Auto-waves in magnetic superconductors

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It is shown that under certain conditions, ferromagnetic superconductors that undergo a transition from the superconducting to the normal state with decreasing temperature may act as an active medium with restoration. Various self-organizing (auto-wave) regimes in such superconductors are considered.

1. Many alloys and certain magnetic superconductors (such as $ErRh_4B_4$, $HoMo_6S_8$, and $TmFe_3Si_5$) lose their superconductivity as the temperature decreases and become normally conducting, ferromagnetically ordered materials (see the review in Ref. 1). The superconductivity is quenched by a first-order phase transition.

In these superconductors the low-temperature state is resistive. When a current passes through the low-temperature normal phase, Joule heating raises the temperature of the material to the point where it may become superconducting again. In the superconducting state, no heat is evolved and the superconductor is cooled to the temperature of the cryostat. If the latter temperature is below the critical value T_0 for the transition to the ferromagnetic phase, the superconductivity is quenched and the cycle is repeated. Although the possibility of auto-waves (self-organizing waves) was noted in Refs. 2, the oscillation period was calculated incorrectly because the latent heat of transition was neglected.

In the present paper we show that auto-waves occur in magnetic superconductors of the above type. Auto-waves are analogs of auto-oscillations in distributed active media; they propagate without attenuation in dissipative materials because an external source provides a constant supply of energy which replenishes the losses. Examples of auto-waves include dark domains in ordinary superconductors.[3] There is particular interest in active media with restoration,[4] in which localized solitary auto-waves propagate, after the solitary wave has passed through, the medium returns to its starting state. A wide variety of auto-waves regimes can occur which are specific to such materials. Auto-wave behavior has so far been observed only in physicochemical and biophysical systems, e.g., for the Belousov-Zhabotinskiĭ chemical reaction, or in heart muscle tissue.[5] It is interesting to note that similar auto-wave behavior may also occur in current-carrying magnetic superconductors. The experimental observation of an inhomogeneous state in a thin superconducting current-carrying $ErRh_4B_4$ film was mentioned in Ref. 6, but unfortunately no details were given.

2. We will analyze the one-dimensional case, i.e., a wire in a magnetic superconductor carrying a current of specified density j; the temperature $\overline{T}$ of the surrounding material is kept constant. The temperature distribution $T(x,t)$ along the wire obeys the heat conduction equation

$$\frac{\partial T}{\partial t} = \chi\, \partial^2 T/\partial x^2 - \gamma (T-\overline{T}) + W, \qquad (1)$$

where χ is the thermal diffusivity and γ is the heat transfer coefficient (for simplicity, we assume that χ, γ, and the specific heat c are the same in the normal and superconducting phases). In the ferromagnetic normal phase $W = j^2\rho/c$, where ρ is the specific resistance, while $W = 0$ in the superconducting phase. Although the phase transition occurs at T_0, the superconducting phase may be supercooled down to $T_0(1-\delta_-)$ and the normal phase superheated to $T_0(1+\delta_+)$. In magnetic superconductors, δ_+ and δ_- are typically $\sim 10^{-1}$–10^{-2} (Ref. 1); for example, in an $ErRh_4B_4$ film $T_0 \approx 1$ K and $\delta_+ \approx \delta_- \approx 0.05$ (Ref. 6).

We consider the quasistationary case, for which local thermal equilibrium is preserved. The interface between the two phases is then at the transition temperature T_0. If the interface moves with velocity v toward the superconducting phase, then the heat balance condition implies that the jump in the derivative is equal to

$$\left.\frac{\partial T}{\partial x}\right|_s - \left.\frac{\partial T}{\partial x}\right|_n = -\frac{\lambda v}{c\chi}, \qquad (2)$$

where λ is the latent heat of transition from the superconducting phase to the normal ferromagnetic phase and c is the specific heat. The sign of the jump is reversed if the interface moves in the opposite direction. The results (2) were derived by I. M. Lifshits in his study of the kinetics of quenching of type-I superconductivity by a field.[7]

In what follows it will be helpful to use dimensionless variables $\tau = \gamma t$, $r = (\gamma/\chi)^{1/2}x$, velocity $s = (\gamma\chi)^{-1/2}v$, and $\Theta = T/T_0$ for the time, length, velocity, and temperature, respectively. The dimensionless parameters

$$w = \rho j^2/c\gamma T_0, \quad q = \lambda/cT_0, \quad \overline{\Theta} = \overline{T}/T_0 \qquad (3)$$

then describe the behavior of the system.

3. If $1 > \overline{\Theta} > 1-\delta_-$ and $w > 1 - \overline{\Theta} + \delta_+$ then the superconducting current-carrying magnetic wire acts as an active medium with restoration. Indeed, in this case the superconducting state is the only uniform stationary state, because the current-induced heating makes the normal ferromagnetic state absolutely unstable. However, an isolated domain of ferromagnetic normal phase may move with a constant velocity against a background of superconducting material.[1)] In order to find a solution of Eq. (1) describing such a propagating wave, we assume a temperature distribution of the form $T(x,t) = T(x-vt)$. Equation (1) then reduces to a second-order ordinary differential equation with constant coefficients; its solution is completely determined

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by the boundary conditions $T = T_0$ and (2) at the superconducting/normal interface, together with the requirement that $T \rightarrow \bar{T}$ as $x \rightarrow \pm \infty$. Because of their complexity, we will not give the general expressions for the temperature distribution but merely mention the most interesting physical properties of the auto-wave structures. A domain consisting of normal phase must be narrow if $w \gg 1$; the width a_n and velocity s_0 are equal to

$$a_n = \left[\frac{6(\bar{\Theta}+2q-1)}{\bar{w}} \right]^{1/2}, \quad s_0^2 = 6w \left(\frac{2q+\bar{\Theta}-1}{1-\bar{\Theta}} \right)^2 - 4. \quad (4)$$

If $1 - w < \bar{\Theta} < 1 - \delta$ but $\bar{\Theta} < 1 + \delta_- - w$, superconducting domains may move against a background of ferromagnetic normal phase.

4. When $1 - \delta_- < \bar{\Theta} < 1 + \delta_+ - w$ we have a bistable active system which may be in either the superconducting or ferromagnetic stationary states (both are homogeneous). Uniform motion of the phase interface, i.e., propagation of a "kink" wave, may occur in addition to traveling domains. If the region of normal phase grows in size, the velocity of the interface can be found easily by solving Eq. (1) with a temperature profile $T(x - v_1 t)$ as in Sec. 3; the result is

$$s_1^2 = 4(1-\bar{\Theta}-w/2)^2/(q+\bar{\Theta}-1)(1+q-w-\bar{\Theta}). \quad (5)$$

The same expression holds if the superconducting phase grows, provided we replace q by $-q$.

5. In the most interesting case $1 + \delta_+ - w < \bar{\Theta} < 1 - \delta_-$, the system has no homogeneous stationary states. The wave solution $T(x - vt)$ of Eq. (1) then describes periodic traveling domains, and the solution can be found as in Secs. 3 and 4 by imposing periodic boundary conditions. This periodic domain structure can move with various velocities s, but only the high-velocity case

$$s \gg (w/q)^{1/2} \text{ and } w \gg 1-\bar{\Theta} \gg q$$

is amenable to analytic treatment. The diameters a_n and a_s of the normal and superconducting regions are given by (see Fig. 1)

$$a_n = qs/(w+\bar{\Theta}-1), \quad a_s = qs/(1-\bar{\Theta}). \quad (6)$$

To first order in $1/s$, the oscillation period is independent of the velocity and is equal to

$$T = qw/(1-\bar{\Theta})(w+\bar{\Theta}-1). \quad (7)$$

In addition to moving domains, a static dissipative domain structure may also exist. The corresponding solution of (1) satisfying periodic boundary conditions can easily be found in the static case. The lengths of the superconductivity and normal regions are related by

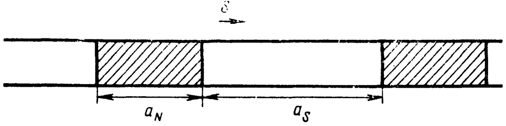

FIG. 1. A periodic traveling-domain structure in a ferromagnetic superconducting wire carrying a specified current J. The hatched regions show the normal ferromagnetic phase.

$$\frac{\text{th}(a_s/2)}{\text{th}(a_n/2)} = \frac{\bar{\Theta}+w-1}{1-\bar{\Theta}}. \quad (8)$$

However, (8) implies that the static domain structure becomes unstable at large currents because a_n tends to zero (for stability, a_n must at least be much larger than the superconducting correlation length).

The parameters in (3) characterizing the behavior of the system vary widely, depending on the specific system and we can give only very rough estimates here. For definiteness, consider a thin $ErRh_4B_4$ ribbon of thickness $d \sim 10^4$ Å on a substrate, as in the experiment described in Ref. 6. For $ErRh_4B_4$ the transition from the superconducting to the normal phase occurs at $T_0 = 1$ K, for which the free energies of the two phases are equal. The latent heat of transition is comparable to the superconducting condensation energy,[1] i.e., $\lambda \sim 10^{21}$ K/cm^3. According to Ref. 8, the resistivity of $ErRh_4B_4$ at $T \sim 1$ K is $\rho \sim 10\ \mu\Omega \cdot$ cm, the specific heat is $c \sim 10^{-1}$ J/K $\cdot$ cm^3, and the thermal conductivity is $\varkappa \sim 0.5$ W/K $\cdot$ m (Ref. 9). We then find that $\chi \sim \varkappa/c \sim 5 \cdot 10^{-2}$ cm^2/s and $q \sim 10^{-1}$. Most of the heat transfer occurs at the contact with the substrate; we will assume that the $ErRh_4B_4$ film and the substrate are separated by an insulating film of roughly the same thickness d and with $\varkappa_i \sim 10^{-1}\varkappa$; we then have $\gamma = 10^{-1}\varkappa/cd^2 \sim 5 \cdot 10^5$ s^{-1}. The characteristic (dimensional) values are: length $(\chi/\gamma)^{1/2} \sim 10^{-3}$–$10^{-4}$; velocity $(\chi\gamma)^{1/2} \sim 10^2$ cm/s; time $\gamma^{-1} \sim 10^{-5}$ s. The dimensionless Joule heat $w \sim 1$ corresponds to a characteristic current density $j \sim 5 \cdot 10^4$ A/cm^2. Clearly, the properties of the dissipative structures will be very sensitive to changes in the film thickness d.

Since (as already noted) Ref. 6 gives few details concerning the formation of the inhomogeneous state, a comparison with experiment is hardly feasible at this time.

6. Extremely complicated and interesting auto-wave structures (guiding centers, helical waves, etc.) are known to occur in two-dimensional active media.[4,5] In particular, two-dimensional active media can be built up from magnetic superconductors, and we now mention two types of possible experiments along these lines.

Assume that a current is fed toward the center of a thin disk of magnetic superconductor and removed along the edge of the disk, i.e., the current flows radially. Then various types of structures can form; in particular, concentric waves of alternating normal and superconducting phases may be generated.

The homogeneity condition is violated in the above experiment because the current density drops in inverse proportion to the distance from the center. Since the current must exceed a critical value (see Sec. 5) for auto-waves to propagate, we conclude that the auto-wave behavior in this case must be confined to a limited region.

However, homogeneous active media can also be built up from magnetic superconductors. For instance, assume that a plate of magnetic superconductor is located between two plates consisting of wires with a highly anisotropic conductivity (in-plane conductivity much less than the conductivity normal to the plane), and that the potential difference between the two plates is kept constant. Then the current density will be the same in all parts of the active medium which are in the normal phase. Such an experiment should make it possible to observe all the auto-wave structures characteristic of the Belousov-Zhabotinskiĭ reaction.

In closing we note that the auto-wave regimes described above might also be observed in distributed systems in which a first-order phase transition from a highly conducting to a poorly conducting state occurs as the temperature drops.

[1)]Of course, a sequence of traveling domains may also be observed under the same conditions. In this paper we do not discuss the boundaries of the regions within which the solutions are stable.

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