Magnetic properties of anisotropic type-II superconductors

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Expressions are derived for the anisotropies of the penetration depth, surface impedance, vortex energy, lower critical field H_{c1}, and magnetization curve of a superconductor with a large Ginzburg-Landau parameter $\varkappa$ in the case when the superconducting phase possesses uniaxial or biaxial anisotropy. The properties of the vortex lattice in the vicinity of H_{c1} and in the case when $H_{c1} \ll H \ll H_{c2}$ are discussed.

1. INTRODUCTION

The investigation reported below was prompted by the experimental and theoretical investigations of the superconductivity occurring in the so-called "heavy-fermion" systems: $CeCu_2Si_2$ (Ref. 1), UBe_{13} (Ref. 2), UPt_3 (Ref. 3), etc. According to Refs. 4 and 5, if the superconductivity in these systems is of the nontrivial type, then the transition into the superconducting phase could be accompanied by a lowering of the symmetry and the appearance of a superconducting state possessing uniaxial anisotropy (as in the case of, for example, cubic UBe_{13}). As emphasized repeatedly in Refs. 4 and 6, the properties of such superconductors in a magnetic field and the nature of their anisotropy should provide qualitative information about the nature of the resulting superconducting phase. According to Ref. 4, such characteristics are to be expected if the superconducting state is realized on the basis of solutions for a superconducting order parameter that corresponds to one of the multidimensional representations of the crystal point group. Attempts to measure the anisotropy of the upper critical field[7,8] H_{c2} have so far not furnished an unambiguous answer. Owing either to crystal imperfection, or to the fact that the representation realized is one-dimensional, or because the superconductivity in UBe_{13} and $CeCu_2Si_2$ is, on the whole, of the BCS type, no unusual anisotropy of the H_{c2} field has thus far been found in the vicinity of T_c. The investigations of the magnetization, penetration-depth, etc., anisotropies discussed below could, in their turn, prove to be different, and might even turn out to be a more convenient tool for the study of the nature of these superconductors. Nevertheless, it must be emphasized that there are among the superconductors of this class such systems as UPt_3, U_6Fe (Ref. 9) and $CeCu_2Si_2$ which, on the one hand, intrinsically possess uniaxial crystal symmetry, and on the other, are type-II superconductors with a large Ginzburg-Landau parameter $\varkappa \sim 20$, and that this property is, apparently, an inherent property of the superconductors themselves, and not connected with the presence of defects in them.

Thus far, the anisotropy of the type-II superconductors and the related field-penetration and Abrikosov-vortex-structure-formation characteristics have been suited in Nb-type systems, for which $\varkappa \sim 1$, and which possess cubic symmetry. The $\varkappa$ values for the uniaxial superconductors $InSn_4$ (Ref. 10) and In_3Sn (Ref. 11) are, apparently, of the same order of magnitude, and this naturally makes the theoretical analysis difficult because of the nonlocality of the electrodynamics. The A-15 compounds (e.g., V_3Si) typically have large $\varkappa$ values, but they are also basically cubic, the slight tetragonality in them being possible only as a result of a low-temperature martensitic transition. We wish to point out here that, regardless of whether the superconductivity in the compounds UPt_3, U_6Fe, and $CeCu_2Si_2$ is of the usual type or pertains to the nontrivial, but nondegenerate type (i.e., the solution to the equation for the superconducting gap corresponds to a one-dimensional representation), these materials constitute a class of highly anisotropic type-II superconductors for which the magnetization, etc., anisotropy effects should be strongly pronounced. Indeed, for the UPt_3 superconductor[12] there is as much as a 200% difference between the value of H_{c2} along the hexagonal axis and the value in the plane perpendicular to this axis. Similar values are observed in $CeCu_2Si_2$ (Ref. 13). There are other examples. Thus, the H_{c2} field is known to be highly anisotropic in tetragonal $Hg_{3-\delta}AsFe_6$ (Ref. 14), the quality of whose samples is, apparently, also quite good. Further, certain layered superconductors belong to the class of uniaxial crystals with large $\varkappa$ (Ref. 15).

Below we present some results obtained in a theoretical analysis of the anisotropies of the impedance, penetration depth, lower critical field, magnetization curve, and some other properties. This analysis is substantially simplified by the locality of the electrodynamics of these superconductors, and the anisotropy of each of the indicated properties can therefore be given in the entire temperature range. For the cubic superconductors with small $\varkappa \sim 1$, the problem of vortex penetration into the crystal is usually treated numerically (see, for example, Ref. 16). The H_{c1} anisotropy in a layered superconductor has been computed by Takanaka,[17] but for only two symmetric orientations of the field. As to the anisotropic properties of heavy-fermion superconductors, let us point out again that a detailed investigation would be extremely important for the determination of which superconducting class they belong to.[4]

2. PENETRATION DEPTH AND SURFACE IMPEDANCE

If the London electrodynamics is applicable and the Ginzburg-Landau parameter is large ($\varkappa \gg 1$), then in weak

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fields the problem reduces to the solution of the electrodynamics equations

$$\operatorname{rot}\mathbf{h}=(4\pi/c)\mathbf{j} \tag{1}$$

(here and below $\mathbf{h}$ is the local magnetic field), with the current given by the expression

$$j_i=\frac{\hbar e}{2m}\hat{n}_{ik}^s\left(\nabla_k\Phi-\frac{2e}{\hbar c}A_k\right), \tag{2}$$

where $\hat{n}_{ik}^s$ is the "superconducting-electron density" tensor. Let nontrivial superconductivity develop in a cubic crystal on the basis of one of the multidimensional representations for the order parameter. Then, according to Ref. 4, the superconducting phase acquires a uniaxial anisotropy, and $\hat{n}_{ik}^s$ can accordingly be written in the form

$$\hat{n}_{ik}^s=n_0\delta_{ik}+n'\nu_i\nu_k, \tag{3}$$

where n_0 and n' are, of course, functions of temperature and $\boldsymbol{\nu}$ is the anisotropy axis. The generalization of (3) for the biaxial model is obvious. Expressing $\nabla\Phi-(2e/\hbar c)\mathbf{A}$ from (2) in terms of the curl $\mathbf{h}$ in the Maxwell equation (1), and applying the curl operation again, we obtain

$$\mathbf{h}+\frac{mc^2}{4\pi e^2}\operatorname{rot}(\hat{\mathbf{n}}_s^{-1}\operatorname{rot}\mathbf{h})=0. \tag{4}$$

Further, we can write Eq. (4) with the aid of (3) in the form

$$\mathbf{h}+\delta_0^2[\operatorname{rot}\operatorname{rot}\mathbf{h}+\lambda\operatorname{rot}(\boldsymbol{\nu}(\boldsymbol{\nu}\operatorname{rot}\mathbf{h}))]=0, \tag{5}$$

where

$$(\hat{n}_{ik}^s)^{-1}=\frac{4\pi e^2}{mc^2}\delta_0^2(\delta_{ik}+\lambda\nu_i\nu_k),$$

$$\delta_0^2=\frac{mc^2}{4\pi n_0e^2},\quad \lambda=-\frac{n'}{n_0+n'}.$$

The solution of this equation offers no difficulty. For example, let the axis $\boldsymbol{\nu}$ make an angle ψ with the normal to the surface, and let the attenuation (screening) of the external field $\mathbf{H}$, which is oriented along the surface, occur in the plane defined by $\boldsymbol{\nu}$ and $\mathbf{H}$. Then, defining the impedance according to the relation[18]

$$E_\alpha=\xi_{\alpha\beta}[\mathbf{Hn}]_\beta,$$

we find that the impedance matrix

$$\xi=-\frac{i\omega}{c}\begin{pmatrix}\delta_0(1+\lambda\sin^2\psi)^{1/2} & 0\\ 0 & \delta_0\end{pmatrix}, \tag{6}$$

where the xy-coordinate system has been chosen in the surface plane such that $\nu_y=0$. Let us now assume that both the axis $\boldsymbol{\nu}$ and the field $\mathbf{H}$ lie in the crystal-surface plane. The solution for the field near the surface is

$$h_x=H_x\exp[-|z|/\delta_0],\qquad h_y=H_y\exp[-|z|/\delta_0(1+\lambda)^{1/2}],$$

the z axis being oriented along the normal to the surface. It is remarkable that the magnetic field, as it attenuates with distance into the sample, turns toward or away from the $\boldsymbol{\nu}$ axis according as λ is smaller or greater than zero.

3. THE LOWER CRITICAL FIELD H_{c1} AND THE MAGNETIZATION CURVES

The formulation and solution for this problem differ little from the well-known problems for isotropic superconductors (see, for example, Refs. 19 and 20). Let us, following Ref. 19, write the expression for the electromagnetic part of the free energy of the superconductor in the form

$$F=\frac{1}{8\pi}\int\left[\mathbf{h}^2+\frac{mc^2}{4\pi e^2}\operatorname{rot}\mathbf{h}(\hat{\mathbf{n}}_s^{-1}\operatorname{rot}\mathbf{h})\right]d^3r. \tag{7}$$

The field $\mathbf{h}$ satisfies the equation

$$\mathbf{h}+\frac{c^2m}{4\pi e^2}\operatorname{rot}(\hat{\mathbf{n}}_s^{-1}\operatorname{rot}\mathbf{h})=\mathbf{l}\Phi_0\sum_i\delta(\mathbf{r}-\mathbf{r}_i), \tag{8}$$

where $\mathbf{l}$ is the unit vector along the vortex axis (the induction $\mathbf{B}$) and Φ_0 is the flux quantum. As usual, we have, in accordance with the assumption that $\delta_0\gg\xi_0=\hbar v_F/\pi\Delta(0)$, neglected the vortex core structure. We shall not, for the moment, specify the lattice configuration. We assume, however, that the vortex cores do not overlap, and that the intervortex spacing d can at the same time be smaller than the penetration depth. This condition encompasses the region of fields stretching from the vicinity of H_{c1}, where the field penetrates the crystal in the form of isolated vortices forming a very sparse lattice, up to fields $H\sim H_{cm}$ (H_{cm} is the thermodynamic critical field) in which the fluxes of the individual vortices practically overlap and the magnetic moment of the superconductor is small.

Going over to the Fourier representation in terms of the vortex-lattice vectors, we have

$$\mathbf{h}(\mathbf{r})=\frac{1}{S}\sum_{\mathbf{k}}\mathbf{h}_{\mathbf{k}}e^{i\mathbf{kr}},\qquad \mathbf{h}_{\mathbf{k}}=\int\mathbf{h}(\mathbf{r})e^{-i\mathbf{kr}}\,d^2\mathbf{r},$$

where S is the area of a unit cell, containing, as usual, one flux quantum. The lattice parameters themselves must, in principle, be determined on the basis of energy considerations. Above $\mathbf{k}=\mathbf{a}_iN_i$, where the $\mathbf{a}_i$ are the vectors forming the unit cell. In the Fourier representation, Eq. (8) has the form

$$\mathbf{h}_{\mathbf{k}}-\frac{c^2m}{4\pi e^2}[\mathbf{k},\hat{\mathbf{n}}_s^{-1}[\mathbf{kh}_{\mathbf{k}}]]=\mathbf{l}\Phi_0. \tag{9}$$

The induction $B=\Phi_0/S$ is oriented along the unit vector $\mathbf{l}$ specifying the vortex axis. It is not difficult to verify that, according to (9), the expression (7) for the free energy of a unit volume can be rewritten in the form

$$F=\frac{\Phi_0}{8\pi S^2}\sum_{\mathbf{k}}\mathbf{lh}_{\mathbf{k}}. \tag{10}$$

If by chance $\hat{n}_{ik}^s$ is the anisotropic tensor (3), then the field $\mathbf{h}_{\mathbf{k}}$ has components parallel and perpendicular to the vortex axis $\mathbf{l}$. From Eq. (9) we easily obtain

$$\mathbf{lh}_{\mathbf{k}}=\Phi_0\frac{1+k^2\delta_0^2+\lambda\delta_0^2k^2\nu_l^2}{(1+k^2\delta_0^2)[1+k^2\delta_0^2+\lambda\delta_0^2(k^2-k_x^2\nu_x^2)]}, \tag{11}$$

$$\mathbf{h}_{\mathbf{k}}[\mathbf{k}\boldsymbol{\nu}]=\Phi_0\frac{k_y\nu_x}{1+k^2\delta_0^2+\lambda\delta_0^2(k^2-k_x^2\nu_x^2)}, \tag{11'}$$

where the orthogonal system of coordinates xyl has been chosen such that $\nu_y=0$ (Fig. 1). We see that we need only the expression (11) for the computation of (10).

Let us begin by finding the energy of a single vortex. This case corresponds to our going over in the formulas (10) and (11) to integration over $\mathbf{k}$, with the dominant logarithmic contribution coming from the region $\delta_0^{-1}\ll|\mathbf{k}|\ll\xi_0^{-1}$.

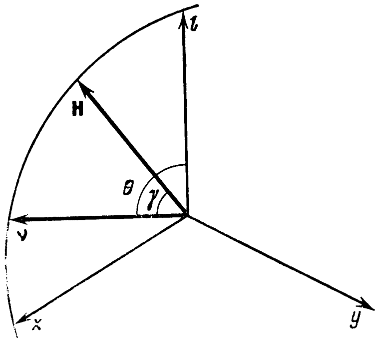

FIG. 1.

By means of a simple change of scale in the d^2k integration, we obtain[1)]

$$\varepsilon=\varepsilon_0\left[1-\frac{\lambda}{1+\lambda}\sin^2\theta\right]^{1/2},\quad \varepsilon_0=\left(\frac{\Phi_0}{4\pi\delta_0}\right)^2\ln\varkappa, \qquad (12)$$

where $\varkappa=\delta_0/\xi_0$ and θ is the angle between **1** and **ν**.

As is well known, to find the relations connecting the macroscopic variables (i.e., the induction **B** and the intensity **H**), it is convenient to use the Gibbs potential $G=F-\mathbf{BH}/4\pi$, and then find the minimum of G for a given **H** field. We immediately obtain as a result the expression for the magnitude of the "critical field":

$$H_{c1}=H_{c1}^0\frac{(1+\bar{\lambda}\sin^2\theta)^{1/2}}{\cos(\theta-\gamma)},$$

$$H_{c1}^0=\frac{\Phi_0}{4\pi\delta_0^2}\ln\varkappa,\qquad \bar{\lambda}=-\frac{\lambda}{1+\lambda}. \qquad (13)$$

Here γ is the angle between **ν** and **H**. The true value of the critical field H_{c1}, i.e., the value at which vortices begin to penetrate the superconductor, corresponds to the minimum value of (13). The relative orientation of the external field and the vortex direction is obtained by varying (13) with respect to the angle θ. As a result, we find

$$\operatorname{tg}\theta_0=\operatorname{tg}\gamma/(1+\bar{\lambda}), \qquad (14)$$

$$H_{c1}(\gamma)=H_{c1}^0\left[\cos^2\gamma+\frac{\sin^2\gamma}{1+\bar{\lambda}}\right]^{-1/2}, \qquad (15)$$

where θ_0 is the angle between **1** and **ν** when $H=H_{c1}(\gamma)$.

Thus, the field **B** at the commencement of its penetration does not coincide in direction with either **ν** or **H**. Evidently, in the opposite limiting case, when the field almost completely penetrates the sample, i.e., when $\xi_0\ll d\ll\delta_0$, the vortices are oriented along the direction of the field **H**. In the logarithmic approximation, we find from the expression (10), (11) for the free energy that

$$\mathbf{H}=\mathbf{B}+\mathbf{1}H_{c1}^0\frac{\ln(d/\xi_0)}{\ln(\delta_0/\xi_0)}(1+\lambda\sin^2\gamma)^{1/2}. \qquad (16)$$

According to (16), in this region of field intensities the magnetic moment $\mathbf{M}=(\mathbf{B}-\mathbf{H})/4\pi$ is small, and is of the order of H_{c1}. Further refinement of this formula, which, in the logarithmic approximation, does not depend on the lattice configuration, requires us to carry out numerical computations for the purpose of comparing the various variants of the vortex lattice, and we did not carry it out.

Let us return to the region of field intensities close to the lower critical field, where the lattice is sparse, and it is sufficient to consider the interaction of only the closest neighbors (see the references cited in Refs. 19 and 20):

$$G=\frac{1}{S}\varepsilon(\theta)-\frac{\mathbf{BH}}{4\pi}+\frac{1}{2}\sum_{ij}U(\mathbf{r}_i-\mathbf{r}_j). \qquad (17)$$

Going over in the formula (10) to the coordinate representation again, and assuming, on account of the linearity of the problem, that the field at a point where a vortex is located is a superposition of the field of the vortex itself and the fields of the nearest neighbors, we see that the computation of the energy of interaction of the closest neighbors requires knowledge of the law according to which the individual-vortex fields fall off at large distances. Going over to the coordinate representation in (11) in the region $r\gg\delta_0$, we obtain (the derivation of this formula is given in the Appendix)

$$\mathbf{lh}=\frac{\Phi_0}{2\pi\delta_0^2}\left(\frac{\pi}{2}\right)^{1/2}\left[\frac{e^{-r/\delta_0}}{(r/\delta_0)^{1/2}}\lambda\nu_l^2\frac{C}{1-C}\right.$$

$$\left.+\frac{1}{\alpha\beta}\frac{e^{-r'/\delta_0}}{(r'/\delta_0)^{1/2}}\left(1-\lambda\nu_l^2\frac{C}{1-C}\right)\right], \qquad (18)$$

where

$$\alpha=(1+\lambda\nu_l^2)^{1/2},\quad \beta=(1+\lambda)^{1/2},$$

$$r=(x^2+y^2)^{1/2},\quad r'=(x^2/\alpha^2+y^2/\beta^2)^{1/2},$$

$$C=\left(\frac{\cos^2\varphi}{\alpha^4}+\frac{\sin^2\varphi}{\beta^4}\right)\left(\frac{\cos^2\varphi}{\alpha^2}+\frac{\sin^2\varphi}{\beta^2}\right)^{-1}.$$

The xy plane is perpendicular to the vortex axis **1**; the anisotropy axis **ν** and the external field **H** lie in the xl plane (see Fig. 2); and the angle ψ is measured from the φ axis.

The choice of one of the asymptotic forms in the expression (18) is dictated by the sign of λ. The simplest case is the one with $\lambda<0$. Then $\exp(-r/\delta_0)\gg\exp(-r'/\delta_0)$, and the second term in the square brackets in (18) can be dropped.

Substituting (18) into the Gibbs potential G, (17), we can find from the condition for a minimum of G the vortex-lattice parameters in the xy plane: the shape of the unit cell, the cell area $S=\Phi_0/B$, as well as the correction $\Delta\theta=\theta-\theta_0$ to the angle between **ν** and **1** {let us recall that, if $H\equiv H_{c1}(\gamma)$, then $\theta=\theta_0$ [see (14)]}. The vortex lattice in the case ($\lambda<0$) under consideration consists, up to distortions of order $(\delta_0/d)\ll1$, of equilateral triangles, with

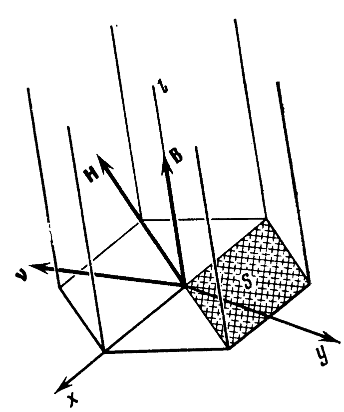

FIG. 2.

$$B=\frac{2\Phi_0}{\sqrt{3}\,\delta_0^2}\left[\ln\frac{H_{c1}(\gamma)}{H-H_{c1}(\gamma)}\right]^{-2}. \tag{19}$$

But the lattice is not degenerate with respect to rotation in the xy plane, and it is oriented as shown in Fig. 2.

The determination of the correction $\Delta\theta$ is greatly facilitated by the fact that θ does not enter into the argument of the exponential function in (18). Consequently, in varying G with respect to θ, we can, to within the same δ_0/d error, drop the term $\partial U/\partial\theta$. Thus, we obtain

$$\theta-\theta_0=\frac{H-H_{c1}(\gamma)}{H_{c1}^0}\sin(\theta_0-\gamma)\,\frac{(1+\lambda\sin^2\theta_0)^{1/2}}{1+\lambda}. \tag{20}$$

Thus, the magnetic field begins to penetrate the crystal at $H=H_{c1}(\gamma)$, and at this point the direction of the vortices, which is given by (14), coincides neither with that of the anisotropy axis $\mathbf{v}$, nor with that of the external field $\mathbf{H}$. As the external-field intensity increases [in the region $H>H_{c1}(\gamma)$], the vortex lattice turns. For small ΔH we have, according to (20), $\Delta\theta\sim\Delta H$, but at $H\gg H_{c1}(\gamma)$ the vortices finally align themselves along the direction of the external field, as follows from (16).

These qualitative results are valid for $\lambda>0$ as well, although this case is somewhat more complicated because of the fact that the anisotropy-related exponential function in (18) predominates. But before we consider the $\lambda>0$ case, it is appropriate to note that the available experimental data on the anisotropy of the upper critical field H_{c2} in UPt_3 (Ref. 12) and $CeCu_2Si_2$ (Ref. 13) allow us to estimate the quantity λ. At low temperatures $\lambda\approx-0.6$. In the vicinity of T_c in $CeCu_2Si_2$ we have, according to Ref. 13, the opposite situation. This case can be considered to be an example of the situations in which the anisotropic superconducting phase with large $\varkappa$ and positive λ should be realized.

Now let $\lambda>0$. Making in (18) (where it is now sufficient to retain only the second term, since $\exp(-r/\delta_0)\ll\exp(-r'/\delta_0)$) the change of variables $x'=x/\alpha, y'=y/\beta$, and accordingly, $S'=S/\alpha\beta$, we find that, in the new variables, the vortex-vortex interaction is isotropic (again up to the pre-exponential factor). Consequently, the vortex lattice in the xy plane can be obtained by stretching a regular hexagonal lattice along the x and y axes and increasing the dimensions along these axes α and β times respectively. The original regular hexagonal lattice should be oriented in the xy plane as shown in Fig. 2. Thus, the vortex lattice in the present case should be made up of not equilateral triangles, but isosceles triangles with a side-to-base ratio equal to $[1+\lambda(3+\cos^2\theta)/4]^{1/2}$. For the induction B we obtain [cf. (19)]

$$B=\frac{1}{\alpha\beta}\frac{2\Phi_0}{\sqrt{3}\,\delta_0^2}\left[\ln\frac{H_{c1}(\gamma)}{H-H_{c1}(\gamma)}\right]^{-2}. \tag{21}$$

The result $\theta-\theta_0\sim H-H_{c1}(\gamma)$ for $H-H_{c1}(\gamma)\ll H_{c1}(\gamma)$ remains valid.

To conclude the present section, let us consider the case of layered superconductors, for which $n_0+n'\ll n_0$ in the "superconducting-electron-density" tensor, or, in terms of the effective-mass tensor of an anisotropic theory of the Ginzburg-Landau type, $m_\perp\gg m_\parallel$, where $m_\parallel$ and $m_\perp$ are the effective electron masses along a layer and in the transverse direction (i.e., in the direction parallel to $\mathbf{v}$) respectively. Notice that in the present case $\lambda=m_\perp/m_\parallel-1\gg1$. In this standard notation the expressions for the vortex energy ε, (12), the lower critical field H_{c1}, (15), and the angle θ_0, (14), at which the field begins to penetrate the crystal in the case when $H=H_{c1}$ have the form

$$\varepsilon=\varepsilon_0\left(1-\frac{m_\perp-m_\parallel}{m_\perp}\sin^2\theta\right)^{1/2}, \tag{12'}$$

$$H_{c1}(\gamma)=H_{c1}^0\left(\cos^2\gamma+\frac{m_\perp}{m_\parallel}\sin^2\gamma\right)^{-1/2}, \tag{15'}$$

$$\operatorname{tg}\theta_0=(m_\perp/m_\parallel)\operatorname{tg}\gamma. \tag{14'}$$

Since $m_\perp/m_\parallel\gg1$, it follows from (14') that $\theta_0\approx\pi/2$, that at $H=H_{c1}$ the vortices are oriented along the layers for practically any direction of the external field. For H slightly higher than H_{c1} the vortices form a lattice in which the mean distance between the nearest neighbors along a layer is much greater than in the perpendicular direction.

4. BIAXIAL CRYSTALS

In this section we present some results concerning crystals whose electron-density tensor possesses biaxial anisotropy. The problem is of some interest, since there could be realized in the transition into the superconducting state in such crystals as $CeCu_2Si_2$ and U_6Fe a two-dimensional representation for the equations for the superconducting gap, and thus give rise to a biaxial symmetry of the new phase.

Let us write the density tensor $\hat{n}_{ik}$ as follows:

$$\hat{n}_{ik}=n_0\delta_{ik}+n_1\mu_i\mu_k+n_2\nu_i\nu_k,\quad \boldsymbol{\mu}\boldsymbol{\nu}=0, \tag{22}$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ are the anisotropy axes. The London equations have the form

$$\mathbf{h}+\operatorname{rot}(\mathbf{\Pi}\operatorname{rot}\mathbf{h})=0,$$

where

$$\Pi_{ij}=\hat{n}_{ij}^{-1}\frac{c^2m}{4\pi e^2}=n_0^{-1}\frac{c^2m}{4\pi e^2}(1+\alpha\mu_i\mu_j+\beta\nu_i\nu_j). \tag{23}$$

From (23) we immediately obtain the penetration-depth anisotropy. By proceeding in much the same way as before, we can easily write out the impedance matrix in coordinates such that $\alpha\mu_x\mu_y+\beta\nu_x\nu_y=0$. It has the form

$$\xi=-\frac{i\omega}{c}\delta_0\begin{pmatrix}(1+\alpha\mu_x^2+\beta\nu_x^2)^{1/2} & 0\\ 0 & (1+\alpha\mu_y^2+\beta\nu_y^2)^{1/2}\end{pmatrix}. \tag{24}$$

Similarly, we can easily generalize the derivation of the formulas for the field H_{c1} and the magnetization. Thus, the field component along the vortex axis, i.e., the component figuring in (10), is given by the expression

$$\begin{aligned}\mathbf{l}\mathbf{h}_k=\Phi_0\{&1+\beta[1-(\mathbf{kv})^2]+\alpha[1-(\mathbf{k}\boldsymbol{\mu})^2]+\alpha\beta[1-(\mathbf{k}\boldsymbol{\mu})^2-(\mathbf{kv})^2]\\&-2(\mathbf{l}[\mathbf{k}\boldsymbol{\mu}])(\mathbf{l}[\mathbf{kv}])\alpha\beta(\mathbf{k}\boldsymbol{\mu})(\mathbf{kv})-\alpha(1+\beta[\mathbf{kv}]^2)(\mathbf{l}[\mathbf{k}\boldsymbol{\mu}])^2\\&-\beta(1+\alpha[\mathbf{k}\boldsymbol{\mu}]^2)(\mathbf{l}[\mathbf{kv}])^2\}\{\delta_0^2k^2+\delta_0^2\beta(k^2-(\mathbf{kv})^2)+\delta_0^2\alpha[k^2\\&-(\mathbf{k}\boldsymbol{\mu})^2]+\alpha\beta[k^2-(\mathbf{k}\boldsymbol{\mu})^2-(\mathbf{kv})^2]\}^{-1},\quad k\delta_0\gg1.\end{aligned} \tag{25}$$

And from (25) we obtain for the vortex energy the expression

$$\varepsilon=\varepsilon_0\{1+\beta\sin^2\varphi_1\cos^2\varphi_2+\alpha\sin^2\varphi_1\sin^2\varphi_2-\alpha\beta\cos^2\varphi_1 +\alpha\beta\cos^2\rho(1-\sin^2\varphi_1\sin^2\varphi_2)(1-\sin^2\varphi_1\cos^2\varphi_2)\}pq, \quad (26)$$

where the anisotropy axes $\mathbf{v}$ and $\boldsymbol{\mu}$ and the direction $\mathbf{z}=[\mathbf{v}\boldsymbol{\mu}]$ are taken to be the coordinate axes. The angles φ_1 and φ_2 are the polar and azimuthal angles of the vector $\mathbf{1}$ in this coordinate system, and the xyl coordinate system, which is fixed to the vortex, has been chosen such that $\mathbf{v}$ lies in the xyl plane. Further,

$$\cos\rho=[(1-\sin^2\varphi_1\cos^2\varphi_2)^{1/2}\cos\varphi_1-\sin^2\varphi_1\cos^2\varphi_2]\times(1-\sin^2\varphi_1\sin^2\varphi_2)^{-1/2},$$

$$p^2=(a\cos^2\varphi_0+b\sin^2\varphi_0+c\sin 2\varphi_0)^{-1},$$

$$q^2=(a\sin^2\varphi_0+b\cos^2\varphi_0-c\sin 2\varphi_0)^{-1},$$

$$\mathrm{tg}\, 2\varphi_0=2c/(a-b),$$

$$a=1+\beta(1+\alpha)\sin^2\varphi_1\cos^2\varphi_2-\alpha(1+\beta)\times(1-\sin^2\varphi_1\sin^2\varphi_2)\sin^2\rho,$$

$$b=1+\beta(1+\alpha)-\alpha(1+\beta)(1-\sin^2\varphi_1\sin^2\varphi_2)\cos^2\rho,$$

$$c=\alpha(1-\beta)\sin\rho\cos\rho(1-\sin^2\varphi_1\sin^2\varphi_2).$$

For $\alpha=0$ we have the results obtained in the preceding sections for uniaxial crystals.

For $H\gg H_{c1}$, the magnetic-field intensity in the superconductor is given by

$$\mathbf{H}=\mathbf{B}+\mathbf{1}H_{c1}{}^0[\ln(d/\xi_0)/\ln(\delta_0/\xi_0)]\{1+\beta\sin^2\varphi_1\cos^2\varphi_2 +\alpha\sin^2\varphi_1\sin^2\varphi_2-\alpha\beta\cos^2\varphi_1+\alpha\beta\cos^2\rho(1-\sin^2\varphi_1\sin^2\varphi_2)(1-\sin^2\varphi_1\cos^2\varphi_2)\}pq, \quad (27)$$

where all the angle-dependent quantities are given in the $\mathbf{l}\|\mathbf{H}$ coordinate system. Similarly to the case of a uniaxial crystal, the angle between the vortices and the external field $\mathbf{H}$ is of the order of $\Delta\theta\sim H_{c1}^{0}/H\ll 1$.

In order to rewrite the results obtained in terms of the mass tensor m_{ik}, we must make the change of variables:

$$m_{[\nu\mu]}=m_0,\quad m_\mu=m_0(1+\alpha),\quad m_\nu=m_0(1+\beta).$$

5. CONCLUSION

Thus, we have obtained for uniaxial and biaxial superconductors a number of results that describe in the London limit the anisotropies of the principal characteristics of a superconductor. These expressions could have been immediately used to describe the anisotropy of, for example, the hexagonal compound UPt_3, or the tetragonal compound $CeCu_2Si_2$. We said above that these compounds are apparently of interest simply because of the fact that they are type-II superconductors, to which the London electrodynamics is applicable, and are therefore materials whose anisotropic characteristics can all be expressed in closed analytic form. Of particular interest, it seems to us, are the vortex-penetration characteristics for these superconductors, the measurement of the magnetization curves, and, perhaps, the direct observation of the vortex lattice. Also, the above-presented formulas should be used to identify the superconducting classes in the "heavy-fermion" systems, since superconducting phases with lowered symmetries could be realized among these classes in the indicated systems. Then all the formulas corresponding to uniaxial crystals would be directly applicable to the unusual phases in the cubic UBe_{13} crystal (let us recall that, according to the classification proposed in Ref. 4, the superconducting phase in a cubic crystal could possess threefold or fourfold axes).

The formulas obtained for biaxial superconductors could, in their turn, be used for the investigation of uniaxial materials, in which two-dimensional representations[4] admit, for example, of even the appearance of twofold axes. We mentioned above that measurements of the anisotropy of the upper critical field H_{c2} in the vicinity of T_c (Refs. 7 and 8) have so far yielded a negative result. It seems to us, however, that this answer ought not to be taken to be final, since the samples used in these investigations were not of very high quality, and the transitions in the majority of cases extended over temperature ranges broader than the region of applicability of the Ginzburg-Landau theory. If we assume that the transition width is connected with the inhomogeneity of the material, then, besides the regions for which the temperature is close to the transition temperature, there can be in the sample regions where the local transition temperature is markedly higher than the experimental temperature, and are therefore already in a developed superconducting phase. These regions, oriented, perhaps, randomly, could, being in one of the anisotropic phases, simulate isotropic behavior. Some "upper-critical-field" anisotropy, defined with respect to the middle of the resistance transition in a magnetic field, has been detected in $CeCu_2Si_2$ by Aliev *et al.*,[8] but its origin and connection with the foregoing ideas remain for the present obscure.

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APPENDIX

The interaction between the vortices is determined by the asymptotic form of the field at $r\gg\delta_0$, i.e., we must find the asymptotic form of the Fourier integral of $\mathbf{1h_k}$ [see (11)]. Let us begin by finding the asymptotic form in coordinate representation for the inverse Fourier transform:

$$\Phi_0\{(1+k^2\delta_0^2)[1+k^2\delta_0^2+\lambda\delta_0^2(k^2-k_x^2\nu_x^2)]\}^{-1}. \quad (\mathrm{A1})$$

As to $\mathbf{1h_k}$, its inverse follows from (A1), since the difference between them amounts to differentiation of the latter with respect to the coordinates.

The asymptotic form in (A1) is given by the poles of

$$\frac{\Phi_0}{2\pi\delta_0^2}\left(\frac{\pi}{2}\right)^{1/2}\frac{e^{-r/\delta_0}}{(r/\delta_0)^{1/2}},\quad \frac{\Phi_0}{2\pi\delta_0^2}\left(\frac{\pi}{2}\right)^{1/2}\frac{1}{\alpha\beta}\frac{e^{-r'/\delta_0}}{(r'/\delta_0)^{1/2}}$$

[the notation is the same as in (18)]. For the field component along $\mathbf{1}$ we have

$$h_l(\mathbf{r})\approx\frac{\Phi_0}{2\pi\delta_0^2}\left(\frac{\pi}{2}\right)^{1/2}\left[M\frac{e^{-r/\delta_0}}{(r/\delta_0)^{1/2}}+N\frac{1}{\alpha\beta}\frac{e^{-r'/\delta_0}}{(r'/\delta_0)^{1/2}}\right],$$

where M and N are, as indicated above, uniquely determined by differentiating with respect to the coordinates and requiring that the usual isotropic result be recovered in the limit as $\lambda\to 0$. Thus, we obtain

$$M=-\frac{C}{1-C},\quad N=\frac{1}{1-C},\quad C=\frac{\alpha^{-4}\cos^2\varphi+\beta^{-4}\sin^2\varphi}{\alpha^{-2}\cos^2\varphi+\beta^{-2}\sin^2\varphi}.$$

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