and in the case of a self-induced transparency.[10,11]

One of the remarkable properties of this equation is the existence of solition solutions.[7] For our purpose, we note that the substitutions $\Phi \to \Phi + \pi$ and $p \to -p$ do not alter the form of Eq. (8). We then have $A_1 \to -A_2$. Since the boundary conditions for electromagnetic waves are generally asymmetric (one of them is the pump and it is characterized by $f_i \not\equiv 0$), it follows that if we know one of the solutions of Eq. (8) we can readily obtain the other (complementary) solution for the case of pumping by the other wave. The substitution $p \to -p$ (p is a nonlinear correction to the time and space variables) determines in the final analysis, the existence of subsonic and supersonic perturbations. The latter provide additional opportunities compared with the Josephson and self-induced transparency systems, in which solitons travel at a velocity somewhat lower than the characteristic velocity (which is the velocity of light in the investigated medium).

We can easily see that in the one-soliton regime, we have

$$\Phi = 4\,\mathrm{arctg}\,\psi, \quad \psi = \exp(bp + q/b), \quad 0 < \Phi < 2\pi,$$

and the solution of Eq. (8) expressed in terms of the initial variables is identical with Eq. (3). The two-soliton regime corresponds to the solution described by the system (5). Multisoliton solutions can be explained by the familiar method.[7] It should be pointed out that a direct comparison of these results with those of Ref. 7 is difficult, because in Ref. 7 the condition $v_1 = v_2$ corresponds to $v_1 = v_3$.

We shall now consider the case when the pump wave is of intermediate frequency $\omega_2 < \omega_1$. It follows from the above analysis that the one-soliton solution is obtained from the system (3) by the substitutions $b^2 \to -b^2$ and $A_1 \leftrightarrow -A_2$. Instead of Eq. (4), the soliton velocity is now described by

$$v = 1 + g^2(b^2 + g^2/v)^{-1} > 1, \tag{9}$$

which corresponds to a supersonic perturbation. The solution describing the formation of a soliton from an acoustic signal

$$A_3(0,\tau) = 2B\,\mathrm{sech}\,2B\tau, \quad A_1(0,\tau) = 0, \quad A_2(0,\tau) = 1,$$

has the same form as the solution (5) (after the substitution $A_1 \leftrightarrow -A_2$) and we find that

$$\psi = B(B^2-1)^{-1/2}\,\mathrm{sh}\,[(B^2-1)^{1/2}x]\,\mathrm{sech}\,B(2\tau - x). \tag{10}$$

In view of the factor $(B^2-1)^{1/2}$, the difference between Eqs. (10) and (5) is considerable: a soliton can form only when the amplitude B exceeds a certain threshold condition $B > 1$. We can easily see that when $(B^2-1)^{1/2}x \gg 1$ and $B\,|2\tau - x| \gg 1$, a soliton forms with the following parameters

$$A_1 = \mathrm{sech}\,\chi, \quad A_2 = \mathrm{th}\,\chi, \quad A_3 = b\,\mathrm{sech}\,\chi,$$
$$b = (B^2-1)^{1/2} + \sigma B, \quad \chi = b(x - v\tau), \tag{11}$$
$$\sigma = \mathrm{sign}\,(2\tau - x), \quad v = 2B\sigma/b.$$

In the asymptotic case described by $x > 2\tau$ and $B \gg 1$, we have $b = -\frac{1}{2}B$, which describes a pulse traveling at a velocity $v = 2B^2 \gg 1$ considerably higher than the velocity of sound. At the same time in the region $x < 2\tau$ ($\sigma = 1$) a second pulse is formed and its amplitude and velocity are $b = 2B$ and $v \gg 1$, respectively. Therefore, an above-threshold perturbation splits into two solitons of larger and smaller amplitudes and the velocities of these solitons are very different, so that the distance between them in space increases.

If $B < 1$, then instead of Eq. (10) we find that

$$\psi = B(1-B^2)^{-1/2}\sin[(1-B^2)^{1/2}x]\,\mathrm{sech}\,B(2\tau - x), \tag{12}$$

which corresponds to a periodically modulated traveling packet known as a breather. We can say that the condition

$B = 1$ is the threshold for the conversion of a breather into solitons.

We shall conclude with the following comments. The equilibrium velocity of solitons v can be given a clear physical meaning if we begin with the obvious conditions of conservation of energy of such excitations:

$$\partial P_{1,2}/\partial x+\partial W_{1,2}/\partial \tau=0, \qquad (13)$$

where

$$P_{1,2}=A_3^2\pm A_{1,2}^2/\nu, \quad W_{1,2}=A_3^2\pm A_{1,2}^2.$$

Then, the velocity v is defined, in accordance with the general Whitham relationship,[12]

$$v=v_{1,2}=P_{1,2}/W_{1,2}=1+A_{1,2}^2(\pm A_3^2+A_{1,2}^2/\nu)^{-1} \qquad (14)$$

by analogy with the definition of the group velocity of a linear wave packet. Since we are interested in the velocity of propagation of localized excitations, it is clear that in the case of a pump A_i, we have to take $v = v_j$, where $i \neq j$ and $i, j = 1$ or 2. For example, in the case of the pump A_1, we have

$$v=v_2=1-A_2^2(A_3^2-A_2^2/\nu)^{-1}. \qquad (15)$$

A direct substitution readily shows that in the case of the soliton (3) the velocity (15) is exactly the same as that given by Eq. (4). A similar result applies also if the pump is A_2. Therefore, acoustooptic soliton envelopes under consideration here are strongly nonlinear excitations consisting of acoustic and electromagnetic subsystems. Such excitations travel as a whole, have a definite velocity and energy, and represent a natural nonlinear mode of the three-wave resonance interaction under consideration.

It therefore follows that in the case of resonance acoustoopic interaction of parallel electromagentic waves with sound we can expect formation of soliton envelopes as a result of injection of an input optical or acoustic signal from the boundary. Formation of a soliton is the result of competition between a nonlinearity associated with the absorption of a pump wave and dispersive spreading of wave packets because of the existence of two group velocities (optical and acoustic) in the system. Modulation of the pump near the entry makes it possible to control the soliton dynamics. If $\omega_1 > \omega_2$, then a soliton-like pulse is formed for any "seed" amplitude at the entry and it is the result of evolution of a perturbation in a convectively unstable medium. However, considerable nonsoliton corrections may also be excited. If $\omega_1 < \omega_2$, then the process of formation of solitons is of threshold nature. In the case of the above-threshold amplitude of the injected signal in a system, two concurrent solitons with different amplitudes and supersonic velocities are formed. In view of the existence of a threshold the contribution of nonsoliton corrections is small in the case of the investigated interaction.

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