Oscillations of chain of Bloch lines in a domain wall

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Oscillations of a periodic chain of Bloch lines in a domain wall are investigated and the band character of these oscillations is determined. Each Bloch line situated in a potential well moves along an ellipse and bends the domain wall. The overlap of these bands determines the principal mechanism of the interaction between Bloch lines. It predominates over the exchange interaction between the Bloch lines, which depends on their topological charge ("kink-kink" and "kink-antikink" interactions), if the domain wall bends over a length considerably exceeding the length of the Bloch line. The spectrum of a chain of Bloch lines is therefore independent of the relative signs of the charges on neighboring lines. The sign of the Bloch-line topological charge does influence, however, the direction of the force acting on the wall in a uniform magnetic field parallel to the magnetization in the subdomains. This can lead to resonance splitting for Bloch lines in such a field.

1. INTRODUCTION

Bloch lines (BL) are regions that separate subdomains with oppositely rotating magnetization **M** in domain walls (DW) of ferromagnetic materials. Investigations of static and dynamic properties of BL were initiated in view of their strong influence on magnetic bubble domains (BD).[1] They are now attracting greater interest in view of the prospects of using BL as information carriers in memory devices, with a possibility of appreciably increasing the information density.[2] A Block line is also an interesting object of purely physical research. It constitutes a topologically stable linear soliton having interesting dynamics. The equation of motion for the BL contains a gyrotropic force similar to the Magnus force for hydrodynamic vortices, since the BL has a topological charge analogous to the circulation of a hydrodynamic vortex,[1,3] This circumstance influences substantially dynamics of BL, which can also be called magnetic vortices.

Oscillations of a BL situated in a potential well made up by forces of magnetostatic origin and the DW elasticity have recently been investigated.[4–7]

In the present paper the oscillation theory developed for one BL in Ref. 5 (see also Ref. 8) is generalized for the case of a regular chain of BL in DW. It is assumed that in the static position the DW coincides with the XZ plane (see Fig. 1 below), and BL are parallel to the Z axis, along which the magnetization distribution is assumed to be uniform. Each BL is contained in its own potential well. Such a well can be due to the fields of magnetic charges on the surface of a film, or can be artificially formed by producing a periodic potential relief on a film containing a DW with a BL. The latter turns out to be necessary for technical applications.[9] The result is a strictly periodic BL chain whose oscillation spectrum should have a band structure, as was indeed found from a theoretical calculations. The principal mechanism for interaction between BL is the overlap of the DW bends produced by BL moving along an ellipse. Thus, while a static BL chain is one-dimensional, its motion is always at least two-dimensional. The "flexural" interaction considered in the present paper turns out to be much more substantial than the usually allowed-for interaction between one-dimensional solitons, an interaction that manifests itself at distances on the order of their width and depends on the sign of the topological charge (interaction of the "kink-kink" or "kink-antikink" type). The BL-oscillation spectrum obtained here is therefore independent of the sign alternation of the topological charges in the chain. This alternation, however, is important when one considers the response, likewise investigated in the present paper, of a BL chain to an alternating uniform magnetic field. The sign of the BL topological charge determines the direction of the force acting on the BL in a uniform magnetic field parallel to the magnetization in the subdomains, and consequently also the direction of the bending of the DW near the BL. The frequency of the resonance of BL oscillations in such a field is therefore independent of the relative signs of the topological charges of neighboring BL. This can qualitatively explain the experimentally observed splitting of the resonant peak of oscillating BL.[6]

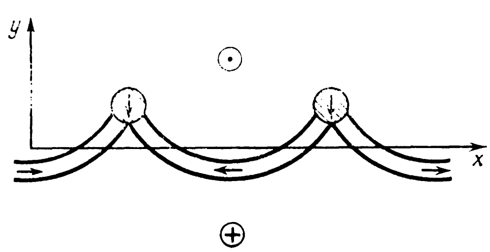

FIG. 1. The shaded circles denote Bloch lines. The arrows indicate the directions of **M** in the central plane of the domain wall.

We disregard in our investigation effects connected with the finite thickness of the film (along the Z axis) and leading to a "twisted" DW [Ref. 1, §8E). These effects call for a special analysis beyond the scope of the present paper. It can be assumed, however, that the fact that the sample is bounded in the direction of the BL can be effectively reduced to a mere renormalization of the parameters of the theory.

No explicit account of the magnetostatic interaction was taken. It was taken partially into consideration, however, by introducing in the theory the forces that return the DS and BL to their equilibrium positions. The magnitude of

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these forces enters in our calculations via the phenomenologically specified parameters K_0 and K_1. In addition, magnetostatic charges can be produced on the DW if the wave vector of the bending wave has a component along the magnetization in the domains. Such an interaction can lead to an effective increase of the DW elasticity, something that can also be taken roughly into account by altering the parameters of our theory. Finally, there exists a long-range magnetostatic interaction between charges that are produced on the BL themselves.[1] This interaction can compete with the flexural interaction considered in the present paper, and the relation between them will be estimated below.

2. EQUATION OF MOTION OF A DOMAIN WALL WITH BLOCH LINES

A chain of BL spaced L apart is shown in Fig. 1. The motion of the DW sections empty of BL is described on the basis of the linearized Slonczewski equations[1] without dissipation:

$$\dot{\psi}=-\gamma H_z\,\mathrm{sign}\left[\frac{\partial M_z}{\partial y}\right]-\gamma\frac{K_0}{2M}q+\frac{\sigma_0\gamma}{2M}\nabla^2 q,$$
$$\frac{\dot{q}}{\Delta_0}=4\pi\gamma M\psi-\frac{2\gamma A}{M}\nabla^2\psi, \qquad (1)$$

where $q(x)$ describes the DW displacement from the equilibrium position, $\psi(x)$ is the angle between the magnetization direction in the central plane of the DW and the X axis, H_z is the field that moves the DW, K_0 is a parameter of a magnetostatic-origin, restoring force acting on the DW, γ is the gyromagnetic ratio, $\sigma_0=4(AK_u)^{1/2}$ is the DW surface energy density, and $\Delta_0=(A/K_u)^{1/2}$ is the effective DW thickness, where A and K_u are the parameters of the inhomogeneous exchange and of the uniaxial anisotropy. Elimination of ψ yields

$$\ddot{q}/\Delta_0=-4\pi\gamma^2 MH_z\,\mathrm{sign}[\partial M_z/\partial y]-2\pi\gamma^2K_0q+2\pi\gamma^2\sigma_0\nabla^2q$$
$$-\gamma^2\sigma_0A\nabla^4q/M^2. \qquad (2)$$

The theory expounded below presupposes that the characteristic wavelength $l_{\mathrm{bend}}^{-1}=(K_0/\sigma_0)^{1/2}$ for flexural oscillations is much larger than the BL width $\Lambda=(A/2\pi M^2)^{1/2}$. We can therefore neglect henceforth the term with the fourth spatial derivative in Eq. (2).

The action of the BL on the DW dynamics was taken into account by introducing in (2) point forces concentrated on the BL, the latter regarded as infinitely thin. Thus, Eq. (2) takes the form

$$m_D\ddot{q}-\sigma_0\frac{\partial^2 q}{\partial x^2}+K_0q=-2MH_z\,\mathrm{sign}\left[\frac{\partial M_z}{\partial y}\right]+\sum_i f_i\delta(x-x_i), \qquad (3)$$

where $m_D=(2\pi\gamma^2\Delta_0)^{-1}$ is the Döring mass of the DW; f_i is the force exerted on the DW by a BL located at x_i; $\delta(x)$ is the delta function. Integrating Eq. (3) over a small vicinity of the BL, we obtain the boundary conditions that must be satisfied by the solutions of the equation of motion (3) for DW sections containing no BL:

$$-(\sigma_0\partial q/\partial x)_{x_i+0}+(\sigma_0\partial q/\partial x)_{x_i-0}=f_i. \qquad (4)$$

To determine f_i we consider the law of BL motion[1,10]

$$-\delta U/\delta\mathbf{r}_i=(M/\gamma)[\mathbf{G}\dot{\mathbf{r}}_i], \qquad (5)$$

where U is the effective free energy of the system, $\mathbf{r}_i$ is the radius vector of the BL in the xy plane, and $\mathbf{G}$ is the gyrotropy vector. We are interested only in one component $G_z=2\pi\nu_i$, where the BL topological charge is

$$\nu_i=\mathrm{sign}\left[\frac{\partial M_z}{\partial y}\right]\mathrm{sign}\left[\frac{\partial M_x}{\partial x}(x_i)\right]\mathrm{sign}[M_y(x_i)].$$

Inside the BL, the magnetization departs from the DW plane, and only there do we have $M_y\neq 0$. It can be stated for the situation shown in Fig. 1 that $\nu_i=1$ for the left-hand BL and $\nu_i=-1$ for the right.

The static forces in the left-hand side of (5) take the form

$$-\partial U/\partial x_i=-2MH_x\Delta_0\pi\,\mathrm{sign}[\partial M_x(x_i)/\partial x]-K_1(x_i-\bar{x}_i),$$
$$-\partial U/\partial y_i=-f_i, \qquad (6)$$

where H_x is the field that propels the BL along the DW, K_1 is a parameter of the elastic restoring force; and $\bar{x}_i$ is the equilibrium position of the BL. It is recognized here that the force exerted on the BL by the DW is equal and opposite to the force f_i applied to the DW. Substitution of (6) in (5) yields the equations of motion of the BL:

$$-(2\pi M/\gamma)\nu_i\dot{y}_i=2MH_x\Delta_0\pi\,\mathrm{sign}[\partial M_x(x_i)/\partial x]+K_1(x_i-\bar{x}_i), \qquad (7)$$
$$(2\pi M/\gamma)\nu_i\dot{x}_i=f_i. \qquad (8)$$

Eliminating x_i, we obtain for f_i the expression

$$f_i=-m_{yBL}\ddot{y}_i-(4\pi^2M^2/K_1\gamma)\Delta_0\nu_i\,\mathrm{sign}[\partial M_x(x_i)/\partial x]\dot{H}_x, \qquad (9)$$

where

$$m_{yBL}=4\pi^2M^2/\gamma^2K_1 \qquad (10)$$

is the DW mass localized on the BL. Substituting (9) in (3) and (4) we ultimately obtain the equation of motion of a DW with a BL:

$$m_D\ddot{q}-\sigma_0\frac{\partial^2 q}{\partial x^2}+K_0q+m_{yBL}\ddot{q}\delta(x-x_i)=-2MH_z\,\mathrm{sign}\left[\frac{\partial M_z}{\partial y}\right]$$
$$-(4\pi^2M^2/K_1\gamma)\Delta_0\nu_i\,\mathrm{sign}[\partial M_x(x_i)/\partial x]\dot{H}_x\delta(x-x_i) \qquad (11)$$

and the boundary conditions for the equation of motion on DW sections free of BL,

$$-\left(\sigma_0\frac{\partial q}{\partial x}\right)_{x_i+0}-\left(\sigma_0\frac{\partial q}{\partial x}\right)_{x_i-0}=-m_{yBL}\ddot{q}(x_i)$$
$$-\frac{4\pi^2M^2}{K_1\gamma}\Delta_0\nu_i\,\mathrm{sign}\left[\frac{\partial}{\partial x}M_x(x_i)\right]\dot{H}_x. \qquad (12)$$

The mass m_{yBL} specified by (10) was introduced in Ref. 11. The parameter K_1 in the equation for the BL mass characterizes the restoring force acting on the BL along the DW. No stipulation is made here concerning the origin of this force. It can be due to magnetostatic interaction, as in Ref. 11, or result from an interaction between the BL and a defect or a potential relief artificially deposited on the film. The mass m_{yBL} must be distinguished from the BL mass introduced in Ref. 5 to describe BL motion along a DW. It should be noted that according to Ref. 5 the BL traces an ellipse as it oscillates. Such an elliptically polarized oscillation can be described in terms of the displacements either across or along the DW, i.e., either excluding the coordinate x_i from the equations of motion, as we did above, or excluding y_i, as in Ref. 12. The value of the BL mass in the corresponding equation of motion depends on the description method. The equation of motion for the BL in terms of the displacement x_i along the wall is obtained in the following manner: in accordance with Ref. 5, we obtain the solution for slow free oscillations of the DW near an individual BL [Eq. (11) without the right-hand side], when the Döring mass of the wall can be neglected, i.e., $q = y_i \exp(-k|x - x_i|)$, where $k = (K_0/\sigma_0)^{1/2}$. Substituting this solution in the condition (4) we obtain an expression for the force f_i, which must next be substituted in (8). Elimination of the variable x_i from the system (7) and (8) yields the equation of motion for a BL with mass m_{yBL}, and by eliminating y_i we obtain the equation of motion in terms of the displacement x_i:

$$m_{xBL}\ddot{x}_i + K_1 x_i = 0, \tag{13}$$

$$m_{xBL} = 2^{-1}(\sigma_0 K_0)^{-1/2}(2\pi M/\gamma)^2. \tag{14}$$

Since the masses m_{yBL} and m_{xBL} are connected ith motion of the BL across and along the wall, respectively, they can be correspondingly called the transverse and longitudinal masses.

3. SPECTRUM OF BLOCH-LINE OSCILLATIONS IN A DOMAIN WALL

Since a DW with BL is a periodic structure with period L, its oscillations are described by the Bloch functions for the DW displacements:

$$q(x,\varkappa) = e^{i\varkappa x}u(x,\varkappa), \tag{15}$$

where $u(x+L,\varkappa) = u(x,\varkappa)$, and $(\varkappa L)$ is the phase shift of the oscillations of the neighboring BL.

It follows from (8) the the displacements $q(x,\varkappa)$ on the DW sections free of BL are described by a combination of two plane waves:

$$q(x,\varkappa) = e^{-i\omega t}[A(\varkappa)e^{ikx} + B(\varkappa)e^{-ikx}], \tag{16}$$

where $k = [(m_D\omega^2 - K_0)/\sigma_0]^{1/2}$. Substitution of (16) in the boundary conditions (12) and (15) yields a dispersion equation that relates ω and $\varkappa$:

$$\cos\left\{\left[\frac{m_D}{\sigma_0}(\omega^2-\tilde\omega^2)\right]^{1/2} L\right\} - \cos\varkappa L = \frac{\omega^2}{\omega_\infty^2}\left(\frac{\omega^2}{\tilde\omega^2} - 1\right)^{-1/2}\sin\left\{\left[\frac{m_D}{\sigma_0}(\omega^2-\tilde\omega^2)\right]^{1/2} L\right\} \tag{17}$$

at $\omega \gg \tilde\omega$ and

$$\mathrm{ch}\left\{\left[\frac{m_D}{\sigma_0}(\tilde\omega^2-\omega^2)\right]^{1/2} L\right\} - \cos\varkappa L = \frac{\omega^2}{\omega_\infty^2}\left(1 - \frac{\omega^2}{\tilde\omega^2}\right)^{-1/2}\mathrm{sh}\left\{\left[\frac{m_D}{\sigma_0}(\tilde\omega^2-\omega^2)\right]^{1/2} L\right\} \tag{18}$$

at $\omega \gg \tilde\omega$. Here $\tilde\omega = (K_0/m_D)^{1/2}$ is the gap in the spectrum of the flexural waves of the free DW, and the frequency

$$\omega_\infty^2 = 2(\sigma_0 K_0)^{1/2}(\gamma/2\pi M)^2 K_1 \tag{19}$$

coincides with the oscillation frequency of an isolated BL if $\tilde\omega \ll \omega_\infty$ (see Ref. 15). As might be expected, the oscillations of a DW with BL have a band spectrum. The lower limits of all but the lowest bands are described by the simple equation

$$\omega_n^2 = \tilde\omega^2 + (\sigma_0/m_D)(\pi n/L)^2, \tag{20}$$

where $n > 0$ is the number of the band. These frequencies correspond to values $kL = \pi n$. It can be seen from (17) that the corresponding values of $\varkappa$ are 0 or π/L, and in each case this is the bottom of the band.

We track now the variation of the oscillation spectrum of a BL chain as a function of the transverse BL mass (m_{yBL} can be verified by changing the parameter K_1). We relate m_{yBL} to the DW Döring mass over the length between the BL.

In the case $m_{yBL}/m_D L \ll 1$ (i.e., in the case of large K_1), narrow forbidden gaps are present between the bands, and their width decreases with decrease of the parameter $m_{yBL}/m_D L$. The corresponding spectrum is shown in Fig. 2a). For high-number bands (such that $\omega \gg \tilde\omega$) the expression for the relative width of the forbidden gap takes the simpler form $\Delta_g\omega/\omega = m_{yBL}/m_D L$. The frequency ω_0 of the bottom of the lowest bands approaches the frequency $\tilde\omega$ as $m_{yBL}/m_D L$ decreases and is described by the expression $\omega^2 = \tilde\omega^2(1 - m_{yBL}/m_D L)$. In the limit as $m_{yBL}/m_D L \to 0$ the spectrum of the oscillations becomes continuous with $\omega \gg \tilde\omega$, corresponding to a free DW without BL.

In the opposite case $m_{yBL}/m_D L \gg 1$ we have low values of K_1. The BL pins the wall, each band is compressed towards its bottom, and the lowest band towards $\omega = 0$ (see Fig. 2b).[1)] at $m_{yBL}/m_D L \gg 1$ we can derive for the relative widths of the high-number bands the simple relation

$$\Delta_b\omega/\omega = (2/kL)^2 m_D L/m_{yBL}.$$

Correct expressions for the lowest band are ($l_{\text{bend}} = (\sigma_0/K_0)^{1/2}$:

$$\Delta_b\omega = 2\tilde\omega\left(\frac{m_D L}{m_{yBL}}\right)^{1/2}\left(\frac{L}{l_{\text{bend}}}\right)^{-1/2}\exp\left(-\frac{L}{2l_{\text{bend}}}\right)\left(\mathrm{sh}\frac{L}{2l_{\text{bend}}}\right)^{-1/2}, \tag{21}$$

$$\omega_0 = \tilde\omega\left(\frac{m_D L}{m_{yBL}}\right)^{1/2}\left(\frac{L}{l_{\text{bend}}}\right)^{-1/2}\left(2\,\mathrm{th}\frac{L}{2l_{\text{bend}}}\right)^{1/2}. \tag{22}$$

In the limit as $m_{yBL}/m_D L \to \infty$ the spectrum becomes strictly discrete.

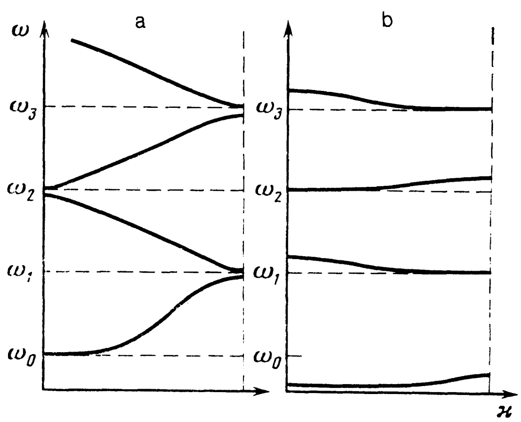

FIG. 2.

Generalizing the foregoing, we can draw the following conclusion: owing to the gyrotropy of the BL motion, if the BL can move quite freely along the DW ($K_1 = 0$) the latter becomes pinned, and if the BL is absolutely pinned ($K_1 = \infty$), on the contrary, the DW becomes free to move. Similar results were obtained in numerial experiments by Matsuyama and Konishi.[9] They have found that a potential relief superimposed on a film and restricting the motion of the BL along the DW increases substantially the mobility of the wall itself.

We examine now the transformation of the spectrum when the distances between the BL are increased and when the BL becomes solitary. In the limit as $L\to\infty$ each band except the lowest one becomes compressed towards its bottom, and all the ω_n ($n>0$) tend simultaneously to the gap frequency, forming a quasicontinuum $\omega > \tilde{\omega}$. The solution of (11) takes the form $q = q(0)\exp(-k|x|)$, where $k = [(1-\omega^2/\tilde{\omega}^2)K_0/\sigma_0]^{1/2}$. Thus the lower band of the spectrum corresponds at $L\to\infty$ to flexural DW oscillations localized on BL, in other words, to oscillations of an isolated BL. These oscillations are described by Eq. (13) for the case $\omega \ll \tilde{\omega}$, so that the wall banding produced by the BL motion can be treated statically. The condition $\omega \ll \tilde{\omega}$ is met if the mass m_{yBL} is high enough compared with m_D/k, i.e., with the Döring mass over the length $1/k$ on which the bending of the DW takes place. While in this case the inertia of the DW over the length of the band (i.e., its kinetic energy) is of little importance, the longitudinal mass of the BL, on the contrary, is connected with the energy accumulated in the bend domain wall. The fact that a DW is bent by a Bloch wall and the role of the bending in BL dynamics were noted also by Matsuyama and Konishi[9] in a numerical modeling of BL motion.

In the limit as $L\to\infty$ the width of the lowest band is given by

$$\Delta_b\omega = 2\omega_\infty e^{-kL}. \tag{23}$$

It is important that the argument of the exponential contains $k = 1/l_{\text{bend}}$, which is the reciprocal of the DW bend length, and not $1/\Lambda$, where Λ is the BL width. Thus the interaction between the BL is not via exchange repulsion of the BL (taken into account by Slonczewski[12] in the determination of the equilibrium period of a BL structure in a DW), but in terms of the DW bend.

The band width (23) determined by the bending interaction can be compared with possible splitting of the natural oscillations of two BL as a result of their magnetostatic interaction. The energy of the magnetostatic interaction of two BL separated by a distance L can be determined from an expression that follows from Eq. (1) of a paper by Khodenkov[13] for L exceeding significantly the film thickness h:

$$E_M = \pm 8(\pi\Delta_0 Mh)^2/L. \tag{24}$$

The coupled oscillations of two magnetostatically interaction BL are given by the following two equations, which are obtained by adding to Eq. (13) for each BL the interaction forces:

$$\begin{aligned} m_{xBL}\ddot{x}_1 + K_1x_1 + \frac{1}{h}\frac{\partial^2 E_M}{\partial L^2}(x_1 - x_2) &= 0,\\ m_{xBL}\ddot{x}_2 + K_1x_2 - \frac{1}{h}\frac{\partial^2 E_M}{\partial L^2}(x_2 - x_1) &= 0, \end{aligned} \tag{25}$$

where x_1 and x_2 are the displacements of the BL from the equilibrium positions. From (25) follows an expression for the magnetostatic splitting of the natural oscillations of the BL:

$$\Delta\omega = \frac{\partial^2 E_M/\partial L^2}{2K_1 h}\omega_\infty = 8\pi^2\frac{(\Delta_0 M)^2 h}{K_1 L^3}\omega_\infty. \tag{26}$$

The splitting (23) due to the bending interaction of the BL is substantial when $kL\sim 1$. To estimate the magnetostatic splitting at these distances we can substitute $L\approx 1/k$ in (26). We find from this that the magnetostatic splitting is small so long as

$$8\pi^2(\Delta_0 M)^2 hk^3/K_1 \ll 1. \tag{27}$$

Substituting the values $\Delta_0 = 5.6\cdot 10^{-6}$ cm, $M = 15.5$ G, $k = (K_0/\sigma_0)^{1/2} = 6.4\cdot 10^3$ cm^{-1}, and $K_1 = 160$ erg/cm^3 used in the numerical experiment of Matsuyama and Konishi,[9] and assuming a film thickness $h = 10^{-4}$ cm, we obtain in the left-hand side of the inequality (27) the value ~ 0.1.

As already mentioned, our theory is valid so long as the length $1/k = (\sigma_0/K_0)^{1/2}$ of the DW bend exceeds the dimension Λ of the Bloch line. The very same case was considered recently by Zvezdin and Popkov[14] for nonlinear motion of one BL. In the linear case their results agree with those of Ref. 5. They considered also nonlinear BL motion in the opposite limiting case $\Lambda \gg (\sigma_0/K_0)^{1/2}$, when the BL causes no noticeable DW bending suppressed by a strong gradient of the field that fixes the DW position. In this case the main contribution to the longitudinal mass of the BL mass is similar in its origin to the Döring mass for DW. On the other hand, the mass of gyrotropic origin becomes insignificant. Just such a case was considered earlier by Ignatchenko and Kim,[15] who likewise investigated linear oscillations of a BL chain.

4. INDUCED OSCILLATIONS OF A DOMAIN WALL WITH BLOCH LINES

Owing to the gyrotropic character of the BL motion, the oscillations of DW with BL can be excited both by the field H_z and by the field H_x. The BL oscillations are always elliptically polarized.

As can be seen from Eq. (11), when an alternating field H_z acts on DW, the oscillations excited in the DW are indifferent with respect to the topological charge ν_i, although the latter determines the direction of the BL motion along the elliptic trajectory. In a uniform field H_z, the BL oscillations are in phase if they have the same signs of ν_i.

However, the force exerted on the BL by the uniform field H_z depends on the sign of the quantity $\nu_i\,\text{sign}[\partial M_x(x_i)/\partial x]$. If this sign alternates along the BL chain, the uniform field H_z excites an oscillation with a wave number $\varkappa = \pi/L$ corresponding to the boundary of the Brillouin zone. If the DW contains sections with a different type of alternation of the topological charge ν_i, splitting of the peaks corresponding to the spectral bands will be observed in the spectrum oscillations excited by the uniform alternating field H_x. Such a picture was qualitatively observed in Ref. 4 (see Fig. 4 of that reference). A noticeable splitting occurs in this case in a field parallel to the magnetization in the subdomains, in agreement with the theory. On the other hand, the splitting turns out to be of the same order as the value predicted by Eq. (23), one assumes the values $k = 0.3 \cdot 10^3$ cm^{-1} determined in Ref. 5 and $L \approx 10^{-2}$ cm. A detailed quantitative comparison with experiment, however, is made difficult by the lack of exact values of the parameters of the theory.

The results of our theory can be compared also with the numerical experiments of Matsayuma and Konishi.[9] In these experiments they modeled the motion of a DW containing BL and acted upon by a magnetic-field step. This caused oscillations of a chain of BL situated in a potential relief. The frequencies of these oscillations can be compared with the values of the frequencies that follow from our analytic theory, using the parameters cited in Ref. 9: $A = 2.63 \cdot 10^{-7}$ erg/cm, $\Delta_0 = 5.6 \cdot 10^{-6}$ cm, $M = 15.5$ G, $K_0 = 7.76 \cdot 10^6$ erg/cm^4, $K_1 = 164$ erg/cm^3, $L = 5.28 \cdot 10^{-5}$ cm, and $\gamma = 1.83 \cdot 10^7 g^{-1/2} \cdot cm^{1/2}$. Solving Eq. (18) for these parameters on the bottom of the lowest band, where $\varkappa = 0$, we obtain a frequency $\omega/2\pi = 7.61$ MHz, 20% higher than the frequency $\omega/2\pi = 6.50$ MHz obtained from the numerical experiment. These small discrepancies can be attributed to the fact that the numerical experiment was carried out for a rather dense BL chain, with spacing $L = 4\Lambda$, and account was taken of damping and nonlinear effects disregarded in the theory developed above.

CONCLUSION

We have constructed in this paper a theory for the oscillations of a regular chain of BL placed in potential wells. The Bloch lines execute oscillations, moving along elliptic orbits, and bending thereby the domain wall. The overlap of bends due to neighboring BL is the main mechanism of the interaction between the BL; this is confirmed by the agreement with the results of the numerical experiments.

It must be emphasized that the applicability of our theory is not confined to materials with high quality factors, when the dimension Λ of the BL is substantially larger than the thickness Δ_0 of the DW. Although the Slonczewski equations are usually derived for the case $\Lambda \gg \Delta_0$, satisfaction of the last inequality is needed only if these equations are to be used inside the BL, where the structure of the wall differs strongly from that of a Bloch wall. In our theory, however, they are used only to describe small DW oscillations in a region free of BL, where a sufficient condition for their validity is that the characteristic length of these oscillations exceed the domain-wall thickness. This can be varified by following the derivation of Slonczewski's equations. Equation (5) for the BL, derived by Tiele,[10] from the Landau-Lifshitz equation (see also Ref. 1, §12F), can be used for any quality factor (i.e., at any ratio Λ/Λ_0). A low quality factor, however, can decrease greatly the range of validity of our theory with respect to nonlinear effects.

[1)]The zero frequency corresponds to the Goldstone mode that stems from translational invariance in the limit as $K_1 \to 0$.

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