Static susceptibility of crystals containing magnetically ordered regions in the neighborhood of dislocations

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We investigate the fluctuating domain structure of cylindrical magnetically ordered regions (MOR) which appear in the deformed regions of a crystal close to dislocations. The MOR partition function is evaluated using a functional integral whose calculation reduces to finding the smallest eigenvalue ε of a one-dimensional Schrödinger equation for a particle in a periodic field with a two-well potential in each unit cell. We obtain an analytic expression for ε in the limiting cases of large and small effective masses of the particles; the intermediate masses are obtained by numerical calculations. This allows us to determine the free energy, magnetic moment, and susceptibility of the MOR as functions of the external magnetic field, temperature and crystal parameters in single-crystal and polycrystalline samples which contain dislocations. It is found that the MOR susceptibility depends strongly on field in comparatively weak fields, and decreases monotonically as the field increases. Near the Curie point, for appreciable dislocation densities, the susceptibility can be comparable to that of an ideal crystal. We discuss the possibility of formation of a dislocation dipole glass in an MOR system in the vicinity of non-rectilinear dislocations, or in the presence of a nonuniform distribution of the dislocation lines.

The deformations of a crystal near dislocations are quite large and affect substantially the exchange interaction of its atoms. As a result, regions with large local values of magnetization can be produced in the crystal near dislocation lines noticeably above the Curie point. For real crystals which contain dislocations, the formation of such regions alters qualitatively the experimental picture as regards second-order phase transitions.[1]

Magnetically-ordered regions can in principle appear near dislocations even in crystals where magnetic order does not occur at all in the absence of dislocations. They can be due to broken bonds between atoms which lie along the dislocation lines,[2] to restructuring of the electronic spectrum near the dislocation cores in paramagnetic metals,[3] and to the magnetic impurity atoms of the Cottrell atmospheres.

The transverse dimensions of an MOR usually turn out to be considerably smaller than other characteristic lengths in the problem under investigation. Therefore these regions are effectively one-dimensional; in the absence of an external magnetic field they are broken up into elongated domains with different magnetization directions. Under the action of an external field, the sizes of these domains and the directions of their magnetization can change, which causes a large magnetic susceptibility corresponding to the giant magnetic moments of the domains (in analogy with super-paramagnetism). Order-of-magnitude estimates of these susceptibilities are given in Ref. 1, where it is shown that despite the small volume of the MOR's, their contributions to the susceptibility near dislocations can be comparable to that of the remaining part of the crystal. In this paper we develop a quantitaive theory of the dislocation contribution to the susceptibility and of its dependence on the field.

Because the model under discussion is one-dimensional, we can calculate the MOR partition function in an external field by using a method, developed for this purpose,[4–7] to calculate a certain related functional integral. In this method the calculations of this integral is formally reduced to determining the ground-state energy for a certain Schrödinger equation. In the present case this equation describes the motion of a particle in a one-dimensional periodic potential which is a function of the external magnetic field. In the limiting cases of weak and strong bonds, this ground state energy can be determined analytically, while in the general case a numerical calculation is needed. This makes it possible to find the partition function, the magnetization, and the magnetic susceptibility over a wide range of values of the parameters of the problem, of the temperature, and of the field. The obtained dependence of the magnetization on the magnetic field intensity turns out to be quite nonlinear even in comparatively weak fields, considerably weaker than in ideal crystals.

In Section 1 below we formulate a model of MOR's with variable directions of magnetization, and obtain an expression for its free energy. In Section 2 the MOR partition function is determined with the help of the functional-integration method mentioned above. In Section 3 the expressions we have obtained are used to study the dependence of the magnetization and susceptibility of a crystal with dislocations on temperature and field. In Section 4 we discuss the possibility of creating a dislocation dipole glass near the Curie point in systems with non-rectilinear dislocation lines or with inhomogeneous dislocation densities.

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1. MODEL AND STRUCTURE OF THE ORDERED REGIONS

The radius r_0 of an MOR produced in the vicinity of a dislocation is quite large near the Curie point, and substantially exceeds the correlation r_c in an ideal magnet.[1] This allows us to introduce a local Curie temperature $T_C(\mathbf{r}) \equiv T_C[u_{ij}(\mathbf{r})]$, which is a function of the strain $u_{ij}(\mathbf{r})$ at a given point. The local values of the thermodynamic quantities in an ideal crystal are functions of $\tau = (T - T_C^0)/T_C^0$ (T_C^0 is the Curie temperature of the ideal crystal); in the case under investigation here, they will be functions of $\tau - \tau_C(\mathbf{r})$, where for a rectilinear dislocation

$$\tau_C(\mathbf{r}) = [T_C(\mathbf{r}) - T_C^0]/T_C^0 = \varkappa u_{ii},\ u_{ii} = bf(\vartheta)/r_\perp. \qquad (1)$$

Here we assume as usual that $T_C(\mathbf{r})$ is linearly dependent on the dilatation u_{ii} (with a proportionality coefficient $\varkappa T_C^0$, b is the length of the Burger's vector, $r_\perp$ is the distance to the dislocation line, and $f(\vartheta)$ is a dimensionless angle-dependent factor [$\max f(\vartheta) \sim 10^{-1}$].

In particular, the magnetization in an MOR varies with radius as $[\tau_C(\mathbf{r}) - \tau]^\beta$, falling to zero at the boundary of the region defined by $r_\perp = r_0(\vartheta)$, where

$$r_0(\vartheta) = r_0 f(\vartheta),\ r_0 = \varkappa b/\tau. \qquad (2)$$

Along the dislocation line (the Z axis) the magnetization fluctuates (see Ref. 1). As will be shown below, the characteristic length l for this fluctuation (i.e., the thickness of a domain wall) can be appreciably larger than $f(\vartheta)r_0$. The direction of the magnetization with an MOR cross section is constant, so that the problem can be regarded as effectively one-dimensional and we can consider the local values of thermodynamic quantities averaged over the MOR cross section, which depend only on z. We develop below a quantitative theory just for such a case: $l \gg r_0 f_m$ [$f_m = \max f(\vartheta)$], in which the direction of magnetization does not depend on the transverse coordinate $\mathbf{r}_\perp$. The case $l \lesssim r_0 f_m$, however, in which the direction $\mathbf{M}(\mathbf{r})$ depends significantly on $\mathbf{r}_\perp$, will be considered only qualitatively.

If only the direction of the vector $\mathbf{M}(z)$ averaged over the magnetization cross section varies along the Z axis, while the vector length remains fixed, then the volume energy density per unit length φ_e calculated along the Z axis increases by an amount

$$Sg(d\mathbf{M}/dz)^2/M^2 = Sg(\nabla\alpha)^2,$$

where S is the cross section area of the MOR, α is the angle between $\mathbf{M}$ and Z, and $g \sim kT_C/r_c$. Here $g(d\mathbf{M}/dz)^2/M^2$ and $\mathbf{M}$ are obtained by averaging $g(\mathbf{r}_\perp)(\nabla\mathbf{M}(\mathbf{r}_\perp))^2/M^2(\mathbf{r}_\perp)$ and $\mathbf{M}(\mathbf{r}_\perp)$ over the MOR cross section, i.e.,

$$S = \xi_1 10^{-2}\varkappa^2 b^2/\tau^2,\ g = \xi_2 (kT_C/r_{c_0})\tau^\nu,\ M = \xi_3 M_0 \tau^\beta, \qquad (3)$$

where $\xi_i \sim 1$; ν and β are the critical exponents, r_{c_0} is the radius of the exchange interaction, and M_0 is the saturation magnetization for $T = 0$.

A deviation of the direction of magnetization of a cylindrical MOR from the Z axis leads also to a growth in the magnetostatic energy density U_m. In the case under investigation here, when $l \gg r_0 f_m$, the magnetic charges connected with variation of the magnetization in directions transverse to the MOR axis give rise to a much stronger field $\mathbf{H}_\perp$ than the charges with density $-dM_z/dz$ which are connected with variation of $\mathbf{M}$ in the longitudinal Z direction. Thus we can neglect the field of these latter charges and use only the field $\mathbf{H}_\perp$ to calculate the energy U_m. Since we are not interested in the precise value of numerical factors in U_m, we will assume that the MOR cross section in the XY plane is ellipsoidal, and neglect the inhomogeneity of $\mathbf{M}(\mathbf{r}_\perp)$ in this cross section. Then

$$U_m = 2\pi(n_x M_x^2 + n_y M_y^2),$$

where n_x and n_y are the demagnetization coefficients for a model with constant $\mathbf{M}(\mathbf{r}_\perp)$ over the MOR cross section; if the inhomogeneity of $\mathbf{M}(\mathbf{r}_\perp)$ in the MOR cross section is included, n_x and n_y include correction factors of order unity. In the case investigated below of cubic ferromagnets near the Curie temperature, the magnetic-anisotropy energy can usually be neglected compared to the magnetostatic energy.

For simplicity, we will henceforth limit ourselves to the case of a strongly anisotropic MOR cross section (caused, e.g., by elastic anisotropy of the crystal), for which $n_x \ll n_y$. In this case,the magnetization fluctuates in practice in the XZ plane (this remains qualitatively correct also when $n_x \sim n_y$). The change in the free energy for an MOR of length $\mathscr{L}$, due to rotation of the vector $\mathbf{M}(z)$ as determined by a definite function $\alpha(z)$, and due to the interaction of the magnetization with the external field $\mathbf{H}$ takes the form

$$\Delta F[\alpha(z)] \equiv \mathscr{H}[\alpha(z)] = S\int_0^{\mathscr{L}} [g(d\alpha/dz)^2 + 2\pi n_x M^2 \sin^2\alpha - \mathbf{MH}]\,dz. \qquad (4)$$

Fluctuations can be due not only to rotation of the vector $\mathbf{M}(z)$, but also to changes in its modulus. These latter fluctuations lead to an increase of φ_e on the order of $SkT_C^0 v^{-1}\tau^\gamma(\delta\eta)^2$, where v is the volume of the unit cell, γ is the critical exponent for the susceptibility, and $\eta = M/M_0$. Thus, the appearance of a domain structure in which the modulus of $\mathbf{M}(\mathbf{r})$ and with $|\delta\eta| \sim \eta$ in the domain walls, will be therodynamically less advantageous than the formation of a structure connected with a rotation of $\mathbf{M}(\mathbf{r})$, if

$$(kT_c\tau^\gamma/\pi n_x v M_0^2) \gg 1. \qquad (5)$$

Henceforth we will investigate cases in which condition (5) holds, and the magnetization inhomogeneities are caused by rotation of $\mathbf{M}(z)$. When the inequality (5) is reversed, the vector $\mathbf{M}(z)$ remains almost collinear with the Z axis and fluctuates in magnitude and sign. A similar problem was investigated in Ref. 7. Equation (4) applies not only to MORs produced near dislocations somewhat above the Curie point of the ferromagnet, but also to MORs in nonmagnetic crystals (where they are due to magnetic impurity atoms or to variations in the local characteristics of the crystal near the dislocation lines). We need only keep in mind that in this case S, g, and M^2 depend weakly on temperature and in (5) we have $\tau \sim 1$, while M_0 and kT_C refer only to the region of the dislocation core.

Expression (4) corresponds to a fixed angular distribution $\alpha(z)$ in the MOR. The possibility of fluctuations of $\alpha(z)$ decreases the free energy of the system. A fluctuating domain structure can therefore arise in the MOR. The character of this domain structure in the absence of an external field ($\mathbf{H}=0$) is determined by the ratio of the first two terms of (4). If U_m is relatively small so that only the first term is significant, the function $\alpha(z)$ varies smoothly over characteristic distances $L \sim gS/kT$ (determined by the condition $gSL^{-1} \sim kT$). Thus, we can neglect the second term in (4) if $2\pi n_x M^2 \ll g/L_c^2$, i.e., if the condition $m \ll 1$ holds, where

$$m = 2\pi n_x M^2 g S^2 (kT)^{-2}. \tag{6}$$

If, however $m \gg 1$, the magnetostatic energy [the second term in (4)] is important. The condition that it be a minimum leads to the MOR breaking up into pronounced antiphase domains with the magnetization parallel to the MOR Z axis (in these domains, $\alpha = 0$ or $\alpha \approx \pi$). The domains are separated by walls whose thicknesses l, in correspondence with (4), are at $l \gg r_0 f_m$ of order $l \sim (g/n_x M^2)^{1/2}$, i.e., far smaller than the average domain lengths $L(\ln(L/l) \sim m^{1/2}$. The magnetization distribution in the domain wall is determined by the Euler equation for the functional (4) (which is the static Landau-Lifshitz equation). Substituting the well-known solution to this equation into (4) (see, e.g., §43 in Ref. 8), we can find the domain-wall energy

$$E_0 = 4S(2\pi n_x M^2 g)^{1/2} = 4m^{1/2}kT \qquad (m \gg 1). \tag{7}$$

Equations (4) and (7) are correct only if $l \gg r_0 f_m$, i.e., $g \gg M^2 S$ (for $n_x \sim 1$). As we approach the Curie point, g can become on the order of or less than $M^2 S$, and thus the thickness l of a domain wall bcomes comparable to the MOR diameter $r_0 f_m$. In this case the magnetic field is caused more by the variation of the magnetization in the longitudinal direction (to the magnetic charges $-dM_z/dz$), than in the transverse direction; the vectors $\mathbf{M}(\mathbf{r})$ in the cross section of the domain wall can be noncollinear, while the expression for the magnetostatic energy U_m will differ in form from the second term in (4). In the limit $g \ll M^2 S$, the basic contribution to the wall energy E_0 comes from U_m, and is of the order of $E_0 \sim (MS)^2/r_0 f_m$, $l \sim r_0 f_m$. This order-of-magnitude estimate of E_0 is implied in the equations presented below [in particular, in (8)], if $l \lesssim r_0 f_m$, $E_0 \gg kT$ (in place of the more exact result (7) for $l \gg r_0 f_m$).

A longitudinal field $\mathbf{H} \| Z$ leads in the volume to a preponderance of domains whose magnetization is parallel to the field, and in comparatively weak (at $m \gg 1$) fields $H \gg H_0 \sim kT/LMS$ (explicit expressions for H_0 are given below) the relative fraction of domains antiparallel to the field tends to zero. The susceptibility connected with the anomalously large magnetic moments of the domains is quite large in the region $H \sim H_0$. In the case of pronounced domains ($m \gg 1$) the free energy φ per unit length of MOR can be found by investigating the statistics of the domain walls. In not very strong fields, where $l \ll L^*$ (L^* is the length of the domains antiparallel to the field), we can neglect interactions among domain walls; such a calculation for $\mathbf{H} \| Z$ leads to the expression

$$\varphi = \text{const} - [\xi[16 n_x M^2 (kT)^2/g] \exp(-2E_0/kT) + M^2 H^2 S^2]^{1/2}, \tag{8}$$

where ξ is a dimensionless parameter ($\xi = 4m^{1/2}$ for $l \gg r_0 f_m$) which is determined below [see (16)]. The derivatives of this expression specify the average magnetization and susceptibility of the MOR.

In the case of a tilted field $\mathbf{H}$, lying in the XZ plane and oriented at an angle ϑ to the $\mathbf{Z}$ axis of the MOR, the directions of the magnetization $\mathbf{M}$ in a domain with $m \gg 1$, i.e., the angles α beween $\mathbf{M}$ and the $\mathbf{Z}$ axis, are determined from the condition that the latter two terms of (4) be minimized with respect to α:

$$\sin\alpha \cos\alpha = h \sin(\vartheta - \alpha), \qquad h = H(4\pi n_x M)^{-1}. \tag{9}$$

For $m \gg 1$, the rotations of the vectors $\mathbf{M}$ make up the principal contribution to the susceptibility in fields $H \sim H_0' = 4\pi n_x M \gg H_0$ (in fields $H \sim 4\pi M$, processes which take $\mathbf{M}$ out of the XZ plane are also important), while for $H \sim H_0$ the fractional change in the volume fraction of domains with differing signs of M_z plays the dominant role. Henceforth we will basically be investigating systems with $H_0 \ll H_0'$, in which these effects can be studied separately, or the case of longitudinal fields $\mathbf{H} \| Z$.

2. PARTITION FUNCTION OF AN MOR

To calculate the partition function $\mathscr{Z}$ of an MOR from first principles, it is necessary to evaluate the functional integral

$$\mathscr{Z} = \int D\alpha(z) \exp\{-\mathscr{H}[\alpha(z)]/kT\} \tag{10}$$

for all possible functions $\alpha(z)$, i.e., for all possible fluctuational rotation of the vector $\mathbf{M}(z)$. The role of the effective Hamiltonian $\mathscr{H}[\alpha(z)]$ in the considered MOR model is played by expression (4). In the case of one-dimensional problems of this kind, evaluation of the integral (10) reduces formally to the problem of finding the smallest eigenvalue ε of a certain operator H_{eff}.[4-7] For an MOR with Hamiltonian (4) this leads to a one-dimensional Schrödinger equation for a particle of mass m [see (6)] in a periodic field at $\hbar = 1$); in the case of a longitudinal field ($\mathbf{H} \| Z$), $\varphi = -\mathscr{L}^{-1} kT \ln \mathscr{Z}$ is determined by the equations

$$\varphi = \pi n_x M^2 S \varepsilon, \qquad H_{eff}\psi = \varepsilon\psi,$$
$$H_{eff} = -(1/2m)(d^2/d\alpha^2) + (2\sin^2\alpha - 4h\cos\alpha). \tag{11}$$

The eigenvalues $\varepsilon(m, h)$ depends on the dimensionless parameters m and h; taking into account (6) and (9), Eq. (11) determines the dependence of the free energy $\varphi\mathscr{L}$ of the MOR on the field $\mathbf{H}$ and on the parameters n_x, M, g, and S.

The Schrödinger equation (11) can be solved analytically by standard methods in the limiting cases of weakly and strongly bound electrons. Let us investigate first the case of weak binding, i.e., small particle masses m. In the limit of small m and arbitrary mh, we can disard $2\psi \sin^2\alpha$ in Eq. (11), which reduces now to Mathieu's equation. Thus, ε can be expressed in terms of the lowest eigenvalue $a_0(q)$ of the standard equation $y'' + (a - 2q\cos 2z)\, y = 0$, for which

the following expansions in q and $1/q$ were obtained (see Ref. 9):

$$\begin{aligned}\varepsilon&=(1/8m)\,a_0(-16mh)=-16mh^2[1-7(2mh)^2\\&+(998/9)(2mh)^4+(68687/36)(2mh)^6-\ldots]\\&\qquad(m\ll 1,\quad 8mh\ll 1),\\\varepsilon&=(1/8m)[-x^2/2+x-1/4-1/16x-3/64x^2\\&\quad-53/1024x^3-\ldots],\quad x=8(mh)^{1/3}\\&\qquad(m\ll 1,\ 16mh\gg 1).\end{aligned}\tag{12}$$

In the case of large masses m, to solve the Schrödinger equation (11) of a particle in a periodic field we can make use of the tight-binding approximation for the electrons. Since in (11) the potential energy in each period 2π has for $h\neq 0$ two minima of unequal depth [at $\alpha=2\pi n$ and $\alpha=\pi(2n+1)$, where n is an integer], the wave function in this approximation is the following linear combination:

$$\psi_k(\alpha)=\sum_{n=-\infty}^{\infty}[C_1\varphi_1(\alpha-2\pi n)\exp(2\pi ink)+C_2\varphi_2(\alpha-\pi-2\pi n)\times\exp(\pi(2n+1)ik)]\tag{13}$$

of "atomic" functions $\varphi_i(\alpha)$ and $\varphi_2(\alpha)$. The lowest eigenvalue ε corresponds to the wave function (13) with $k=0$. Substituting it into (11), we find that the corresponding eigenvalues are determined by just the same equation

$$\begin{vmatrix}\varepsilon_1-\varepsilon & W\\ W & \varepsilon_2-\varepsilon\end{vmatrix}=0,$$

$$\varepsilon_i=\int_{-\infty}^{\infty}\varphi_i(\alpha-(i-1)\pi)\,H_{eff}\varphi_i(\alpha-(i-1)\pi)\,d\alpha,\qquad i=1,2,$$

$$W=2\int_{-\infty}^{\infty}\varphi_1(\alpha)\,(H_{eff}-\varepsilon)\,\varphi_2(\alpha-\pi)\,d\alpha,\tag{14}$$

as in the double-well potential problem (see, e.g., Ref. 10). The approximation (19) is applicable if the condition

$$4h\ll m^{-1/3}\ll 2,\tag{15}$$

is fulfilled, allowing us to neglect the contributions to $\psi_k(\alpha)$ from excited states of the wells and limit ourselves in (14) to the nearest-neighbor approximation. When condition (15) holds, the basic dependence of $\varepsilon_1-\varepsilon_2$ on the field h is connected with the shift of the potential minima, and $\varepsilon_1-\varepsilon_2=8h$, while the dependence of W on h can be neglected in the region of interest ($4h\lesssim W$). In this limiting case we can determine W by using the quasiclassical approximation for the wave-functions in the region below the barrier (see §50 in Ref. 11). Choosing the energy origin such that $\varepsilon_1+\varepsilon_2=0$ [when condition (15) holds, $\varepsilon_1+\varepsilon_2$ is practically independent of h], we obtain

$$\varepsilon=-[(4h)^2+W^2]^{1/2},\qquad W=8\left(\frac{2}{\pi}\right)^{1/2}m^{-1/4}\exp(-4m^{1/2}).\tag{16}$$

In the limiting case under study here, we can likewise define $4mW$ at $h=0$ as the difference between the two lowest eigenvalues corresponding to symmetric and antisymmetric solutions of the Mathieu equation for large values of the parameter $q=m$. Using the asymptotic expression 20.2.31 in Ref. 9 for this difference, we obtain

$$W\sim m^{-1/4}\exp(-4m^{1/2})$$

with the numerical coefficient given in (16) (the quasiclassical-approximation equation leads to the less precise numerical factor $4(2e/\pi^2)^{1/2}$, because this approximation is correct only in the barrier region, and is inapplicable in the potential wells). The expression obtained for φ from (11) and (16) agrees at $4m^{1/2}\gg 1$ with Eq. (8) for a structure with well-defined domain walls.

In the general case of arbitrary masses m, $\varepsilon(m,h)$ can be found numerically. For this purpose, it is convenient to seek the eigenfunction for the ground state of (11) in the form

$$\psi_0(\alpha)=\exp(2m^{1/3}\cos\alpha)\sum_{k=0}^{\infty}a_k\cos k\alpha.\tag{17}$$

Then ε is determined as a root of the characteristic equation $D_0(\varepsilon)=0$, where D_0 is a codiagonal operator with matrix elements

$$\begin{aligned}&d_{n,n}=2m\varepsilon-(n-1)^2,\quad d_{2,1}=2m^{1/3}(4m^{1/3}h-1),\\&d_{n\pm1,n}=2m^{1/3}[(4m^{1/3}h-1)/2\mp(n-1)]\qquad(n=2,\ 3\ldots).\end{aligned}\tag{18}$$

If we denote by D_p the determinant obtained from D_0 by deleting the first p rows and columns, it is easy to show that

$$D_p=d_{p+1,\,p+1}D_{p+1}-d_{p+2,\,p+1}d_{p+1,\,p+2}D_{p+2},\tag{19}$$

i.e., D_0/D_1 can be expressed as a continued fraction

$$D_0/D_1=d_{11}-d_{12}d_{21}/(d_{22}-d_{23}d_{32}/(d_{33}-d_{34}d_{43}/(d_{44}-\ldots.\tag{20}$$

In the particular cases where the continued fraction is truncated, the roots of the equation $D_0(\varepsilon)=0$, including the smallest root, can be found exactly (Ref. 12). For arbitrary m and h this equation is easily solved on a computer by equating expression (20) to zero. The dependence of $\varepsilon(m,h)$ on h so obtained for small m is in good agreement with the analytic formulas (12), and agrees with (16) for large m.

3. DEPENDENCE OF THE SUSCEPTIBILITY ON TEMPERATURE, EXTERNAL FIELD, AND CRYSTAL PARAMETERS

The susceptibility of a crystal with dislocations can depend appreciably on the dislocation density and orientation relative to the external field. Let us first investigate the simplest case of rectilinear dislocations parallel to each other in a longitudinal magnetic field, and discuss the magnetization and susceptibibity of an individual MOR near a dislocation.

The average magnetization $\bar{\mathbf{M}}$ of a MOR in a longitudinal magnetic field and the specific suceptibility $\chi_{\parallel}$ of the MOR are defined by the derivatives of $\varepsilon(m,h)$ with respect to h:

$$\bar{\mathbf{M}}=-\frac{1}{S}\frac{\partial\varphi}{\partial\mathbf{H}}=-\frac{1}{4}M\frac{\partial\varepsilon}{\partial h}\frac{\mathbf{H}}{H},\tag{21}$$

$$\chi_{\parallel}=\partial\bar{M}/\partial H=-\frac{1}{16\pi n_z}\frac{\partial^2\varepsilon}{\partial h^2}$$

For the large- and small-m regimes, the differentiation of expressions (12) and (16) for $\varepsilon(m, h)$ can be performed analytically. For example, for $4m^{1/2} \gg 1$ we have according to (16) and (21),

$$\chi_{\parallel} = \frac{1}{4\pi n_x} \frac{h_0^2}{(h^2+h_0^2)^{1/2}} \qquad h_0 = \frac{W}{4}. \tag{22}$$

In the region of intermediate m, the differentiation in (21) must be done numerically. A plot of the zero-field susceptibility $\chi_{\parallel}^{(0)}$ so obtained as a function of the parameter m is shown in Fig. 1; plots of the function $\chi_{\parallel}(h)$ versus external field inensity h for several values of m are shown in Fig. 2.

From Fig. 1 it is clear that $\chi_{\parallel}(0)$ is not large for small m [according to (12) and (21), $\chi_{\parallel}(0) = 2m/\pi n_x$ for $m \ll 1$], but grows rapidly with increasing m, going over to an exponential dependence.

$$\chi_{\parallel}(0) = (\pi n_x W)^{-1} = (m^{1/2}/128\pi n_x^2)^{1/2} \exp(4m^{1/2}) \tag{23}$$

for large m [see (22) and (16)]. This rapid rise in $\chi_{\parallel}(0)$ is due to the appearance of the giant magnetic moments $M^0 \sim \exp(4m^{1/2})$ of the domains at $4m^{1/2} \gg 1$. Comparatively weak fields, for which $M^0 H \sim kT$, lead to reconstruction of the domain pattern. Consequently, as is clear from Fig. 2, the susceptibility depends significantly on field, decreasing monotonically as the field increases. Whereas for small m Eq. (12) implies that this dependence appears only for rather strong fields $h \sim h_0 = 1/8m$, for large m [as is clear from (22) and (16)] it is already significant even in very weak fields $h \sim h_0 = W/4 \sim m^{-1/4} \exp(-4m^{1/2})$ [according to (9), h is measured in units of the characteristic field $H_0' = 4\pi n_x M$]. In strong fields $h \gg h_0$, according to (21), (12), and (16), we have

$$\chi_{\parallel} = (64\pi n_x m^{1/2} h^{3/2})^{-1}$$

for $m \ll 1$, and

$$\chi_{\parallel} = (64\pi n_x)^{-1} W^2 h^{-3}$$

for $(8h)^{-2} \gg m \gg 1$. The average magnetization of an MOR in weak fields ($h \ll h_0$) equals

$$\bar{M} = \chi_{\parallel}(0) H_0' h,$$

while in strong fields it is close to M:

$$\bar{M} = M - M/8(mh)^{1/2}$$

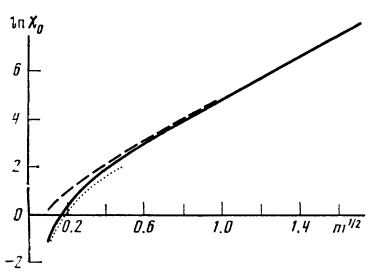

FIG. 1. Dependence of $\chi_0 = 16\pi n_x \chi_{\parallel}(0)$ on $m^{1/2}$; the dotted curve is based on Eqs. (2) and (21), the dashed curve on (16) and (21), and the continuous curve on the results of numerical calculations.

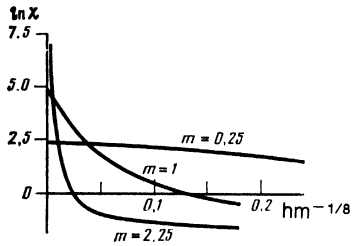

FIG. 2. Dependence of $\chi = 16\pi n_x \chi_{\parallel}(h)$ on the magnetic field for various values of m (temperature). The abscissas $hm^{-1/8}$ are proportional according to (3), (6) and (9) to external field and do not depend on temperature at $\beta = 1/3$ and $\nu = 2/3$.

for $m \ll 1$ and

$$\bar{M} = M - MW^2(32h^2)^{-1}$$

for $m \gg 1$. These effects were discussed qualitatively in Ref. 1.

The dimensionless parameter m which plays the role of the effective mass in the auxiliary Schrödinger problem (11), can be expressed in terms of the real crystal parameters and temperature by using formula (6). If the MOR near the dislocation are formed at temperatures close to the Curie point, we get, taking (3) into account,

$$m = 10^{-4}\xi \cdot 2\pi n_x \varkappa^4 p/\tau^{4-2\beta-\nu}, \qquad p = M_0^2 v/kT_c,$$
$$\xi = \xi_1^2 \xi_2 \xi_3^2 b^4/r_{c0} v \sim 1. \tag{24}$$

The quantity m grows rapidly when we get close to the Curie point (as $\tau^{2\beta+\nu-4}$, i.e., as $\tau^{-8/3}$ for $\beta = 1/3$, $\nu = 2/3$), which leads to appreciable growth of the susceptibility and of its dependence on the field. For example, for $M_0 = 10^3$ G, $v = 10^{-23}$ cm^3, $T_C = 1000$ °K ($p = 7\times 10^{-5}$), $\xi = 1$, $2\pi n_x = 1$, $\varkappa = 1$ we have $m = 0.7$ for $\tau = 10^{-3}$ and $m = 4.4$ for $\tau = 5\times 10^{-4}$.

The total per-uit susceptibility χ_t of a paramagnet with MOR's near dislocations is determined by the formula

$$\chi_t = \chi_{id} + \chi_d, \qquad \chi_d = n_d S \chi_{\parallel}, \tag{25}$$

where χ_{id} is the susceptibility of an ideal crystal (for short-range exchange forces $\chi_{id} \sim p\tau^{-\gamma}$) and n_d is the length of the dislocation lines per unit volume (dislocation density). At appreciable dislocation densities and large enough m, it is clear from (3), (23) and (24) that the second term in (25), which is due to MOR's near dislocations, can be comparable to or even larger than the first term. For example, for $n = 10^8$ cm^{-2}, $\tau = 5\times 10^{-4}$, $m = 4.4$, $p = 7\times 10^{-5}$, and $S = 10^{-11}$ cm^2, we have $\chi_d/\chi_{id} \sim 1$.

We now investigate MOR's with $m > 1$ in the general case when the magnetic field has an arbitrary orientation relative to the dislocations. Since for $m > 1$ (i.e., $h_0 \ll 1$) the characteristic field $H_0' = 4\pi n_x M$ which can cause rotation of the vectors M in the domains is far larger than the field $H_0 = H_0' h_0 \ll H_0'$ in which a substantial fraction of the domains change sign, the processes of growth and rotation of domains occur at greatly different field values. In the region of fields $H \ll H_0'$ we can neglect the growth and rotation of the magnetization, so that the basic contribution to the

change in magnetic moment and in susceptibility, as previously noted, is due to the influence of the longitudinal field component on the domain structure of the MOR. If the dislocations are parallel, the results above are correct if we replace h by $h\cos\vartheta$ in them, where ϑ is the angle between $\mathbf{H}$ and the dislocation lines.

In a strained crystal there are usually several differently oriented systems of dislocation lines, and to calculate χ_d it is necessary to average over these orientations. It is simplest to do this average for a polycrystaline sample, in which there is no crystal texture and all dislocation directions are equally probable (taking into account the different orientations of the grains). In the low-field region ($h \ll h_0$), the result of this averaging is a factor $\langle \cos^2\vartheta \rangle = 1/3$ in χ_d:

$$\chi_d = {}^1/_3 n_d S \chi_{\parallel}(0) \quad \text{at} \quad H \ll H_0 = H_0' h_0 \ll H_0' = 4\pi n_x M, \quad h_0 \ll 1. \tag{26}$$

For $h_0 \ll 1$ (large $4m^{1/2}$), the condition $H \ll H_0'$ is compatible with the condition $h \gg h_0$ of the strong fields $H \gg H_0$. For such fields, within the overwhelming portion of almost all MOR the magnetization is oriented along the field component $\mathbf{H}_{\parallel}$ parallel to the dislocations. Only for dislocation orientations $\vartheta \sim \pi/2$ can reorientation of domains in the field continue. Taking (16) into account and averaging over the angles, it is not difficult to show that for $H_0 \ll H \ll H_0'$ and $m \gg 1$ we have

$$\begin{aligned} &\bar{M}_d = {}^1/_2 n_d M S [1 - (h_0/h)^2 \ln (2h/e^{1/2}h_0)], \\ &\chi_d = (n_d S h_0^2/4\pi n_x h^3) \ln (2h/e h_0), \\ &h_0 = W/4, \end{aligned} \tag{27}$$

i.e., the susceptibility is proportional to h^{-1} as in the case of longitudinal fields, but because a certain fraction of the MOR's are almost transverse to the field the susceptibility increases also by a logarithmic factor.

According to (27), in systems with large $4m^{1/2}$ and for strong enough fields $h \gg h_0$ the susceptibility χ_d which arises from rearrangement of the domain structure will be small. In this region of fields, an important contribution to the change in the average magnetization vector $\bar{\mathbf{M}}$ of the MOR's and in their susceptiblities comes from processes which rotate the magnetization of the MORs in an oblique field from the direction of $\mathbf{H}_{\parallel}$ to that of $\mathbf{H}$. In the case under study here, when $n_x \ll n_y$, fields $H \ll H_0'' = 4\pi n_y M$ rotate $\bar{\mathbf{M}}$ in XZ planes and $\bar{\mathbf{M}}$ be taken out of these planes only for $H \ll H_0''$. The direction of $\bar{\mathbf{M}}$ in each MOR is determined by Eq. (9), in whch we replace H by $H\cos\phi$, where ϕ is the angle between $\mathbf{H}$ and the XZ plane for a given dislocation. Averaging over the orientations of the XZ planes (i.e., over ϕ) and over the dislocation directions in these planes, we find that for $h \gg h_0$ (almost all the MOR's are practically single-domain) and at $H \ll H_0''$ the processes of rotation lead to the following contribution to χ_d:

$$\chi_d = \frac{n_d S}{4\pi n_x}\left(\frac{1}{3} - \frac{3}{8}h + \frac{2}{5}h^2\right) \quad (h_0 \ll h \ll 1), \tag{28}$$

$$\bar{\mathbf{M}}_d = \frac{\pi}{4} M n_d S \left(1 - \frac{1}{8h^2}\right)\frac{\mathbf{H}}{H},$$

$$\chi_d = \frac{n_d S}{64 n_x h^3} \quad \left(1 \ll h \ll \frac{n_y}{n_x}\right).$$

In stronger fields comparable to $H_0'' \gg H_0'$, the rotation processes which take the magnetization $\bar{\mathbf{M}}$ of the MOR out of the XZ plane become important. Allowing for these processes, $\bar{\mathbf{M}}$ and χ_d in such fields are given by the expressions

$$\begin{aligned} &\bar{\mathbf{M}} = \frac{\pi}{4} M n_d S \left(1 + \frac{4}{3\pi}h' - \frac{1}{8h^2}\right)\frac{\mathbf{H}}{H}, \\ &\chi_d = \frac{n_d S}{12\pi n_y}\left(1 + \frac{3\pi n_y}{16 n_x h^3}\right) \\ &\qquad \left(h' = \frac{H}{4\pi n_y M} \ll 1\right), \\ &\bar{\mathbf{M}} = M n_d S \left(1 - \frac{1}{15h'^2}\right)\frac{\mathbf{H}}{H}, \quad \chi_d = \frac{n_d S}{30\pi h'^3} \quad (h' \gg 1). \end{aligned} \tag{29}$$

From (27)–(29) it is clear that in the region of weak fields, where $h \ll h_0^{2/3}$, the susceptiblity of MOR's is due essentially to the domain-growth processes considered above, which lead to a sharp falloff of χ_d in the region $h \sim h_0$; however, in larger fields at $h \ll h_0^{2/3}$, the processes which rotate the magnetization play a greater role. In this case, according to (28), χ_d within a certain interval $h_0^{2/3} \ll h \ll 1$ depends weakly on the field and has a value smaller than χ_d for $h \to 0$ by a factor h_0^{-1} [see (26) and (22)]; subsequently at $h \gg 1$, as is clear from (28) and (29), χ_d once again decreases rapidly.

In MOR's with $4m^{1/2} \lesssim 1$, the fields H_0 and H_0' are comparable, and the processes of domain growth and rotation of the magnetization take place simultaneously. In this case the determination of χ_d for polycrystalline samples requires numerical methods to average over the directions. However, in the limit $m \to 0$, when we can neglect the magnetostatic energy, the susceptibility in general does not depend on the orientation of $\mathbf{H}$ relative to the MOR. Thus, as in a single crystal, we have according to (12, (21), and (25)

$$\begin{aligned} &\chi_d = (2 n_d S m/\pi n_x)(1 - 168 m^2 h^2 + \ldots) \quad (16mh \ll 1), \\ &\chi_d = \frac{n_d S}{64\pi n_x m^{1/2} h^{3/2}} \quad (16mh \gg 1). \end{aligned} \tag{30}$$

In this limiting case of small m, the MOR contribution to the total susceptibility of the crystal is usually small.

4. DISCUSSION OF RESULTS: THE DISLOCATION SPIN GLASS

From the results presented here, it follows that in the presence of a sizable dislocation density n_d, fluctuating domains in MOR's close to the dislocations can add to the susceptibility substantial contribution χ_d that depends strongly on the field and rises rapidly as the Curie point T_C^0 is approached. These results were obtained for an isolated rectilinear MOR and are correct only if the domain length L is small compared to the lengths L' between sharp bends in the dislocation lines and the lengths L_1 of the links in the network of intersecting dislocations [$(L_1 \sim r_d^2/r_0 f_m)$ in the case of a random network, and $L_1 \sim r_d$ in the case of a Frank network]. We also require that the thickness $r_0 f_m$ of a MOR

be small compared to the average spacing r_d between dislocations. According to Ref. 1, as the temperature falls, for $L \sim L_1$ the one-dimensional picture of a MOR goes over into a three-dimensional picture, and as a result of a phase transition in the network at $T = T_f$, in place of fluctuating domains there arises a magnetization of one sign. If (for very large n_d) we still have not reached $r_0 f_m \sim r_d$ as $T \to T_C^0$, then a percolation phase transition must take place (Ref. 13). Below these transition temperatures the results we have obtained will obviously be inapplicable. In systems with small $\varkappa$, even for small n_d the formation of MOR's at $\tau \sim \tau_0$ can lead at once to the appearance of a network ($\tau_f \sim \tau$; see Ref. 1) and the theory developed here is nowhere valid.

The character of the magnetic sturcture of a MOR will be significantly altered when we take into account non-rectilinear dislocation lines, in particular their sharp bends which often appear in real dislocation ensembles. Below the network phase-transition point T_f (or above this point if $L > L'$), when certain conditions are fulfilled, the dipole interactions between portions of the MOR network can make thermodynamically advantageous formation of a dislocation dipole glass consisting of parts with differing directions of magnetization **M**. In the case of thick domain walls, when $l \gg r_0$ ($g^{1/2} \gg M r_0 f_m$), the exchange energy E_e and the total energy $E_0' = E_e + E_m$ of a kink in the MOR are minimal if the directions of **M** on both sides of the kink make an acute angle. In this case, below the network phase transition point T_f, in a sufficiently homogeneous MOR network, a new magnetization $\mathbf{M}_f$ can appear, which gradually diffuses throughout the volume of the entire crystal (Ref. 1).

However, if $g^{1/2} \ll M r_0 f_m$, then the basic contribution to the kink energy E_0' comes from the magnetic energy E_m, and in the case of sharp kinks (with angles $> \pi/2$), E_0' decreases sharply (by $\sim M^2 S r_0 f_m \gg kT$), when the directions of **M** on both sides of the kink make not an acute, but an obtuse angle. Here, there arises a cancellation of the magnetic moments of different parts of the MOR of a random network with $L \gg L'$ and $L_1 \gg L'$, $\mathbf{M}_f = 0$, and a dislocation glass results. The presence of portions of this glass with different directions of magnetization is not connected with thermal fluctuations, as in the dislocation superparamagnetism investigated earlier, but is rather due to the greater thermodynamic advantage (due to dipole forces) associated with the corresponding kink energy, so that the $\mathbf{M}(r)$ distribution turns out to be determined by the random structure of the dislocation ensemble. Only when the temperature gets low enough to make $r_0 f_m \sim r_d$ can contact between the transverse walls of the MOR's lead to a thermodynamically advantageous ferromagnetic state with $\mathbf{M}_f \neq 0$, which extends over the whole volume of the crystal.

Even in the absence of sharp kinks and for $g^{1/2} \gg M r_0 f_m$, i.e., $1 \gg r_0$, a dislocation glass can be energetically advantageous if, as often happens in real systems, the dislocation network is essentially not one-dimensional. Thus, if in the crystal there are sections with dimensions $\sim R$ and with a high density of dislocations $n_d' \sim (r_d')^{-2}$. i.e., appreciably larger than the mean density n_d, then the total dipole moment $\mu \sim M r_0^2 f_m^2 n_d' R^3$ of all the MOR's is connected with these sections. The energy $\mu^2 (R')^{-3}$ of the dipole interaction of two neighboring sections separated by a distance R' increases the energy $n_d' R^2 E_0'$ of the domain walls of the MOR's produced by rotation of the magnetization of one of the sections if the conditions $R^4 r_0 f_m \gg r_d'^2 R'^3$ for $g^{1/2} \ll M r_0 f_m$ or $M R^4 r_0^2 f_m^2 \gg r_d' g^{1/2} R'^3$ for $g^{1/2} \ll M r_0 f_m$ are met. In these cases, for sufficiently random locations of such sections of the crystals, and for $L(T) \gg R$ (i.e., close to the point T_C^0), the dipole interactions should lead to the appearnace of dislocation glass. Depending on the crystal parameters, this glass is formed when the temperature decreases out of the paramagnetic (i.e., above T_f) or ferromagnetic (below T_f) states of the MOR's near the dislocations; subsequently, at $T \approx T_C^0$, it goes into the ferromagnetic phase which extends over the entire crystal. A dislocation glass can also arise as a nonequilibrilum state, if e.g., the time for rotation of the magnetization directions accidently produced in large sections of the MOR network is extremely long.

The formation of dislocation glass near the Curie point can substantially influence various magnetic properties of a perfect crystal, particularly the dynamic susceptibility $\chi = \chi' - i\chi''$. As is well-known, in spin glasses $\chi''(\omega)$ is almost constant over a broad range of the frequency ω, and $\chi' \sim \ln(\text{const}/\omega)$ (see, e.g., Refs. 14–16). The same dependences were observed in studies of $\chi(\omega)$ in $CdCr_2S_4$ near the Curie point (Ref. 17). One possible explanation for these results could be based on the assumption that MORs near dislocations (which in ordinary real crystals always have appreciable densities) form a dislocation dipole glass, for which the dependences of χ', χ'' on ω are of the kind described above.

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