# Phase slippage lines in wide superconducting films

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The resistive state produced in wide superconducting films by formation of phase slippage lines (PSL), which are two-dimensional analogs of phase slippage centers in narrow channels, is investigated. The onset of PSL, on which superconductivity is locally suppressed and an electric field is generated, permits the major part of the film to remain in a superconducting state at a current higher than critical, in view of tendency of the current to spread out evenly over the film width. The current distribution in a film with a periodic PSL structure is calculated and its current-voltage characteristic is found.

## 1. INTRODUCTION

The resistive state of superconducting films (i.e., the state in which the film resistance differs from zero but is less than the normal value) can be produced by various mechanism. When the current in a film of width $b$ less than the coherence length $\xi$ (superconducting channel) exceeds the critical pair-breaking current, phase-slippage centers (PSC) are produced in the film[1] by the voltage drop. In a wider film, Abrikosov vortices can be formed on its edges by the current's magnetic field and their motion leads to the appearance of resistance.[2]

Experiments[3] on wide films (of width $b$ larger than the effective depth $\lambda_{\rm eff}$ of penetration of the magnetic field into the film) yielded current-voltage characteristics (IVC) similar to the IVC of narrow channels. This is evidence that systems similar to PSC, phase-slippage lines (PSL), can be produced in wide films. These are PSC "stretched" across the current over the entire width of the film. The present paper is devoted to an analysis of such a picture of the resistive state and to a calculation of the corresponding IVC.

In the absence of an electric field (in the Meissner state) the current is not uniformly distributed in the film, and its density is increased with increase of distance from its center. It reaches at the film edge the critical pair-breaking current density $j_c$ when the total current is

$$I_c=\frac{(15\pi)^{1/2}}{2}J_c(b\lambda_{\rm eff})^{1/2},\qquad J_c=j_c d. \tag{1}$$

This current determines the upper stability limit of the Meissner state[2] ($d$ is the film thickness and is assumed small compared with the London penetration depth $\lambda$; in this case $\lambda_{\rm eff}=\lambda^2/d\gg\xi$).

If the current exceeds the critical value $I_c$, PSL can be produced. The current flowing through the PSL is normal, but its density decreases with increasing distance from the line. This equalizes the current distribution over the film width. The film can thus carry a current $I>I_c$ without going into the normal state.

## 2. BASIC EQUATIONS AND STRUCTURE OF PHASE-SLIPPAGE LINE

Consider a film located in the $xy$ plane; the $y$ axis is directed along the film, and $x$ is reckoned from its middle. The current density $\mathbf{j}$ in the film and the vector potential $\mathbf{A}$ are connected by the Maxwell equation

$$\operatorname{rot}\operatorname{rot}\mathbf{A}=4\pi\mathbf{j}d\delta(z)\theta[(b^2/4)-x^2]. \tag{2}$$

This equation must be supplement by the system of microscopic equations for the superconducing order parameter, $\mathbf{j}$, $\mathbf{A}$, and the scalar potential $\varphi$. This system reduces near $T_c$ to the nonstationary Ginzburg-Landau equations[4]

$$-\frac{\pi}{4T}\tau_{ph}\Delta\frac{\partial\Delta}{\partial t}+\frac{\pi}{8T}D(\nabla^2\Delta-4e^2\mathbf{Q}^2\Delta)$$
$$+\frac{T_c-T}{T_c}\Delta-\frac{7\zeta(3)}{8\pi^2}\frac{\Delta^3}{T^2}=0, \tag{3}$$

$$\mathbf{j}=-\sigma\left(\frac{\partial\mathbf{Q}}{\partial t}+\nabla\Phi\right)-\frac{\pi\sigma}{2T}\Delta^2\mathbf{Q}, \tag{4}$$

$$D\nabla(\Delta^2\mathbf{Q})+\frac{\Delta}{2\tau_{ph}}\Phi=0, \tag{5}$$

where $\Delta$ is the modulus of the order parameter,

$$\mathbf{Q}=\mathbf{A}-\frac{1}{2e}\nabla\chi,\qquad \Phi=\varphi+\frac{1}{2e}\frac{\partial\chi}{\partial t}$$

are the guage-invariant potentials of the field ($\chi$ is the phase of the order parameter, $D$ is the electron-diffusion coefficient, $\sigma$ is the conductivity of the metal in the normal state, $\tau_{ph}$ is the time of the electron-phonon collisions, and $\hbar=c=1$. The first term in (4) corresponds to the current of the normal electrons, and the second to the superconducting current. Equation (5) describes the pentration of the field $\mathbf{E}=-(\nabla\Phi+\partial\mathbf{Q}/\partial t)$ into the superconductor. Equations (3) to (5) are valid in the temperature region

$$(T_c\tau_{ph})^{-2}<1-T/T_c<(T_c\tau_{ph})^{-1},$$

but their qualitative corollaries given below are valid also at a large deviation from $T_c$.

The system (3)–(5), with all the quantities dependent on a single variable (meaning a one-dimensional channel), was used in Refs. 4 and 5 to investigate the construction of the PSC and to calculate the IVC of the channel. It was shown in these references that the nonstationary processes associated with the phase slippage occur mainly inside a core of size $\xi=[\pi D/8(T_c-T)]^{1/2}$, in which the superconductivity is locally suppressed. At the center of the core, the

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modulus of the order parameter vanishes periodically with a frequency $\omega(j)$, and its phase changes jumpwise by $2\pi$ to each side. The current in the core region is carried mainly by normal electrons; the potential $\Phi$ that determines the imbalance of the electrochemical potential $s$ of the pairs and quasiparticles oscillates already weakly at the core boundaries and is close to $\pm\omega/4e$. With increasing distance from the core, at distances on the order of the electric-field penetration depth

$$l_E=\pi^{-1}(D\tau_{ph})^{1/2}(14\zeta(3)T/T_c-T)^{1/4}\gg\xi,$$

$\Phi$ decreases and the current turns from normal to superconducting.

The construction of the PSL can be described by generalizing the one-dimensional solutions to the case of a film of finite width. All the points of the line are PSC that oscillate at the same frequency but with some phase shift. Analysis of Eqs. (2)–(5) shows that the behavior of the order parameter and of the field potential in every place on the line is analogous to the one-dimensional case. A constant oscillation frequency along the PSL corresponds to a constant potential $\Phi$ and the core boundaries. On the other hand, it follows from (4) and (5) that $\Phi$ is proportional to the superconducting current divergence which, as will be seen from the results, is determined mainly by the longitudinal component of the current (the only possible exception is in a small region near the film edge). The transformation of the normal current into a superconducting current outside the strong-oscillations interval is therefore described by the known one-dimensional solutions for the given frequency $\omega(j)$, and the time-averaging current density turns out to be uniformly distributed along the line. The dependence of the oscillation phase $\alpha(x)$ on the transverse coordinate is determined by the difference of the components of the vector potential $Q_x$ on the two sides of the nonstationary region, which are determined in turn from the external (relative to the PSL) electrodynamic equations:

$$d\alpha/dx=2e\delta Q_x. \tag{6}$$

The transition from the PSC to the PSL is thus similar to the transition from a Josephson point junction to a distributed one; the only difference is that the processes in our "junction" are essentially nonequilibrium and the superconducting current here is smaller than the dissipative one.

To calculate the IVC we must find the current and electric-field distributions in the entire film. We transform Eq. (2) with account taken of (3) and (4). Far from the PSL cores, which are narrow compared with the distances between the lines, the current density is given by

$$\mathbf{j}=-\sigma\nabla\Phi-\frac{1}{4\pi\lambda^2}\mathbf{Q},\quad \lambda=\frac{1}{4\pi^2}\left[\frac{7\zeta(3)}{\sigma(T_c-T)}\right]^{1/2}, \tag{7}$$

where the potentials $\mathbf{Q}$ and $\Phi$ are independent of time. For the component of the linear current density $J_{x,y}=j_{x,y}d$ in the regions between the lines we obtain then the set of equations

$$4\pi\lambda_{\text{eff}}\left\{\frac{\partial J_y}{\partial x}-\frac{\partial J_x}{\partial y}+\sum_n\delta J_x{}^n(x)\delta(y-y_n)\right\}$$
$$+\int_{-\infty}^{\infty}dy'\int_{-b/2}^{b/2}dx'\frac{J_y(x',y')(x'-x)-J_x(x',y')(y'-y)}{[(x'-x)^2+(y'-y)^2]^{3/2}}=0, \tag{8}$$

$$\frac{\partial J_x}{\partial x}+\frac{\partial J_y}{\partial y}=0 \tag{9}$$

with boundary conditions

$$J_x(\pm b/2)=0. \tag{10}$$

Equation (8) contains $\delta$-function terms because the phase of the order parameter has vortex-like singularities in the cores of the PSL's located at $y=y_n$. The transverse current density $J_x$ inside the cores is "rotated" by an amount $\delta J_x^n(x)$. In turn, after expressing the current densities $J_{x,y}$ in terms of the $\delta J_x^n(x)$ with the aid of Eqs. (8)–(10), we obtain from the self-consistency condition

$$J_y(x,y_n)=I/b, \tag{11}$$

which corresponds to invariance of the oscillation frequency along the line. The electric field $E$, which is directed along the film in the regions between the PSL, satisfies the equation

$$\partial^2E/\partial y^2=E/l_E^2, \tag{12}$$

whose boundary conditions

$$E(y_n)=I/bd\sigma \tag{13}$$

correspond to the absence of a superconducting current in the PSL. Equations (8)–(13) determine completely the electrodynamics of the film and permit its IVC to be determined.

## 3. CURRENT-VOLTAGE CHARACTERISTIC OF A FILM CONTAINING PHASE SLIPPAGE LINES

To find the IVC of a film we must specify the position of the PSL in it. If the film is assumed homogeneous, it is reasonable to assume a periodic arrangement of the lines. We choose then the period $l$ of the structure such as to satisfy the minimum entropy production condition, which corresponds obviously to the maximum possible $l$ at a given current $I$.

We expand the current-density components $J_{x,y}$ in Fourier series in the coordinate $y$

$$\begin{gathered}J_x(x,y)=2\,\mathrm{Re}\sum_{m=1}^{\infty}J_{xm}\exp(ik_my),\qquad k_m=\frac{2\pi m}{l}\\ J_y(x,y)=\bar{J}_y(x)+\tilde{J}_y(x,y),\\ \tilde{J}_y(x,y)=2\,\mathrm{Re}\sum_{m=1}^{\infty}J_{ym}\exp(ik_my),\end{gathered} \tag{14}$$

where $\bar{J}_y(x)$ is the mean value, along the film, of the longitudinal current density ($J_y$ is an even function in $x$ and $y$, while $J_x$ is odd). We put $y_n=nl$ in (8) and take its Fourier transform. Using the relation

$$J_{ym}=\frac{i}{k_m}\frac{\partial J_{xm}}{\partial x},\tag{15}$$

which follows from Eq. (9), we obtain equations for the Fourier components of the transverse current density

$$4\pi\lambda_{\rm eff}\left\{\frac{i}{k_m}\frac{\partial^2 J_{xm}}{\partial x^2}-ik_mJ_{xm}+\frac{1}{l}\delta J_x\right\}$$
$$-\int_{-b/2}^{b/2}J_{xm}(x')R(x'-x,k_m)\,dx'=0,\tag{16}$$

$$R(x'-x,k_m)$$
$$=\int_{-\infty}^{\infty}\exp(ik_mu)$$
$$\times\left\{\frac{u}{[(x'-x)^2+u^2]^{3/2}}+\frac{\partial}{\partial x'}\frac{(x'-x)}{[(x'-x)^2+u^2]^{3/2}}\frac{i}{k_m}\right\}du$$

and also of the mean value $\bar{J}_y$

$$4\pi\lambda_{\rm eff}\left\{\frac{\partial \bar{J}_y}{\partial x}+\frac{1}{l}\delta J_x\right\}=-2\int_{-b/2}^{b/2}\frac{\bar{J}_y(x')}{x'-x}dx'.\tag{18}$$

The results will show that the period $l$ of the structure is almost always small compared with the film width $b$. In this case the current density varies little over distances $\sim l$ in practically the entire width of the film, except for a narrow region near its edge. This allows us to take $J_{xm}$ outside the integral sign and neglect the first term in the curly brackets. Taking (15) into account, we then obtain for the current density the expressions

$$J_x=\frac{4\lambda_{\rm eff}}{l}\delta J_x\sum_{m=1}^{\infty}\frac{\sin(2\pi my/l)}{1+4\pi m\lambda_{\rm eff}/l},\tag{19}$$

$$\tilde{J}_y=\frac{2}{\pi}\lambda_{\rm eff}\frac{d}{dx}\delta J_x\sum_{m=1}^{\infty}\frac{\cos(2\pi my/l)}{(1+4\pi m\lambda_{\rm eff}/l)m},\tag{20}$$

which are valid if $(b/2)-|x|\gg\max(l,\lambda_{\rm eff})$.

It follows at the same time from the self-consistency condition (11) that the longitudinal current density averaged over $y$ is equal, almost over the entire film width, to

$$\bar{J}_y=I/b.\tag{21}$$

In fact, let us substitute this value in (19) and obtain the jump of the transverse current density

$$\delta J_x(x)=\frac{1}{2\pi}\frac{l}{\lambda_{\rm эфф}}\frac{I}{b}\ln\frac{(b/2)+x}{(b/2)-x}.\tag{22}$$

Calculating now $\tilde{J}_y$ from (20), we find that the ac component of the longitudinal density becomes equal to $I/b$ (and Eqs. (21) and (22) no longer hold) at

$$(b/2)-|x|\sim l\ln(l/\lambda_{\rm eff}),$$

if $l\gg\lambda_{\rm eff}$, and at a smaller distance from the edge in the opposite case. As the edge is further approached, $J_x$ and $\delta J_x$ vanish rapidly; it can also be shown with the aid of (18) that at the edge of the film

$$\tilde{J}_y(b/2)\sim(I/b)\max\{(l/\lambda_{\rm eff})^{1/2},1\}.\tag{23}$$

We see thus from (19)–(23) that the transverse current density component reaches a maximum near the PSL at a distance

$$\delta x_{\max}\sim\max(l,\lambda_{\rm eff})$$

from the edge. This value of

$$^1/_2\delta J_x((b/2)-\delta x_{\max}),$$

which is given with logarithmic accuracy by (22), increases with increasing $l$. Therefore, by equating it to the critical current density $J_c$ we can find the optimal, from the standpoint of minimum dissipation, distance between the PSL at a given current $I$:

$$l(I)\approx4\pi\lambda_{\rm eff}\frac{J_cb}{I\ln(I/\lambda_{\rm eff}J_c)}.\tag{24}$$

We now calculate with the aid of (12) and (13) the dependence of the average electric field in the film, and hence of the voltage drop $V$ across it, on the total current, i.e., the IVC of the film:

$$V=IR_N\frac{2l_E}{l}\,\mathrm{th}\frac{l}{2l_E},\tag{25}$$

where $R_N$ is the resistance of the film in the normal state, and $l$ is given by (24). When the current $I$ exceeds the value $J_cb$, if

$$l(J_cb)\gg l_E,\ \text{i.e.,}\ l_E\ll\lambda_{\rm eff}/\ln(b/\lambda_{\rm eff}),$$

the IVC of the film is determine by the equations obtained for a one-dimensional channel.[5] In the opposite case the structure period determined for a narrow channel is larger than that given by (24) for the same current density. The IVC of the film is therefore determined by expressions (24) and (25) also at $I>J_cb$; the excess current that is typical of the channel at $I\gg J_cb$ is absent in this case.

We consider now the influence of a weak perpendicular magnetic field $H$ on the resistive state of the film. Turning on such a field leads to the appearance of a term $H$ in the right-hand sides of Eqs. (8) and (18), and hence in the denominator of Eq. (24) for $l$. The structure period $l$ altered in this manner determines, via Eq. (25), the IVC of the film in a magnetic field.

## 4. DISCUSSION OF RESULTS

A resistive state can thus be produced in a film of width $b\gg\lambda_{\rm eff}$ by creating a periodic structure of phase-slippage lines arranged transversely in the film and producing in it a voltage drop. The period $l$ of such a structure [Eq. (24)] is small compared with the film width at all currents $I>I_c$, where $I_c$ is the critical current [Eq. (1)]. Formation of the PSL is accompanied by a jumpwise appearance of voltage on the film; the current-voltage characteristic, determined by (25) with (24) taken into account, is in the main quadratic if $l$ exceeds the electric-field penetration depth $l_E$:

$$V\approx R_N\frac{l_E}{\lambda_{\rm eff}}\frac{I^2}{J_cb}\ln\frac{b}{\lambda_{\rm eff}};\tag{26}$$

at $l\ll l_E$ the IVC of the film is close to Ohm's law. Whereas relation (26) remains valid up to currents $I\sim J_cb$, it must be replaced by an expression corresponding to the PSC structure in a narrow superconducting channel if the average current density $I/b$ exceeds the critical density $J_c$. It can be shown at the same time that the minimum average current density at which PSL can be produced is small compared with $J_c$ in proportion to the parameter $\xi/l_E \ll 1$.

The initial equations (3)–(5) are valid only in a narrow temperature region near $T_c$. Nonetheless, the results obtained for the IVC of the film are valid qualitatively in a rather wide range, and quantitatively in all cases when the penetration of the electric field into the superconductor is described by Eqs. (12) and (13) (with a suitable penetration depth $l_E$ that exceeds the coherence length $\xi$).

The resistive state connected with PSL formation is accompanied by a higher energy dissipation (it produces a larger effective film resistance) than the motion of Abrikosov vortices.[2] Therefore, starting with the principle of minimum entropy production, the resistance state should set in primarily in those cases when the vortex mechanism cannot maintain, for one reason or another, the stability of the superconducting state in the greater part of the film. This situation occurs, for example, when the total current exceeds the value

$$I_m \sim J_c b \ln^{-1/2}(b/\lambda_{\text{eff}}) < J_c b,$$

at which the current density reaches the critical value simultaneously on the edges and at the center of the film.[2] This can occur also when the current density in the greater part of the film exceeds the value $J^*$ corresponding to the maximum viscous-friction force,[6] and also in the case of strong pinning of the vortices. The resistive state at $I = I_c$ can then begin with motion of the vortices, and as the current increases a PSL structure is produced, a fact accompanied by a jump of the voltage on the IVC (by a group of closely spaced jumps if the finite length of the film is taken into account).

This is approximately the picture observed in the experience in which PSL were observed.[3] A quantitative comparison of the theory with experiment is made difficult, however, by the fact that under real conditions the PSL can coexist with a dynamic state, and their arrangement need not necessarily coincide with that considered in the present paper, but be determined, for example, by the film inhomogeneities, whose influence was disregarded by us.

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