Recombination of electrons in semiconductors under the influence of triple collisions

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A study is made of the electron recombination due to electron-electron scattering in the field of a singly charged Coulomb center. It is shown that the energy of a trapped electron relaxes more slowly than the other integrals of motion and the form of the energy distribution function of such electrons is found. The electron gas temperature is calculated and the formula for the recombination time is confirmed.

1. INTRODUCTION. FORMULATION OF THE PROBLEM

The process of recombination regarded as motion in the space of total energy has been studied by Pitaevskiĭ[1] and by Pitaveskiĭ and Gurevich.[2,3] The inelasticity of the scattering processes in a weakly ionized plasma was attributed to collisions with neutral atoms.[1] However, if a plasma is sufficiently dense,[2,3] energy may be transferred by electron-electron collisions. Abakumov, Perel', and Yassievich[4–6] used a similar method to calculate the trapping of carriers by attractive centers in semiconductors. In these investigations the inelasticity of the scattering processes was linked to acoustic phonon emission. An increase in the carrier density in an energy band results in predominance of the inelastic electron-electron scattering processes in recombination. The case of multiply charged centers is considered in Ref. 2. The approach employed in Refs. 1–6 is inapplicable to singly charged centers. This is because the equation for the distribution of the total electron energy E is derived by averaging the collision integral describing the scattering of free particles in terms of the coordinates of a trapped electron, which is incorrect in the case of electron-electron scattering. The interaction between trapped and free electrons cannot generally be reduced to the interaction of "free" particles, i.e., the orbital motion of a trapped electron is significant during the interaction time. The kinetic equation for the distribution function of the total electron energy, obtained by formal application of the method of Refs. 1–6, has no physically meaningful solutions.

It is therefore necessary to determine the probability $W(E,E')$ of a transition from an energy E to an energy E' for a trapped electron under the influence of free electrons. The corresponding mechanical problem of a singly charged center ($Z = 1$) is considered in Sec. 2. The form of the function $W(E,E')$ can be used to draw the conclusion that the Fokker–Planck approximation can give only the order of magnitude, because the scattering processes involving the transfer of an energy $|E' - E| \sim E$ make a considerable contribution to the recombination flux for any value of E.

We shall now formulate the problem in general terms. We shall assume that a stationary (steady) state is established in a system and that in this state electrons are excited optically from lower levels of centers to the conduction band. Collisions of the released free electrons with one another establish a Maxwellian energy distribution in a gas, with its own temperature T_e and density n. Collisions may result in the trapping of one of the electrons by a high level of a Coulomb center. The energy of such a trapped electron changes subsequently because of the interaction with the gas of free electrons. This interaction may result in a repeat ionization of the trapped electron. However, the probability of this event decreases as the electron drops to increasingly deeper levels so that in the interval $-T_e \lesssim E < 0$ this probability is of the order of unity, whereas for $|E| \gg E_e, E < 0$ the ionization probability is low. Therefore, a recombination flux forms at $E \sim -T_e$. If the electron temperature T_e is much smaller than the ionization energy of the Coulomb centers $|E_1|$, the motion of the trapped electrons is usually quasiclassical.[6] We shall assume hereafter that the electron can again be excited optically, and so on. We shall postulate that a repeat photoexcitation occurs only if $|E| \gg T_e, E < 0$ and also that the rate of generation of electrons G ($\text{cm}^{-3}\cdot\text{sec}^{-1}$) is not known. In our problem the density of free electrons and their temperature can be expressed in terms of the generation rate G, photoexcitation energy ε_0 (in the case of monochromatic photoexcitation), and crystal lattice parameters.

In the case of a semiconductor plasma the charge e and the mass m of an electron should be replaced by their effective values.

The plasma is assumed to be almost ideal so that

$$\nu \equiv e^2 N^{1/3}/T_e \ll 1, \quad e^2 n^{1/3}/T_e \ll 1, \tag{1}$$

where N is the concentration of attractive centers. We shall show below that it follows from the conditions of Eq. (1) that the recombination time τ_R is long compared with the energy relaxation time τ_ε, so that free electrons do indeed comprise a Maxwellian gas. Moreover, it follows from the conditions of Eq. (1) that the energy relaxation time is long compared with the time of motion of a free electron from one center to another, i.e,

$$\tau_\varepsilon \gg \max(N^{1/3}/v_{Te};\ N^{2/3}/v_{Te}^2 \tau_p), \tag{2}$$

where v_{Te} is the velocity of a thermal electron and τ_p is the momentum relaxation time, which is of the order of τ_ε for a

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gas of free electrons. In spite of the fact that $\tau_p \sim \tau_\varepsilon$, the distribution of free electrons is isotropic because of the condition (2). In the case of trapped electrons a transition to a state with the distribution of the total energy is possible if the other parameters of the orbit (or the integrals of motion) relax faster than the energy. We shall show that this situation does indeed occur in the specific case of electron-electron (ee) scattering in the field of a Coulomb center.

Furthermore, the conditions of Eq. (1) imply also that wells overlap only weakly, i.e., that the overlap energy is $E_0 \sim e^2 N^{1/3} \ll T_e$, so that the motion of an electron in the region of formation of a recombination flux occurs in the field of one center.

We shall now consider the kinetic equation for free and trapped electrons. The distribution of the total energy $f_+(E)$ of free electrons is (apart from very high and very low energies) given by[7]

$$f_+(E) = f_M(E) - j(E), \quad f_M(E) = \frac{4}{\pi^{1/2}} \frac{n}{\tilde{n}} e^{-E/T_e},$$
$$j(E) = \frac{\pi^{1/2} G \tilde{\tau}_{ee}(T_e, n)}{4\tilde{n}\gamma(3/2, E/T_e)} \quad \frac{1}{\tilde{\tau}_{ee}(T_e, n)} = \frac{8\pi n e^4 \Lambda}{m^{1/2} T_e^{3/2}}, \quad \tilde{n} = T_e g(T_e), \tag{3}$$

where $g(\varepsilon)$ is the density of states of an electron with an energy ε; Λ is the Coulomb logarithm; $\gamma(\alpha, x)$ is an incomplete gamma function. The nonequilibrium distribution function f describes a recombination flux in a gas of free electrons and we find that $j \ll n/\tilde{n}$ when $E \sim T_e$.

In the case of negative total energies the distribution function is far from equilibrium. This distribution function $f_-(E)$ satisfies the following kinetic equation:

$$C_{ee}(f_M, f_M | E) + C_{ee}(j, f_M | E)$$
$$+ C_{ee}(f_-, f_M | E) + C_{ee}(f_-, f_- | E) + I(E) = 0. \tag{4}$$

In Eq. (4), C_{ee} represents an electron-electron collision integral in which the first function describes the distribution of the particles being scattered, whereas the probability of scattering is given by the second function. If $E > 0$, then in the zeroth approximation the functional is $C_{ee}(f_M, f_M | E) = 0$, whereas for $E < 0$ it represents a source which supplies [together with $C_{ee}(j, f_M | E)$] electrons to the region of negative energies. The term $I(E)$ describes photoexcitation. If the problem of recombination is solved, then the complete kinetic equation for electrons of all energies yields, subject to allowance for the scattering by phonons, the balance equation describing the electron temperature. Since the electron-electron scattering does not alter the total energy of the electrons, all the entire pump power $G(\varepsilon_0 - E_1)$ is dissipated by transfer to the lattice:

$$G(\varepsilon_0 - E_1) = Q + \tilde{Q}. \tag{5}$$

The term Q describes the rate of energy loss due to the scattering of free electrons by phonons. If, as is assumed, free electrons are weakly disturbed by the scattering on trapped electrons, the term Q can be calculated without allowance for such scattering. The required result can be found in Refs. 9, 8, and 7. The rate $\tilde{Q}$ of the energy losses due to the scattering of trapped electrons by phonons will be calculated in Sec. 3.

2. MECHANICS OF THE INTERACTION BETWEEN FREE AND TRAPPED ELECTRONS

A study of the probability $W(E, E')$ of a transition of a trapped electron from a state of energy E to a state of energy E' under the influence of a Maxwellian gas of electrons (triple collisions) will be studied by considering first the mechanical problem of the interaction of a free electron of energy ε, arriving from infinity at a point represented by the impact parameter ρ, with a given trapped electron. The result of the interaction is very different for different parts of the plane (ρ, ε) shown in Fig. 1. We shall begin with the regions characterized by $\varepsilon \ll |E|$ and identified by the index a in Fig. 1. It is clear from this figure that the trajectory of an incident slow electron may be of two types. It is found that trajectories of the first type (Ia) characterized by $\rho \gg \rho^*$ lie far from an orbit, so that irrespective of the nature of arrival, the minimum approach distance is $r_{\min} \gg e^2/|E|$. We shall show that for these trajectories the transferred energy $|E' - E|$ is exponentially small.

Perturbation of a Kepler system by a slow electron traveling at a considerable distance from the system results in a slow change in the parameters and orientation of the orbit. Averaging over the revolution period, we obtain the following equations for the evolution of the integrals of motion

$$\dot{\mathbf{L}} = \frac{3e^2}{2mr^3}[\mathbf{M} \times \mathbf{r}], \quad \dot{\mathbf{M}} = \frac{3e^2}{4|E|r^3}[\mathbf{L} \times \mathbf{r}],$$
$$\dot{E} = \frac{e^2(\delta+2)}{4\delta|E|r^3}\left(\frac{3(\mathbf{r}, \mathbf{L})(\mathbf{r}, \mathbf{v})}{r^2} - (\mathbf{v}, \mathbf{L})\right), \tag{6}$$

where $\mathbf{r}$ and $\mathbf{v}$ are the radius vector and the velocity of a free electron; $\mathbf{L}$ is the Lenz vector; $\mathbf{M}$ is the angular momentum; δ is the eccentricity of the orbit of a trapped electron. The first two equations of the system (6) yield the orbit precession frequency

$$\Omega = \frac{e^2}{r^2}\left(\frac{9}{8m|E|}\right)^{1/2}. \tag{7}$$

The following relationship between the transit time of a free electron t_0, the precession frequency Ω, and the revolution period T applies to trajectories of the first type:

$$T \ll t_0 \ll \Omega^{-1}. \tag{8}$$

In fact, if the condition (8) is assumed to be satisfied, it follows from the system of equations (6) that a free electron moves in the field of a fixed dipole formed by a Coulomb center and by the charge of a trapped electron averaged over a revolution period. The time dependence of the distance to the dipole is then

$$r^2 = \rho^2 - \frac{ep\cos\theta_0}{\varepsilon} + \frac{2\varepsilon}{m}t^2, \tag{9}$$

where $\mathbf{p}$ is the average dipole moment; θ_0 is the angle between the vector $-\mathbf{p}$ and the initial velocity $\mathbf{v}_\varepsilon$. It is clear from Eq. (9) that if $\varepsilon\rho^2 \gg ep\cos\theta_0$, then the trajectories lie far from the center, at a distance of the order of ρ, so that

$$t_0^2 \sim \rho^2 \frac{m}{\varepsilon} \gg \frac{e^4 m}{|E|e^2} \gg T^2;$$

on the other hand, we find that

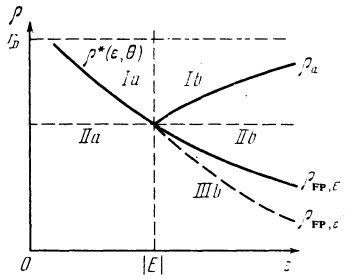

FIG. 1. Regions (ρ,ε) in which different methods of describing the scattering of a free electron of an energy ε by a trapped electron E can be used (see Sec. 2). The transferred energy is exponentially small in the regions Ia and Ib, it is of the order of $|E|$ in the region IIa, and of the order of $e^4/\varepsilon\rho^2$ in the regions IIb and IIIb. The Fokker–Planck (FP in the subscripts) approximation for trapped electrons is valid only in the region IIb. The impact parameter $\rho^*(\varepsilon,\theta)$ for the capture by an orbit is defined by Eq. (11) and the adiabatic size is $\rho_a = v_\varepsilon T$; $\rho_{FP,E}$ and $\rho_{FP,\varepsilon}$ are the impact parameters (distances) at which the transferred energy is of the order of $|E|$ and ε, respectively; r_D is the Debye radius (static screening).

$$\Omega^{-1}\sim(\rho^2/e^2)\,(m|E|)^{-1/2}\gg\rho\,(m/\varepsilon)^{1/2}\sim t_0.$$

Therefore, Eq. (9) can indeed be used and we can assume that trajectories of the (Ia) type do not intersect the orbit. We can estimate the transferred energy by expanding the orbital velocity $\mathbf{v}(t)$ as a three-dimensional Fourier series, the coefficients and period of which are slow functions of time with a characteristic scale Ω^{-1}:

$$\mathbf{v}(t)=\sum_{n=-\infty}^{\infty}\mathbf{u}_n(t)\exp[2\pi nit/T(t)],$$

and we shall estimate the integral

$$\Delta E=\int_{-\infty}^{\infty}(\mathbf{F},\mathbf{v})\,dt$$

$$=\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^2\,dt}{r^3(t)}(\mathbf{r}(t),\mathbf{u}_n(t))\exp\left[-i\frac{2\pi nt}{T(t)}\right]. \qquad (10)$$

We shall deform the integration contour to the upper $(n>0)$ or lower $(n<0)$ half-plane of the complex quantity t so that it passes through saddle points. If $n\neq 0$, it follows from the condition of Eq. (8) that in each half-plane there are two saddle points located at distances of the order of t_0 and Ω^{-1} from the real axis, and they make an exponentially small contribution to ΔE compared with nt_0/T and $n/\Omega T$, respectively. The term with $n=0$ makes zero contribution to ΔE for the following reason. The direction of the vector $\mathbf{u}_0(t)$ is identical with the direction of the vecor $\dot{\mathbf{L}}$ if we can ignore a change in the energy. It then follows from the right-hand side of the expression in the system (6) that $\mathbf{u}_0(t)\perp\mathbf{r}(t)$ and the term with $n=0$ vanishes, which justifies the initial assumption of the smallness of the change in the energy of an orbital electron.

It therefore follows that a significant transfer of energy occurs if the incident electron is captured by an orbit. If, as before, we still assume that the dipole is immobile, this occurs at

$$\varepsilon\rho^2<\varepsilon\,(\rho^*)^2\equiv ep\cos\theta_0\sim\varepsilon\,[(e^2/|E|)\,(|E|/\varepsilon)^{1/2}]^2\cos\theta_0. \qquad (11)$$

In the opposite limiting case when the dipole precession angle of $\varphi=\int_{-\infty}^{\infty}\Omega(t)dt$ is large compared with unity, the condition of Eq. (11) again selects those trajectories which are captured by an orbit. The self-consistent motion of the electrons can then be represented as follows. The orbit of a bound electron precesses rapidly and the value of the Lenz vector $\langle\mathbf{L}\rangle$ averaged over a precession period differs little from $(L_0\cos\theta_0,0,0)$. The x axis is assumed directed toward the free electron, L_0 is the initial amplitude of the Lenz vector, and the angle θ_0 is between the vector $-\mathbf{L}_0$ and the direction $\mathbf{v}_\varepsilon$. Therefore, the motion of a free electron can be regarded as taking place in a central field with a potential $U(r)=-L_0\cos\theta_0/r^2$. The center captures an electron if the condition (11) is satisfied. We can expect the condition (11) to be valid, in order of magnitude, also in the intermediate case when $\varphi\sim 1$.

When an electron is captured by an orbit (trajectory IIa) and if the distances between all three particles are of the same order of magnitude $(e^2/|E|)$ and their kinetic energies are of the order of $|E|$, then there is no other quantity with the dimension of energy in the problem apart from $|E|$, so that the transfer of energy is also of the order of the binding energy of a trapped electron. It follows from Eq. (11) that the cross section for inelastic scattering accompanied by the transfer of an energy $\Delta E\sim|E|$ is

$$\sigma\sim\rho^2\sim ep/\varepsilon\sim e^4/\varepsilon|E|. \qquad (12)$$

The interaction should result in one of the electrons leaving a center because the initially infinite phase space of the system cannot become finite. We might expect that exceptional situations of formation of a bound state in a system of three bodies attracting each other in pairs and arriving from infinity[10] have no bearing on our case. It should be stressed that the initial simplifying assumptions ($\varepsilon\ll|E|$ and even $\rho\gg e^2/|E|$) are unimportant in the sense that a rigorous calculation of the cross section (12) still requires a solution of this type of problem for three bodies ($m_1=m_2$, $m_3=\infty$, $Z=1$) in its general form, because the moving electrons are in a situation when their energies as well as mutual distances are of the same order of magnitude. Figure 2 shows, by way of illustration, a computer experiment on collisions between two electrons.

We shall now consider the limit when a free electron of energy ε is fast and its velocity v_ε is high compared with the velocity of orbital motion of a trapped electron V_E ($\varepsilon\gg|E|$). The trajectories of a free electron can be divided arbitrarily into three types (Fig. 1). In the first (Ib) the time taken to travel a distance equal to the impact parameter ρ is long compared with one revolution period T: $\rho\gg\rho_a\equiv v_\varepsilon T\propto(e^2/|E|)\,(\varepsilon/|E|)^{1/2}$. In this case the interaction of a free electron with the other two charges is adiabatic. A direct calculation shows that the transferred energy ΔE is small:

$$\Delta E\sim\exp(-\rho/v_\varepsilon T), \qquad (13)$$

in spite of the fact that the Kepler system is degenerate and the changes in the action variables during a scattering event are generally power-law small. The transfer of energy is possible only in the case of the trajectories IIb and IIIb. We must

state more precisely that if $\rho \lesssim e^2/|E|$ we have to distinguish two impact parameters ρ_- and ρ_+, measured from a trapped electron (which is almost immobile during the motion of a free electron in a sphere of radius ρ_a) and from a center which has captured an electron, respectively. (The division into regions in Fig. 1 in the case when $\rho \lesssim e^2/|E|$ corresponds to the use of ρ_- as the ordinate.) If $\rho_+ \gg e^2/\varepsilon$, $\rho \ll \rho_a$, then during the scattering (inside a sphere ρ_a) both electrons may be regarded as free and then for trajectories of type II*b* when

$$\rho_- \gg \rho_{FP,E} \equiv (e^2/|E|)\,(\varepsilon/|E|)^{-1/2},$$

the transferred energy is $e^4/\varepsilon\rho_-^2 \ll |E|$. In the region III*b*, when $\rho_- \lesssim \rho_{FP,E}$ the transferred energy is of the order of or greater than $|E|$. In addition to these trajectories, there is one more case $\rho_+ \lesssim e^2/\varepsilon$, but among all the trajectories with $\rho \sim \rho_- \sim e^2/|E|$ an allowance for those which are additionally curved by the field of the center, is an exaggeration of the accuracy.

It therefore follows that in the limit $\varepsilon \gg |E|$ the scattered electrons interchange energy "irreversibly" only within a sphere of radius ρ_a The relaxation of other integrals of motion of a trapped electron occurs already for $\rho \lesssim r_D$ (r_D is the Debye radius). The condition $r_D \gg \rho_a, \rho^*$ ensures—as shown below—that the distribution function of the trapped electrons is isotropic.

3. KINETIC EQUATION

We shall consider the kinetic equation bearing in mind that a change in the energy E alters greatly the relative values of the various terms in (4). We shall consequently divide the region of negative energies into three subregions: A, B, C. In the subregion A defined by $-T_e \lesssim E < 0$ it is almost always possible (with the exception of energies very close to zero) to drop the term $C_{ee}(f_-, f_-)$. A recombination flux forms in the subregion and, as pointed out earlier, photoexcitation can be ignored so that only the first three terms remain in Eq. (4):

$$C_{ee}(f_+,\ f_M|E) + C_{ee}(f_-,\ f_M|E) = 0. \tag{14}$$

In the subregion B we have $|E| \gg T_e$, $E_1 < E < -T_e$, where E_I is the characteristic energy at which photoexcitation may become significant; it is now possible to ignore all the processes with the exception of the interaction between trapped and free electrons, so that we are left with

$$C_{ee}(f_-,\ f_M|E) = 0. \tag{15}$$

The linear integral equation of Eq. (15) should be supplemented by a boundary condition which, in the case of such a division, is the requirement of matching of the distribution functions $f_-(E)$ found separately from Eqs. (14) and (15) at $E \sim -T_e$.

We shall not consider the subregion C, where $E < E_I$, and we shall assume a strong disequilibrium of $f_-(E)$, so that the flux from C to B is small compared with the reverse flux. This is always true if T_e is sufficiently small compared with E_I.

Beginning from the subregion A, we shall show that Eq.

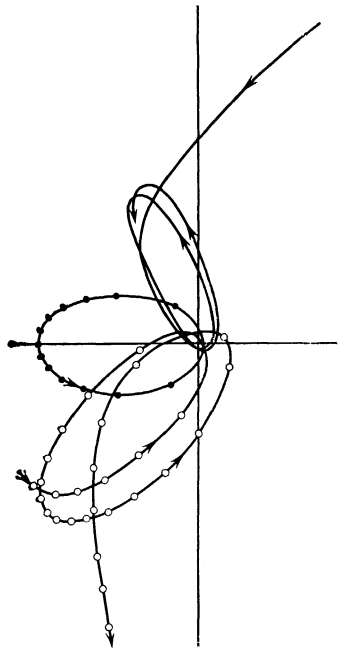

FIG. 2. Example of a computer calculation of the trajectory of an electron (planar projection). The black dots represent the initial orbit of a trapped electron. The free electron is far outside the figure on the right. The initial data are selected so as to satisfy the condition (11). The open circles and the continuous curve represent the trajectories of an initially trapped and an initially free electron in the course of charge exchange. We can see that up to that time the precession turns the orbit by an angle of about $\pi/4$. In the course of charge exchange the emerging particle acquires a considerable energy (the points are plotted at equal time intervals). We can see the initial precession of the orbit of a newly trapped electron.

(4) is indeed an integral equation. We shall do this using the results of Sec. 2 and estimate the rate of change of the energy $A(E,\varepsilon)$ of a trapped electron with the energy E under the influence of free electrons with an energy ε and a density $n_\varepsilon = f_M(\varepsilon)g(\varepsilon)$:

$$A(E,\varepsilon) = \int \Delta\varepsilon(\rho)\, n_\varepsilon v_\varepsilon d\sigma_\rho.$$

The use of the Rutherford cross section valid in the case of free electrons yields the following expressions for the regions $\varepsilon \gg |E|$ (Fig. 1):

$$A_{IIb}(E,\varepsilon) \sim \frac{\varepsilon}{\tau_{ee}(n_\varepsilon,\varepsilon)} \ln\left(\frac{\rho_a}{\rho_{FP,E}}\right) \sim \frac{\varepsilon}{\tau_{ee}(n_\varepsilon,\varepsilon)} \ln\left(\frac{\varepsilon}{|E|}\right)$$

$$A_{IIIb}(E,\varepsilon) \sim \frac{\varepsilon}{\tau_{ee}(n_\varepsilon,\varepsilon)} \ln\left(\frac{\rho_{FP,E}}{\rho_{FP,\varepsilon}}\right) \sim \frac{\varepsilon}{\tau_{ee}(n_\varepsilon,\varepsilon)} \ln\left[\left(\frac{\varepsilon}{|E|}\right)^{1/2}\right] \sim A_{IIb}(E,\varepsilon),$$

$$\tau_{ee}(n_\varepsilon,\ \varepsilon) \equiv ne^4/(m^{1/2}\varepsilon^{3/2}). \tag{16}$$

The contribution of electrons with $\varepsilon \ll |E|$ does not contain a logarithm and it can be ignored. It follows from the relationships in Eq. (16) that calculation of the first moment of the function $W(E,E')$ gives contributions of the same order of magnitude to the integral with respect to $E' - E$ and these contributions are made by all the transferred energies beginning from $e^4/\varepsilon\rho_a^2$ and right down to ε. We recall that the derivation of the free-electron Landau collision integral uses

essentially the circumstance that

$$\ln(r_D/\rho_\varepsilon) \gg \ln(\varepsilon/\varepsilon'),$$

where $\rho_\varepsilon = e^2/\varepsilon$; ε and ε' are the energies of the scattered free electrons, $\varepsilon \gg \varepsilon'$. This inequality makes it possible to use, with a logarithmic precision, the Fokker–Planck equation.[11,12] In this case the cutoff radius is ρ_a, but $\ln(\rho_a/\rho_\varepsilon) \sim \ln(\varepsilon/\varepsilon')$.

In the region $|E| \ll T_e$ the distribution function $f_-(E)$ can be represented in the form $f_-(E) = f_-(0) + \delta f(E)$. The correction δf can be estimated only symbolically by solving in this region the equation with the Landau collision integral (which is valid in respect of the order of magnitude). The coefficient of diffusion in the space of total energy [$D_M(\varepsilon)$, see Refs. 12, 9, and 7] is defined by the relationship:

$$\tilde{g}(E) D_M(E) = \langle g(\varepsilon) D_M(\varepsilon) \rangle$$

$$= -\frac{2}{3} \int \frac{d^3r}{V} \left(\frac{\varepsilon g(\varepsilon) d\varepsilon}{\tilde{\tau}_{ee}(\varepsilon)} \right) \frac{1}{n} \delta\left(E - \varepsilon - \frac{e^2}{r} \right)$$

$$\times \left[\int_0^\varepsilon d\varepsilon' \varepsilon' g(\varepsilon') f_M(\varepsilon') \right.$$

$$\left. + \varepsilon g(\varepsilon) \int_\varepsilon^\infty d\varepsilon' f_M(\varepsilon') \right], \qquad (17)$$

where the density of states at $E < 0$ is

$$\tilde{g}(E) = \pi^2/4 g(T_e) \nu^3 (T_e/|E|)^{3/2}. \qquad (18)$$

We find from Eqs. (17) and (18), assuming that $f_M(E) = 0$ when $E < 0$, that the diffusion coefficient is described by

$$D_M(E) = \frac{8|E| T_e}{3\sqrt{\pi} \tilde{\tau}_{ee}(n_M, T_e)}, \quad \Lambda = \ln\left(\frac{T_e}{|E|} \right). \qquad (19)$$

The equation

$$\tilde{g}(E) D_M(E) \, (\partial/\partial E) f_-(E) = G \qquad (20)$$

yields then the distribution function

$$f_-(E) = f_M(0) \, [1 + (|E|/T_e)^{3/2}], \quad |E| \ll T_e. \qquad (21)$$

The constant of integration $f_M(0)$ is found from the condition of matching to a Maxwellian distribution at $E = 0$; this condition must be introduced in the adopted approximation in order to represent the source $C_{ee}(f_M, f_M|E)$ [see Eq. (14)]. The function (21) is identical with that found by Gurevich[3] for $|E| \ll T_e$ (when $Z \gg 1$).

We shall now consider the subregion B and we shall do this by writing down Eq. (15) in its full form:

$$\int [W(E, E') f_-(E) - W(E', E) f_-(E')] \tilde{g}(E') dE' = 0; \qquad (22)$$

here, the probability of energy transfer $W(E,E')$ in the scattering by slow free electrons differs from zero for $E \sim E'$ and $E' < E$, and the energy relaxation time [see Eq. (12)] can be estimated at

$$\frac{1}{\tau_E} = \int_{-\infty}^{0} W(E, E') \tilde{g}(E') dE' \sim n v_{Te} \sigma \sim \frac{T_e}{|E| \tau_{ee}(n, T_e)}. \qquad (23)$$

Integrating Eq. (22) and equating the result to the constant flux along the energy axis, we obtain the required equation for the distribution function $f_-(E)$, which is estimated to be

$$f_-(E) = \frac{G \tau_{ee}}{\nu^3 \tilde{n}} \left(\frac{E}{T_e} \right)^{3/2}. \qquad (24)$$

It should be noted that if $Z > 1$, the exponent in the above expression is 3/2 (Ref. 3). Finally, it follows from the condition $f(-T_e) \sim f_M(0)$ that

$$n \sim e^{-5} (m T_e^9)^{1/4} (G/N)^{1/2}. \qquad (25)$$

The quantities n and N are linked by the linear relationship $n + N_A = N$, where $N_D > N_A$; N_D and N_A are the numbers of donor and acceptor impurities. If $n \gtrsim N_A$, then

$$n \sim (G^2 m T_e^9 / e^{20})^{1/6}. \qquad (26)$$

The recombination time τ_R is defined by

$$\tau_R = n/G \sim \tau_{ee}(n, T_e)/\nu^3. \qquad (27)$$

This is a simple Thomson-like formula[13,6]: an electron can recombine if it is—in the energy sense—above a center (probability ν^3) and it loses an energy $\sim T_e$ by electron-electron scattering in a time $\tau_{ee}(n, T_e)$. Multiplying $f_-(E)$ by $\tilde{g}(E)$, we obtain the number of particles which have an energy E:

$$n_-(E) \sim \frac{n\nu^3}{T_e} \left[1 + \left(\frac{T_e}{|E|} \right)^{3/2} \right]. \qquad (28)$$

At $|E| \sim E_0$, Eq. (23) matches the number of free particles $n_+(E_0) \sim g(E_0) f_M(E_0) \sim (n/T_e) \nu^{1/2}$. The divergence of the integral for the total number of trapped particles makes it necessary to truncate the integral at deep energies $E = E_1$, where $E_1 < 0$. The above formulas are valid if the total number of electrons is less than the concentration of all the centers (including nonionized centers). The energy E_1 may be, for example, the ionization potential.

The result (27) is predicted in Ref. 2. In our case we have $Z = 1$, so that the scattering is essentially of the three-body type and the transfer of energy does not reduce to a series of small-angle collisions between the trapped and free electrons. Although the diffusion approximation is no longer valid here, Eq. (26) is identical (apart from a coefficient of the order of unity) with the result obtained in Ref. 2.

The Thomson approach[13] used in Refs. 14 and 15 is also accurate only in order of magnitude. In general, recombination is not due to single scattering and the contribution of such scattering to the recombination flux is of the order of the diffusion contribution, so that the results of Refs. 2 and 14–16 are identical with Eq. (27). As pointed out in Ref. 14, the coefficient in Eq. (27) should be selected carefully. For example, in Ref. 14, where the upper limit for the recombination coefficient is calculated, the relevant integral diverges linearly and is truncated when the interaction becomes adiabatic. This choice of the integration limit is only approximate and the required integral is in fact near this limit. One should also point out that the location of a "bottleneck" at a depth $(5/2)T_e$ (Ref. 15) is not justified, since typical energy transfers are of the order of T_e.

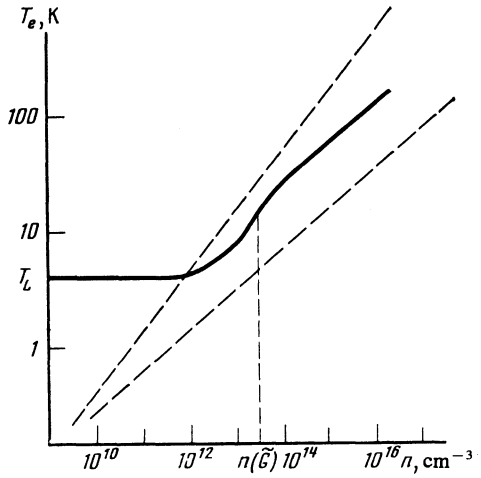

FIG. 3. Solution of the balance equations. In Eqs. (32) and (25) the unknown numerical coefficients are assumed to be unity. The upper dashed line represents the equation $\tau_{ee} = \tau_A$. The lower dashed line is $\nu = 1$. Our solution is valid between these two straight lines. $\varepsilon_0 - E_1 = 200$ K and $T_L = 4.2$ K. $\tilde{Q}$ reaches the value $G\alpha ms^2$ when $\tilde{G} \sim 10^{22}$ cm$^{-3}\cdot$sec^{-1}.

Finally, the theoretical and numerical calculations[17,18] based on the Chandrasekhar formulas for binary collisions also provide only an order-of-magnitude description of the recombination process. The numerical coefficient in Eq. (27) is not known. One of the possible ways of finding this coefficient is a numerical solution of the integral equation (4). In such a calculation it is necessary to average the transferred energy found from the equations of motion over all possible ways of approach and to obtain $W(E,E')$ for Eq. (4).

We shall now calculate the rate $\tilde{Q}$ of the energy losses due to the scattering of trapped electrons by phonons. We shall assume that energy is lost only because of the spontaneous emission of deformation acoustic phonons, i.e., that the lattice temperature is sufficiently low. An allowance for the acoustic phonons in the dynamic friction coefficient of electrons has the effect that the kinetic equation for the distribution function of the electrons in the region $|E| \gg T_e$, $E < 0$, is

$$\tilde{g}(E)\,(A_{ee}+A_{ac})\,f(E)=G, \qquad f(-T_e)\sim f_M(0). \tag{29}$$

In our case we have $A_{ee}(T_e) \gg A_{ac}(T_e)$, so that the recombination time is governed by the electron-electron scattering processes. Nevertheless, allowance for A_{ac} in the kinetic equation is sometimes important. The expression for $A_{ac}(E)$ can be obtained as follows. When the emission of a phonon occurs in the same way as in the case of free electrons, we then have

$$A_{ac}(E)=\frac{16}{3\pi}\frac{T_e}{\tau_{ac}(T_e)}\left(\frac{|E|}{T_e}\right)^{1/2}, \qquad \frac{1}{\tau_{ac}(T_e)}=\frac{2\sqrt{2}}{\pi}\frac{C^2 m^{5/2} T_e^{1/2}}{\rho_c \hbar^4}, \tag{30}$$

where C and ρ_c are, respectively, the deformation-potential constant and the density of the crystal.[4,6] The expressions in Eq. (30) are valid if $ms^2 \ll |E| \ll \alpha ms^2$, where $\alpha = e^2/\hbar s$, and s is the velocity of sound. The left-hand inequality means that the electron velocity v is higher than the velocity of sound, whereas the right-hand inequality shows that a typical energy of the emitted phonons is greater than the gap between levels. We can show that in the region $E \gg \alpha ms$ the probability of phonon emission falls rapidly and the contribution of this region to the integral of $\tilde{Q}$ can be ignored. Then, substituting Eq. (30) and $A_{ee} = |E|/\tau_E$ [for τ_E see Eq. (23)] into Eq. (28), we obtain

$$\tilde{Q}=\frac{n v^3 T_e}{\tau_{ac}}\int_0^{\alpha ms^2/T_e}\frac{z^{3/2}\,dz}{1+z^{3/2}[\tau_{ee}(T_e)/\tau_{ac}(T_e)]}, \qquad z=\frac{|E|}{T_e}. \tag{31}$$

The selection of the lower limit is unimportant if $\alpha ms^2 \gg T_e$. This is precisely the case we shall consider below because in the case of the opposite inequality the value of $\tilde{Q}$ is known to be small against the background Q (in terms of the parameter ν). It is clear from Eq. (31) that the maximum value

$$\tilde{Q}=(n v^3/\tau_{ee})\,\alpha ms^2 = G\alpha ms^2$$

is reached when, in the vicinity of $E = -\alpha ms^2$, the distribution function is governed by phonons.

Substituting in Eq. (5) the resultant formula for $\tilde{Q}$ as well as the expression for $Q = n(T_e - T_L)/\tau_{ac}(T_e)$, and applying Eq. (25), we obtain the following equation for the electron temperature as a function of the pump rate:

$$G(\varepsilon_0-E_1)=\frac{m^{11/4}T_e^{15/4}\,G^{1/2}C^2}{e^5 N^{1/2}\rho\hbar^4}\left[\frac{T_e-T_L}{T_e}+\frac{Ne^6}{T_e^3}\int_0^{\alpha ms^2/T_e} dz z^{3/2}\left(1+\frac{z^{3/2}em^{11/4}N^{1/2}C^2}{G^{1/2}T_e^{11/4}\rho\hbar^4}\right)^{-1}\right]. \tag{32}$$

At high pump rates we have $N \approx n$, and, moreover, we also find that $\tilde{Q} \ll Q$, so that subject to Eq. (26), we find that

$$T_e = G^{2/9}(\varepsilon_0-E_1)^{1/2}e^{10/9}\rho^{1/3}\hbar^{4/3}/C^{2/3}m^{8/3}. \tag{33}$$

If $n \ll N$, but $\tilde{Q} \ll Q$, then

$$T_e = G^{2/15}(\varepsilon_0-E_1)^{4/15}N^{2/15}\rho^{4/15}\hbar^{16/15}e^{4/3}/C^{8/15}m^{11/15}; \tag{34}$$

$\tilde{Q}$ then reaches the value $G\alpha ms^2$ if

$$\frac{\tau_{ee}(n,T_e)}{\tau_{ac}(T_e)}\left(\frac{\alpha ms^2}{T_e}\right)^{3/2} \gtrsim 1.$$

Solving the above inequality subject to Eq. (34), we obtain

$$n \lesssim (\alpha ms^2)^{18/11}(\varepsilon_0-E_1)^{1/11}m^3\rho\hbar^4N^{1/11}/e^{38/11}C^2. \tag{35}$$

At these electron densities the electron temperature is given by

$$T_e = G^{2/15}(\varepsilon_0-E_1-\alpha ms^2)^{4/15}N^{2/15}\rho^{4/15}\hbar^{16/15}e^{4/3}C^{8/15}m^{11/15}. \tag{36}$$

If the semiconductor is sufficiently pure, i.e., if

$$N_A < \tilde{N} = \rho^{11/10}\hbar^{44/10}m^{33/10}(\alpha ms^2)^{9/5}(\varepsilon_0-E_1)^{11/10}/C^{22/10}e^{19/5},$$

then $\tilde{Q}$ becomes important when $N_A \lesssim n \lesssim \tilde{N}$ and the formula which defines the electron temperature is similar to Eq. (33):

$$T_e = G^{9/2}(\varepsilon_0-E_1-\alpha ms^2)^{1/3}\rho^{1/3}\hbar^{4/3}e^{10/9}/C^{2/3}m^{8/9}. \tag{37}$$

This is precisely the case illustrated in Fig. 3. The transition

from Eq. (33) to Eq. (37) is manifested in logarithmic scale as an inflection. It should be pointed out that if $\alpha ms^2 \ll |E_1|$, then the contribution of $\tilde{Q}$ to the balance equation is always small.

We shall now show that the hypothesis that the distribution is isotropic when $E<0$ (and also when $E>0$) does not result in additional restrictions. We shall estimate the relaxation time τ of the integrals of motion **M** and **L** of the trapped particles. The free electrons passing by at distances equal to the impact parameter ρ, $r_D \gtrsim \rho \gg e^2/|E|$, cause precession of the orbit at the frequency given by Eq. (7). The angle of rotation of the Lenz vector (or moment) of the orbit during the passage of a single electron at a distance equal to the impact parameter is

$$\Delta\varphi = \int_{-\infty}^{\infty} \Omega(t)\,dt \sim \frac{e^2}{(m|E|)^{1/2}} \int_{-\infty}^{\infty} \frac{dt}{r^2(t)} \sim \frac{e^2}{(m|E|)^{1/2}} \frac{1}{\rho v_e}.$$

The diffusion coefficient of angular deviation is

$$D \sim \int_{e^2/|E|}^{r_D} [\Delta\varphi(\rho)]^2 n_e v_e \rho\, d\rho;$$

the same quantity represents the relaxation of the orbit eccentricity. Therefore, the required relaxation time of the integrals of motion (apart from the energy) is

$$\tau \sim D^{-1} \sim \tau_{ee}(n,\ T_e)\,(|E|/T_e)\ln^{-1}(r_D|E|/e^2). \tag{38}$$

On the other hand, for the energy relaxation time, we get from Eqs. (16) and (23)

$$\tau_E \sim \begin{cases} \tau_{ee}(n, T_e)\ln^{-1}(T_e/|E|), \\ |E| \ll T_e \\ \tau_{ee}(n, T_e)\,(|E|/T_e), \\ |E| \gg T_e \end{cases} \tag{39}$$

Comparing Eqs. (38) and (39), we can see that $\tau \ll \tau_E$. Finally, comparing Eq. (21) with a similar expression for the recombination time in the case of scattering by phonons (in semiconductors) or neutral atoms (in a gas plasma), we find that the mechanism in question predominates at pump rates

$$G \gg G^* \equiv m/\tilde{\tau}^3(T_e)e^2,$$

where $\tilde{\tau}(\varepsilon)$ is the energy relaxation time in the case of scattering on the thermostat phonons or the neutral atoms in a plasma.

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