Multiphoton resonant ionization of atoms in an intense electromagnetic radiation field

A. D. Gazazyan and R. G. Unanyan

Institute of Physics Research, Armenian Academy of Sciences
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We investigate resonant ionization of atoms in an external intense electromagnetic radiation field, when the upper state of the atom comprises many closely spaced levels that are at multiphoton resonance with the lower level. The multilevel structure of the upper state can either consist of many close levels of highly excited states of the atom or be a multiplet. The photoelectron distribution in energy and the probability of resonant ionization of the atom in the intense electromagnetic field are calculated as functions of time. The case of doubly split states of the upper level is investigated. A computer calculation is carried out of the photoelectron distribution for one-photon resonance of the fine-splitting levels $3^2S_{1/2}$ with $3^2P_{1/2}$ and $3^2P_{3/2}$ of the sodium atom, and the results are plotted.

§1. INTRODUCTION

A theoretical and experimental investigation of resonant ionization of atoms is not only of fundamental importance for the elucidation of the laws governing multiphoton detachment of an electron from an atom, but also of practical significance for isotope separation, electron polarization, study of highly excited states, and others. Resonant ionization of atoms is the subject of many works in which correct account is taken of transitions between discrete levels and continuum states. The temporal evolution of resonant ionization of atoms was investigated in Ref. 1 under conditions when both the resonant and the ionizing fields are turned on instantaneously, and with neglect of the transitions between all spontaneous and nonresonant transitions as well as transitions between continuum states. This reference contains also a generalization of the Fano's familiar configuration-interaction method[2] for a constant perturbation to the case of a periodic one.

Of particular interest in resonant ionization is the case of intermediate two-photon resonance. Turning-on the resonant field adiabatically in the inversion regime[3] leads to substantially new results for the resonant-ionization probability. The influence of self-induced inversion on resonant ionization was considered in Refs. 4–6. A more complete discussion of resonant processes in multiphoton ionization of atoms is given in Ref. 7. The polarization-angular distribution of photoelectrons was investigated by perturbation theory in Refs. 7 and 8. Three- and four-photon ionization of an atom in the presence of a multiplet resonance was investigated in Ref. 9, but a final expression for the probability was given only for four-photon ionization and was obtained by perturbation theory. Two-photon ionization of an atom in the presence of an intermediate resonant doublet was considered also in Ref. 10, where the ionization probability as $t\to\infty$ was obtained for instantaneous application of an external field. The expression obtained there for the probability is, however, very unwieldy and the authors confined themselves to the locations and widths of the maxima in strong fields.

We consider in the present paper resonant ionization of atoms in an intense external electromagnetic radiation field, when the upper state of the atom constitutes many closely spaced levels that are at multiphoton resonance with the lower level. The multilevel structure of the upper state may be either many close levels of highly excited states of the atom or a multiplet. Fano's generalized configuration-interaction method for periodic perturbation[2] is used to calculate the photoelectron distribution in energy and the probability of resonant interaction of an atom in an intense electromagnetic radiation field as a function of time. The computer-determined photoelectron distribution for one-photon resonance of the sodium-atom fine-splitting levels $3^2S_{1/2}$ with $3^2P_{1/2}$ and $3^2P_{3/2}$ is reported.

§2. PROBABILITY OF MULTIPHOTON IONIZATION OF AN ATOM IN THE CASE OF *N* CLOSE INTERMEDIATE RESONANCES

We consider the ionization of an atom in the field of intense electromagnetic radiation, when the upper level consists of many close levels that are at multiphoton resonance with the lower level (Fig. 1). For the basis wave functions for the discrete spectrum of the atom in the electromagnetic-wave field we choose quasi-energy wave functions written in the approximation of multiphoton resonance with adiabatic turning-on of the periodic perturbation

$$V(t)=V^-e^{-i\omega t}+V^+e^{i\omega t}. \tag{1}$$

These functions take the form

$$\Phi_0(t)=\exp\left[-\frac{i}{\hbar}\lambda_0 t\right]\left(C_0{}^0\psi_0+e^{-in\omega t}\sum_{i=1}^{N}C_i{}^0\psi_i\right),$$

$$\Phi_k(t)=\exp\left[-\frac{i}{\hbar}(\lambda_k-n\hbar\omega)t\right]\left(C_0{}^k\psi_0+e^{-in\omega t}\sum_{i=1}^{N}C_i{}^k\psi_i\right), \tag{2}$$

$$k=1,\ldots,N,$$

where λ_ν ($\nu=0, 1,\ldots,N$) are the atom quasienergies in the radiation field; when the interaction is turned off, i.e., as

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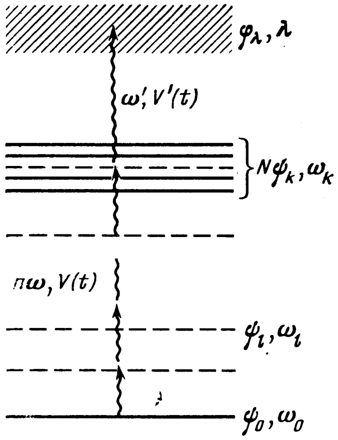

FIG. 1.

$V(t) \to 0$, they go over into the free-atom energy levels, and the wave functions (2) go over into the free-atom functions

$$\Phi_\nu(t) \to \psi_\nu e^{-i\omega_\nu t}.$$

Turning-on an ionizing field, the interaction with which we denote by

$$V'(t) = V'^{+} e^{i\omega' t} + V'^{-} e^{-i\omega' t}, \tag{3}$$

we represent the complete solution of the Schrödinger equation, with the continuum taken into account, in the form

$$\Psi(t) = a_0(t)\Phi_0(t) + \sum_{k=1}^{N} a_k(t)\Phi_k(t) + \int a_\lambda(t)\varphi_\lambda(t)\,d\lambda, \tag{4}$$

where $\varphi_\lambda(t) = \varphi_\lambda \exp[-i\lambda t/\hbar]$ is the unperturbed wave function of the continuous spectrum of an atom of energy λ.

Substituting the expansion (4) in the Schrödinger equation, we obtain a system of differential equations for the coefficients $a_\nu(t)$ and $a_\lambda(t)$, which are reducible by the Fourier transformation

$$a_0(t) = \exp\left[\frac{i}{\hbar}(\lambda_0 + n\hbar\omega + \hbar\omega')t\right]\int dE\, a_0(E)\exp\left[-\frac{i}{\hbar}Et\right],$$
$$a_k(t) = \exp\left[\frac{i}{\hbar}(\lambda_k + \hbar\omega')t\right]\int dE\, a_k(E)\exp\left[-\frac{i}{\hbar}Et\right], \tag{5}$$
$$a_\lambda(t) = \exp\left[\frac{i}{\hbar}\lambda t\right]\int dE\, a_\lambda(E)\exp\left[-\frac{i}{\hbar}Et\right]$$

to the following system of algebraic equations for the Fourier coefficients $a_\nu(E)$ and $a_\lambda(E)$:

$$(E - E_\nu)a_\nu(E) = \int d\lambda\, \vartheta_{\lambda\nu}^* a_\lambda(E), \qquad \nu = 0, 1, \ldots, N,$$
$$(E - \lambda)a_\lambda(E) = \sum_{\nu=0}^{N} \vartheta_{\lambda\nu} a_\nu(E), \tag{6}$$

where

$$E_0 = \lambda_0 + n\hbar\omega + \hbar\omega', \quad E_k = \lambda_k + \hbar\omega', \quad k = 1, 2, \ldots, N, \tag{7}$$

$$\vartheta_{\lambda\nu} = \sum_{k=1}^{N} C_k^\nu V_{\lambda k}'^{-}, \quad \nu = 0, 1, \ldots, N. \tag{8}$$

We solve the system (6) by the methods cited in Ref. 2. We represent the solution of the second equation of the system (6) in the form

$$a_\lambda(E) = [\mathrm{P}/(E-\lambda) + z(E)\delta(E-\lambda)]\sum_{\nu=0}^{N}\vartheta_{\lambda\nu}a_\nu(E), \tag{9}$$

where P stands for the principal value and $z(E)$ is some arbitrary function that will be determined. After substituting (9) in the first equation of (6) we get

$$\sum_{\nu'=0}^{N}[E_{\nu'}\delta_{\nu\nu'} + F_{\nu\nu'}(E)]a_{\nu'}(E) + z(E)\vartheta_{E\nu}^*\sum_{\nu'=0}^{N}\vartheta_{E\nu'}a_{\nu'}(E) = Ea_\nu(E), \tag{10}$$

where

$$F_{\nu\nu'}(E) = \mathrm{P}\int\frac{\vartheta_{\lambda\nu}^*\vartheta_{\lambda\nu'}}{E-\lambda}d\lambda \quad (\nu, \nu' = 0, 1, \ldots, N). \tag{11}$$

With the aid of the unitary transformation

$$a_\nu(E) = \sum_{\nu'=0}^{N} A_{\nu\nu'}\tilde{a}_{\nu'}(E), \tag{12}$$

where A is a unitary matrix, we diagonalize the matrix $E_\nu\delta_{\nu\nu'} + F_{\nu\nu'}(E)$. The set of equations for the unitary matrix and the eigenvalues $\tilde{E}_\nu$ takes the form

$$E_\nu A_{\nu\nu'} + \sum_{\mu=0}^{N} F_{\nu\mu}(E)A_{\mu\nu'} = A_{\nu\nu'}\tilde{E}_{\nu'} \tag{13}$$

and the system (10) reduces to the following set of algebraic equations:

$$(E - \tilde{E}_\nu)\tilde{a}_\nu(E) = z(E)\tilde{\vartheta}_{E\nu}^*\sum_{\nu'=0}^{N}\tilde{\vartheta}_{E\nu'}\tilde{a}_{\nu'}(E), \tag{14}$$

whose solutions are

$$\tilde{a}_\nu(E) = z(E)\frac{\tilde{\vartheta}_{E\nu}^*}{E - \tilde{E}_\nu}B(E), \tag{15}$$

where

$$\tilde{\vartheta}_{E\nu} = \sum_{\nu'=0}^{N}\vartheta_{E\nu'}A_{\nu'\nu}, \tag{16}$$

while

$$B(E) = \sum_{\nu=0}^{N}\tilde{\vartheta}_{E\nu}\tilde{a}_\nu(E) = \sum_{\nu=0}^{N}\vartheta_{E\nu}a_\nu(E) \tag{17}$$

is determined from the condition for orthonormalization of the complete quasienergy function to a δ function.

Multiplying Eq. (14) by $\tilde{\vartheta}_{E\nu}/(E - \tilde{E}_\nu)$ and summing over ν, we get

$$\sum_{\nu=0}^{N}\tilde{\vartheta}_{E\nu}\tilde{a}_\nu(E) = z(E)\sum_{\nu=0}^{N}\frac{|\tilde{\vartheta}_{E\nu}|^2}{E - \tilde{E}_\nu}\sum_{\nu=0}^{N}\tilde{\vartheta}_{E\nu}\tilde{a}_\nu(E) \tag{18}$$

and, taking (17) into account,

$$\left[z(E)\sum_{\nu=0}^{N}\frac{|\tilde{\vartheta}_{E\nu}|^2}{E - \tilde{E}_\nu} - 1\right]B(E) = 0.$$

This yields for $z(E)$ the expression

$$z(E)=2\pi\left[\sum_{\nu=0}^{N}\frac{\Gamma_\nu(E)}{E-\tilde{E}_\nu}\right]^{-1}, \tag{19}$$

where

$$\Gamma_\nu(E)=2\pi|\tilde{\vartheta}_{E\nu}|^2. \tag{20}$$

Substituting (15) in (9), we get

$$a_\lambda(E)=B(E)z(E)\left(\frac{\mathrm{P}}{E-\lambda}\sum_{\nu=0}^{N}\frac{\tilde{\vartheta}_{\lambda\nu}\tilde{\vartheta}_{E\nu}^*}{E-\tilde{E}_\nu}+\delta(E-\lambda)\right). \tag{21}$$

Substitution of (15) and (21) in the Fourier expansion (5) yields for the completely orthonormalized quasienergy wave function (4) the final expression

$$\Psi_E(t)=z(E)\left(\frac{1}{z^2(E)+\pi^2}\right)^{1/2}$$
$$\times\exp\left[-\frac{i}{\hbar}Et\right]\left\{\sum_{\nu=0}^{N}\left(\sum_{\nu'=0}^{N}\frac{A_{\nu\nu'}\tilde{\vartheta}_{E\nu'}^*}{E-\tilde{E}_{\nu'}}\right)\right.$$
$$\times\exp\left[\frac{i}{\hbar}\tilde{E}_\nu t\right]\Phi_\nu(t)+\int d\lambda\exp\left[\frac{i}{\hbar}\lambda t\right]$$
$$\left.\times\left[\frac{\mathrm{P}}{E-\lambda}\sum_{\nu=0}^{N}\frac{\tilde{\vartheta}_{\lambda\nu}\tilde{\vartheta}_{E\nu}^*}{E-\tilde{E}_\nu}+\delta(E-\lambda)\right]\varphi_\lambda(t)\right\}. \tag{22}$$

The atom-ionization amplitude

$$A(\lambda,t)=\int\langle\Psi_E(0)|\Phi_0(0)\rangle\langle\varphi_\lambda(t)|\Psi_E(t)\rangle dE$$

reduces upon substitution of the wave functions (2) and (22) to the expression

$$A(\lambda,t)=\exp\left[\frac{i}{\hbar}\lambda t\right]\int dE\frac{z^2(E)}{z^2(E)+\pi^2}\exp\left[-\frac{i}{\hbar}Et\right]$$
$$\times\left(\sum_{\nu=0}^{N}\frac{A_{0\nu}\tilde{\vartheta}_{E\nu}^*}{E-\tilde{E}_\nu}\right)\left(\sum_{\nu=0}^{N}\frac{\tilde{\vartheta}_{\lambda\nu}\tilde{\vartheta}_{E\nu}^*}{E-\tilde{E}_\nu}\right)\left[\frac{\mathrm{P}}{E-\lambda}+z(\lambda)\delta(E-\lambda)\right]. \tag{23}$$

All the matrix elements, as well as $F_{\nu\nu'}(E)$, are assumed hereafter to depend little on the energy E and are regarded as constants. As noted in Ref. 1, these functions are indeed slowly varying in E, since the characteristics interval of their variation ($\sim$Ry) is large compared with their value. The ionization amplitude (23) can then be rewritten as

$$A(\lambda,t)=\exp\left[\frac{i}{\hbar}\lambda t\right]\int_{-\infty}^{+\infty}dE\exp\left[-\frac{i}{\hbar}Et\right]\frac{g(E)}{E-\lambda-i0}$$
$$+\frac{z(\lambda)}{z(\lambda)+i\pi}\left(\sum_{\nu=0}^{N}\frac{A_{0\nu}\tilde{\vartheta}_\nu^*}{E-\tilde{E}_\nu}\right), \tag{24}$$

where

$$g(E)=\beta(E)\Big/\prod_{\nu=0}^{N}(E-s_\nu^*), \tag{25}$$

$$\beta(E)=\alpha(E)/(z(E)-i\pi),$$
$$\alpha(E)=\sum_{\nu=0}^{N}A_{0\nu}\tilde{\vartheta}_\nu^*\left[\prod_{\substack{\mu=0\\(\mu\neq\nu)}}^{N}(E-\tilde{E}_\mu)\right]. \tag{26}$$

In (25), s_ν^* ($\nu=0,1,\ldots,N$) are roots of a complex equation of degree $N+1$:

$$\prod_{\nu=0}^{N}(E-\tilde{E}_\nu)+\frac{i}{2}\sum_{\nu=0}^{N}\Gamma_\nu\left[\prod_{\substack{\mu=0\\(\mu\neq\nu)}}^{N}(E-\tilde{E}_\mu)\right]=0, \tag{27}$$

which is in turn the characteristic equation of the matrix

$$D=\left\|\tilde{E}_\nu\delta_{\nu\mu}-\frac{i}{2}(\Gamma_\nu\Gamma_\mu)^{1/2}\right\|\qquad(\nu,\mu=0,1,\ldots,N). \tag{28}$$

The eigenvalues of the matrix (28) are the roots of the characteristic equation (27). This matrix is uniquely expressed in terms of two Hermitian matrices B and C:

$$D=B+iC,\quad B=\frac{1}{2}(D+D^+),\quad C=\frac{1}{2i}(D-D^+). \tag{29}$$

We denote the eigenvalues of the Hermitian matrix $C=-(1/2)\|(\tilde{\Gamma}_\nu\tilde{\Gamma}_\mu)^{1/2}\|$ by λ_c; they are the roots of the equation

$$|\lambda_c\delta_{\nu\mu}+{}^1/_2(\tilde{\Gamma}_\nu\tilde{\Gamma}_\mu)^{1/2}|=0. \tag{30}$$

The equation (30) of degree $N+1$ is of the form

$$\lambda_c^{N+1}+\frac{1}{2}\left(\sum_{\nu=0}^{N}\Gamma_\nu\right)\lambda_c^N=0, \tag{31}$$

and its roots are

$$\lambda_c^{(k)}=0\qquad(k=0,1,\ldots,N-1),\qquad\lambda_c^{(N)}=-\frac{1}{2}\sum_{\nu=0}^{N}\Gamma_\nu.$$

The imaginary parts of the eigenvalues of the matrix D, and hence the imaginary parts $\operatorname{Im}s_\nu^*$ of the characteristic equation (27), lie in the interval from $\min\{\lambda_c\}$ to $\max\{\lambda_c\}$, i.e.,

$$-\frac{1}{2}\sum_{\nu=0}^{N}\Gamma_\nu<\operatorname{Im}s_\nu^*<0. \tag{32}$$

The roots of Eq. (27) of degree $N+1$ are thus located on the lower complex E half-plane and $\operatorname{Im}s_\nu^*\neq0$.

Integrating with respect to E in the first term of (24) we obtain for the ionization amplitude the expression

$$A(\lambda,t)=\sum_{\nu=0}^{N}f_\nu\frac{1-\exp[i\hbar^{-1}(\lambda-s_\nu^*)t]}{\lambda-s_\nu^*}, \tag{33}$$

where

$$f_\nu=\alpha(s_\nu^*)\Big/\prod_{\substack{\mu=0\\(\mu\neq\nu)}}^{N}(s_\nu^*-s_\mu^*). \tag{34}$$

Consider the case when all the roots of (27) are different. The presence of multiple roots calls for a separate investigation.

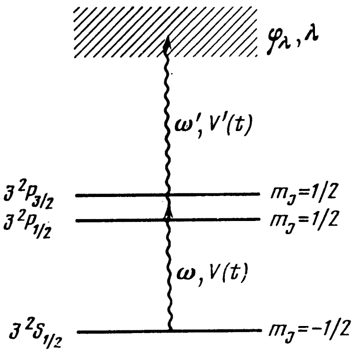

FIG. 2.

At $t=0$ the ionization amplitude vanishes: $A(\lambda,0)=0$. As $t\to\infty$ we obtain for the electron-distribution amplitude the expression

$$A(\lambda,\infty)=\sum_{\nu=0}^{N}\frac{f_\nu}{\lambda-s_\nu^*},\qquad (35)$$

which shows that the photoelectron-distribution amplitude has $N+1$ different maxima located at the points $\lambda=\mathrm{Re}\,s_\nu^*$ with respective widths $2\,\mathrm{Im}\,s_\nu^*$.

Confining ourselves to weak ionization, when $\Gamma_\nu/\Delta E_\nu\sim\Delta_\nu/E_\nu\ll1$, where ΔE_ν is the distance from the quasienergy level E_ν to the nearest quasienergy level, and

$$\Gamma_\nu=2\pi|\vartheta_\nu|^2,\qquad \Delta_\nu=F_{\nu\nu}=\mathrm{P}\int\frac{|\vartheta_\nu|^2}{E-\lambda}d\lambda$$

are respectively the width and shift of this level, approximate solutions of Eqs. (13) and (27) yield for s_ν^* the values

$$s_\nu^*=E_\nu+\Delta_\nu-i\Gamma_\nu/2,\qquad (36)$$

where E_ν is defined in (7).

The total ionization probability at the instant t is given by

$$W(t)=\int_{-\infty}^{+\infty}|A(\lambda,t)|^2d\lambda.$$

Substituting here the amplitude $A(\lambda,t)$ (33), we obtain for the time dependence of the total ionization probability

$$W(t)=\pi\left\{\sum_{\nu=0}^{N}|f_\nu|^2\frac{1-\exp\left[-\frac{2}{\hbar}\mathrm{Im}\,s_\nu t\right]}{\mathrm{Im}\,s_\nu}-4\,\mathrm{Im}\sum_{\mu>\nu=0}^{N}f_\nu^*f_\mu\frac{1-\exp\left[\frac{i}{\hbar}(s_\nu-s_\mu^*)t\right]}{s_\nu-s_\mu^*}\right\}.\qquad (37)$$

In the case of weak ionization $A_{\nu\nu'}\approx\delta_{\nu\nu'}$ and $f_0\approx\vartheta_0^*, f_\nu\approx0$ $(\nu\neq0)$, we get the following approximate expression for the photoelectron-distribution amplitude:

$$A(\lambda,t)\approx\vartheta_0^*\frac{1-\exp[i(\lambda-E_0-\Delta_0)t/\hbar]\exp(-\Gamma_0t/2\hbar)}{\lambda-E_0-\Delta_0+i\Gamma_0/2},\qquad (38)$$

where ϑ_0 is determined from (8) and the width [Eq. (20)], is $\Gamma_0=2\pi|\vartheta_0|^2$. For the photoelectron distribution in energy we get therefore

$$\frac{dW(\lambda,t)}{d\lambda}=\frac{\Gamma_0}{2\pi}\frac{1+\exp(-\Gamma t/\hbar)-2\exp(-\Gamma_0t/2\hbar)\cos((\lambda-E_0-\Delta_0)t/\hbar)}{(\lambda-E_0-\Delta_0)^2+\Gamma_0^2/4}\qquad (39)$$

and the total ionization probability is

$$W(t)=1-\exp(-\Gamma_0t/\hbar).\qquad (40)$$

At relatively short times $t\ll\hbar/\Gamma_0$ and at $t\gg\hbar/(\lambda-E_0-\Delta_0)$ we get from (39)

$$\frac{dW(\lambda,t)}{d\lambda}=\frac{\Gamma_0}{\hbar}t\delta(\lambda-E_0-\Delta_0),\qquad (41)$$

i.e., the ionization regime becomes linear in time in accord with perturbation theory.

We confine ourselves henceforth for simplicity to the case when the upper resonant state comprises two close levels. This is the situation, for example, for fine splitting of an atomic level. The case of an intermediate resonance multiplet was considered in Ref. 9 for three- and four-photon ionization of the atom. Final results, however, were obtained only by perturbation theory for four-photon ionization. The case of a one-photon resonant multiplet in an external electromagnetic field switched-on instantaneously was considered in Ref. 10, but the final expression for the ionization probability, obtained for $t\to\infty$, was very unwieldy. The authors confined themselves therefore only to an analysis of the locations and widths of the maxima in strong fields. The simplest example of the situation considered here occurs at two-photon resonance of a singlet $S_{1/2}$ level with a doublet $D_{1/2}$ and $D_{3/2}$. In this case the quasienergies have a particularly simple form for strong and weak resonant fields.

§3. NUMERICAL RESULTS AND DISCUSSIONS

To investigate the influence of the multiplet structure of a resonant level on multiphoton ionization of an atom, we used computer calculations for only the simplest case, that of one-photon resonance between the fine-splitting levels $3^2S_{1/2}$ and $3^2P_{1/2}$, $3^2P_{3/2}$ of the sodium atom. Figure 2 shows the scheme of two-photon ionization through fine-splitting sublevels of the sodium atom by right-polarized radiation. The ground state $3^2S_{1/2}$ was chosen to have $m_J=-1/2$; the ionization is therefore only through the resonant fine-splitting doublet $3^2P_{1/2}$ and $3^2P_{3/2}$. We consider atom ionization by a radiation field stronger by one or two orders than the resonant field, and turned-on instantaneously at a certain instant $t=0$. The instantaneous turning-on of the field cannot influence substantially the result for the ionization, since the latter links the discrete level with the continuum, where there are no resonances at all.

If the atom can be in the lower state on a level with $m_J=1/2$, ionization by right-hand polarization from this state is possible only through the upper state $3^2P_{3/2}$ of the doublet, with $m_J=3/2$. This leads to two additional maxima, which were investigated in detail in Ref. 1 for an instantaneously applied external field and in Ref. 6 for adiabatically applied resonant radiation.

The matrix elements between the discrete levels of the

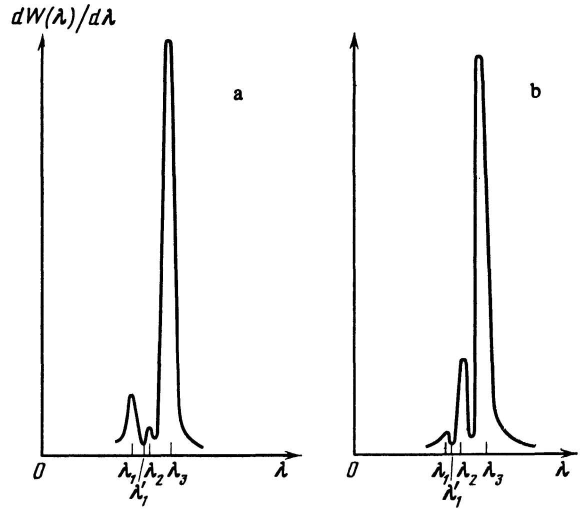

FIG. 3.

ground state $3^2S_{1/2}$ and the fine-splitting doublet $3^2P_{1/2}$ and $3^2P_{3/2}$ are related as follows:

$$\langle 3\,^2S_{1/2},\ m_J=-1/2|V^-|3\,^2P_{1/2},\ m_J=1/2\rangle$$
$$=\sqrt{2}\langle 3\,^2S_{1/2},\ m_J=-1/2|V^-|3\,^2P_{3/2},\ m_J=1/2\rangle \qquad (42)$$

and

$$\langle 3\,^2S_{1/2},\ m_J=-1/2|V^-|3\,^2P_{1/2},\ m_J=1/2\rangle=-i\mathscr{E}d/3, \qquad (43)$$

where $\mathscr{E}$ is the resonant-radiation field strength and d is the radial part of the matrix element (we neglect spin-orbit interaction). For sodium, $d\approx 3.98$ a.u. (Ref. 13).

Figure 3 shows plots of the photoelectron distribution as $t\to\infty$ for two values[1)] of the resonant-radiation intensity, $\mathscr{E}=3\cdot10^{-6}$ (Fig. 3a) and $\mathscr{E}=3\cdot10^{-5}$ (Fig. 3b) at a frequency $\omega=\omega_{P_{3/2}}-\omega_s+\Delta\omega/2$, where $\Delta\omega$ is the fine splitting of the doublet, $\Delta\omega=7.8\cdot10^{-5}$ for sodium. The matrix elements of the transition to the continuous spectrum is based on Seaton's formula,[13,14] which yields the value

$$\langle 3\,^2P_{1/2}|V'^-|\lambda\rangle\approx\langle 3\,^2P_{3/2}|V'^-|\lambda\rangle\approx 5\mathscr{E}', \qquad (44)$$

where $\mathscr{E}'$ is the intensity of the ionizing field. At $\mathscr{E}'=3\cdot10^{-4}$ the ionization width, calculated with the aid of (44), yields $\Gamma_i=1.5\cdot10^{-5}$. The chosen ionizing-radiation frequency is $\omega'=(|E_{P_{1/2}}|+0.01)$. Since Fig. 3 could not be drawn to scale, we present some numbers for the plots. As can be seen from Fig. 3, three maxima with respect to energy are obtained. In Fig. 3a the respective maxima are $dW/d\lambda=3\cdot10^3, 2\cdot10^3, 3\cdot10^4$ with widths $1.3\cdot10^{-5}$, $1.1\cdot10^{-6}$ and $1.9\cdot10^{-5}$. The respective minima are $dW/d\lambda=7\cdot10^2, 9\cdot10^2$. In Fig. 3b the respective maxima are $dW/d\lambda=10^3, 7\cdot10^3$ and $3\cdot10^4$ with widths $1.9\cdot10^{-5}$, $8.8\cdot10^{-7}$ and $1.1\cdot10^{-5}$. The respective minima are $dW/d\lambda=10^2, 8\cdot10^2$.

If the resonant-radiation field strength is further increased so that the interaction energy exceeds considerably the fine-splitting energy and the ionization width, a major role is assumed by one Lorentzian maximum. This means that the ionization goes mainly from the upper resonant doublet, which in this case acts as one level.

In conclusion, we are grateful to M. L. Mikaelyan and M. V. Fedorov for a number of helpful remarks.

1)The quantities that follow are in atomic units.

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