Hydrodynamic theory of echoes in a highly inhomogeneous plasma

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A surface-wave echo of a hydrodynamic nature can arise in a semi-infinite cold plasma with an inhomogeneous transition layer. Information on external perturbations applied to the plasma at the times $t = 0$ and $t = \tau$ is retained by undamped plasma waves excited in the transition layer. The echo response to the external perturbations arises at the time $t = 2\tau$. Expressions are derived for the shape of the echo signal in the limiting cases of short and long waves.

INTRODUCTION

A surface wave in a plasma with a diffuse boundary (with an inhomogeneous transition layer) undergoes collisionless damping as a result of transferring its energy to longitudinal plasma waves.[1,2] These waves are excited near the plasma resonance point, where the frequency of a surface wave is equal to the local plasma frequency. The initial-value problem of natural waves in a bounded plasma with an inhomogeneous transition layer was analyzed in Ref. 3 for the case in which the width of this layer is small in comparison with the wavelength. It was found that the energy of a surface wave, which is initially distributed over a broad region of k-space with a width on the order of the reciprocal of the wavelength, is eventually pumped entirely into plasma oscillations of the electric field component parallel to the density gradient. In a cold, collisionless plasma, these oscillations are undamped and are localized in the vicinity of the plasma-resonance point. The width of the localization region is much smaller than the width of the transition layer. In a plasma with a diffuse boundary, the resonant damping of a surface wave is thus not a consequence of an irreversible dissipation of the wave energy. Undamped oscillations of the electric field in the plasma-resonance region may be thought of as a hydrodynamic analog of Van Kampen waves[4] in such a system.

Undamped oscillations of the field in the plasma-resonance region give rise to plasma-echo effects.[5] A short pulse of an external perturbation, applied to the plasma at the time $t = 0$ in the form of a surface wave with a wavenumber k_1, in the direction in which the plasma is homogeneous, decays exponentially over time, leaving undamped oscillations in the electric field component parallel to the density gradient in the plasma-resonance region. These oscillations are modulated by a second external perturbation which is applied to the plasma at $t = \tau$ and which has a wavenumber k_2. The phase evolution of a nonlinear microscopic perturbation gives rise to the excitation, at the time $t = 2\tau$, of a macroscopic surface charge with sum and difference wavenumbers, $k_2 \pm k_1$.

In the present paper we offer a theory for echoes in a plasma with a diffuse boundary. This theory incorporates retardation of the surface waves, in contrast with Ref. 5. The shape of the echo signal is found in the limiting cases of strong and weak slowing of the waves.

LINEAR DISPERSION OF A HIGHLY NONUNIFORM PLASMA

Let us assume that the plasma occupies the region $x > 0$ and is homogeneous along y and z. We assume that the equilibrium plasma density, $n_0(x)$, increases monotonically in the transition region $0 < x < a$ from zero to a constant value $n_0(a)$ and that it remains at the value $n_0(a)$ at $x > a$. Restricting the discussion to a cold plasma with immobile ions, we begin with the hydrodynamic equations for electrons and Maxwell's equations for the electromagnetic field:

$$\begin{gathered}\frac{\partial n}{\partial t}+\operatorname{div}(n\mathbf{v})=0,\qquad \frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}=-\frac{e}{m}\mathbf{E},\\ \operatorname{rot}\mathbf{B}=\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}-\frac{4\pi e}{c}n\mathbf{v},\qquad \operatorname{div}\mathbf{B}=0,\\ \operatorname{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},\qquad \operatorname{div}\mathbf{E}=-4\pi e[n-n_0(x)].\end{gathered}\tag{1}$$

Here e and m are the charge and mass of the electron, n is the density, $\mathbf{v}$ is the hydrodynamic velocity of the electron fluid, and c is the velocity of light.

We solve system (1) by a method of successive approximations, writing the variables in the form

$$A=A_0+A^{(1)}+A^{(2)}+\ldots,$$

where A_0 is the equilibrium value, and $A^{(1)}$ and $A^{(2)}$ are linear and quadratic perturbations, respectively. If the electromagnetic field does not depend on the spatial coordinate z, the system (1) can be broken up into two independent systems for the field components (E_z, B_x, B_y) and (E_x, E_y, B_z). Here we will analyze only the system of equations for (E_x, E_y, B_z), which describes a *TM* wave, for which there may be solutions in the form of surface oscillations. In discussing the results of the linear theory of the dispersion properties of surface waves in a plasma with a diffuse boundary, we follow Ref. 3. Using Laplace time transforms and Fourier transforms in the coordinate y, we find the following equations for the components of the electromagnetic field from system (1) in first-order perturbation theory:

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$$\frac{\partial}{\partial x}\left(\frac{\varepsilon}{N+\varepsilon}\frac{\partial}{\partial x}E^{(1)}_{ykp}\right)-\frac{p^2}{c^2}\varepsilon E^{(1)}_{ykp}=G_1, \tag{2}$$

$$E^{(1)}_{xkp}=-i\frac{N}{k(N+\varepsilon)}\frac{\partial}{\partial x}E^{(1)}_{ykp}+G_2, \tag{3}$$

$$B^{(1)}_{zkp}=-\frac{c}{p}\frac{\varepsilon}{N+\varepsilon}\frac{\partial}{\partial x}E^{(1)}_{ykp}+G_3, \tag{4}$$

where

$$N=(kc/p)^2,\quad \varepsilon(x,p)=1+\omega_{Le}^2(x)/p^2,$$

$$\omega_{Le}^2(x)=4\pi e^2 n_0(x)/m,$$

$$G_1(x)=-i\frac{k}{p}\left[\frac{\partial}{\partial x}\frac{1}{N+\varepsilon}\left(E^{(1)}_{xk}+\frac{1}{p}\frac{\partial}{\partial t}E^{(1)}_{xk}\right)+i\frac{k}{N}\left(E^{(1)}_{yk}+\frac{1}{p}\frac{\partial}{\partial t}E^{(1)}_{yk}\right)\right]_{t=0},$$

$$G_2(x)=-\frac{1}{p(N+\varepsilon)}\left(E^{(1)}_{xk}+\frac{1}{p}\frac{\partial}{\partial t}E^{(1)}_{xk}\right)_{t=0}, \tag{5}$$

$$G_3(x)=-\frac{1}{p}\left[B^{(1)}_{zk}+i\frac{kc}{p}\frac{1}{N+\varepsilon}\left(E^{(1)}_{xk}+\frac{1}{p}\frac{\partial}{\partial t}E^{(1)}_{xk}\right)\right]_{t=0}.$$

Expressions (5) describe initial perturbations of the electromagnetic fields, which we will treat below as external perturbations. The linear perturbations of the directed velocity and density of the electrons are expressed in terms of the electromagnetic field in the following way:

$$v^{(1)}_{x,ykp}=-\frac{e}{mp}E^{(1)}_{x,ykp},$$
$$n^{(1)}_{kp}=\frac{\partial}{\partial x}\left(\frac{en_0}{mp^2}E^{(1)}_{xkp}\right)+ik\frac{en_0}{mp^2}E^{(1)}_{ykp}. \tag{6}$$

A solution of Eq. (2) in the region $0<x<a$ is

$$E^{(1)}_{ykp}(x)=E^{(1)}_{ykp}(0)\left[1+\frac{p^2}{c^2}\int_0^x dx'\frac{N+\varepsilon(x')}{\varepsilon(x')}\int_0^{x'}dx''\varepsilon(x'')\right]$$
$$+\frac{1}{1+N}\frac{\partial}{\partial x}E^{(1)}_{ykp}\Big|_{x=0}\int_0^x dx'\frac{N+\varepsilon(x')}{\varepsilon(x')}$$
$$+\int_0^x dx'\frac{N+\varepsilon(x')}{\varepsilon(x')}\int_0^{x'}dx''G_1(x''). \tag{7}$$

In the region $x<0$, where $n_0(x)=0$, we have $\varepsilon(x,p)=1$. Assuming that there are no initial perturbations of the electromagnetic fields in this region, and imposing the requirement that the field component E_y vanish in the limit $x\to-\infty$, we find from (2)

$$E^{(1)}_{ykp}(x<0)=E^{(1)}_{ykp}(0)\exp(\varkappa_0 x), \tag{8}$$

where

$$\varkappa_0^2=k^2+p^2/c^2,\quad \mathrm{Re}\,\varkappa_0>0.$$

At $x>a$, the dielectric constant of the plasma does not depend on x, since here we have $n_0(x)=n_0(a)=\text{const}$. In the absence of initial perturbations of the fields in the region $x>a$, we find from (2) the following solution, which vanishes in the limit $x\to+\infty$:

$$E^{(1)}_{ykp}(x>a)=E^{(1)}_{ykp}(a)\exp[-\varkappa(x-a)], \tag{9}$$

where

$$\varkappa^2=k^2+(p^2/c^2)\varepsilon(a),\quad \varepsilon(a)=1+\omega_{Le}^2(a)/p^2,\quad \mathrm{Re}\,\varkappa>0.$$

Assuming that the plasma at $0<x<a$ is highly nonuniform,

$$\left|\frac{\partial n_0}{\partial x}\right|\gg kn_0,$$

i.e., assuming $ka\ll 1$, we can omit from (7) the terms which contain a double integral over x. Joining solutions (7)–(9), and requiring that the field component E_y and its derivative be continuous at the points $x=0$ and a, we find

$$E^{(1)}_{ykp}(x)=-\frac{N+\varepsilon(a)}{\varkappa D(p,k)}\left(1+\frac{\varkappa_0}{1+N}\int_0^x dx'\frac{N+\varepsilon(x')}{\varepsilon(x')}\right)\int_0^a G_1(x)dx. \tag{10}$$

Here $D(p,k)$ is the dispersion function, which describes surface oscillations of this semi-infinite plasma with a very nonuniform layer:

$$D(p,k)=\varepsilon(a)\left(1+\frac{\varkappa_0}{1+N}\int_0^a dx\frac{N+\varepsilon(x)}{\varepsilon(x)}\right)+\frac{N+\varepsilon(a)}{\varkappa}\left(\frac{\varkappa_0}{1+N}+\frac{p^2}{c^2}\int_0^a\varepsilon(x)dx\right). \tag{11}$$

Equating (11) to zero, and setting $p=-i\omega_0+\nu$, we find the following expressions for the natural frequency ω_0 and the damping rate ν:

$$\omega_0^2(k)=k^2c^2+\frac{\omega_{Le}^2(a)}{2}\pm\left(k^4c^4+\frac{\omega_{Le}^4(a)}{4}\right)^{1/2}, \tag{12}$$
$$\nu(k)=\frac{\alpha^2(\alpha-1)\pi\varkappa h}{(2\alpha-1)(2\alpha^2-2\alpha+1)}|\omega_0|,$$

where

$$\alpha=k^2c^2/\omega_0^2,\quad h^{-1}=\frac{d}{dx}[\omega_{Le}^2(x)/\omega_0^2]|_{\omega_{Le}(x)=\omega_0}\sim a^{-1}.$$

It can be seen from (10) that the time evolution of the field component $E_y^{(1)}$ is determined by the poles of the dispersion function $D(p,k)$ in the complex p plane. According to (12) these poles describe exponential damping of $E_y^{(1)}$ at a rate ν.

Substituting (10) into (3), we find an expression for the electric field component $E_x^{(1)}$:

$$E^{(1)}_{xkp}(x)=\frac{i}{k}\frac{N[N+\varepsilon(a)]}{\varkappa D(p,k)\varepsilon(x)}\left(\frac{\varkappa_0}{1+N}+\frac{p^2}{c^2}\int_0^x\varepsilon(x')dx'\right)\int_0^a G_1(x)dx$$

$$-i\frac{N}{k\varepsilon(x)}\int_0^x G_1(x')dx'+G_2(x). \qquad (13)$$

Expression (13), along with the poles of $D(p,k)$, which describes oscillations which are damped over time, contains poles of $\varepsilon(x,p)$. These poles give rise in the transition layer $0<x<a$ to oscillations at the local plasma frequency $\omega_{Le}(x)$; in the cold collisionless plasma, these are undamped oscillations.

Since the function $\varepsilon(x,p)$ appears in the numerator in expression (4) for the magnetic field $B_z^{(1)}$, the oscillations of $B_z^{(1)}$ are determined by the poles of $D(p,k)$; i.e., they are damped in time.

ECHO RESULTING FROM UNDAMPED OSCILLATIONS IN THE TRANSITION LAYER

Let us examine the nonlinear response of a plasma to the initial perturbations in (5), produced in the transition layer $0<x<a$. In second-order perturbation theory, system (1) takes the following form after Laplace transforms in time and Fourier transforms in y:

$$\frac{\partial}{\partial x}E_{xkp}^{(2)}+ikE_{ykp}^{(2)}=-4\pi en_{kp}^{(2)}, \qquad (14)$$

$$\frac{\partial}{\partial x}E_{ykp}^{(2)}-ikE_{xkp}^{(2)}=-\frac{p}{c}B_{zkp}^{(2)},$$

$$ikB_{zkp}^{(2)}=\frac{p}{c}E_{xkp}^{(2)}-\frac{4\pi en_0}{c}v_{xkp}^{(2)}-\frac{4\pi e}{c}\int\frac{dk'}{2\pi}\int\frac{dp'}{2\pi i}n_{k-k',p-p'}^{(1)}v_{xk'p'}^{(1)},$$

$$\frac{\partial}{\partial x}B_{zk,p}^{(2)}=-\frac{p}{c}E_{yk,p}^{(2)}+\frac{4\pi en_0}{c}v_{yk,p}^{(2)}+\frac{4\pi e}{c}\int\frac{dk'}{2\pi}\int\frac{dp'}{2\pi i}n_{k-k',p-p'}^{(1)}v_{yk',p'}^{(1)},$$

where

$$v_{xk,p}^{(2)}=-\frac{e}{mp}E_{xkp}^{(2)}-\frac{1}{2p}\frac{\partial}{\partial x}\times\int\frac{dk'}{2\pi}\int\frac{dp'}{2\pi i}(v_{xk-k',p-p'}^{(1)}v_{xk',p'}^{(1)}+v_{yk-k',p-p'}^{(1)}v_{yk',p'}^{(1)}),$$

$$v_{yk,p}^{(2)}=-\frac{e}{mp}E_{yk,p}^{(2)}-i\frac{k}{2p}\times\int\frac{dk'}{2\pi}\int\frac{dp'}{2\pi i}(v_{xk-k',p-p'}^{(1)}v_{xk',p'}^{(1)}+v_{yk-k',p-p'}^{(1)}v_{yk',p'}^{(1)}),$$

$$n_{k,p}^{(2)}=-\frac{1}{p}\frac{\partial}{\partial x}(n_0v_{xk,p}^{(2)})-i\frac{kn_0}{p}v_{yk,p}^{(2)}-\frac{1}{p}\int\frac{dk'}{2\pi}\int\frac{dp'}{2\pi i}\left[\frac{\partial}{\partial x}(n_{k-k',p-p'}^{(1)}v_{xk',p'}^{(1)})-ikn_{k-k',p-p'}^{(1)}v_{yk',p'}^{(1)}\right].$$

From system (14) we easily find equations for the components of the electromagnetic field.

$$\frac{\partial}{\partial x}\left(\frac{\varepsilon}{N+\varepsilon}\frac{\partial}{\partial x}E_{yk,p}^{(2)}\right)-\frac{p^2}{c^2}\varepsilon E_{yk,p}^{(2)}=-i\frac{2\pi ek}{p^2}\int\frac{dk'}{2\pi}\int\frac{dp'}{2\pi i}\left\{ik\frac{2p}{N}n_{k-k',p-p'}^{(1)}v_{yk',p'}^{(1)}-\frac{\partial}{\partial x}\frac{2p}{N+\varepsilon}n_{k-k',p-p'}^{(1)}v_{xk',p'}^{(1)}+\left(\frac{\partial}{\partial x}\frac{n_0}{N+\varepsilon}\frac{\partial}{\partial x}-\frac{p^2n_0}{c^2}\right)(v_{xk-k',p-p'}^{(1)}v_{xk',p'}^{(1)}+v_{yk-k',p-p'}^{(1)}v_{yk',p'}^{(1)})\right\}, \qquad (15)$$

$$E_{xk,p}^{(2)}=-\frac{i}{k}\frac{N}{N+\varepsilon}\frac{\partial}{\partial x}E_{yk,p}^{(2)}-\frac{4\pi e}{N+\varepsilon}\int\frac{dk'}{2\pi}\int\frac{dp'}{2\pi i}\left[\frac{n_0}{2p^2}\frac{\partial}{\partial x}v_{xk-k',p-p'}^{(1)}v_{xk',p'}^{(1)}-\frac{ik}{p}n_{k-k',p-p'}^{(1)}v_{xk',p'}^{(1)}\right], \qquad (16)$$

$$B_{zkp}^{(2)}=-\frac{c}{p}\frac{\partial}{\partial x}E_{ykp}^{(2)}+i\frac{kc}{p}E_{xkp}^{(2)}. \qquad (17)$$

As in the integration of Eq. (2), we find the solution of Eq. (15) in the spatial regions $x<0$, $0<x<a$, and $x>a$. We find the integration constants from the conditions that $E_y^{(2)}$ and $\partial E_y^{(2)}/\partial x$ are continuous at the points $x=0$ and a. Since the resulting expression is extremely lengthy, we will retain in it only a single term, which is the term which dominates the echo oscillations:

$$E_{yk,p}^{(2)}(x)=i\frac{2\pi e^3k}{m^2p^2}\frac{\varepsilon(a,p)}{D(p,k)}\left(1+\frac{\varkappa_0}{1+N}\int_0^x dx'\frac{N+\varepsilon(x',p)}{\varepsilon(x',p)}\right)\times\int_0^a dx\frac{n_0(x)}{\varepsilon(x,p)}\frac{\partial}{\partial x}\int\frac{dk'}{2\pi}\int\frac{dp'}{2\pi i}\frac{E_{xk-k',p-p'}^{(1)}(x)E_{xk',p'}^{(1)}(x)}{(p-p')p'}. \qquad (18)$$

The nonlinear electric field in (18) is the result of the interaction of undamped linear electric fields $E_x^{(1)}$. The macroscopic response to initial perturbations should be manifested in this system as a nonlinear surface charge

$$\sigma_{kp}^{(2)}=-e\int_0^a dxn_{kp}^{(2)}(x). \qquad (19)$$

Expressing the nonlinear density perturbation $n^{(2)}$ in terms of the electric fields with the help of the first equation in system (14), we find

$$\sigma_{kp}^{(2)}=\frac{1}{4\pi}(E_{xkp}^{(2)}(a)-E_{xkp}^{(2)}(0))+\frac{ik}{4\pi}\int_0^a dxE_{ykp}^{(2)}(x). \qquad (20)$$

It is not difficult to see that the first term on the right side of

(20) does not contain a pole of $\varepsilon(x,p)$ and does not describe an echo.

Let us examine the situation in which the nonlinear surface charge is the result of the application to the plasma of two external perturbations, with wave numbers k_1 and k_2, in the direction perpendicular to the density gradient, at the times $t=0$ and $t=\tau$, respectively:

$$E^{\text{ext}}(y,\ t)=E_1\exp(\mp ik_1y)\delta(\omega_0 t)+E_2\exp(ik_2y)\delta[\omega_0(t-\tau)]. \qquad (21)$$

The constant ω_0 has the dimensionality of a frequency. To simplify the equations, we replace expression (13) for the field $E_x^{(1)}$ by the expression

$$E_{xkp}^{(1)}(x)=\frac{i\omega_0}{p\varepsilon(x,p)D(p,k)}E_{kp}^{\text{ext}}. \qquad (22)$$

We now substitute (21) and (22) into (18) and take the inverse Fourier and Laplace transforms. We carry out the integration over p' and p by closing the integration contour in the left-hand half-plane, taking into account the contributions to the integrals only from the poles of $\varepsilon(x,p')$ and $\varepsilon(x,p)$, which describe undamped oscillations. Substituting the result into (20), and integrating by parts in the integral over x ($k_\pm=k_2\pm k_1$), we find

$$\sigma_\pm^{(2)}(y,t)=-i\frac{ek_\pm{}^2a}{96\pi m}E_1E_2(t-\tau)e^{ik_\pm y}\times\int_0^{\omega_{Le}(a)}\frac{d\omega}{\omega^4}\frac{[\omega_{Le}{}^2(a)-\omega^2]\exp[-i\omega(t-2\tau)]}{D(-i\omega,k_\pm)D(-2i\omega,k_2)D(i\omega,\mp k_1)}+\text{c.c.} \qquad (23)$$

The integration variable $\omega=\omega_{Le}(x)$ has been introduced in (23), and only the single term describing the echo surface charge has been retained. The integrand in (23) contains the rapidly oscillating function $\exp[-i\omega(t-2\tau)]$, so that the value of the integral tends toward zero at all times except near the point $t=2\tau$. At this time, a macroscopic surface charge arises in the transition layer at $0<x<a$ and leads to the excitation of an echo surface wave with wavenumbers $k_\pm$.

Expression (11) for the dielectric function $D(-i\omega,k)$ is quite complicated in the general case. It is therefore convenient to evaluate the integral in (23) in the limits of long and short oscillations, in which expression (11) simplifies substantially.

ECHOES IN THE LIMITS OF LONG AND SHORT WAVES

We can formally take the short-wave limit, $(kc)^2\gg\omega_{Le}^2(a)$, by letting $c\to\infty$. In this case, the surface oscillations are quasistatic, and the dielectric function (11) becomes

$$D(-i\omega,k)=\frac{2}{\omega^2}(\omega+\omega_0+i\nu_k)(\omega-\omega_0+i\nu_k), \qquad (24)$$

where [cf. (12)]

$$\omega_0=\omega_{Le}(a)/2^{1/2},\quad \nu_k={}^1/_4\pi kh|\omega_0|.$$

We can now carry out the integration in (23) by switching to the complex ω plane and closing the integration contour in the upper ($t<2\tau$) or lower ($t>2\tau$) half-plane. The contribution to the integral which comes from the corresponding line segments $\mathrm{Re}\,\omega=0$, $\omega_{Le}(a)$, is small in comparison with the contribution from the poles of dielectric functions (24). As a result we find from (23)

$$\sigma_\pm^{(2)}(y,t)=\frac{ek_\pm{}^2a}{576m}\frac{E_1E_2(t-\tau)}{\nu_1+\nu_\pm}\sin\left[k_\pm y-\frac{\omega_{Le}(a)}{\sqrt{2}}(t-2\tau)\right]\times\begin{cases}\exp[\nu_1(t-2\tau)], & t<2\tau,\\ \exp[-\nu_\pm(t-2\tau)], & t>2\tau,\end{cases} \qquad (25)$$

where $\nu_i\equiv\nu(k_i)$. We see from (25) that the echo signal reaches a maximum at the time $t=2\tau$ and decays exponentially on both sides of the maximum. The shape of the signal is asymmetric in time. The surface charge in (25) excites an echo quasistatic surface wave with the following potential at the boundary:

$$\Phi_\pm^{(2)}(0)\approx\Phi_\pm^{(2)}(a)\approx4\pi\sigma_\pm^{(2)}/k_\pm{}^2a. \qquad (26)$$

In the opposite limit, $(kc)^2\ll\omega_{Le}^2(a)$, the long surface waves are quasitransverse, and dielectric functions (11) can be written

$$D(-i\omega,k)=\frac{\omega_{Le}{}^2(a)}{2\omega^6}(\omega+kc+i\tilde{\nu}_k)(\omega-kc+i\tilde{\nu}_k), \qquad (27)$$

where

$$\tilde{\nu}_k=\pi hk^3c^2/\omega_{Le}(a).$$

As in the quasistatic limit, we carry out the integration in (23) in the complex ω plane. In this case, the contribution from the integrals along the line segments $\mathrm{Re}\,\omega=0$, $\omega_{Le}(a)$ is small in comparison with the contribution from the poles of the integrand under the conditions $|t-2\tau|\omega_{Le}(a)\gg1$. The result given below thus describes the behavior of the tails of the echo signal far from its maximum value:

$$\sigma_\pm^{(2)}(y,t)=-\frac{8ek_\pm{}^2a}{3m}\frac{E_1E_2c^9}{\omega_{Le}{}^{10}(a)}(t-\tau)\begin{cases}-k_1{}^{13}(k_1{}^2-k_\pm{}^2)^{-1}(k_1{}^2-{}^1/_4k_2{}^2)^{-1}\cos[k_\pm y-k_1c(t-2\tau)]\\ \times\exp[\tilde{\nu}_1(t-2\tau)], \quad t\ll2\tau,\\ k_\pm{}^{13}(k_\pm{}^2-{}^1/_4k_2{}^2)^{-1}(k_\pm{}^2-k_1{}^2)^{-1}\cos[k_\pm y-k_\pm c(t-2\tau)]\exp[-\tilde{\nu}_\pm(t-2\tau)]\\ +({}^1/_2k_2)^{13}({}^1/_4k_2{}^2-k_\pm{}^2)^{-1}({}^1/_4k_2{}^2-k_1{}^2)^{-1}\cos[k_\pm y-{}^1/_2k_2c(t-2\tau)]\\ \times\exp[-{}^1/_2\tilde{\nu}_2(t-2\tau)], \quad t\gg2\tau.\end{cases} \qquad (28)$$

Substituting (16) and (18) into (17), we find the following expression for the echo magnetic field at the point $x = 0$:

$$B_z^{(2)}(0, y, t) = \frac{e}{m}\frac{32\pi E_1 E_2}{3c^5\omega_{Le}^{10}(a)}\frac{k_\pm(t-\tau)}{k_\pm^2-k_1^2}\begin{cases} (k_1c)^{14}(k_1^2-{}^1/_4k_2^2)^{-1}(k_\pm^2-k_1^2)^{-1/2}\cos[k_\pm y-k_1c(t-2\tau)]\exp[\tilde{\nu}_1(t-2\tau)], \\ t\ll 2\tau, \\ (k_\pm c)^{12}c\omega_{Le}(a)(k_\pm^2-{}^1/_4k_2^2)^{-1}\cos[k_\pm y-k_\pm c(t-2\tau)]\exp[-\tilde{\nu}_\pm(t-2\tau)], \\ t\gg 2\tau. \end{cases} \quad (29)$$

The reason for the difference in the time evolutions of echo signals (28) and (29) is that in the case of nonelectrostatic oscillations the surface charge is a forced perturbation which accompanies resonant oscillations of the electromagnetic field of the surface wave.

CONCLUSION

It has been assumed in this theory that the surface wave is damped in a collisionless fashion over a time shorter than the time interval τ between the external perturbations, as it excites three-dimensional plasmons at the plasma-resonance point $x = x_0$. On the other hand, the time interval τ should be too short for damping of plasmons, which retain information on the external perturbation. This damping could result only from binary collisions or a thermal removal of plasmons from the plasma-resonance region.[2] The condition for the applicability of these results is therefore

$$\min\left\{\frac{1}{\nu_{ei}};\frac{1}{\omega_{Le}}\left(\frac{a}{r_{De}}\right)^{2/3}\right\}\Bigg|_{x=x_0}\gg\tau\gg\frac{1}{\nu}, \quad (30)$$

where ν_{ei} is the electron-ion collision rate, and ν is the damping rate of the surface wave, given in (12),

This analysis has shown that an echo in a plasma is not necessarily the result of a modulation of the charged-particle distribution function by external perturbations; i.e., it is not necessarily of a kinetic nature. For an echo to appear there must be undamped microscopic oscillations of some quantity; the phase focusing of these oscillations gives rise to the excitation of a macroscopic signal. In the case at hand, these oscillations are oscillations of the electric field component parallel to the density gradient near the plasma-resonance point. These oscillations are plasma oscillations at the local plasma frequency. An echo in a highly inhomogeneous plasma may therefore be of a hydrodynamic nature; i.e., it may be determined by oscillations of a charged fluid, rather than by oscillations of resonant particles of an ionized gas.

The echo in a bounded plasma with an inhomogeneous transition layer is analogous to an oscillatory echo in a system with a continuous spectrum.[6]

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