Photon splitting in a strong electromagnetic field

V. N. Baĭer, A. I. Mil'shteĭn, and R. Zh. Shaĭsultanov

Nuclear Physics Institute, Siberian Division, USSR Academy of Sciences
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Splitting of a photon in a strong electromagnetic field of general form (with both field invariants different from zero) is considered by using the operator-diagram technique. Explicit expressions are obtained for all the amplitudes of the process at arbitrary values of the parameters. An expression is derived for the cross section of the process under photon-absorption conditions. A representation of the amplitudes in a quasiclassical approximation is obtained. The possibility of observing the field of the axes of a single-crystal is analyzed.

1. INTRODUCTION

Virtual creation and annihilation of electron-positron pairs is known to induce nonlinear self-action of an electromagnetic field. A characteristic process of nonlinear quantum electrodynamics is the scattering of light by light. In external fields, a photon can be split into two ($\gamma \to \gamma_1 + \gamma_2$) as well as deflected (coherently scattered, $\gamma \to \gamma'$). At low photon energies ($\omega \ll m$) the splitting process can be analyzed by using the Heisenberg-Euler effective Lagrangian (see, e.g., Ref. 1, §§129, 130). For arbitrary photon energies and field intensities, an exact calculation (in terms of the field) is required. Photon splitting in a constant and uniform external field was considered in Refs. 2–6, where earlier works, containing errors, are cited. Photon splitting was considered by *Z* and *I*. Bialynicki-Birula[2] and by Adler *et al.*,[3] who used an effective Heisenberg-Euler Lagrangian; polarization selection rules, especially with allowance for dispersion, were also obtained in Ref. 3. Adler[4] analyzed in detail the process in an external magnetic field and obtained the allowed-transition amplitude for the general case of an arbitrary photon energy; the expressions for this amplitude turned out to be too unwieldy for further use. He used a Green's function for an electron in an external magnetic field in the Schwinger proper-time representation.[7] Papanyan and Ritus[5,6] considered photon splitting in a crossed field $\mathbf{E} \perp \mathbf{H}$, $E = H$, using likewise the electron Green's function in the proper-time representation.

In the general case, the photon-splitting probability depends on three dimensionless invariants[1)]

$$\frac{E}{F} = \frac{e}{m^2}[(\mathscr{F}^2+\mathscr{G}^2)^{1/2}-\mathscr{F}]^{1/2}, \quad \frac{H}{H_0} = \frac{e}{m^2}[(\mathscr{F}^2+\mathscr{G}^2)^{1/2}+\mathscr{F}]^{1/2}$$
$$\varkappa = \frac{}{m^3}(|F_{\mu\nu}k^\nu|^2)^{1/2}, \tag{1.1}$$

where $\mathscr{F}$ and $\mathscr{G}$ are the field invariants:

$$\mathscr{F} = -{}^1/_4 F_{\mu\nu}F^{\mu\nu} = {}^1/_2(\mathbf{E}^2-\mathbf{H}^2),$$
$$\mathscr{G} = -{}^1/_4 F_{\mu\nu}^* F^{\mu\nu} = (\mathbf{EH}), \qquad F_{\mu\nu}^* = {}^1/_2 \varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}, \tag{1.2}$$

while E_0 and H_0 are the quantum-electrodynamics critical fields:

$$H_0 = m^2/e = 4.41\cdot 10^{13}\ \text{Oe}, \quad E_0 = m^2/e = 1.32\cdot 10^{16}\ \text{V/cm}. \tag{1.3}$$

The fields E and H in (1.1) are the electric and magnetic field in a special reference frame in which $\mathbf{E} \| \mathbf{H}$. If the invariants H/H_0 and E/E_0 are small compared with unity and with the photon-energy-dependent invariant $\varkappa$, they can be neglected. This is equivalent to transforming to the case of a crossed field (or to the quasiclassical approximation, see Ref. 8, p. 68). For the results obtained in a constant and uniform field to be valid, it is necessary that the field change little over the length (duration) of the process formation. For the process considered, the formation length is[8]

$$l_f \sim \min\left(\frac{1}{m}\frac{E_0}{E}, \frac{1}{m}\frac{H_0}{H}\right). \tag{1.4}$$

Photon splitting in a Coulomb (substantially inhomogeneous) field was considered in Ref. 9.

We derive here a general expression for the amplitude for splitting of a photon into two photons in a constant and uniform electromagnetic field at arbitrary values of the invariant (1.1). We use for this purpose the operator diagram technique developed by Katkov, Strakhovenko, and one of us.[10] It was found that the solution of this (technically quite cumbersome) problem can be substantially simplified. The amplitudes obtained in the particular case $\mathscr{G} = 0$, when the magnetic (or electric) field strength is zero, was found to be noticeably more compact than those obtained in Ref. 4.

The process discussed is not solely of general theoretical interest. A number of practical observations have been proposed for it. In Refs. 3 and 4 the process was considered as a possible mechanism for production of linearly polarized photons in pulsar fields (assuming that the pulsar fields $H \sim H_0$). It was also proposed to observe this process in interaction between hard (tens of GeV) photons with a laser wave.[5] We analyze here one other possibility, viz., observation of photon splitting in the fields of axes made up of atom chains or crystal planes in a single crystal. Production of electron-positron pairs by high-energy photons has by now been investigated in considerable detail (see, e.g., Ref. 11) and it has been established that the constant-field approximation can be used if the photon entry angle (the angle between the momentum **k** and the single-crystal axis) is $\vartheta_0 \ll V_0/m$, where V_0 is the scale of the potential of the axis made up of a chain of single-crystal atoms. The first experiment on the on pair production in the field of an axis has already been performed.[12] Photon splitting in a single crystal

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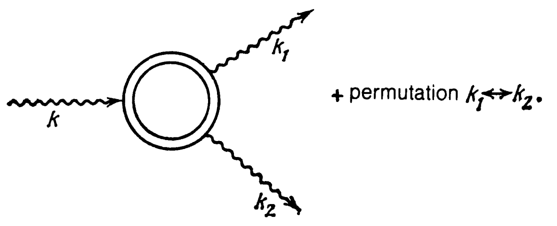

FIG. 1. Diagram of photon splitting in an external electromagnetic field.

is analogous to pair production, although its probability is smaller by a factor $(\alpha/\pi)^2$. Since the fields of single-crystal axes amount to 10^{10}–10^{11} V/cm, the photon energies needed to reach a value $\varkappa \gtrsim 50$ (when the effect is a maximum) are of the order of 100–1000 GeV.

2. CALCULATION OF THE PHOTON-SPLITTING AMPLITUDE

Consider the amplitude of photon splitting $(k \to k_1 + k_2)$ in a constant and uniform electromagnetic field $F_{\mu\nu}$ for which $k = k_1 + k_2$. On the mass shell we have $k_1^2 = k_2^2 = k^2 = 0$, $k = (\omega, \mathbf{k})$ etc. Strictly speaking, dispersion takes place in an external electromagnetic field, i.e., the photon acquires a mass that depends on its polarization. This mass is determined from the polarization operator in the given field (see Refs. 13 and 10). The mass acquired, however, turns out to be small and is manifested in fact in the polarization selection rules. This question will be analyzed below. The analysis can be carried out (cf. Refs. 1–5) on the mass shell in the approximation called collinear $(\mathbf{k} \| \mathbf{k}_1 \| \mathbf{k}_2)$; it is useful then to introduce the vector

$$\lambda = \frac{k}{\omega} = \frac{k_1}{\omega_1} = \frac{k_2}{\omega_2}, \quad \boldsymbol{\lambda}^2 = 1, \quad \lambda^2 = 0. \tag{2.1}$$

It is conveneient to use for the analysis a special reference frame in which $E \| H$ [the fields E and H in this frame are given by Eqs. (1.1) and (1.2)]. We choose the common direction of these vectors to be the 3-axis of a Cartesian system. The field tensor can then be represented in the form (see Ref. 14)

$$F_{\mu\nu} = C_{\mu\nu} E + B_{\mu\nu} H. \tag{2.2}$$

Here

$$C_{\mu\nu} = g_\mu{}^0 g_\nu{}^3 - g_\mu{}^3 g_\nu{}^0, \quad B_{\mu\nu} = g_\mu{}^2 g_\nu{}^1 - g_\mu{}^1 g_\nu{}^2, \tag{2.3}$$

where $g_{\mu\nu}$ is the metric tensor.

We use the operator diagram technique developed in Ref. 10. In the lowest order of perturbation theory in the interaction with photons, represented by the diagram of Fig. 1 where the double line is the electron propagator in an external field, we write the amplitude of the process in the form[2)]

$$T_1 = -e^3 \,\mathrm{Sp} \langle 0 | \hat{e} \frac{1}{\hat{P} - m} \hat{e}_1^* \frac{1}{\hat{P} + \hat{k}_1 - m} \hat{e}_2^* \frac{1}{\hat{P} + \hat{k} - m} | 0 \rangle, \tag{2.4}$$

where $\hat{P} = \gamma^\mu (i\partial_\mu - eA_\mu)$, A_μ is the vector potential of the external field, $e^\mu = e^\mu(k)$, $e^\mu(k_n) = e_n^\mu$ are the photon polarization vectors $\hat{e} = \gamma_\mu e_\mu$. It is necessary to add to the amplitude (2.4) the amplitude $T_2 = T_1(k_1 \leftrightarrow k_2, e_1 \leftrightarrow e_2)$. The total decay amplitude is $T = T_1 + T_2$. In this approach, the main problem is to calculate the mean value over the states $x = 0$: $\langle 0 | ... | 0 \rangle$, which includes an aggregate of noncommuting operators P_μ, and the calculation procedure is based on the closure of the algebra of the operators $[P_\mu, P_\nu] = -ieF_{\mu\nu}$. Direct application of the operator-diagram-technique[10] rules to the calculation of the amplitude (2.4) leads to an extremely cumbersome expression. An essential element of the present paper is therefore a transformation of (2.4). We square the electron propagators and use the identities

$$\begin{gathered} \hat{e}_2^* (\hat{P} + \hat{k} + m) = -(\hat{P} + \hat{k}_1 - m) \hat{e}_2^* + \hat{e}_2^* \hat{k}_2 + 2(e_2^* P) + 2(e_2^* k_1), \\ \hat{e}_1^* (\hat{P} + \hat{k}_1 + m) = -(\hat{P} - m) \hat{e}_1^* + \hat{e}_1^* \hat{k}_1 + 2(e_1^* P); \end{gathered} \tag{2.5}$$

the amplitude (2.4) then takes the form

$$\begin{aligned} T_1 = -e^3 \,\mathrm{Sp} \Big[& \langle 0 | \hat{e} \hat{P} \frac{1}{\hat{P}^2 - m^2} (\hat{e}_1^* \hat{k}_1 + 2(e_1^* P)) \frac{1}{(\hat{P} + \hat{k}_1)^2 - m^2} \\ & \times (\hat{e}_2^* \hat{k}_2 + 2(e_2^* P)) \frac{1}{(\hat{P} + \hat{k})^2 - m^2} | 0 \rangle \\ & - \langle 0 | \hat{e} \hat{P} \frac{1}{\hat{P}^2 - m^2} \hat{e}_1^* \hat{e}_2^* \frac{1}{(\hat{P} + \hat{k})^2 - m^2} | 0 \rangle \\ & - \langle 0 | \hat{e} \hat{e}_1^* \frac{1}{\hat{P}^2 - m^2} (\hat{e}_2^* \hat{k}_2 + 2(e_2^* P)) \frac{1}{(\hat{P} + \hat{k}_2)^2 - m^2} | 0 \rangle \Big], \end{aligned} \tag{2.6}$$

where terms with odd numbers of γ-matrices have been omitted and it is recognized that $e_i k_j = 0$ in the collinear approximation. This transformation was aimed at decreasing the number of γ matrices in the terms containing the operator P_μ in the numerator; since these terms make the most cumbersome contribution to the amplitude of the process. As a result, the expression for the amplitude (2.6), while seemingly more complicated, is noticeably simplified and the calculation can be continued directly with the aid of the rules of the diagram technique of Ref. 10.

We present explicit expressions for the mean values that appear in the calculation following an exponential parametrization of the propagators,

$$(N, N_\mu, N_{\mu\nu}, N_{\mu\nu\lambda}) = \langle 0 | (1, P_\mu, P_\mu P_\nu, P_\mu P_\nu P_\lambda) \Theta | 0 \rangle, \tag{2.7}$$

where

$$\Theta = \exp(iP^2 s_1) \exp[i(P + k_1)^2 s_2] \exp[i(P + k)^2 s_3]. \tag{2.8}$$

We introduce a notation that permits a noticeable simplification of the expressions:

$$\begin{gathered} y = eEt_3, \quad y_{1,2} = eE(2t_{1,2} - t_3), \quad y_3 = y + y_1 - y_2; \\ x = eHt_3, \quad x_{1,2} = eH(2t_{1,2} - t_3), \quad x_3 = x + x_1 - x_2, \end{gathered} \tag{2.9}$$

where $t_1 = s_1$, $t_2 = s_1 + s_2$, $t_3 = s_1 + s_2 + s_3$. The quantity N was calculated in Ref. 10 [Eqs. (2.43) and (2.47)]:

$$\begin{gathered} N = -iRe^{i\varphi}, \quad R = R_E R_H, \\ R_E = \frac{eE}{4\pi \,\mathrm{sh}\, y}, \quad R_H = \frac{eH}{4\pi \sin x}. \end{gathered} \tag{2.10}$$

The phase is here

$$\psi=\psi_E+\psi_H,$$

$$\psi_E=\frac{\sigma^2}{2eE\,\mathrm{sh}\,y}[(\omega^2-\omega_1\omega_2)\,\mathrm{ch}\,y-\omega\omega_1\,\mathrm{ch}\,y_1-\omega\omega_2\,\mathrm{ch}\,y_2+\omega_1\omega_2\,\mathrm{ch}\,y_3],$$

$$\psi_H=\frac{\sigma^2}{2eH\sin x}[(\omega^2-\omega_1\omega_2)\cos x-\omega\omega_1\cos x_1-\omega\omega_2\cos x_2+\omega_1\omega_2\cos x_3], \tag{2.11}$$

where $\sigma^2=\lambda C^2\lambda=\lambda B^2\lambda=1-\lambda_3^2$.

The quantities N_μ, $N_{\mu\nu}$, and $N_{\mu\nu\lambda}$ are expressed in terms of N in accordance with Eqs. (2.26)–(2.28), (2.14), and (2.46) of Ref. 10:

$$\begin{gathered}N_\mu=Q_\mu N,\qquad N_{\mu\nu}=[Q_\mu Q_\nu-iU_{\nu\mu}{}^{-1}]N,\\ N_{\mu\nu\lambda}=[Q_\mu Q_\nu Q_\lambda-iQ_\mu U_{\lambda\nu}{}^{-1}-iQ_\nu U_{\lambda\mu}{}^{-1}-iQ_\lambda U_{\nu\mu}{}^{-1}]N,\\ Q_\mu=-[U^{-1}(U(t_3-t_1)k_1+U(t_3-t_2)k_2)]_\mu,\\ U=U(t_3),\qquad U(s)=(e^{-2eFs}-1)/eF.\end{gathered} \tag{2.12}$$

The procedure of calculating the amplitude (2.6) consists of exponential parametrization of the electronic propagators, transformation of the expressions with the aid of formulas

$$\exp(isP^2)P_\mu\exp(-isP^2)=[\exp(-2eFs)P]_\mu, \tag{2.13}$$

$$\exp\left(\frac{i}{2}e\sigma Fs\right)\gamma_\mu\exp\left(-\frac{i}{2}e\sigma Fs\right)=[\gamma\exp(2eFs)]_\mu,$$

where $\sigma F=\sigma^{\mu\nu}F_{\nu\mu}$, $\sigma^{\mu\nu}=\frac{1}{2}i[\gamma_\mu,\gamma_\nu]$, and calculation of the traces of the matrices followed by the use of Eqs. (2.7)–(2.12). The second and third terms of (2.6) contain only two electron propagators, and the corresponding equation can be obtained from the general one by equating one of the parameters s_k to zero.

To calculate the photon-splitting amplitudes for arbitrary polarizations it suffices to find two amplitudes:

$$\begin{aligned}&T_B=T_{B\to CC}: && e_\mu=(\lambda B)_\mu/\sigma, \quad e_{1\mu}=e_{2\mu}=(\lambda C)_\mu/\sigma,\\ &T_C=T_{C\to CC}: && e_\mu=e_{1\mu}=e_{2\mu}=(\lambda C)_\mu/\sigma.\end{aligned} \tag{2.14}$$

All the remaining amplitudes are obtained from these by interchanging the frequencies and fields

$$\begin{aligned}&T_{C\to BC}=T_{B\to CC}(\omega\leftrightarrow-\omega_1),\\ &T_{C\to BB}=T_{B\to CC}(E\leftrightarrow iH,\ \sigma\to i\sigma),\\ &T_{B\to BC}=T_{C\to BB}(\omega\leftrightarrow-\omega_2),\\ &T_{B\to BB}=T_{C\to CC}(E\leftrightarrow iH,\ \sigma\to-i\sigma).\end{aligned} \tag{2.15}$$

After rather laborious calculations we get for the amplitude

$$T_{B(C)}=-4e^3\sigma^3\int\limits_0^\infty dt_3\int\limits_0^{t_3}dt_2\,Re^{-im^2t_3}\left\{\int\limits_0^{t_2}dt_1\,e^{i\psi}I_{B(C)}+\frac{2i}{\sigma^2}I_{B(C)}^{(0)}\right\}, \tag{2.16}$$

where we have for the $B\to CC$ transition

$$I_B=\frac{1}{\sin x}[(\omega_1\cos x_1+\omega_2\cos x_2)\cos x-\omega]\times\left[\frac{2ieE\,\mathrm{cth}\,y}{\sigma^2}\mathrm{ch}\,y_3+\omega_1\omega_2 g\right]+\omega(\omega_1^2f_1+\omega_2^2f_2)+\omega\omega_1\omega_2f_3+\omega_1\omega_2(\omega_1\cos x_1+\omega_2\cos x_2)\,\mathrm{ch}\,y_3\sin(x_2-x_1);$$

$$I_B{}^0=\frac{\omega\,\mathrm{ch}\,y}{\sin x}\sin^2\left(\frac{x-x_2}{2}\right)\exp\{i\psi_1(\omega)\},$$

$$\psi_1(\omega)=\psi_1{}^E(\omega)+\psi_1{}^H(\omega),$$

$$\psi_1{}^E(\omega)=\frac{\sigma^2\omega^2(\mathrm{ch}\,y-\mathrm{ch}\,y_2)}{2eE\,\mathrm{sh}\,y},$$

$$\psi_1{}^H(\omega)=\frac{\sigma^2\omega^2(\cos x-\cos x_2)}{2eH\sin x},$$

$$f_1=\frac{(\mathrm{ch}\,y_3-\mathrm{ch}\,y_2)}{\mathrm{sh}\,y}\times\left[\sin x_1\,\mathrm{sh}\,y_1+\frac{(1-\cos x\cos x_1)(1-\mathrm{ch}\,y\,\mathrm{ch}\,y_1)}{\sin x\,\mathrm{sh}\,y}\right] \tag{2.17}$$

$$f_2=f_1(x_1\leftrightarrow x_2,\ y_1\leftrightarrow y_2),$$

$$f_3=\sin x_1\,\mathrm{ch}\,y_1+\frac{\sin x_1\,\mathrm{sh}\,y_1}{\mathrm{sh}\,y}(\mathrm{ch}\,y-\mathrm{ch}\,y_2)+\frac{(\mathrm{ch}\,y_3-\mathrm{ch}\,y_2)}{\sin x\,\mathrm{sh}^2y}(1-\cos x\cos x_2)(1-\mathrm{ch}\,y\,\mathrm{ch}\,y_1)+\text{members }(x_1\leftrightarrow-x_2,\ y_1\leftrightarrow-y_2),$$

$$g=\frac{1}{\mathrm{sh}^2y}[2\,\mathrm{ch}\,y_3-(1+\mathrm{ch}^2y_3)\,\mathrm{ch}\,y]+\mathrm{ch}\,y_3\cos(x_2-x_1),$$

and for the $C\to CC$ transition

$$\begin{aligned}I_C={}&\omega\omega_1\omega_2\left(\mathrm{sh}\,y_1\cos x_1+\frac{\cos x_3\,\mathrm{sh}(y_2-y_1)}{2\,\mathrm{ch}\,y}\right)+\frac{1}{\mathrm{sh}\,2y}\\ &\times[\mathrm{ch}\,y(\omega_1\,\mathrm{ch}\,y_1+\omega_2\,\mathrm{ch}\,y_2)-\omega][\cos x\,\mathrm{ch}\,y(\omega_1\omega_2+h_1h_2)-\omega_1\omega_2\cos x_3\,\mathrm{ch}\,y_3\\ &-2\omega_1h_1\cos x\,\mathrm{sh}\,y]+\omega\omega_1\left(\cos x_1\,\mathrm{ch}\,y_1-\frac{\cos x}{\mathrm{ch}\,y}\right)\\ &-2ieE\omega_1\frac{\cos x}{\mathrm{sh}^2y}(\mathrm{ch}\,y_3-\mathrm{ch}\,y_2)\\ &\times(2\,\mathrm{ch}\,y\,\mathrm{ch}\,y_1-1)+\text{members }(x_1\leftrightarrow-x_2,\ y_1\leftrightarrow-y_2,\ \omega_1\leftrightarrow\omega_2);\end{aligned}$$

$$I_C{}^0=\frac{\cos x}{\mathrm{sh}\,y}\mathrm{sh}^2\left(\frac{y-y_2}{2}\right)(\omega e^{i\psi_1(\omega)}-\omega_1e^{i\psi_1(\omega_1)}-\omega_2e^{i\psi_1(\omega_2)}), \tag{2.18}$$

$$h_1=\frac{1}{\mathrm{sh}\,y}(\omega_2\,\mathrm{ch}\,y+\omega_1\,\mathrm{ch}\,y_3-\omega\,\mathrm{ch}\,y_2),$$

$$h_2=\frac{1}{\mathrm{sh}\,y}(\omega_1\,\mathrm{ch}\,y+\omega_2\,\mathrm{ch}\,y_3-\omega\,\mathrm{ch}\,y_1).$$

The amplitudes (2.16) and (2.18), with relations (2.15) taken into account, solve the problem of photon splitting in a uniform and constant electromagnetic field at arbitrary values of the invariants (1.1).

We proceed now to discuss the dispersion effects due to the acquisition of mass by a photon in an external field. Photon propagation in an external field is described by the Dyson equation, which we write in the form

$$(k^2_{(\lambda)}g_{\mu\nu}-\Pi_{\mu\nu})e^{\nu}_{(\lambda)}=0 \qquad (\lambda=\mathrm{I, II, III, IV}), \tag{2.19}$$

where $e_{(\lambda)}$ are the corresponding polarization vectors, $k^2_{(\lambda)}$ is the photon mass, and $\Pi_{\mu\nu}$ is the polarization operator in the given field. A solution corresponding to physical polarization can be written for this equation in the form

$$\begin{aligned} e_{\mathrm{I}}^{\mu} &= \frac{1}{(1+a^2)^{1/2}}[(\lambda B)^{\mu}+a(\lambda C)^{\mu}], \\ e_{\mathrm{II}}^{\mu} &= \frac{1}{(1+a^2)^{1/2}}[-a(\lambda B)^{\mu}+(\lambda C)^{\mu}]. \end{aligned} \tag{2.20}$$

Explicit expressions for $k^2_{\mathrm{I,II}}$ and a are given in Ref. 10. Note that if only a magnetic field is present ($E=0$) then $a\to 0$, and in the presence of only an electric field ($H=0$) we have $a\to\infty$:

$$E=0: \quad e_{\mathrm{I}}^{\mu}=(\lambda B)^{\mu}, \quad e_{\mathrm{II}}^{\mu}=(\lambda C)^{\mu}; \tag{2.21a}$$

$$H=0: \quad e_{\mathrm{I}}^{\mu}=(\lambda C)^{\mu}, \quad e_{\mathrm{II}}^{\mu}=-(\lambda B)^{\mu}. \tag{2.21b}$$

In the collinear approximation the probability of photon splitting per unit time is expressed in terms of its amplitude as follows (up to the pair-production threshold):

$$dW=\frac{1}{32\pi}|T|^2\frac{d\omega_1}{\omega^2}\vartheta\left(\frac{k^2}{\omega}-\frac{k_1^2}{\omega_1}-\frac{k_2^2}{\omega_2}\right). \tag{2.22}$$

The polarization selection rules follow, in particular, from the ϑ function contained in this expression. It is customary to use a refractive index n, defined by the relation

$$k^2_{(\lambda)}=\omega^2(1-n^2_{(\lambda)}); \tag{2.23}$$

the allowed transition should then satisfy the relation

$$\omega_1 n_1^2+\omega_2 n_2^2-\omega n^2\geqslant 0. \tag{2.24}$$

Here n_1 and n_2 are the refractive indices for the frequencies ω_1 and ω_2 that depend on the photon polarization.

We present also the asymptotic forms of the amplitudes (2.16)–(2.18) for relatively weak fields and low photon energies:

$$\begin{aligned} T_B&=a_0\sigma^3 H(13H^2-2E^2), \\ T_C&=a_0\sigma^3 E(39H^2+24E^2), \quad a_0=e^6\omega\omega_1\omega_2/315\pi^2m^8. \end{aligned} \tag{2.25}$$

The remaining amplitudes are obtained from these with the aid of the rules (2.15). In a magnetic field ($E=0$), these amplitudes are transformed into those obtained in Ref. 3. As noted above, the physical polarizations (2.20) must be used in the analysis of the process. In the case considered we have $e_{\mathrm{I}}\propto F\lambda$ and $e_{\mathrm{II}}\propto F^*\lambda$, and if we transform from the amplitudes (2.25) to amplitudes in terms of e_{I} and e_{II} we get

$$T_{\mathrm{I}\to\mathrm{II\,II}}=13a_0(\varkappa/\omega)^3, \quad T_{\mathrm{I}\to\mathrm{I\,I}}=24a_0(\varkappa/\omega)^3, \tag{2.26}$$

where $\varkappa$ is defined in (1.1). Equations (2.26) are valid if $\varkappa\ll 1$.

Finally, substituing the asymptotic values of the amplitudes in expression (2.22) for the probability (disregarding the ϑ function!), averaging over the initial-photon polarizations, and summing over the polarizations of the final ones, we obtain for the probabilities the expressions given in Refs. 2 and 5.

3. PHOTON SPLITTING IN A MAGNETIC OR ELECTRIC FIELD

We proceed now to the important particular case when only a magnetic field ($E=0$) or only an electric field ($H=0$) is present. Only two types of selection rules are known[1,3] for a magnetic field. The first is governed by *CP* invariance of the electromagnetic interaction. In terms of the polarizations (2.21a) (at $E=0$) the *CP*-allowed transitions are

$$e_{\mathrm{I}}\to e_{\mathrm{II}}e_{\mathrm{II}}, \quad e_{\mathrm{I}}\to e_{\mathrm{I}}e_{\mathrm{I}}, \quad e_{\mathrm{II}}\to e_{\mathrm{I}}e_{\mathrm{II}}. \tag{3.1}$$

Three other transitions (e.g., $e_{\mathrm{II}}\to e_{\mathrm{I}}e_{\mathrm{I}}$) are forbidden. These selection rules are valid, naturally, for all energies. The second type entails satisfaction of the condition (2.24). It is shown in Refs. 1–3 that the only (3.1) transition that satisfies relation (2.24) in the range where the effective Lagrangian can be used is $e_{\mathrm{I}}\to e_{\mathrm{II}}$. Using the dispersion relations, it was shown in Ref. 4 that this conclusion is valid for all $\omega<2m$. At $\omega>2m$ a channel is opened for pair production by a photon in an external field. The refractive index [see 2.23)] acquires therefore an imaginary part. As a result, the spatial component of the photon wave vector acquires a negative imaginary part and the incident wave attenuates as it propagates in the field. The integral with respect to the coordinate along k must therefore be taken between finite limits, and the probability $dW(L)$ of photon splitting after negotiating a length L in the field becomes meaningful (these questions are analyzed also in Ref. 6). The expression for $dW(L)$, which must now be used in lieu of (2.22), is

$$dW(L)=\frac{1}{32\pi}|T|^2\frac{d\omega_1}{\omega^2}\rho(L), \tag{3.2}$$

where

$$\begin{gathered} \rho(L)=\frac{1}{\pi}\int_0^\infty\frac{1+e^{-2\gamma L}-2e^{-\gamma L}\cos((\beta-x)L)}{\gamma^2+(\beta-x)^2}dx, \\ \beta=\mathrm{Re}\left(\frac{k^2}{\omega}-\frac{k_1^2}{\omega_1}-\frac{k_2^2}{\omega_2}\right), \quad \gamma=-\mathrm{Im}\left(\frac{k^2}{\omega}+\frac{k_1^2}{\omega_1}+\frac{k_2^2}{\omega_2}\right). \end{gathered} \tag{3.3}$$

In the limit $\gamma L\ll 1$, $\beta L\gg 1$ (the photon absorption is negligibly small, the effect is considered over large lengths L), we have $\rho(L)\propto L\vartheta(\beta)$, so that $dW(L)=LdW$ [see (3.2) and (2.22)] in accordance with the definition of dW as the probability of the process per unit length (time). In another limiting case $\gamma L\gg 1$ (under conditions of strong absorption),

$$\rho(L)=\frac{1}{\pi\gamma}\left(\frac{\pi}{2}+\mathrm{arctg}\,\frac{\beta}{\gamma}\right) \tag{3.4}$$

and in the limit $|\beta/\gamma|\to\infty$ we have asymptotically

$\rho(L)\propto\vartheta(\beta)/\gamma$. It is clear therefore that in the limit as $\gamma\to 0$ the function $\rho(L)$ is proportional to $\vartheta(\beta)$ and the selection rules based on the inequality (2.24) are valid. In the general case, however, $\rho(L)\neq 0$ regardless of the sign of β so that all the transitions (3.1) are possible.

Expressions for the photon-splitting amplitude, valid for arbitrary fields H and photon energies ω, follow from (2.17), (2.18) and (2.15) in which we let $E\to 0$ (naturally, a similar conclusion follows from (2.6) if we substitute there directly the mean values (2.7) and (2.12) calculated in the magnetic field). To obtain the explicit form of the amplitude in a magnetic field we must substitute in (2.16) [see (2.10), (2.17)]

$$R\to\frac{1}{4\pi}R_H,\quad \psi=\psi_H+\psi_0,$$

$$\psi_0=\frac{\sigma^2}{t_3}[\omega\omega_1 t_1(t_1-t_3)+\omega\omega_2 t_2(t_2-t_3)-\omega_1\omega_2(t_1-t_2)(t_1-t_2+t_3)];$$

for the I→II II transition

$$I_B\to I^H_{\mathrm{I}\to\mathrm{II\,II}}=\frac{2i}{\sigma^2 t_3\sin x}[\cos x(\omega_1\cos x_1+\omega_2\cos x_2)-\omega]+\frac{4\omega_1\omega_2}{\sin x}\sin^2\left(\frac{x_2-x_1}{2}\right)\times\left[\omega_1\sin^2\left(\frac{x_1+x_3}{2}\right)+\omega_2\sin^2\left(\frac{x_2-x_3}{2}\right)\right],\tag{3.5}$$

$$\psi_1(\omega)\to\sigma^2\omega^2 t_2(t_3-t_2)/t_3+\psi_1^H(\omega),$$

we must let $\cosh y\to 1$ in I_B^0. The expression obtained is much simpler than that given by Adler[4] [Eq. (2.5)]. The amplitude $T^H_{\mathrm{II}\to\mathrm{I\,II}}$ of the II→I II transition can be determined from (3.5) by making the substitution $T^H_{\mathrm{II}\to\mathrm{I\,II}}=T^H_{\mathrm{I}\to\mathrm{II\,II}}(\omega\leftrightarrow-\omega_1)$ [see (2.15)], while the amplitude of the $T^H_{\mathrm{I}\to\mathrm{II}}$ transition can be determined from (2.16) and (2.18), viz., $T^H_{\mathrm{I}\to\mathrm{II}}=T_C(E\leftrightarrow iH,\ \sigma\to i\sigma)$, after which one must put $E=0$.

If only an electric field is present ($H=0$), the polarization CP-allowed transitions are (3.1). The amplitude $T^E_{\mathrm{I}\to\mathrm{II\,II}}$ of the transition follows from (3.5) when the substitutions $E\to iH$, $\sigma\to i\sigma$, are made, the amplitude $T^E_{\mathrm{II}\to\mathrm{I\,II}}$ is obtained from $T^E_{\mathrm{I}\to\mathrm{II\,II}}$ by the interchange $\omega\leftrightarrow-\omega_1$, while the $T^E_{\mathrm{I}\to\mathrm{I\,I}}$ amplitude is obtained from (2.16) if $R\to R^E/4\pi$, $\psi=\psi_E+\psi_0$, and the limit $H\to 0$ is taken in (2.16) (all $\cos x$, $\cos x_n\to 1$)

The calculations with the obtained amplitudes T^H (T^E), which are relatively simple, are not too complicated and can be used to determine the photon splitting in single crystals. Recognizing that $H=0$ and $\mathbf{k}\perp\mathbf{E}$, we have from (1.1)

$$\varkappa=\frac{\omega}{m}\frac{E}{E_0},\tag{3.6}$$

where E is the electric field at a given distance from the axis (plane). As already noted, to reach the region $\varkappa\gtrsim 1$ in which the splitting is a maximum, the photon energies must be very high, $\omega\gg m$. We must thus consider the quasiclassical approximation in an electric field. Carrying out the standard expansions (see Ref. 14) in the amplitudes in the quasiclassical approximation

$$T_Q=\frac{e^3}{4\pi^2}m\varkappa\int_0^\infty dt\int_0^1 dt_2\left[\int_0^{t_2}dt_1\exp(i\varphi_Q)I_Q+I_Q^{(0)}\right]e^{-it},$$

$$\varphi_Q=-\frac{\varkappa^2t^3}{3}[\nu_1t_1^2(1-t_1)^2+\nu_2t_2^2(1-t_2)^2-\nu_1\nu_2(t_2-t_1)^2(t_1-t_2+1)^2],\tag{3.7}$$

in which, for the polarization (2.21b),

$$I_Q^{\mathrm{I}\to\mathrm{II\,II}}=i(1+\eta)-4\varkappa^2t^3\nu_1\nu_2(t_2-t_1)^2(\nu_1t_1^2+\nu_2(1-t_2)^2),$$

$$I_Q^{(0)\mathrm{I}\to\mathrm{II\,II}}=-2it_2^2\exp(i\varphi_0),$$

$$I_Q^{\mathrm{I}\to\mathrm{I\,I}}=i[8t_1(1-t_2)-1+\eta]-\frac{\varkappa^2t^3}{2}\{2\nu_1\nu_2(t_1-t_2)[(2t_1-1)(2t_2-1)+1+2(t_1+t_2-1)(\eta_1-\eta_2)]+(1+\eta)[2\nu_1\nu_2(t_2-t_1)(t_1-t_2+1)+\eta_1\eta_2]\},\tag{3.8}$$

$$I_Q^{(0)\mathrm{I}\to\mathrm{I\,I}}=2it_2^2[\nu_1\exp(i\nu_1^2\varphi_0)+\nu_2\exp(i\nu_2^2\varphi_0)-\exp(i\varphi_0)].$$

In (3.7) and (3.8) we used the notation

$$\nu_{1(2)}=\omega_{1(2)}/\omega,\quad \eta=\nu_1(1-2t_1)^2+\nu_2(1-2t_2)^2,$$
$$\eta_{1(2)}=2[\nu_{1(2)}(t_1-t_2)(t_1-t_2+1)+t_{2(1)}(1-t_{2(1)})],\tag{3.9}$$
$$\varphi_0=-\frac{\varkappa^2t^3}{3}t_2^2(1-t_2)^2.$$

The third amplitude follows from (3.7) and (3.8) in accordance with (2.15):

$$T_Q^{\mathrm{II}\to\mathrm{I\,II}}=T_Q^{\mathrm{I}\to\mathrm{II\,II}}(\omega\leftrightarrow-\omega_1).\tag{3.10}$$

Another representation of quasiclassical amplitudes was obtained in Ref. 5. The expressions obtained here for T_Q are in our opinion more compact. At $\varkappa\ll 1$ we have from (3.5)–(3.10) the known expressions (see Refs. 1–5, as (2.26)). At $\varkappa\gg 1$ the asymptotes of the amplitudes T_Q agree with those obtained in Ref. 5.

The amplitudes (3.7)–(3.10) were calculated numerically in the region $\varkappa\gtrsim 1$, in which they reach their maximum values. Figures 2 and 4 whose plots of $|T|^2/\omega$ for the transitions I→II II and I→I I for which the distribution in ν_1 is symmetric under the substitution $\nu_1\leftrightarrow\nu_2$, and for the transition II→I II, for which there is no such symmetry. The value of $|T|^2/\omega$ is measured in units of $\xi=4\alpha^3eE/\pi m$. The sets of frequencies for which the plots were drawn are given in the figure captions. Denoting the parameter $\varkappa$ corresponding to the maximum $|T|^2/\omega$ by $\varkappa_m$, we have $\varkappa_m\approx 80$, ≈ 30, and ≈ 200 for I→II II, I→I I, and II→I II, respectively. It can be seen that the probability of the transition II→I II is always

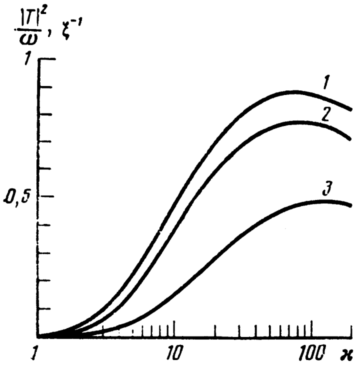

FIG. 2. The value of $|T|^2/\omega$ for the transition I→II II in units of $\xi = 4\alpha^3 eE/\pi m$. Curve 1—for $\nu_1 = \nu_2 = 0.5$; curve 2—for $\nu_1 = 0.35$, $\nu_2 = 0.65$; curve 3—for $\nu_1 = 0.2$, $\nu_2 = 0.8$.

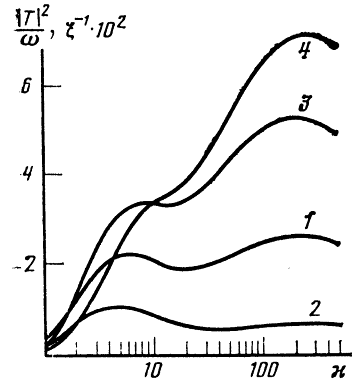

FIG. 4. Value of $|T|^2/\omega$ for the transition II→I II in units of ξ: curve 1—$\nu_1 = 0.35$, $\nu_2 = 0.65$; curve 2—$\nu_1 = \nu_2 = 0.5$; curve 3—$\nu_1 = 0.65$, $\nu_2 = 0.35$; curve 4—$\nu_1 = 0.8$, $\nu_2 = 0.2$.

smaller than for the others, and in the region of its maximum the dominant transition is I→II II. We note also that at $\varkappa \gg 1$ all the cited amplitudes reach their asymptotes most smoothly, so that the asymptotes can be used only at very large values of the parameter.

To find the probability of the process we must substitute $|T|^2/\omega$ and the function $\rho(L)$ (3.3) in Eq. (3.2). To find $\rho(L)$ we must first find β and γ [see (3.3)], i.e., the real and imaginary parts of the refractive index [see (2.23)]. In the general case one must use the results of Ref. 10. Expressions for the refractive index in the quasiclassical approximation can be found in Ref. 8 (cf. also Ref. 15):

$$n_{I,II}^2 = 1 + \frac{\alpha}{12\pi}\left(\frac{eE}{m^2}\right)^2 g_{I,II}, \qquad (3.11)$$

where

$$g_{I,II} = -\int_0^1 dv\, a_{I,II} \int_0^\infty ds\, s \exp[-i(s+\varkappa^2 f^2 s^3/48)],$$

$$a_I = f(2-f/2), \quad a_{II} = f(2+f), \quad f = 1-v^2. \qquad (3.12)$$

At $\varkappa \ll 1$ we have (see Refs. 1 and 8)

$$g_I = {}^{16}/_{15}, \quad g_{II} = {}^{28}/_{15}, \qquad (3.13)$$

and at $\varkappa \gg 1$ the function $g_{I,II}$ decrease (see Ref. 8):

$$g_{I,II} = \frac{c_0}{\varkappa^{4/3}} e^{2i\pi/3} b_{I,II},$$

$$b_I = 2, \quad b_{II} = 3, \quad c_0 = \frac{4\cdot 6^{2/3}\Gamma^2(2/3)}{7\Gamma(7/6)} \pi^{1/2} \approx 6.6. \qquad (3.14)$$

Plots of the real and imaginary parts of the functions $g_{I,II}$ are shown in Fig. 5. Calculation of the refractive index (3.11) yields β and γ (3.3), after which we can obtain $\rho(L)$ at all values of the parameters, whose choice is dictated by the experimental conditions. Note that β and γ depend on $\nu_{1(2)}$.

It is useful to consider the region $\gamma L \ll 1$, from which we can estimate the upper bound of the splitting probability (for realistic experimental conditions). In this region $\rho(L)$ is given by the elementary function (3.4). The analysis is particularly simple at $\varkappa \gg 1$. In this case

$$\gamma_{I\to II\,II} = \frac{3^{1/2}\alpha}{12\pi} c_0 \frac{eE}{m} \frac{1}{\varkappa^{1/3}}\left[1 + \frac{3}{2}(\nu_1^{-1/3} + \nu_2^{-1/3})\right], \qquad (3.15)$$

and with increasing $\varkappa$ the imaginary part γ decreases: $\gamma \propto \varkappa^{-1/3}$ Since $|T|^2/\omega \propto \varkappa^{-1/3}$ in this region, the quantity $dW(L)$ (3.2) tends to a finite limit at asymptotically large values $\varkappa \gg 1$. This result can be easily understood: actually, the length over which the photons have not yet been converted into e^+e^- pairs increases in proportion to $\varkappa^{1/3}$ and the probability $dW(L)$ turns out to be independent of energy in view of this effective lengthening of the region where the photon splitting takes place:

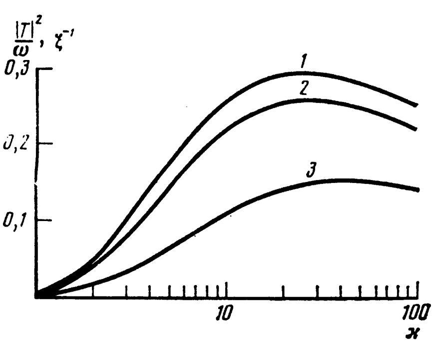

FIG. 3. Value of $|T|^2/\omega$ for the transition I→II; the callouts are the same as in Fig. 2.

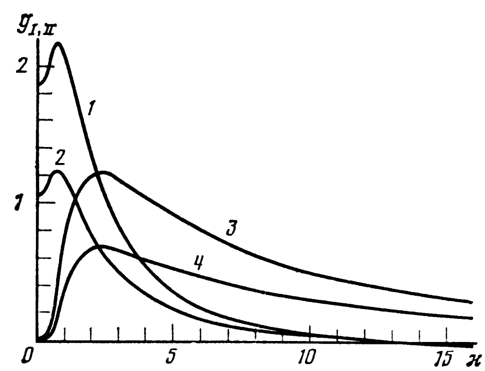

FIG. 5. Plots of the functions $g_{I,II}$: Curve 1—Re g_{II}; curve 2—Re g_I; curve 3—Im g_{II}; curve 4—Im g_I.

$$W_{\mathrm{I}\to\mathrm{II\,II}}(L)\approx 0.15\alpha^2/\pi^2,\quad W_{\mathrm{I}\to\mathrm{I\,I}}(L)\approx 0.024\alpha^2/\pi^2. \qquad (3.16)$$

4. CONCLUSION

As noted in the Introduction, at $\vartheta_0 \ll V_0/m$ the process of pair photoproduction in the fields of axes (or planes) can be treated in the approximation in which the field is constant (over the pair-production length). This means that the process takes place at a given distance ρ from the axis (we consider hereafter, for the sake of argument, the axial case). The axis field is $E=0$ at $\rho=0$, increases to a maximum at $\rho\sim u_1$ (u_1 is the amplitude of the thermal oscillations), and then decreases. To obtain the probability for a single crystal we must therefore evaluate the integral

$$S^{-1}\int d^2\rho\, W(\rho),$$

where S the cross-section area per axis. At $\vartheta_0 \gg V_0/m$ the pair production is coherent, i.e., the Born approximation can be used. A general theory of pair production has been developed, is valid for all angles ϑ_0, and applies in limiting cases to the cases mentioned above.[16,17]

The dependence of the probability on photon splitting on the angle of its entry into the discussed quasiclassical region $(E/E_0)\ll 1$ is perfectly similar to the dependence of the pair-photoproduction probability described above. The equations derived in Sec. 3 are therefore applicable at $\vartheta_0 \ll V_0/m$. The range of validity of the Born approximation was discussed in Ref. 18.

We consider now the possibility of observing photon splitting in single crystals. We must bear in mind here the difference from the case of a magnetic field. There the photon splitting can be dominant at $\varkappa \ll 1$ (pair photoproduction is exponentially suppressed), and for its observation it would suffice to use an energy interval corresponding to long paths. In single crystals, on the other hand, pair production in the field of the axes can be accompanied also by pair photoproduction on individual nuclei (the Bethe-Heitler mechanism). The following circumstances must therefore be taken into account when planning experiments on photon splitting in single crystals:

1. The photon mean free path must not exceed the radiation length (disregarding the technical difficulties entailed in the production of large single crystals).

2. The experiment must be performed under absorption conditions (i.e., Eqs. (3.2) and (3.3) must be used), and the main absorption process is pair photoproduction in the field of the axes (or via the Bethe-Heitler mechanism).

3. Under these conditions it is necessary to take into account, generally speaking, all the amplitudes of the process. To obtain a maximum effect, the experiment must be performed at an energy corresponding to maximum $W_{\gamma\to\gamma\gamma}$ (recall that the dominant transition in this case is I→II II).

4. The principal background process will be photon emission by the particles of the produced pair $\gamma\to e^+e^-\to e^+\gamma e^-\gamma$ in the case when the photons carry away almost the entire energy of the particles. To suppress this background the photons used must likewise have in the crystal shorter mean free paths than the radiation lengths.

5. For electrons and photons moving near crystal axes, the radiation lengths at the energies in question are shorter by 10–100 times (depending on the material on the axis) than in the corresponding amorphous substance.[16,17]

6. Just as in the case of pair production, photon splitting in the field of the axis will be accompanied by photon splitting on individual nuclei (9), with a probability that we denote by $W^{\gamma\to\gamma\gamma}_{\text{nucl.}}$. The ratio $r^{\gamma\to\gamma\gamma}\equiv W^{\gamma\to\gamma\gamma}_{\text{field}}/W^{\gamma\to\gamma\gamma}_{\text{nucl.}}$ can exceed unity substantially only in the region of the maximum $W^{\gamma\to\gamma\gamma}_{\text{field}}$. From this standpoint, the effect can be separated only in the region of a maximum.

In the region where $|T|^2/\omega$ is a maximum we have $\varkappa \gg 1$. In this case, as can be seen from Fig. 5, $\operatorname{Im} g_{\mathrm{I,II}} \gg \operatorname{Re} g_{\mathrm{I,II}}$. When these conditions are satisfied and at $\gamma L \ll 1$ (the photon absorption is small) we have $\rho(L)=L$ in (3.2) and (3.3), and then $dW(L)/L$ is the standard probability. Note that in this limiting case the expression for the probability of the process does not contain a ϑ function.

8. To obtain actual estimates we used an axis potential in the form assumed in Refs. 11, 16, and 17. The calculation was carried out for tungsten ($\langle 111\rangle$ axis, $T=77$ K) at an energy $\omega=400$ GeV. Under this condition $r^{\gamma\to\gamma\gamma}\approx 1.3$, but when the energy is increased the value of $r^{\gamma\to\gamma\gamma}$ in tungsten can exceed 2. In other substances, say germanium (at an energy corresponding to the maximum $|T|^2/\omega$), $r^{\gamma\to\gamma\gamma}$ can exceed 10. Under the indicated conditions we have $W^{\gamma\to\gamma\gamma}_{\text{field}}/W^{\gamma\to e^+e^-}_{\text{field}}\approx 0.3\alpha^2/\pi^2$. Note that $W^{\gamma\to e^+e^-}_{\text{field}}$ in tungsten is 10 times larger than the probability of pair production on individual nuclei.

1) We use a system of units with $\hbar=c=1$, a metric $ab=a^0b^0-\mathbf{ab}$, $\alpha=e^2/4\pi$, and an electron mass m.

2) Note that Eq. (2.1) of Ref. 10 was written for the case when all the photons are incoming; we included the factor $(2\pi)^{-4}$ in the definition of the mean value $\langle 0|...|0\rangle$.

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