Self-similar solutions for the compression of a plasma and a magnetic field by a liner

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The compression of a plasma and a magnetic field by a cylindrical shell (or liner) is analyzed. The formation of a thin boundary layer with a dense plasma near the liner substantially accelerates the loss of magnetic flux from the plasma. The structure of this boundary layer is found. It is described by self-similar solutions of the transport equations in a plasma with a magnetic field. Analytic solutions are derived for the problem with an arbitrary (quasistatic) evolution of the plasma compression. Possibilities for generating ultrastrong pulsed magnetic fields in such systems are discussed.

INTRODUCTION

The possibilities of the method proposed by Bogomolov *et al.*[1] for producing ultrastrong magnetic fields (on the order of 100 MG) by compressing a plasma with a frozen-in magnetic field by a cylindrical shell (a liner) have been discussed in several papers.[1–3] In this approach, the magnetic field is intensified by currents induced in the plasma (so that the liner may be totally nonconducting), in contrast with the conventional method for generating megagauss fields, which involves the compression of magnetic flux by a liner.[4] This distinction basically frees us from one of the most serious limitations imposed on the magnetic fields which can be reached: the explosion of a current-carrying skin in the liner.[4]

The primary question now is how rapid the compression should be if we wish to keep the magnetic field frozen in the plasma. Bogomolov *et al.*[1] assumed that this condition could be met by achieving a large magnetic Reynolds number R_m:

$$R_m = R(t)\, u_L(t) / D_m(t) \gg 1. \tag{1}$$

Here R is the radius of the liner, $u_L = |\dot{R}|$ is its velocity, $D_m = c^2/4\pi\sigma$ is the magnetic viscosity of the plasma, and σ is the conductivity of the plasma. As was pointed out in Ref. 2, however, the magnetic flux trapped by the plasma is lost from this system far more rapidly than would be implied by the simple estimates which lead to condition (1). The reason is that as the magnetic field is building up in the main volume of the plasma, the plasma is pushed toward the liner wall, where it forms a thin boundary layer of dense plasma. The flow which arises carries the magnetic field off to the liner wall in a convective manner and simultaneously causes a pronounced compression of the current layer at the wall. As a result, the condition which must be satisfied in order to achieve effective compression of the magnetic field is considerably stiffer than (1), specifically,

$$R_{eff} \equiv R u_L / D_{eff} \gg 1, \tag{2}$$

where D_{eff} is the effective magnetic viscosity coefficient, which is far larger than D_m. The coefficient D_{eff} was evaluated in Ref. 2.

The accelerated loss of magnetic flux from a plasma compressed by a liner has also been studied by Velikovich *et al.*[3] The results of their numerical simulation of the compression confirm the features which we have just described in the diffusion of the magnetic field. As for the analytic solutions of the hydrodynamic equations of the plasma which were derived in Ref. 3, we note that they do not give us a basis for formulating a condition—to replace condition (1)—under which the magnetic field remains frozen in the plasma. The reason is that Velikovich *et al.*[3] studied self-similar solutions, in which all the scale lengths are proportional to the liner radius. In other words, the plasma compression is uniform over the volume. In contrast, the accelerated loss of magnetic flux in which we are interested here stems from a redistribution of the plasma over the cross section, which unfolds continuously against the background of the uniform compression of the plasma.

In the present paper we show that the problem of the evolution of the magnetic field in a liner-compressed plasma is amenable to analytic solution for an arbitrary compression law $R(t)$. Our solution is based on the circumstance that the thickness of the boundary layer at the liner, in which the plasma and magnetic field gradients are important, remains at all times small in comparison with the radius of the system. Consequently, the time evolution of the parameters of the homogeneous plasma can be found from simple equations describing the balance of the number of particles, the energy, and the magnetic flux between the bulk of the plasma and the boundary layer. Correspondingly, in Section 1 of this paper we find the structure of the boundary layer. As we will see, this structure is described by self-similar solutions of the transport equations in a plasma with a magnetic field. In section 2 we derive equations describing the behavior of the magnetic field during the compression of the plasma. These results can be used to evaluate the effectiveness of this method for generating strong magnetic fields.

1. BASIC EQUATIONS AND STRUCTURE OF THE BOUNDARY LAYER

We assume a cylindrically symmetric compression of a plasma with a magnetic field directed along the axis of the

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cylinder (the z axis). We assume that the problem is one-dimensional, so that all quantities are functions of only the radius r and the time t. We assume that there is initially (before the compression) a homogeneous plasma with a density n_0, a temperature T_0, and a magnetic field H_0 inside the liner, whose radius is R_0. The very basis of the problem (the conversion of the kinetic energy of the liner into magnetic-field energy) implies that the thermal energy of the plasma is much smaller than the magnetic energy, so that the condition $\beta_0 = 8\pi n_0 T_0 / H_0^2 \ll 1$ holds. During the subsequent compression of the plasma, the magnetic field in the plasma becomes stronger than the field at the liner wall (if the liner is nonconducting, the magnetic field at it does not change at all; it remains equal to its initial value). On the other hand, the compression velocities which are of practical interest are considerably lower than the Alfvén velocity in the plasma, so that a process of this sort is approximately adiabatic, and there is time for equalization of the total pressure in the system:

$$\frac{\partial}{\partial r}\left(nT + \frac{H^2}{8\pi}\right) \approx 0. \tag{3}$$

Under the condition $\beta_0 \ll 1$, this equation implies that although the plasma pressure in the main volume remains low in comparison with the magnetic-field pressure ($\beta_i = 8\pi n_i T_i / H_i^2 \ll 1$), a boundary layer in which the plasma pressure is $nT \sim H_i^2/8\pi$ and its density satisfies $n \gg n_i$ forms near the liner (here and below, H_i, T_i, and n_i are the magnetic field, temperature, and density of the homogeneous plasma at the center). Figure 1 shows qualitative profiles of the magnetic field and the plasma pressure during the compression.

The thickness Δ of the boundary layer of dense plasma is small in comparison with the radius of the system ($\Delta \ll R$; see Subsection 1 in the Appendix), so that we may regard finding the structure of this layer as a two-dimensional problem. We transform to a coordinate system which is moving with the liner; we put the plane $x = 0$ at the wall, and we put the plasma in the region $x > 0$. The transport equations in the plasma given in the review by Braginskiĭ[5] and the equilibrium condition in (3) can then be written

$$nT + H^2/8\pi = H_i^2/8\pi, \tag{4}$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0, \tag{5}$$

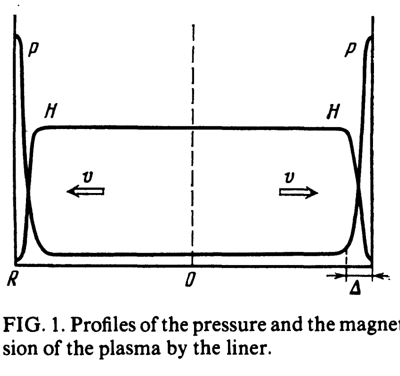

FIG. 1. Profiles of the pressure and the magnetic field during the compression of the plasma by the liner.

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left(\frac{c^2}{4\pi\sigma}\frac{\partial H}{\partial x} + \frac{c}{e}\beta_\wedge \frac{\partial T}{\partial x} - vH\right) = -\frac{\partial q_H}{\partial x}, \tag{6}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}nT + \frac{H^2}{8\pi}\right) = \frac{\partial}{\partial x}\left\{\varkappa_\perp \frac{\partial T}{\partial x} + \frac{cT}{4\pi e}\beta_\wedge \frac{\partial H}{\partial x} - \frac{5}{2}nTv + \frac{H}{4\pi}\left(\frac{c^2}{4\pi\sigma}\frac{\partial H}{\partial x} + \frac{c}{e}\beta_\wedge\frac{\partial T}{\partial x} - vH\right)\right\} = -\frac{\partial q_W}{\partial x}. \tag{7}$$

Here we are using the notation of Ref. 5; $\varkappa_\perp$ is the thermal conductivity of the plasma in the direction across the magnetic field, and $\beta_\wedge$ is the thermoelectric coefficient [the last term on the right side of (7) incorporates the Poynting contribution to the energy flux]. The boundary conditions on system (4)–(7) are imposed at the liner surface (at $x = 0$). There the magnetic field is equal to the external field, $H(0,t) = H_0$, and the plasma flow velocity vanishes, $v(0,t) = 0$. We also assume that the heat capacity of the liner is substantial, so that the plasma temperature remains constant at the liner surface: $T(0,t) = T_L$. Outside the boundary layer (i.e., at $x \gg \Delta$) the plasma and the magnetic field are both uniform, so that in the solutions of equations (4)–(7) of interest there we will have $H = H_i$, $n = n_i$, and $T = T_i$ in the limit $x \to +\infty$.

The pronounced compression of the boundary layer which we mentioned above has the consequence that the magnetic field flux q_H and the energy flux q_W carried to the boundary layer by the plasma flow are far greater than the changes in the magnetic flux and the energy inside the layer (Subsection 2 in the Appendix):

$$\Delta q_H = -\int_\Delta \frac{\partial H}{\partial t}\,dx \ll q_H,$$

$$\Delta q_W = -\int_\Delta \frac{\partial}{\partial t}\left(\frac{3}{2}nT + \frac{H^2}{8\pi}\right)dx \ll q_W.$$

We can then replace Eqs. (6) and (7) by the conditions that the total fluxes of the energy and of the magnetic field are constant:

$$q_H = vH - \frac{c^2}{4\pi\sigma}\frac{\partial H}{\partial x} - \frac{c}{e}\beta_\wedge\frac{\partial T}{\partial x} = v_i H_i, \tag{6'}$$

$$q_W = -\varkappa_\perp\frac{\partial T}{\partial x} - \frac{cT}{4\pi e}\beta_\wedge\frac{\partial H}{\partial x} + \frac{5}{2}nTv + q_H\frac{H}{4\pi} = v_i\frac{H_i^2}{4\pi}, \tag{7'}$$

where $v_i = v(+\infty)$ is the velocity with which the homogeneous plasma flows into the boundary layer. In Eqs. (4), (6'), and (7') it is convenient to transform to the dimensionless variables

$$h = H/H_i, \quad \rho = n/n_i, \quad u = v/v_i, \quad \theta = T/T_H,$$

where

$$T_H = H_i^2/8\pi n_i, \quad \xi = x|v_i|/D_H, \quad D_H = c^2/4\pi\sigma(T_H),$$

and D_H is the magnetic viscosity of the plasma, with temperature $T = T_H$. The equations can then be written

$$\rho\theta + h^2 = 1, \tag{8}$$

$$uh + \theta^{-3/2}dh/d\xi + \alpha d\theta/d\xi = 1, \tag{9}$$

$$\varkappa d\theta/d\xi+\alpha\theta dh/d\xi+{}^5/_4\rho\theta u+h=1. \quad (10)$$

An important aspect of this problem is that although the plasma in the main volume is strongly magnetized (the condition $\omega_H\tau \gg 1$ holds, where ω_H is the cyclotron frequency, and τ is the particle scattering time) the decrease in the magnetic flux and the increase in the plasma density in the boundary layer lead to demagnetization of this plasma, so that at the liner wall the condition $\omega_H\tau \ll 1$ holds. At the same time, the kinetic coefficients $\beta_\wedge$ and $\varkappa_\perp$ which appear in Eqs. (6') and (7') depend in completely different ways on the parameters of the plasma and the magnetic field in the regions of magnetized and unmagnetized plasma.[5] We will therefore use some simple model expressions which give a qualitatively correct description of the behavior of the plasma transport coefficients in both limiting cases. We will accordingly study separately three regions of the plasma parameters: region I in which the plasma ions are magnetized, and the condition $(\omega_H\tau)_i > 1$ holds; region III in which the plasma is unmagnetized with $(\omega_H\tau)_e < 1$; and an intermediate region II, where the electrons are magnetized, but the ions are not. Using

$$(\omega_H\tau)_i=\mu^{1/2}(\omega_H\tau)_e, \quad \mu\equiv m_e/m_i\ll 1,$$

where μ is the ratio of the electron and ion masses, we can describe the boundaries of region II by $\mu^{1/2} < (\omega_H\tau)_i < 1$. The dimensionless coefficients $\varkappa$ and α which appear in Eqs. (9) and (10) can now be written in the following form[5]:

$$\varkappa=\begin{cases}\mu^{-1/2}\rho^2/h^2\theta^{1/2}\,(\mathrm{I}), & h\theta^{3/2}/\rho>\mu^{-1/2}\delta^{-1},\\ \delta\rho\theta/h\,(\mathrm{II}), & \delta^{-1}<h\theta^{3/2}/\rho<\mu^{-1/2}\delta^{-1},\\ \delta^2\theta^{5/2}\,(\mathrm{III}), & h\theta^{3/2}/\rho<\delta^{-1},\end{cases}$$
$$\alpha=\begin{cases}\rho/h\theta^{3/2}\,(\mathrm{I,\ II}),\\ \delta^2h\theta^{3/2}/\rho\,(\mathrm{III}).\end{cases} \quad (11)$$

Here $\delta\equiv\omega_{He}\tau_e(n_i, H_i, T_H)\gg 1$ is a large parameter. As we will see below, the existence of this large parameter gives rise to an anomalously rapid loss of magnetic flux in the plasma. Let us formulate the boundary conditions in terms of the new variables. Since the quantity H_i increases during the compression of the plasma and becomes much greater than the initial field H_0 (and thus much greater than the field at the liner), we may assume $h(\xi=0)\approx 0$. The condition $\beta_i \ll 1$ means that the temperature of the homogeneous plasma satisfies $T_i \ll T_H = H_i^2/8\pi n_i$, so that we have $\theta(\xi\to\infty)\approx 0$. On the other hand, the temperature T_H increases during the compression of the plasma ($T_H \propto H_i$) and ultimately becomes much higher than the temperature T_L of the liner shell. We thus also have $\theta(\xi=0)\approx 0$. As a result, the structure of the boundary layer is described in terms of the dimensionless variables defined above by the universal equations (8)–(10) with the boundary conditions

$$\begin{gathered}\rho(+\infty)=h(+\infty)=u(+\infty)=1,\\ u(0)=h(0)=\theta(0)=\theta(+\infty)=0.\end{gathered} \quad (12)$$

This result means that at all times the profiles of the density n, the magnetic field H, and the plasma temperature T differ only through changes in the scale of these quantities and in the length scale. In other words, the boundary layer evolves in a self-similar way.

Since the plasma flow velocity u appears in Eqs. (8)–(10), these equations will generally have to be solved jointly with the continuity equation (5). In the particular case with which we are concerned here, however, this situation simplifes considerably, since the velocity is related in an unambiguous way to the plasma density ($u = 1/\rho$; see Subsection 3 in the Appendix) in that part of the boundary layer in which the convection components of the fluxes of the magnetic field and the energy are important.

We first consider Eqs. (8)–(10) in the unmagnetized plasma near the liner wall (region III). Since the magnetic field is weak here (at the wall itself we have $h = 0$), we conclude from (8) that the plasma pressure satisfies $\rho\theta\sim 1$. In this case the fluxes of the energy and the magnetic field are dominated by the thermal conductivity of the plasma and magnetic diffusion, so that we can write, according to (9)–(11)

$$\varkappa\frac{d\theta}{d\xi}=\delta^2\theta^{5/2}\frac{d\theta}{d\xi}\sim 1,\qquad \theta^{-3/2}\frac{dh}{d\xi}\sim 1. \quad (13)$$

Also using the boundary conditions (12), we then see that the distance from the liner wall is described by $\xi\sim\delta^2\theta^{7/2}$, and the increase in the magnetic field is described by $dh\sim\delta^2\theta^4 d\theta$. This solution holds up to $\theta\lesssim\theta_1\sim\delta^{-2/5}$, where the electrons become magnetized (and now the magnetic field is $h\sim 1$). The thickness of this region is $\Delta\xi=\xi_1\sim\delta^2\theta^{7/2}\sim\delta^{3/5}$. At $\xi>\xi_1$ there is a transition to region II, where the electrons are magnetized ($\omega_{He}\tau_e > 1$), while the ions are not yet magnetized. Here it is convenient to write Eqs. (9) and (10) in the following form, where we are using (8) and (11):

$$\begin{gathered}\theta^{-3/2}\frac{dh}{d\xi}=-\frac{1}{2h\theta^{3/2}}\frac{d(\rho\theta)}{d\xi}=1,\\ \varkappa\frac{d\theta}{d\xi}=\delta\frac{\rho\theta}{h}\frac{d\theta}{d\xi}=1-h-\alpha\theta\frac{dh}{d\xi}=\rho\theta\left(\frac{1}{1+h}-\frac{1}{h}\right).\end{gathered} \quad (14)$$

We see from the last equation that in region II the condition $d\theta/d\xi<0$ holds; i.e., the plasma temperature reaches its maximum, $\theta_{max}\sim\theta_1$ at $(\omega_H\tau)_e\sim 1$. As the temperature then decreases, $\Delta\theta\sim\theta_1$, the plasma pressure falls off from a value $\rho\theta\sim 1$ to a value $\rho\theta\sim\mu^{1/2}$, where the plasma ions also become magnetized (so that there is a transition in region II from a plasma with $\beta\sim 1$ to a plasma with $\beta\ll 1$). In deriving (14) we assumed that the condition $\rho\gg 1$ holds everywhere in region II, so that the convective components of fluxes (9) and (10) are still unimportant here. This assumption imposes a restriction on the parameter δ: $\mu^{1/2}\theta_1^{-1}\gg 1$, i.e.,[1)] $\delta\gg\mu^{-5/4}$. In the magnetized plasma (region I), the relatively high ion thermal conductivity has the consequence that the plasma pressure falls off rapidly, while the plasma temperature remains nearly constant, $\theta\sim\theta_1$:

$$d\ln(\rho\theta)/d\ln\theta\sim\mu^{-1/2}\gg 1.$$

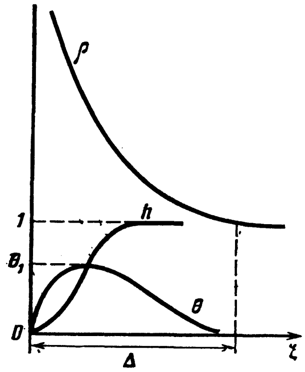

FIG. 2. Structure of the boundary layer.

Events proceed in this direction until the plasma density reaches $\rho > 1$. Further from the wall we have $\rho \approx h \approx 1$, and the plasma temperature falls in accordance with $\theta \sim \mu^{-1}\xi^{-2}$. The characteristic thickness $\Delta\xi$ of the boundary layer is of order $\delta^{3/5}$. Figure 2 shows qualitative profiles of the θ, ρ, and h.

Below we will need to know the amount of plasma in the boundary layer:

$$\Pi_\Delta = \int_0^\infty (\rho - 1)\, d\xi.$$

Using the solutions found above, we easily see that this integral is dominated by that part of the boundary layer in which the electron magnetization parameter satisfies $\omega_{He}\tau_e \sim 1$, and the plasma pressure is $\rho\theta \sim 1$:

$$\Pi_\Delta \sim \int_\Delta \rho\, d\xi \sim \theta_1^{-1}\xi_1 \sim \delta \gg 1. \qquad (15)$$

Returning to the dimensional variables, and making use of the cylindrical geometry of the problem, we can write the total number N_Δ of plasma particles in the boundary layer as follows:

$$N_\Delta = 2\pi R n_i \frac{D_H}{v_i}\Pi_\Delta = 2\pi R n_i \frac{D_H}{v_i}\alpha\delta = \alpha RcH_i/2ev_i, \qquad (16)$$

where the numerical factor α is of order unity. To determine this factor we need to solve Eqs. (6′) and (7′) with the exact values of the transport coefficients in the plasma.[5]

2. INCREASE OF THE MAGNETIC FIELD DURING THE PLASMA COMPRESSION

Let us find the time evolution of the parameters of the homogeneous plasma as it is compressed by the liner. The magnetic flux trapped by the plasma, $\Phi = \pi R^2 H_i$, decreases as a result of convective removal of magnetic flux with the flow, so we can write

$$d\Phi/dt = -2\pi R q_H = -2\pi R v_i H_i. \qquad (17)$$

We find yet another equation from the condition describing the balance in the number of particles between the homogeneous plasma and the boundary layer, which may be written in the form

$$dN_\Delta/dt = 2\pi R v_i n_i. \qquad (18)$$

Now specifying a definite plasma compression law[2)] $R(t)$, and making use of the fact that the magnetic field is frozen in the homogeneous plasma,

$$H_i/n_i = \text{const} = H_0/n_0,$$

we find from Eqs. (16)–(18) the time dependence of the magnetic field in the plasma, $H_i(t)$. We introduce the dimensionless quantities

$$\tau = t\frac{u_L}{R_0}, \quad r(\tau) = R/R_0,$$

$$x(\tau) = \Phi/\pi R_0^2 H_0, \quad y(\tau) = N_\Delta/\pi R_0^2 n_0,$$

where u_L is a characteristic velocity of the liner. In terms of these variables, Eqs. (16)–(18) become

$$\frac{dx}{d\tau} = -\frac{1}{R_{eff}^{(0)}}\frac{x^2}{r^2 y}, \quad \frac{dy}{d\tau} = \frac{1}{R_{eff}^{(0)}}\frac{x^2}{r^2 y}. \qquad (19)$$

The quantity $R_{\text{eff}}^{(0)}$ which we have introduced here has the meaning of an effective magnetic Reynolds number, specifically,

$$R_{eff}^{(0)} = \pi n_0 e R_0 u_L/\alpha c H_0. \qquad (20)$$

It can be seen from (19) that we have $x + y = \text{const}$; since initially we have $x = 1$ and $y = 0$, we find $y = 1 - x$. Using this relation, we can easily integrate the first equation in (19):

$$\frac{1}{x} - 1 + \ln x = \frac{1}{R_{eff}^{(0)}}\int_0^\tau \frac{dt}{r^2(t)}. \qquad (21)$$

It follows from expression (20) for $R_{\text{eff}}^{(0)}$ that the loss of magnetic flux from the plasma occurs with an effective magnetic diffusion coefficient

$$D_{eff} = \alpha c H_0/\pi e n_0. \qquad (22)$$

Interestingly D_{eff} is totally independent of the electron collision rate in the plasma, although the loss of magnetic flux results from the finite conductivity of the plasma in the boundary layer. The situation here is analogous in many ways to that of shock waves in gases.[6] There the thickness of the shock front adjusts to a value such that the necessary dissipation occurs. In the case at hand, the electrical conductivity and the thermal conductivity of the plasma regulate the width of the boundary layer so that the magnetic flux carried to this layer by the plasma flow can diffuse to the wall.

To illustrate the results we show in Fig. 3 solutions of Eq. (21) for various values of the effective magnetic Reynolds number $R_{\text{eff}}^{(0)}$. This figures shows the behavior of the magnetic field in the plasma when the plasma is compressed by a factor of ten along the radius by a liner moving at a constant velocity, with $r(\tau) = 1 - \tau$. We see that for effective compression of the magnetic flux, i.e., for an increase of the initial field by two orders of magnitude, we need $R_{\text{eff}}^{(0)}$ to be about 30.

An important distinction between the condition derived here of the freezing in of the magnetic field in the plas-

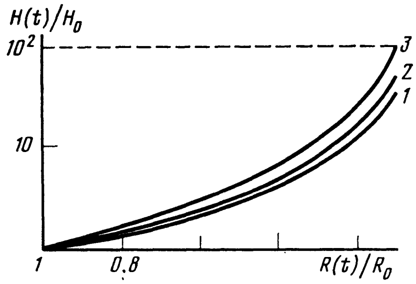

FIG. 3. Profiles of the magnetic field in the plasma for various values of the effective magnetic Reynolds number $R_{\text{eff}}^{(0)}$: 1—10; 2—30; 3—∞.

ma, on the one hand, and condition (1), on the other, is the way in which R_{eff} depends on the plasma density [the magnetic viscosity D_m of the plasma, which figures in (12) does not depend on the plasma density]. It now follows from expression (20) that we would like to increase the initial plasma density n_0 in order to reduce the compression velocity which we will need. Choosing roughly the same parameters for the system as were discussed in Refs. 1 and 3, i.e., $R_0 \approx 10^{-1}$ cm and $H_0 \approx 2\cdot 10^5$ G, we see from (20) that in order to reach $R_{\text{eff}}^{(0)} \approx 30$, even at $n_0 \approx 10^{19}$ cm^{-3}, we would need a liner velocity[3)] $u_L \approx 5\cdot 10^7$ cm/s. Such velocities are considerably higher than the compression velocities Bogomolov *et al.*[1] had in mind when they originally suggested this approach. It should also be noted that the requirements on the initial velocity of the liner may become even more severe when we take into account the slowing of the liner in the final stage of the compression.

APPENDIX

1. The solutions derived here lean heavily on the circumstance that the thickness Δ of the boundary layer at the wall is small in comparison with the plasma radius R. Does this condition acutally hold? According to the estimates in Section 1, we have $\Delta \sim \delta^{3/5} D_H / v_i$. Using expression (16) and the dimensionless magnetic flux x which we introduced in Section 2, we easily find

$$\Delta \sim \frac{D_H}{v_i}\delta^{3/5} \sim \frac{R}{\delta^{2/5}}\frac{1-x}{x}. \tag{A1}$$

We thus see that the thickness of the boundary layer increases monotonically with decreasing magnetic flux in the plasma, but by the time a significiant fraction of the initial magnetic flux has been lost [i.e., $x \sim (1-x) \sim 1$] the quantity Δ is still small:

$$\Delta \sim R/\delta^{2/5} \ll R.$$

2. We can show that the change in the magnetic flux in the boundary layer satisfies $\Delta q_H \ll q_H = v_i H_i$. From Eq. (6) we find

$$\Delta q_H = -\int_\Delta \frac{\partial H}{\partial t}dx \sim \frac{d}{dt}(H_i\Delta).$$

On the other hand, the flux is

$$q_H = v_i H_i = -\frac{1}{2\pi R}\frac{d\Phi}{dt} = -\frac{1}{2\pi R}\frac{d}{dt}(\pi R^2 H_i)$$

[see (17)], so that we have $\Delta q_H / q_H \sim \Delta / R \ll 1$. The same relation can be proved for the energy flux in an analogous way: $\Delta q_W / q_W \sim \Delta/R \ll 1$.

3. In the quasistatic approximation, in which plasma equilibrium condition (3) holds, the plasma flow velocity is determined from the continuity equation (5). Integrating this equation along the coordinate x, and noting that at the liner wall ($x = 0$) there is no plasma flow, we find

$$nv = \frac{\partial}{\partial t}\int_0^x n\,dx'. \tag{A2}$$

As follows from the results of Section 1, however, the amount of plasma in the boundary layer is determined by the region with the coordinate $x \sim x_1 \sim \xi_1 (D_H / v_i)$ [see (15)]. Consequently, the integral on the right side of (A2) becomes independent of the upper limit of the integration at $x > x_1$. This circumstance means that at $x \gg x_1$ there is a constant plasma flow $nv = n_i v_i$ or, in terms of dimensionless variables, $u = 1/\rho$ (and at $x \lesssim x_1$ the condition $u \lesssim 1/\rho$ holds). Consequently, the convective components of the magnetic field flux in (9) and the energy flux in (10) play a role only in that part of the boundary layer in which the plasma density satisfies $\rho \sim 1$.

1)The ultimate result remains the same in the case $\delta < \mu^{-5/4}$. The only change is in the structure of the boundary layer.
2)The time dependence of the plasma radius, $R(t)$, should be determined from the equation of motion of the liner. The solution of this equation is a separate problem.
3)The numerical factor α in (20) was found by comparing the curves in Fig. 3 with the results of a numerical integration of the transport equations in the plasma, taken from Ref. 3. The result is $\alpha \approx 0.4$.

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