Low-temperature properties of ferromagnets with different-ion anisotropy and arbitrary site spin

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(Submitted 31 July 1985)
Zh. Eksp. Teor. Fiz. **90**, 724–732 (February 1986)

The Holstein-Primakoff formalism is used to calculate the low-temperature properties of easy-plane ferromagnets with different-ion anisotropy in an external magnetic field perpendicular to the preferred plane. The small parameter of the problem is the ratio of the anisotropy and exchange constants. The formulas obtained make explicit allowance for a finite value of the site spin S. This approach is also used to study the properties of antiferromagnets and the XY model in the neighborhood of the spin-flip transition. In particular, a temperature renormalization of the spin wave spectrum is found for fields above the critical field.

1. The improving accuracy of experimental studies of the low-temperature properties of magnetically ordered materials and the increasing number of computer "experiments" are providing a stimulus for theoretical studies of the properties of magnets outside the customary framework of the quasiclassical approximation, i.e., without the assumption of large site spin S. The clearest finite-spin effects are seen in systems in which the classical ground state (which is known to agree with the exact ground state as $S\rightarrow\infty$) is not an eigenstate of the spin Hamiltonian, and therefore, on account of quantum effects, the magnetization does not reach its maximum value even for $T=0$. Such magnets are commonly called "canted" (noncollinear). Their ground state in general is substantially different from the classical ground state and is still not known exactly.[1)]

In even the simplest anisotropic ferromagnets with an arbitrary ratio of the anisotropy and exchange constants, the structure of the ground state can be quite complex.[1] On the other hand, for a rather wide class of systems the exchange interaction is considerably stronger than the relativistic interaction, and so the amplitude of the zero-point oscillations is small and the ground state (at least in three-dimensional systems) should be close to the classical ground state. We emphasize, however, that the fact that the canting is small means only that the ground state is equivalent to a state with a low density of quasiparticles, while the interaction between quasiparticles, because of its exchange nature, is by no means small.

The only parameter regulating the strength of the exchange interaction is the value of the atomic spin S It is clear that the larger the spin, the smaller the role of quantum effects and the closer the ground state to the classical. For $S\gg 1$ the amplitudes of all the anharmonic processes are small, since they contain factors with positive powers of $1/S$. Having $1/S$ small makes it possible to take the anharmonicities into account with only second-order perturbation theory.[2]

Real magnets, however, almost always have a spin $S\sim 1$, and for such systems the exchange anharmonicities must be taken into account exactly. In this paper the formulas are derived without using an expansion in powers of $1/S$ and are therefore valid for arbitrary values of the site spin.

2. As the object of study let us taken an easy-plane Heisenberg ferromagnet in an external magnetic field directed along the preferred axis. The corresponding spin Hamiltonian is written

$$\mathcal{H}=-{}^1/_2\sum_{1,\Delta}J_\Delta \mathbf{S}_1\mathbf{S}_{1+\Delta}+{}^1/_2\sum_{\Delta,1}g_\Delta S_1^z S_{1+\Delta}^z-2\mu H\sum_1 S_1^z, \qquad (1)$$

where Δ is the vector distance between atoms, and $J_\Delta>0$.

We assume that $g_\Delta>0$ and $g_\Delta/J_\Delta\ll 1$. These conditions ensure that the canting is slight (i.e., that the density of quasiparticles[2)] is low at $T=0$) and permit an analytical solution of the problem for arbitrary S.

For the exchange part of the Hamiltonian it is natural to set $J_{\Delta=0}=0$, since an exchange interaction can occur only between different atoms. The anisotropy energy, on the other hand, generally contains a term with $g_{\Delta=0}\neq 0$. This term describes the single-ion anisotropy. Quantum effects due to this term appear only if $S\neq 1/2$. Such effects have been studied in a paper by one of the authors.[3] In the present paper we assume that $g_{\Delta=0}=0$ and concentrate on the role of the different-ion anisotropy. This model differs qualitatively from that of a ferromagnetic with single-ion anisotropy in that the anisotropy energy in the present case has the same structure as the exchange energy,[3)] and therefore, first, the problem cannot be treated as a single-particle problem even when the anisotropy is large, and second, the $S=1/2$ case has no special status among the spin values.

3. The standard approach to the study of quantum effects in magnets, which we also take here, consists of replacing the spin operators by bosonic operators, so that the spin Hamiltonian can be represented as the Hamiltonian of a nonideal Bose gas of quasiparticles, and the well-developed diagram technique for bosonic operators can be used. At a low quasiparticle density only the lowest anharmonicities are important, and so the transition from spin operators to bosons is conveniently done with the aid of the Hermitian Holstein-Primakoff transformation.[4] It is well known[2] that this transformation is not an identity. However, analysis shows (see Refs. 5 and 6) that at a low density of quasiparticles the inaccuracy of the transformation can be neglected for arbitrary values of the spin S.

 0038-5646/86/020422-05$04.00

4. The main difficulty in performing the calculation by perturbation theory is, as we have said, the presence of anharmonicities of an exchange origin, which do not have a small parameter. To take these anharmonicities into account it is necessary to sum an infinite series of ladder diagrams every time that two (or more) magnons are simultaneously created in the interaction process.[3,6–8] This summation can be done explicitly, since the corresponding integral equations have factorable kernels.[3,6] Without dwelling on the details of the calculations, which are analogous to those done in Ref. 3, let us give a few results.

a) The spin wave dispersion relation at small quasiparticle momenta remains qualitatively the same as in the classical description:

$$\varepsilon_{\mathbf{k}}=U(\mathbf{n})k, \quad \mathbf{n}=\mathbf{k}/k, \tag{2}$$

but the spin wave velocity $U(\mathbf{n})$ turns out (when the anharmonicity is taken into account) to depend both on the temperature and, in a rather complicated way, on the value of the spin S. The complicated dependence on the spin of the atom arises in spite of the fact that for $g_{\Delta=0}=0$ the transition to normal products does not generate additional factors of the type $1-1/2S$ as are characteristic for models with single-ion anisotropy[3,7,8] (we note in this regard the special status of $g_{\Delta=0}$ in the function of the discrete variable g_Δ). In explicit form, the expression for the spin wave velocity is[4)]

$$U^2(\mathbf{n})=J(0)g(0)S^2B_{ij}n_in_j, \tag{3}$$

where

$$B_{ij}=A_{ij}\left[\sin^2\theta_0+\frac{g(0)\sin^4\theta_0}{2J(0)S}\lambda-\frac{2}{S}\varphi_v(T)\right]-A_{ij}^*\frac{g(0)}{4J(0)S}\sin^4\theta_0. \tag{4}$$

Here we have used the notation

$$g(\mathbf{k})=\sum_\Delta g_\Delta e^{i\mathbf{k}\Delta}, \quad J(\mathbf{k})=\sum_\Delta J_\Delta e^{i\mathbf{k}\Delta}$$

$$\cos\theta_0=H/H_c^{(0)}, \quad \mu H_c^{(0)}={}^1/_2 g(0)S, \tag{5}$$

$$A_{ij}=\frac{1}{2J(0)}\sum_\Delta J_\Delta\Delta_i\Delta_j.$$

The complicated dependence on the atomic spin S resides in the coefficient λ and in the matrix A_{ij}^*. These quantities are written explicitly as

$$A_{ij}^*=\sum_\Delta C_\Delta\Delta_i\Delta_j, \quad \lambda=\frac{1}{N}\sum_\mathbf{p}\frac{f_\mathbf{p}\nu_\mathbf{p}}{1-\nu_\mathbf{p}}, \tag{6}$$

where

$$\frac{1}{N}\sum_\mathbf{p}(\ldots)=\frac{v_0}{(2\pi)^3}\int d^3p(\ldots), \quad \nu_\mathbf{p}=\frac{J(\mathbf{p})}{J(0)},$$

and the function $f_\mathbf{p}$ satisfies the integral equation

$$f_\mathbf{p}-\frac{1}{4NS}\sum_\mathbf{q}\frac{f_\mathbf{q}}{1-\nu_\mathbf{q}}(\nu_{\mathbf{p}+\mathbf{q}}+\nu_{\mathbf{p}-\mathbf{q}}-2\nu_\mathbf{p}\nu_\mathbf{q})=1, \tag{7}$$

which has the solution

$$f_\mathbf{p}=1+\frac{1}{J(0)S}\sum_\Delta C_\Delta(1-\cos\mathbf{p}\Delta). \tag{8}$$

The coefficient C_Δ in Eqs. (6) and (8) are determined from the system of linear equations

$$\sum_{\Delta'}C_{\Delta'}\Phi_{\Delta,\Delta'}=\frac{J_\Delta}{N}\sum_\mathbf{q}\frac{\cos\mathbf{q}\Delta-\nu_\mathbf{q}}{1-\nu_\mathbf{q}} \tag{9}$$

with the matrix

$$\Phi_{\Delta,\Delta'}=\delta_{\Delta,\Delta'}-\frac{J_\Delta}{J(0)S}\frac{1}{N}\sum_\mathbf{q}\frac{(1-\cos\mathbf{q}\Delta')(\cos\mathbf{q}\Delta-\nu_\mathbf{q})}{1-\nu_\mathbf{q}}. \tag{10}$$

The number of equations is equal to the number of neighbors of each atom for which the exchange integral J_Δ is nonzero.

For a simple cubic lattice with a nearest-neighbor interaction we have $C_\Delta=0$ [as follows from Eq. (9)], and therefore the renormalizations due to the zero-point oscillations do not have a complicated dependence on the atomic spin S (in this case $A_{ij}^*=0$, $\lambda=W-1$, where

$$W=\frac{1}{N}\sum_\mathbf{p}(1-\nu_\mathbf{p})^{-1}\approx 1.51$$

is the Watson integral).

The temperature dependence of the spin wave velocity is determined by the function $\varphi_v(T)$, the values of which we shall write out in the limiting cases of extremely low or relatively high temperatures. For $J(0)S\gg g(0)S\sin^2\theta_0\gg T$ we have

$$\varphi_v(T)=\frac{4}{15}\pi^6\left(\frac{T}{4\pi J^*S}\right)^4\left(\frac{4J^*}{g(0)\sin^2\theta_0}\right)^{5/2}\times\left[1+18\cos^2\theta_0\left(\ln\frac{T}{g(0)S\sin^2\theta_0}+O(1)\right)\right] \tag{11}$$

and for $J(0)S\gg T\gg g(0)S\sin^2\theta_0$ we have

$$\varphi_v(T)=\left(\frac{T}{4\pi J^*S}\right)^{5/2}\zeta\left(\frac{3}{2}\right)(1+\cos^2\theta_0). \tag{11'}$$

Here $J^*=J(0)(A_1A_2A_3/v_0^2)^{1/3}$, where the A_i are the principal values of the matrix A_{ij} [see Eq. (5)].

We note that for $T\ll g(0)S\sin^2\theta_0$ the temperature dependence of the spin wave velocity is different for $H=0$ [$\varphi_v(T)\propto T^4$] and for a finite value of the external magnetic field [$\varphi_v(T)\propto T^4\ln T$]. This occurs because when a field is applied along the Z axis, the Hamiltonian acquires three-particle anharmonicities, which give an additional logarithmic contribution to the temperature renormalization of the spin wave energy. We also note that the behavior $\varphi_v(T)\propto T^4\ln T$ is universal for all quantum Bose liquids with a linear Goldstone spectrum and a nonzero amplitude for ternary processes.[9] For easy-plane ferromagnets in the approximation $S\gg 1$ such a temperature dependence was first obtained in Ref. 10. For $H=0$ temperature renormalization of the spin wave velocity for the case $S=1/2$ was found in Ref. 11.

b) The angle θ between the magnetic moment and the Z

axis also depends on the temperature and on the value of the spin:

$$\cos\theta=\cos\theta_0\left[1-\frac{g(0)\lambda}{4J(0)S}\sin^2\theta_0+\frac{2}{S}\varphi_\theta(T)\right], \quad (12)$$

with the following asymptotic forms of the function $\varphi_\theta(T)$ for the cases corresponding to (11) and (11′):

$$\varphi_\theta(T)=\frac{\pi^2}{6}\left(\frac{T}{4\pi J^*S}\right)^2\left(\frac{4J^*}{g(0)\sin^2\theta_0}\right)^{1/2},$$
$$\varphi_\theta(T)=\left(\frac{T}{4\pi J^*S}\right)^{3/2}\zeta(3/2). \quad (13)$$

c) The longitudinal (M_z) and transverse (M_x) components of the magnetization are given by

$$M_z=\frac{2\mu S}{v_0}\cos\theta_0\left[1-\frac{g(0)\lambda}{4J(0)S}\sin^2\theta_0+\frac{1}{S}\varphi_M(T)\right],$$
$$M_x=\frac{2\mu S}{v_0}\sin\theta_0\left[1-\frac{1}{S}\left(2\varphi_\theta(T)-\varphi_M(T)\right)\right]. \quad (14)$$

The asymptotic form of the function $\varphi_M(T)$ differs from that of the function $\varphi_v(T)$ [see Eq. (11)] only in the absence of a logarithmic factor for $T\ll g(0)S\sin^2\theta_0$. For the case corresponding to (11) and (11′) we have

$$\varphi_M(T)=\frac{4\pi^6}{15}\left(\frac{T}{4\pi J^*S}\right)^4\left(\frac{4J^*}{g(0)\sin^2\theta_0}\right)^{5/2},$$
$$\varphi_M(T)=\left(\frac{T}{4\pi J^*S}\right)^{3/2}\zeta(3/2). \quad (15)$$

We note that the temperature dependence is different for the longitudinal and transverse components of the magnetization for $T\ll g(0)S\sin^2\theta_0$:

$$(M_z(T)\propto T^4, \quad M_x(T)\propto T^2).$$

This difference is most apparent in a two-dimensional space: the temperature correction to the magnetization M_x diverges logarithmically, in complete agreement with the Mermin-Wagner theorem,[12] while the temperature corrections to M_z and to the spin wave velocity $U(\mathbf{n})$ remain finite, again in agreement with the familiar ideas about the structure of the low-temperature phase of two-dimensional systems with two-component order parameters.[13]

Of course, expressions (3), (12), and (14) incorporate only the leading (temperature and quantum) renormalizations. We have dropped from these expressions terms of order

$$\left(\frac{g(0)}{J(0)}\sin^2\theta_0\right)^{3/2}, \quad \left(\frac{g(0)}{J(0)}\right)^{1/2}\varphi_{v,\theta,M}(T).$$

As H approaches the critical field H_c, the spin wave velocity goes to zero:

$$U^2(\mathbf{n})\approx J(0)g(0)S^2A_{ij}n_in_j[1-(H/H_c)^2], \qquad H\leqslant H_c. \quad (16)$$

The renormalization of the value of the critical field has only a temperature part, since above the phase transition point (in the high-field region) the ground state is completely ferromagnetic and there are no zero-point oscillations:

$$H_c\approx H_c^{(0)}\left[1-\frac{2}{S}\left(\frac{T}{4\pi J^*S}\right)^{3/2}\zeta(3/2)\right]. \quad (17)$$

The absence of zero-point oscillations in the collinear phase is a characteristics feature of systems in which the Hamiltonian commutes with the operator for the Z component of the total spin.[14]

At finite temperatures in the immediate vicinity of the phase transition point (within the fluctuation region for $H<H_c$) the square-root decay law predicted for $U(n)$ by Eq. (15) should be replaced by the scaling law $U(n)\propto(H_c-H)^r$. The exponent r, according to an analysis of the scaling dimensionalities,[15] is equal to $\beta-\nu\approx 0.65$ (γ and ν are the exponents of the transverse susceptibility and the correlation length). The fluctuation region, however, remains outside our purview, since its width $(\Delta H/H_c)\propto[T/J(0)S]^2[g(0)/J(0)]$ is narrower than the error limits on the determination of the phase transition point [see Eq. (71)].

5. For an arbitrary relationship between g_Δ and J_Δ, model (1) is called the Heisenberg-Ising model. With the aid of this model one can describe the properties of a number of familiar systems. For example, the case $g_\Delta=J_\Delta$ corresponds to the XY model. If $g_\Delta\geqslant 2J_\Delta$, Hamiltonian (1) describes a uniaxial antiferromagnet.[5)] In fact, at such values of g_Δ it is energetically favorable for the spins to order antiferromagnetically along the Z axis, while the sign of the exchange integral for the transverse components of the spins is unimportant because of the invariance of the spin system with respect to rotations about the Z axis.[16] The value $g_\Delta\equiv 2J_\Delta$ corresponds to the case of an isotropic antiferromagnet.

We note that in this description of an antiferromagnet there is only one branch of oscillations. The transition to the standard description based on the introduction of sublattices and two branches of the oscillation spectrum is accomplished by reducing the unit cell in $\mathbf{k}$ space by one-half (Fig. 1).

For an arbitrary relationship between g_Δ and J_Δ the density of quasiparticles at $T=0$ is by no means small, and a proper perturbation theory can be constructed only on the assumption $S\gg 1$. An exception is the region of strong mag-

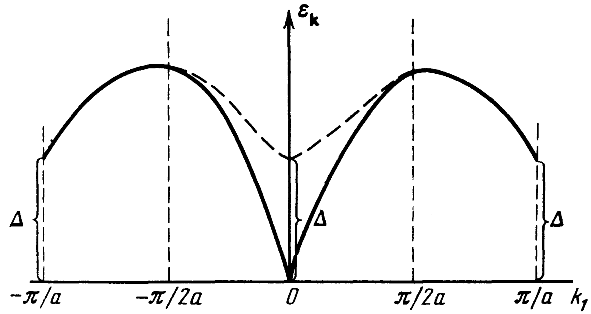

FIG. 1. Spin wave energy $\varepsilon_\mathbf{k}$ in an antiferromagnet in a nonzero external field versus one of the wave-vector components k_1 ($k_2=k_3=0$). The dashed curve shows a second branch of the spectrum, the activational branch, which arises on going over to the reduced cell in k space. The gap Δ is proportional to the applied magnetic field.

netic fields, comparable in strength to the exchange field, in which case the orienting effect of the field is so important that it is favorable for the spins to align approximately along the Z direction. The density of quasiparticles is low in this case because of the proximity of the system to the phase transition to the collinear phase: $(H_c - H)/H_c \ll 1$. It follows that the formulas obtained earlier for the quantum renormalizations apply here with less stringent restrictions on the parameters of the Hamiltonian than the requirement that the ratio of the anisotropy and exchange constants be small: for Eqs. (3), (12), and (14) to apply, it is sufficient that

$$[g(0)/J(0)]\sin^2\theta_0 \ll 1. \qquad (18)$$

In the neighborhood of the transition point, where $\theta_0 \ll 1$, inequality (18) is satisfied at an arbitrary relationship between $g(0)$ and $J(0)$, and the formulas obtained above for $T=0$ are therefore valid near $H_c^{(0)}$ for any systems, including, in particular, the XY model and the antiferromagnet.

At room temperature the interaction between spin waves is substantial in both canted and collinear ferromagnets, and therefore in the case $g(0) \propto J(0)$ the formulas found for the temperature renormalizations should be revised. Specifically, this means that the correct description of the magnetic properties of ferromagnets having a large anisotropy requires the summation of a series of ladder diagrams not only for the exchange but also for the relativistic anharmonicities. Here we consider only the temperature shift of the phase transition point. This quantity is most simply determined from the condition that the renormalized (by the anharmonicities) gap in the spin wave spectrum go to zero in the collinear phase, for which the ground state is well known. The ladder diagrams in this case can be summed in the same way as Dyson did for the isotropic Heisenberg ferromagnet.[17] The result is

$$H_c = H_c^{(0)}\left[1 - \frac{2}{S}\,\frac{1-P/8S^2}{1+g(0)/g_c(0)}\left(\frac{T}{4\pi J' S}\right)^{3/2}\zeta(3/2)\right], \qquad (19)$$

where

$$g_c(0) = 2J(0)S/(W-1),$$

and

$$P = \frac{g(0)}{J(0)}\left(1-\frac{g(0)}{J(0)}\right)\left(1+\frac{g(0)}{g_c(0)}\right)^{-1} \times \frac{1}{N^2}\sum_{\mathbf{p},\mathbf{q}} \frac{(\nu_{\mathbf{p}-\mathbf{q}}+\nu_{\mathbf{p}+\mathbf{q}}-2\nu_{\mathbf{p}}\nu_{\mathbf{q}})\nu_{\mathbf{p}}}{(1-\nu_{\mathbf{p}})(1-\nu_{\mathbf{q}})}. \qquad (20)$$

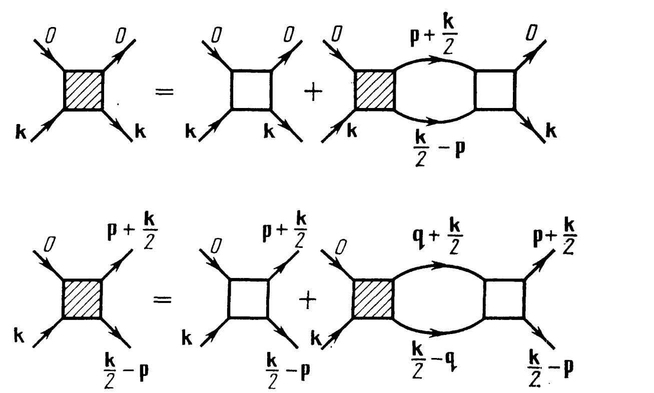

FIG. 2. Graphical equation for the total scattering amplitude (vertex) Γ. The lines denote the Green function $G(\mathbf{k},\omega) = (\varepsilon_{\mathbf{k}} - i\omega)^{-1}$. The shaded squares are the total (renormalized) vertices.

In the limiting case $g(0)/J(0) \to 0$ expression (17) and (19) agree. For a simple cubic lattice with a nearest-neighbor interaction, the integral over wave vectors in (20) goes to zero, and therefore $P \equiv 0$. For such a geometry Eq. (19) has been obtained previously for a spin $S=1/2$ antiferromagnet[18-20] and for the XY model with arbitrary spin.[21] We note that $P\equiv 0$ for $g(0)=J(0)$.

6. The collinear phase that exists in model (1) in the region of strong magnetic fields $H > H_c$ for an arbitrary ratio $g(0)/J(0)$ has a number of properties that distinguish it from the usual collinear phase that occurs in a Heisenberg exchange ferromagnet. The difference is that in the present case the Adler principle does not hold (this phase does not admit a transition to the limit $H=0$) and therefore the amplitude for the scattering of magnons with zero momentum is nonzero. The consequences of this include, first, the presence of temperature renormalization of the gap, as we mentioned earlier, and second, a different kind of temperature dependence of the free energy and spin wave spectrum than in the case of an exchange ferromagnet.[22,23]

To find the leading temperature renormalizations it is sufficient to know the value of the total scattering amplitude $\Gamma_{\mathbf{k},0}^{\mathbf{k},0}$. The corresponding integral equation is depicted graphically in Fig. 2. Its solution for a small wave vector $\mathbf{k}$ is of the form

$$\Gamma_{\mathbf{k},0}^{\mathbf{k},0} = -\frac{g(0)}{J(0)}\left\{(1+\nu_{\mathbf{k}})\left[1+\frac{g(0)}{2J(0)S}\left((1+\nu_{\mathbf{k}})\frac{1}{N}\sum_{\mathbf{p}}\frac{1}{I_{\mathbf{p}}}-1\right)\right]^{-1}+O(k^4)\right\}, \qquad (21)$$

where

$$I_{\mathbf{p}} = 1+\nu_{\mathbf{k}}-\nu_{\mathbf{p}-\mathbf{k}/2}-\nu_{\mathbf{p}-\mathbf{k}/2}.$$

To simplify the derivation of Eq. (21) we confined ourselves to the case of a simple cubic lattice with a nearest-neighbor interaction. Accurate $O(k^2)$, the principal value of the integral in Eq. (21) is

$$\frac{1}{N}\sum_{\mathbf{p}}\frac{1}{I_{\mathbf{p}}} = \frac{W}{2}+0.36(1-\nu_{\mathbf{k}}). \qquad (22)$$

From Eqs. (21) and (22) it is easy to obtain an expression for the temperature renormalization of the spin wave energy:

$$\Delta\varepsilon_{\mathbf{k}} = -\frac{2}{S}\left(\frac{1}{1+g(0)/g_c(0)}\right)\left(\frac{T}{4\pi JS}\right)^{3/2} Z_{3/2}\left(\frac{\varepsilon_0}{T}\right) \times \left[g(0)S - \frac{J(0)S}{2}(1-\nu_{\mathbf{k}})\frac{g(0)}{J(0)}\left(\frac{1}{1+g(0)/g_c(0)}\right) \times \left(1+\frac{g(0)}{J(0)S}0{,}22\right)\right]. \quad (23)$$

Here we have used the notation

$$Z_{3/2}(x) = \sum_{n=1}^{\infty} e^{-nx}/n^{3/2}$$

($Z_{3/2}(x) \simeq e^{-x}$ for $x \gg 1$ and $Z_{3/2}(x) \simeq \zeta(3/2)$ for $x \ll 1$).

7. In conclusion, let us briefly discuss the general properties of spin systems in external magnetic fields. It is known that for many real magnets (e.g., for many-sublattice ferrites or antiferromagnets) there is no small parameter that ensures proximity to collinearity, and for this reason such systems have until now been studied in the quasiclassical approach or in the mean-field approximation.

Nevertheless, in the region of magnetic fields stronger than the exchange field it is always possible to go beyond the quasiclassical approach. In this case all the spins of the magnet perforce have the same quantization axis. This does not mean that the spin structure is always collinear in sufficiently strong fields: collinearity comes about if the magnet initially has only a single preferred axis and if the magnetic field is directed along this axis. However, at large values of H the amplitude of the zero-point oscillations will be small, since the case $H\to\infty$ corresponds to complete ferromagnetic ordering. We note that the longitudinal magnetization for the systems considered here, as in all the known cases, approaches the nominal value by the square-root law[4,6,14]

$$\frac{M(H=\infty)-M(H)}{M(H=\infty)} \propto \frac{1}{H^{1/2}}.$$

We believe that our results can be applied to the description of both nearly isotropic Heisenberg ferromagnets with a weak inter-ion anisotropy (such as $CoCl_2$) and to antiferromagnets in which the values of the spin-flip transition fields are experimentally accessible (EuTe, for example[24]). It should be noted, however, that real magnets (with spin $S \neq 1/2$) have both single-ion and different-ion anisotropies, and their correct description will therefore require taking a superposition of the results of this study and the results of Ref. 3.

1) For instance, the ground state of the Heisenberg antiferromagnet is unknown.

2) In speaking of quasiparticles we mean those defined with respect to to classical ground state.

3) For these reasons the combination of the pure exchange and different-ion anisotropy is often called the anisotropic exchange interaction. We feel that this term is not quite accurate and do not use it.

4) Here and below we confine ourselves to the case $g_\Delta \propto J_\Delta$. This, of course, is a model assumption, which makes it possible to obtain transparent analytical expressions.

5) Antiferromagnets can be described by model (1) if the exchange interaction extends only to nearest neighbors.

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Translated by Steve Torstveit